Structure of the percolation cluster and excess 1/f noise in systems with an exponentially broad spectrum of resistances

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The effective conductivity σ_e and excess $1/f$ noise in a random lattice with an exponentially broad spectrum of resistances is considered. A large number of problems, including, e.g., the problem of the description of hopping conduction at high temperatures (i.e., in the case when the spread of the energy levels of the doping impurity can be neglected[1]), reduce to the problem of the determination of σ_e with an exponentially broad spectrum of resistances. To describe the percolation of current through such a system the weak-link model is applied in the region of smearing, and on the basis of this model a structure for the percolation cluster is proposed and σ_e and the relative spectral density of the excess $1/f$ noise[2] are calculated. Numerical modeling is performed and a value is obtained for the percolation-scaling index of the relative spectral density of the excess $1/f$ noise in the three-dimensional case. It is shown that the structure of the percolation cluster differs from that in the two-phase case. © *1995 American Institute of Physics.*

1. FORMULATION OF THE PROBLEM

We shall consider a cubic network with bond length a_0. If the resistances between the nodes of the network are written in the form $r_i = r_0 \exp(-\lambda x)$, where $x \in (0,1)$ is a random variable with a smooth distribution $D(x)$, and $\lambda \gg 1$, then, disregarding the pre-exponential factor,[1,3–5] we have $\sigma_e \sim \sigma_0 \exp(\lambda x_c)$. Here, x_c is defined in terms of the value of the percolation threshold of the standard two-phase problem on this lattice.

Recently,[5–10] progress has been made in the more precise determination of the dependence of σ_e on λ: The percolation-scaling index y that determines the dependence on λ of the pre-exponential factor via

$$\sigma_e = \sigma_0 \lambda^{-y} \exp(\lambda x_c), \tag{1}$$

has been found analytically and numerically. (For the analytical determination of y it is necessary to use some particular model of the percolation structure.)

To determine the relative spectral density C_e of the excess $1/f$ noise it is also necessary to know the local distribution of fields and currents, and, naturally, the nonuniformity of the distribution of fields and currents is more important for the determination of the excess $1/f$ noise (the fourth moment of the current distribution) than for the determination of σ_e (the second moment).

To determine σ_e and C_e the so-called hierarchical weak-link model has been used, which makes it possible to describe in a unified way the critical behavior of the various kinetic phenomena in two-phase media near the percolation threshold—the thermopower,[11] galvanomagnetic effects,[12,13] the relative spectral density of $1/f$ noise,[14] and also other properties.[15]

In accordance with the technique proposed in Refs. 3–5, a problem with a continuous distribution of resistances can be reduced to a two-phase percolation problem. Here, one introduces a critical value x_c of the random variable, related in a simple way to the percolation threshold p_c of the standard two-phase percolation problem on the same network:

$$\int_{x_c}^{1} D(x)dx = p_c, \quad \int_{x}^{1} D(x)dx = p. \tag{2}$$

All resistances with $x > x_c$ are conventionally regarded as "metallic" (the "black" phase), while those with $x < x_c$ are regarded as "dielectric" (the "white" phase). In this approach (the replacement of a problem with a continuous spectrum of resistances by a two-phase problem with concentration p of the "metallic" phase) the basic question that arises is: What is the value of this concentration? We note immediately that (in contrast to two-phase systems) the concentration p is introduced by convention and is not a free parameter whose value can be chosen at will. Naturally, its value should not appear in the final result.

2. MODEL OF THE PERCOLATION STRUCTURE FOR A SYSTEM WITH AN EXPONENTIALLY BROAD SPECTRUM OF RESISTANCES

In Refs. 2–9 it was shown that for a consistent description of the conduction it is necessary to assume that the system is in the region of smearing. For the two-phase case this

