Precessing structures and spin waves in a spin-polarized Fermi liquid

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For a spin-polarized Fermi liquid the transition from a localized spin wave to a coherently precessing structure as the wave amplitude increases is traced. The frequency spectrum of the small oscillations of the structure thus formed is found, and its stability is demonstrated. The possibility of the existence of coherently precessing nonlinear structures in other physical objects is discussed.

1. INTRODUCTION

The Fermi-liquid interaction in Landau theory is manifested in different ways, depending on the magnitude of the product of the characteristic frequency ω of the phenomenon under investigation and the time τ between collisions of quasiparticles. In the hydrodynamic regime, i.e., when the condition $\omega\tau \ll 1$ is fulfilled, the interaction leads only to a change of the magnitudes of the susceptibilities that characterize the response of the system to small external perturbations. In the collisionless regime ($\omega\tau \gg 1$) the Fermi liquid acquires properties that are qualitatively new in comparison with those of the ideal gas. Since in a pure Fermi liquid we have $\tau \propto 1/T^2$, the condition $\omega\tau \gg 1$ can always be realized in such a liquid by lowering the temperature.

In a spin-polarized Fermi liquid, because of the degeneracy of its states with respect to the spin directions, even a weak Fermi-liquid interaction can lead to appreciable effects. For example, in the collisionless regime the character of the spin transport changes in addition to the spin diffusion a nondissipative spin current arises. This current leads, in particular, to the existence of spin (Silin) waves.[1] Such waves have been detected experimentally in metals,[2] in liquid ^{3}He,[3] and in solutions of ^{3}He in ^{4}He.[4,5] In effect, resonance spin-wave modes were observed. In particular, in the experiments of Ref. 3 with liquid ^{3}He and in the experiments of Ref. 6 with solutions of ^{3}He in ^{4}He a continuous NMR method was used to observe spin-wave modes localized at the container wall by means of a nonuniform magnetic field, i.e., by means of a gradient of the Larmor frequency. It was recently discovered that in solutions of ^{3}He in ^{4}He (Refs. 6, 7, 8) and in pure ^{3}He (Ref. 9), in conditions similar to those in which localized spin-wave modes are observed, as the power of the high-frequency pumping increases and one goes from continuous to pulsed NMR a coherently precessing spin structure is formed. The structure consists of two domains. In one of the domains the magnetization is parallel to the magnetic field, while in the other it is antiparallel. Although the deviations of the magnetization from equilibrium are not small, the entire structure precesses with the same frequency, i.e., it remains a well defined mode. The spin density **S** rotates through angle π over the width of the transition region—the domain wall. As it rotates, **S** remains in the same plane. The entire structure precesses with the same frequency ω_p, equal to the Larmor frequency at the position of the domain wall. The above properties of the structure follow from analysis of that solution of the equations of the spin dynamics of a normal Fermi liquid (the Leggett equations[10]) which describes a coherently precessing structure.[11,12] This solution is generated by the nonuniformity of the field, and cannot be obtained by a small modification of the previously found nonlinear stationary solutions for the case of a uniform field.[13]

In this paper we perform a further theoretical investigation of the precessing structure. In particular, we establish its relationship to the spin waves and trace the transformation of the fundamental standing-spin-wave mode in a nonuniform magnetic field into the precessing structure as the wave amplitude increases. It is also shown that, besides the fundamental structure, in which the rotation of the field within the domain wall is through angle π, there may also exist other modes, corresponding to rotation through $(2m+1)\pi$, where m is an integer. For the "fundamental" structure, corresponding to $m=0$, we find the spectrum of the frequencies of its small oscillations. All the frequencies are real, indicating that the structure is stable. Analysis of small oscillations of the structure is also of practical interest. Periodic variations of the amplitude and frequency of the induction signal are practically always observed when the structure forms in pulsed NMR experiments. The variation of the frequencies of the observed oscillations and comparison of them with the calculated frequencies can serve as a means of measuring the Fermi-liquid parameters.