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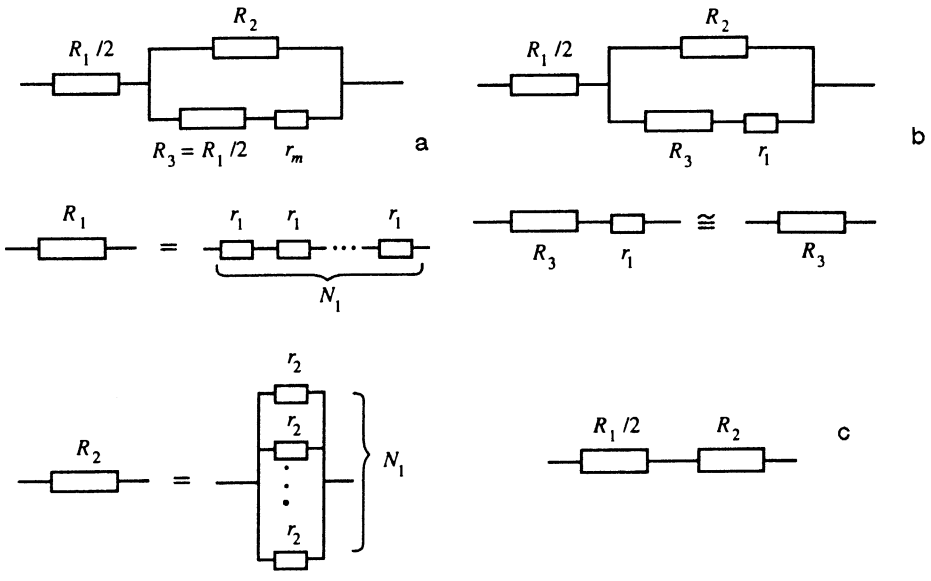

FIG. 1. a—Model of the percolation structure of a two-phase system consisting of randomly distributed resistances r_1 (the strongly conducting, "black" phase), with concentration p, and r_2 (the weakly conducting, "white" phase): $r_1/r_2 \ll 1$. The main current flows across a bridge with resistance R_1 and a layer with resistance R_2. The bridge consists of a set of series-connected resistances r_1 (a single connected bond-see the second scheme), and the layer consists of parallel-connected resistances r_2 (a single disconnected bond-see the third scheme). The resistance r_m takes the values r_1 and r_2 with different probabilities, depending on the concentration p of the strongly conducting phase. The model describes the behavior of the system above the percolation threshold, below it, and in the region of smearing. b— Model of the percolation structure above the percolation threshold outside the region of smearing ($p>p_c$, $\tau>\Delta$). The resistance $r=r_1$, and the scheme goes over into the weak-link model for $p>p_c$ (a bridge with a layer connected in parallel). c—Model of the percolation structure below the percolation threshold outside the region of smearing ($p<p_c$, $|\tau|>\Delta$). The resistance $r_m=r_2$, and the scheme goes over into the weak-link model for $p<p_c$ (a layer with a bridge connected in series).

implies that $-\Delta<\tau<+\Delta$, where $\tau=(p-p_c)/p_c$ is the distance from the percolation threshold and Δ is the magnitude of the region of smearing (the analog of the region of smearing of a second-order phase transition[16,17]), equal[18] to $\Delta=(\sigma_2/\sigma_1)^{1/(t+q)}\ll 1$, where σ_1 and σ_2 are the conductivities of the "black" and "white" phases, and t and q are the critical indices of the conductivity. For systems with an exponentially broad spectrum of resistances,[2]

$$\Delta\approx-\frac{\ln[x_c(1-x_c)\lambda^{+2-(\alpha_1+\alpha_2)}]}{2(1-x_c)\lambda}, \tag{3}$$

where the α_i are the critical indices that characterize the number N_1 of so-called single connected bonds (SCB) and the number N_2 of single disconnected bonds (SDB):

$$N_i\sim|\tau|^{-\alpha_i}. \tag{4}$$

We shall consider first the structure of a conducting cluster in the region of smearing for a two-phase medium[10] (Fig. 1a). In this figure, R_1 is the resistance of a set of SCB (a "bridge") and R_2 is the resistance of a set of SDB (a "layer"):

$$R_1\approx\frac{N_1}{\sigma_1 a_0^{d-2}},\quad R_2\approx(\sigma_2 a_0^{d-2}N_2)^{-1}, \tag{5}$$

where it has been taken into account that the SCB are connected in series and the SDB in parallel. If the system is above the percolation threshold and outside the region of smearing ($\tau>\Delta$), then $r_m=r_1$ and the scheme in Fig. 1a is transformed into the hierarchical weak-link model (Fig. 1b) (see Ref. 19)—the current flows mainly through the bridge from the "black" phase, and the resistance of the bridge is determined from (4) and (5). Below the percolation threshold ($\tau<0$), outside the region of smearing, r_m is equal to r_2 and is connected in parallel with R_2 (we recall that R_2 consists of N_2 resistances r_2 connected in parallel), i.e., with a layer of the "white" phase, whose resistance is determined from (4) and (5) (Fig. 1c). Outside the region of smearing the N_i cease to depend on τ, and take fixed values $N_i(|\tau|<\Delta)=N_i(\Delta)$. We note that here there is no contradiction with "geometrical" percolation. If we treat the structure of a "black" percolation cluster above the percolation threshold purely "geometrically" ($\sigma_2=0$), then we have $\Delta=0$ and as the percolation threshold is approached ($\tau\to 0$) the length $N_i a_0$ of the SCB tends to infinity. But in the case when we are considering a two-phase medium with a finite (nonzero) ratio of phases and finite N_1, both the length of the single connected bonds and the correlation length $\xi=a_0|\tau|^{-\nu}$ remain finite for arbitrarily small values of τ, since $\xi_{\max}=a_0\Delta^{-\nu}$.

If inside the region of smearing the N_i cease to depend on τ, the resistance r_m now depends on τ. With probability $P(\tau)$ (Fig. 2) we have $r_m=r_1$, and with probability $1-P(\tau)$ we have $r_m=r_2$, where

$$P(\tau)=\frac{\Delta+\tau}{2\Delta}. \tag{6}$$

Calculation using this model (Fig. 1a) gives in the region of smearing the familiar expression[18]

$$\sigma_e\approx(\sigma_1^q\sigma_2^t)^{1/(t+q)}(1+\text{const}|\tau|),$$

where $\text{const}|\tau|\ll 1$.

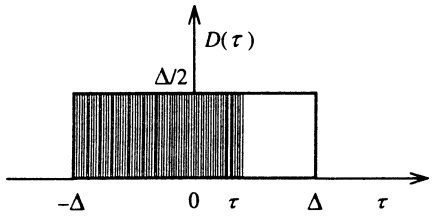

FIG. 2. Distribution function of the resistance r_m. The probability that $r_m = r_1$ is equal to the shaded area.

3. CRITICAL INDICES α_i AND THE STANLEY "COLORED" MODEL

On the one hand, in order that the model of the percolation structure above the percolation threshold (Fig. 1b), below it (Fig. 1c), and at the percolation threshold itself (inside the region of smearing) (Fig. 1a) give values for the critical indices of the conductivity of two-phase systems that agree with those given by numerical methods, it is necessary to set

$$\alpha_1 = \alpha_1' = t - \nu(d-2), \quad \alpha_2 = \alpha_2' = q + \nu(d-2). \tag{7}$$

In this case, from (4), (5) and the definition of the correlation length ξ, for σ_e there immediately follow the well known expressions

$$\sigma_e \approx \sigma_1 \tau^t, \quad \tau > 0, \quad \tau > \Delta;$$
$$\sigma_e \approx \sigma_2 |\tau|^{-q}, \quad \tau < 0, \quad |\tau| > \Delta, \tag{8}$$

in which we have retained only the first terms. On the other hand, it has been shown by rigorous probability-theoretical methods that the critical indices determining the numbers of single connected bonds[20–22] and single disconnected bonds[23,24] are

$$\alpha_1 = \alpha_1'' = 1, \quad \alpha_2 = \alpha_2'' = 1. \tag{9}$$

The contradiction between (7) and (9) ($\alpha_1' \geqslant \alpha_1''$, $\alpha_2' > \alpha_2''$) is explained by the fact that the calculation (e.g., of α_1'') that leads to (9) rested on the assumption that an infinite cluster consists only of SCB with nonmultiple bonds. In fact, it incorporates both single (SCB) bonds and double, triple, etc., bonds. The statistics of such a cluster was considered by Stanley[25] and Pike and Stanley[26]—the so-called colored model (Fig. 3). The singly connected bonds in this model are called red bonds, the multiply connected bonds are called blue bonds, and the dangling ends (along which current does not flow) are called yellow bonds. The numbers of each are determined by the critical indices γ_r, γ_b, and γ_y:

$$N_r \sim \tau^{-\gamma_r}, \quad N_b \sim \tau^{-\gamma_b}, \quad N_y \sim \tau^{-\gamma_y}. \tag{10}$$