2. TRANSITION FROM A SPIN WAVE TO A PRECESSING DOMAIN

The spin dynamics of a Fermi liquid in the collisionless regime is described by the system of equations obtained by Leggett[10] for the spin density **S** and spin-current density $\mathbf{J}_i$. For what follows it is convenient to write out the Leggett equations using the following notation: $\boldsymbol{\omega}_L$ is the Larmor frequency, $\boldsymbol{\sigma}=\gamma^2\mathbf{S}/\chi$, and $\mathbf{j}_i=\gamma^2\mathbf{J}_i/\chi$, where χ is the magnetic susceptibility of the liquid under consideration and γ is the corresponding gyromagnetic ratio. The subscript $i=1$, 2, 3 labels the space components of the spin-current tensor, and the bold print denotes spin vectors. In addition, in the equations we have introduced abbreviated notation for combinations of the Fermi-liquid parameters: $u^2=v_F^2(1+F_0^a)(1+F_1^a/3)/3$, $\tau_1=\tau/(1+F_1^a/3)$, and $\kappa=-(F_0^a-F_1^a/3)/(1+F_0^a)$, where v_F is the Fermi velocity, τ

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is the time between collisions of quasiparticles, and F_0^a and F_1^a are the coefficients of the first two harmonics of the exchange part of the Fermi-liquid interaction. In this notation,

$$\frac{\partial \boldsymbol{\sigma}}{\partial t}+\frac{\partial \mathbf{j}_i}{\partial x_i}=[\boldsymbol{\sigma}\boldsymbol{\omega}_L], \tag{1}$$

$$\frac{\partial \mathbf{j}_i}{\partial t}+u^2\frac{\partial}{\partial x_i}(\boldsymbol{\sigma}-\boldsymbol{\omega}_L)=[\mathbf{j}_i\boldsymbol{\omega}_L]+\kappa[\mathbf{j}_i\boldsymbol{\sigma}]-\frac{\mathbf{j}_i}{\tau_1}. \tag{2}$$

For Eqs. (1) and (2) to be applicable it is required that the characteristic distances over which $\boldsymbol{\sigma}$ and $\mathbf{j}_i$ vary be large in comparison with the smallest of the lengths $\xi_\omega=u/\kappa\sigma$ and $l_D=u\tau_1$. In the case $l_D\ll\xi_\omega$ the collision term in the right-hand side of Eq. (2) is the most important term, and diffusive transport of spin occurs, with diffusion coefficient $D_0=u\tau_1$. In the opposite case, which is realized when $\kappa\sigma\tau_1\gg 1$, the most important term is the term proportional to κ. It is this case (the high-frequency limit) that we shall consider below. Even in this case, however, the collision term cannot be omitted—as before, it determines the transport of the spin magnitude, as becomes obvious when we take the scalar product of Eq. (2) with $\boldsymbol{\sigma}$, when the term containing κ vanishes. In Eq. (1) there is no collision term; this corresponds to neglect of the spin-orbit interaction of the quasiparticles. For the normal phase of liquid ^{3}He and for solutions of ^{3}He in ^{4}He at temperatures of the order of a millikelvin this is a very good approximation. As a consequence of the neglect of the spin-orbit interaction the component of the spin along the direction of the magnetic field is conserved. This leads to the result that Eqs. (1), (2) have the stationary solutions $\mathbf{j}_i=0$, $\boldsymbol{\sigma}=\boldsymbol{\sigma}_0=\boldsymbol{\omega}_L+\mathbf{b}$, where $\mathbf{b}$ is a constant vector parallel to $\boldsymbol{\omega}_L$. The solution with $\mathbf{b}=0$ describes the equilibrium state.

In the limit $\tau\to\infty$ Eqs. (1), (2) for a weakly nonuniform field have solutions $\boldsymbol{\sigma}^0$ and $\mathbf{j}_i^0$, corresponding to steady precession of the spin and spin current.[11,12] The time dependence of $\boldsymbol{\sigma}^0$ and $\mathbf{j}_i^0$ for these solutions is determined by the equations

$$\frac{\partial \boldsymbol{\sigma}^0}{\partial t}=[\boldsymbol{\sigma}^0\boldsymbol{\omega}_p] \quad\text{and}\quad \frac{\partial \mathbf{j}_i^0}{\partial t}=[\mathbf{j}_i^0\boldsymbol{\omega}_p]. \tag{3}$$

We shall assume that the z axis of the rotating coordinate frame is parallel to ω_p and that the Fermi liquid is placed in a container with walls that are impenetrable for spin current. In fact, it is sufficient to have one such wall, perpendicular to the direction of the magnetic field. On this wall the condition

$$\mathbf{j}_i n_i=0 \tag{4}$$

is fulfilled, where $\mathbf{n}$ is the normal to the wall. Solving Eqs. (1), (2) under the conditions (3), (4) makes it possible to determine the dependence of $\boldsymbol{\sigma}^0$ and $\mathbf{j}_i^0$ on the coordinate z. Because of the uniformity of the external conditions, $\boldsymbol{\sigma}^0$ and $\mathbf{j}_i^0$ do not depend on the coordinates x and y transverse to the field. By virtue of the condition (4), $\boldsymbol{\sigma}^0$ remains in the same plane, which can be conveniently taken to be the yz plane of the rotating coordinate frame. To describe the entire structure it is sufficient to specify two functions $\sigma(z)=|\boldsymbol{\sigma}|$ and the angle $\theta(z)$ between $\boldsymbol{\sigma}$ and the z axis. In the most interesting case, when the variation of the Larmor frequency within the volume under consideration is small in comparison with the Fermi-liquid field, i.e., when the strong inequality $|\kappa\sigma|\gg|\omega_L-\omega_p|$ is fulfilled, we obtain for θ the simple equation

$$\frac{u^2}{\kappa\sigma_0}\frac{d^2\theta}{dz^2}+(\omega_L-\omega_p)\sin\theta=0. \tag{5}$$

The dependence of σ on z is determined by the equation

$$\frac{d\sigma}{dz}=\frac{d\omega_L}{dz}\cos\theta-\frac{\omega_L-\omega_p}{\kappa}\frac{d\cos\theta}{dz}. \tag{6}$$

We shall assume, for definiteness, that $\kappa\sigma_0>0$, that ω_L increases in the direction of positive z, that the vessel wall is at $z=0$, and that the liquid occupies the region $z<0$. The Larmor frequency can be written conveniently in the form $\omega_L(z)=\omega_L(0)+z(d\omega_L/dz)$. The combination $\lambda=[u^2/(\kappa\sigma_0(d\omega_L/dz))]^{1/3}$ defines the characteristic length in Eq. (5). According to Eq. (6), the quantity σ varies little over distances of the order of λ, and in Eq. (5) we can assume that σ=const. Having solved Eq. (5) we can then find σ by integrating Eq. (6). The only nonzero component of the spin current is expressed in terms of the derivative of θ by $j_3^x=-(u^2/\kappa)(d\theta/dz)$. Introducing the dimensionless coordinate $\zeta=z/\lambda$ and the dimensionless frequency shift $\nu_p=(\omega_p-\omega_L(0))/(\lambda(d\omega_L/dz))$, we bring Eq. (5) to the form

$$\frac{d^2\theta}{d\zeta^2}+(\zeta-\nu_p)\sin\theta=0. \tag{7}$$

We are interested in solutions of this equation that satisfy the boundary conditions $d\theta/d\zeta=0$ at $\zeta=0$ and $d\theta/d\zeta\to 0$ as $\zeta\to-\infty$. Making the change of independent variable $\zeta=\nu_p+s$, we eliminate the parameter ν_p from Eq. (7):