The critical index is $\gamma_r = \alpha_1''$, and, according to Refs. 20–22, $\gamma_r = 1$. Numerical calculations have shown that, e.g., for the two-dimensional case, $\gamma_b = 1.7$. Thus, in the model of a cluster with α_i', in the calculation of the resistance one takes into account only the single (red) bonds of the black phase, but the number of them is chosen by taking the blue bonds into account—α_i' takes a certain effective value. Instead of the "actual" cluster with red and blue bonds, in the model one considers only red bonds, but with a different number of them ($\gamma_r < \alpha_i' < \gamma_b$).

This approach has turned out to be successful, and has been used to obtain for the critical indices of $1/f$ noise[14] and for higher current moments[19] analytical expressions that agree well with the data from numerical modeling (see, e.g., Refs. 27–29).

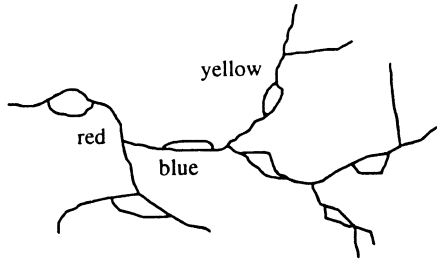

FIG. 3. The colored model of Stanley and Pike. In the terminology of this model, the red bonds are singly connected (backbone) bonds (SCB). The blue bonds are bonds with doubling, and the yellow bonds are dangling ends that do not take part in the conduction.

4. EFFECTIVE CONDUCTIVITY AND EXCESS $1/f$ NOISE IN SYSTEMS WITH AN EXPONENTIALLY BROAD SPECTRUM OF RESISTANCES

On the basis of the model considered above (see also Ref. 2), σ_e and the relative spectral density of the excess $1/f$ noise in a random lattice with an exponentially broad spectrum of resistances have been found to an accuracy that includes the pre-exponential factors. In systems with an exponentially broad spectrum of resistances a "white" phase and a "black" phase are introduced conventionally. It is assumed, however, that the geometrical structure of a percolation cluster remains the same: For a given p, the number of SCB is, as before, $N_i \sim \tau^{-\alpha_i}$. Of course, an SCB (a bridge) now consists not of resistances r_1, but of a set of resistances $r(x) = r_0 \exp(-\lambda x)$, where x runs over values from 1 to $x_1 = x_c + (1 - x_c)\Delta$, while in an SDB (a layer) x runs over values from 0 to $x_2 = x_c - (1 - x_c)\Delta$. As shown in Ref. 2, for a consistent description it is necessary to assume that the system is in the region of smearing, which for such systems has the form $\Delta \sim \ln(\lambda^{2-(\alpha_1+\alpha_2)})/\lambda$. Thus, to determine the effective conductivity and effective noise it is necessary to use the model of a percolation structure inside the region of smearing (the scheme of Fig. 1a). Calculations on the basis of this model make it possible to obtain the percolation-scaling index of the effective conductivity:

$$y = \frac{\alpha_1 - \alpha_2}{2} + \nu(d-2). \tag{11}$$

In the case when the local spectral density of the excess $1/f$ noise $s_g = \{\delta g, \delta g\}$ from a given volume V is proportional to the conductance of this volume g, $s_g \sim g$, on the basis of this model (see also Ref. 2) we obtain for the relative spectral density C_e of the excess $1/f$ noise (the local relative spectral density C of the noise is given by $C = s_g V/g^2$):

TABLE I. Numerical values of the critical-scaling indices.

t	q	ν	y'	y''	$m'-2y'$	$m''-2y''$
1.94[a]	0.75[b]	0.89[b]	0.595	0.89	1.185	0.89
Numerical modeling			$\frac{0.6\pm0.1^{c}}{0.76\pm0.04^{*}}$		0.78	$^{+0.09^{*}}_{-0.08}$

a—from Ref. 29; b—from Ref. 30; c—from Refs. 6 and 31; *) results of this paper.