$$\frac{d^2\theta}{ds^2}+s\sin\theta=0, \tag{8}$$

and the boundary condition $d\theta/ds=0$ must now be imposed at $s=-\nu_p$. The other boundary condition does not change: $d\theta/ds\to 0$ as $s\to-\infty$. Equation (8) has solutions independent of s that also satisfy the boundary conditions $\theta=n\pi$, where $n=0, \pm1, \pm2,\ldots$. These are the previously mentioned stationary solutions, for which the vector $\boldsymbol{\sigma}$ is parallel or antiparallel to $\boldsymbol{\omega}_L$. We shall linearize Eq. (8) about these values of θ. For small deviations ϕ_0 from the "even" points, setting $\theta=2m\pi+\phi_0$ we have

$$\frac{d^2\phi_0}{ds^2}+s\phi_0=0. \tag{9}$$

For the "odd" points, after the substitution $\theta=(2m+1)\pi+\phi_1,\phi_1\ll 1$, we obtain an equation that differs from (9) by a sign:

$$\frac{d^2\phi_1}{ds^2}-s\phi_1=0. \tag{10}$$

Since the angle θ is defined to within $2\pi m$, we can stipulate that as the even stationary solution we shall always take $\theta=0$. The bounded solutions of Eqs. (9), (10) are the familiar Airy functions. In the case of even n the solution $\phi_0=C_0\mathrm{Ai}(s)$ (where C_0 is a constant) decreases like

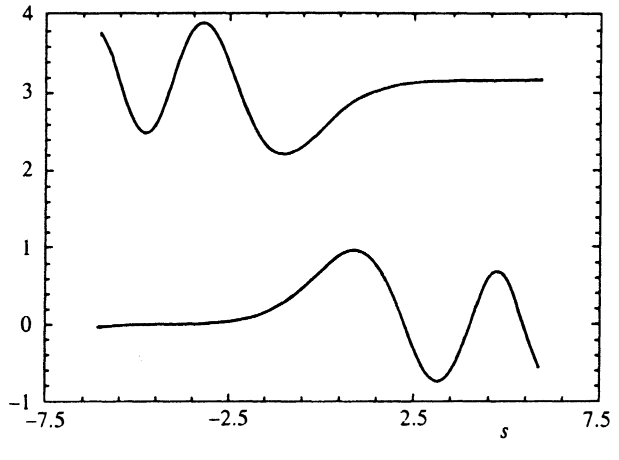

FIG. 1. Bounded solutions of Eq. (8) for values of θ close to $\theta=0$ (lower curve) and $\theta=\pi$ (upper curve).

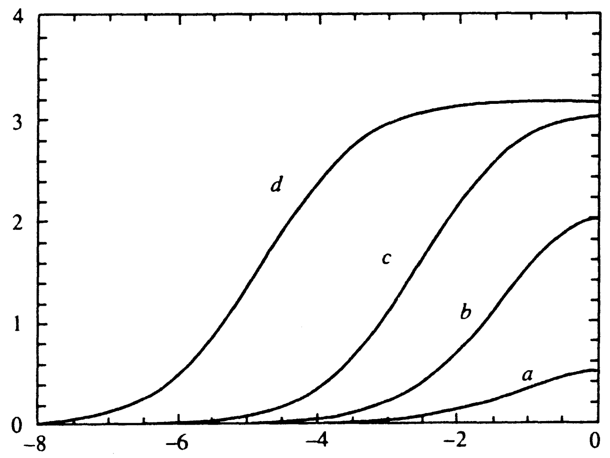

FIG. 2. Transformation of a standing spin wave into a coherently precessing spin structure with increase of the amplitude of the wave. The parameter here is the shift of the wave frequency from the Larmor frequency at the coordinate origin: $\varepsilon=[\omega_p-\omega_L(0)]/\lambda\nabla\omega_L$. The curves depict the dependence of the angle θ on z/λ: curve a) $\varepsilon=-1.0349$; b) $\varepsilon=-1.3540$; c) $\varepsilon=-2.5550$; d) $\varepsilon=-4.8018$.