$$c_e \sim \frac{1}{\sigma_0} e^{-\lambda x_c} \lambda^m = \frac{\lambda^{2\nu}}{\sigma_e}, \tag{12}$$

$$m = -\frac{\alpha_2 - \alpha_1}{2} + \nu d. \tag{13}$$

Comparison of the index y with numerical values[6] and upper and lower bounds[6,7] exhibits satisfactory agreement both in the case $\alpha_i = \alpha_i'$ and in the case $\alpha_i = \alpha_i''$:

$$y(\alpha_i = \alpha_i') = y' = \frac{t-q}{2}, \quad y(\alpha_i = \alpha_i'') = y'' = \nu(d-2). \tag{14}$$

In our view, in the three-dimensional case the agreement is better for $\alpha_i = \alpha_i'$ (see Table I), but with the available accuracy of the values of the critical indices ν, t, and q it is hard to give preference to either variant ($\alpha_i = \alpha_i'$ or $\alpha_i = \alpha_i''$). Therefore, it is natural to turn to the relative spectral density of the excess $1/f$ noise:

$$m(\alpha_i = \alpha_i') = m' = \frac{t-q}{2} + 2\nu, \quad m(\alpha_i = \alpha_i'') = m'' = \nu d \tag{15}$$

since the fourth moment of the current distribution should be more sensitive to the details of the structure.

5. NUMERICAL MODELING

In order to check the expressions obtained for the critical-scaling index m of the relative spectral density of the excess $1/f$ noise we modeled a network of resistances with a random distribution of resistances with an exponentially broad spectrum ($r_i = r_0 \exp(-\lambda x_i)$). In each stage of the calculations we constructed a simple cubic lattice with linear dimension L and considered a possible distribution of resistances. After the creation of the lattice the resistances of the bonds were preserved in a matrix of network equations and unit potential difference was applied to opposite faces of the lattice. The solution of the system of Kirchhoff equations gave the voltages across all the bonds of the lattice. This made it possible to determine the conductance G and the $1/f$ noise $S_G = \Sigma s_{g_i} U_i^4$ of the network of resistances, where U_i is the voltage drop across the i-th bond, $s_{g_i} = \{\delta g_i, \delta g_i\}$ is a temporal correlation function, g_i is the conductance of the i-th bond, and S_G is related to C_e by $C_e = G^{-2} S_G V$, where V is the volume of the system (in the given case, $V = L^3$). In this notation, in the three-dimensional case, Eq. (12) has the form $S_G \sim \lambda^{m-2y} \exp(-\lambda x_c) L^{-1}$. The modeling was performed for different values of the parameter λ from 6 to 80 and for the largest possible size (in our case) $L/a_0 = 15$ (the memory for lattice sizes $L/a_0 > 15$ is greater than 6 MB, and this is important for programming with Windows). For each value of the parameter λ several hundred realizations were modeled, and their conductances and noise were averaged arithmetically (G_a and S_{G_a}) and harmonically (G_h and S_{G_h}). The data obtained are presented in Fig. 4. The averages begin to differ from each other at $\lambda \sim 30$, and this implies that for $\lambda > 30$ the system possesses a fractal structure. In fact, according to (3), the correlation length $\xi \sim \Delta^{-\nu} \sim \lambda^{\nu}$, and $L/a_0 = 15$ corresponds to $\lambda \approx 20$, while for $\lambda \approx 40$ the correlation volume is already twice the chosen size of the system ($\xi/a_0 \approx 30$). Up to this value the arithmetic and harmonic averages practically coincide, and on a double logarithmic

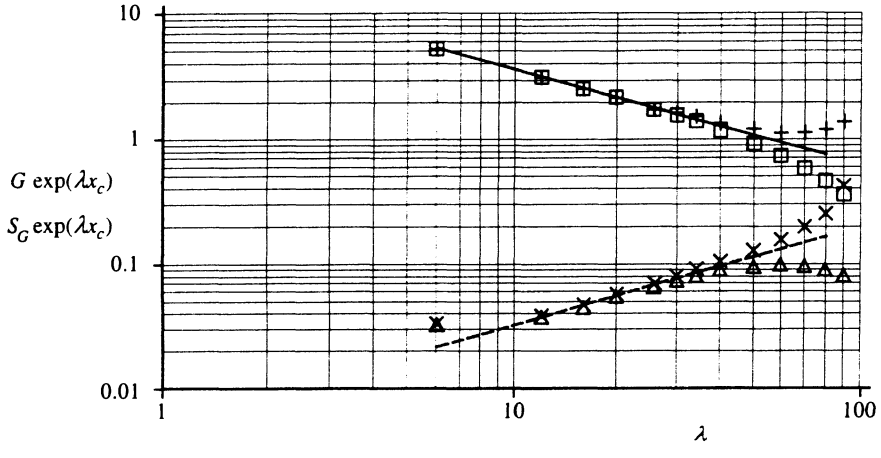

FIG. 4. Results of the numerical modeling, and comparison with data in the literature. The dependence on λ of the conductance G and noise S_G [multiplied by $\exp(\lambda x_c)$] of a network of resistances, plotted in double logarithmic coordinates. The critical-scaling indices are determined by the slopes of the lines, which are plotted by the method of least squares, applied separately for the arithmetic averages (subscript a) and the harmonic averages (subscript h) over realizations of the random spread of resistances: +) G_a; □) G_h; ×) S_{Ga}; △) S_{Gh}.

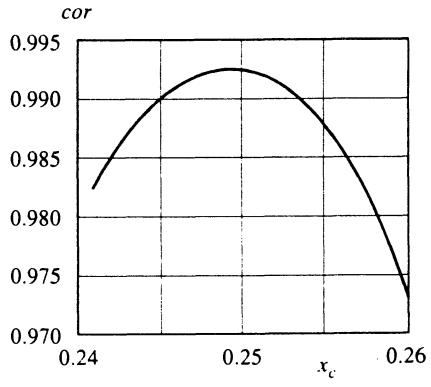

FIG. 5. Dependence of the correlation coefficient *cor* (the product of the four correlation coefficients describing the quality of the approximation of the given G_a, G_h, S_{G_a}, and S_{G_h}) on the choice of values of x_c. The maximum value of *cor* is reached at $x_c=0.2492$, which coincides with the x_c of Ref. 33.

scale they lie on straight lines with slopes $-y$ and $m-2y$. The method of least squares (Fig. 4) for the given interval ($\lambda\sim 40$) gives

$$y=0.76\pm 0.04, \quad m-2y=0.76-0.08+0.09. \quad (16)$$

To determine y and $m-2y$ it is necessary to use a specific value of x_c, and, in accordance with Ref. 33, this was chosen to be $x_c=0.2492$. In the literature, however, other values of x_c are known (from 0.24 to 0.2488),[34–37] and, therefore, the method of least squares was repeated for values of x_c lying in the range from 0.24 to 0.26. To find bounds on the linearity for different values of x_c we used a general correlation coefficient *cor* in the form of a product of four correlation coefficients, each of which describes the quality of the approximation of the given G_a, G_h, S_{G_a} and S_{G_h}. As can be seen from Fig. 5, the correlation coefficient cor takes its greatest value for $x_c=0.2492$. This fact increases the reliability of our estimates of the numerical values of y and $m-2y$.

6. CONCLUSION

Comparison of $m-2y$ given in Eq. (16) with the data from analytical calculation on the basis of the weak-link model (the last two columns of Table I) shows that the value $m''-2y''$ is substantially closer to the result of the numerical modeling. This implies that in the problem of current percolation, in contrast to the two-phase case, in systems with an exponentially broad spectrum of resistances, only the SCB are important in the consideration of the "black" phase. An analogous conclusion can also be formulated for the "white" phase.

Qualitatively, this result can be explained by the fact that in systems with an exponentially broad spectrum of resistances we can neglect the probability of encountering a set of multiply connected resistances that have equal numerical values of the individual resistances in the pairs. One of the resistances in a pair will be exponentially larger than the other, and the pair will degenerate into an SCB resistance.

Thus, the results of numerical modeling make it possible to postulate that the current-carrying cluster in systems with an exponentially broad spectrum of resistances differs from the cluster in two-phase systems (at least for the fourth current moment). In the latter case both the red and the blue bonds are important, while in the former case only the red bonds (the SCB) are important. It is evident that the higher the moment of the current distribution, the more substantial is the difference between these cases. It would be interesting to perform an analytical calculation and numerical modeling for the next (sixth) moment of the current distribution.

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