$\exp[-(2/3)|s|^{3/2}]$ as $s\to-\infty$, and oscillates for $s>0$ (the lower curve in Fig. 1). For sufficiently small values of the constant C_0 this function is also a solution of Eq. (8). The condition $d\phi_0/ds=0$ is fulfilled at $s=s_j$, where s_j are the roots of the derivative of the Airy function. This implies that the boundary condition $d\theta/ds=0$ at $s=0$ can be satisfied only for the discrete values $\nu_p=\nu_p^j=-s_j$. The function ϕ_0, taken in the intervals $(-\infty, s_j)$, describes the changes of the angle θ for the successive modes of standing spin waves, while the eigenvalues s_j determine the frequencies of these modes in units of $\lambda d\omega_L/dz$. Below we shall need the first few values of s_j: $s_0=1.01188$, $s_1=3.2482$, $s_2=4.8201$. In an analogous way the function $\phi_1=C_1\mathrm{Ai}(-s)$ in the intervals $(-s_j,\infty)$ depicts successive spin-wave modes in the case when the liquid occupies the region $s>0$ and the magnetization is antiparallel to the field. The functions ϕ_0 and ϕ_1 are real. In application to the spin waves, this means that for each of the modes all the spins precess in phase about the direction of the magnetic field.

As the amplitudes C_0 and C_1 increase the functions ϕ_0 and ϕ_1 cease to be approximate solutions of Eq. (8). The effects of the nonlinearity should be manifested earliest of all on the behavior of the solutions in the region $s\approx 0$, where the values of ϕ_0 and ϕ_1 are a maximum. Here, ϕ_0 and ϕ_1 can remain a good approximation to the true solution in those regions in which they are small. At a certain amplitude, the solution for which θ tends monotonically to 0 as $s\to-\infty$ in the region of small s goes over continuously into the solution for which θ tends monotonically to π as $s\to+\infty$. As a result, a domain wall is formed in which $\boldsymbol{\sigma}$ rotates through angle π.

Because of the nonlinearity, as the amplitude of the spin-wave mode increases the frequency of the mode also changes. In what follows it is convenient to regard this frequency as a parameter characterizing the solution. Figure 2 shows how this mode is transformed into a domain wall in response to a continuous change of the frequency of the fundamental spin-wave mode. After the wall has been formed, as the frequency further decreases it moves into a region of weaker fields while remaining of practically the same shape. The two-domain structure thus formed can be considered as a stable spin-wave mode of arbitrarily large amplitude. An analysis of the stability of the two-domain structure against small perturbations will be performed in Sec. 4.

3. MULTIPLE DOMAIN WALLS

For sufficiently large amplitudes of the solution that decays at $-\infty$, this solution can be joined with solutions for which θ tends at $+\infty$ to 3π, 5π,..., with the formation of the corresponding domain walls. The existence of walls with rotation through angles 3π, 5π,... is confirmed by numerical solution of Eq. (8). For fixed positive orientations of $\boldsymbol{\sigma}$ at $+\infty$ and $-\infty$, these structures cannot be transformed into each other without taking $\boldsymbol{\sigma}$ out of the plane, and this indicates that they are topologically stable. From the above arguments it is also clear that domain walls with rotation of $\boldsymbol{\sigma}$ through an even multiple of π do not exist.

The observation of multiple walls may be made difficult, first, because they possess very high energy, so that it is difficult for them to be formed, and, second, because such walls should relax faster than the "fundamental" structure. The relaxation of a two-domain structure occurs differently in a completely isolated volume (where the longitudinal component of the spin is conserved) and in a volume in contact with an equilibrium reservoir. In both cases the shape of the domain wall appears in the relaxation rate in the form of the dimensionless coefficient $J=\lambda\int_{-\infty}^{+\infty}(d\theta/dz)^2dz$. Formulas describing the relaxation of the structure in an isolated volume are given in Refs. 11 and 12. The derivation performed in those papers is easily reformulated for a semi-infinite volume. In this case the longitudinal spin component can change as a consequence of the influx of spin from the reservoir, while the magnitude σ_0 of the spin density at the center of the domain wall remains constant. Using the expression for the energy-dissipation rate as before, we obtain for the rate of change of the wall coordinate z_0:

$$\frac{dz_0}{dt}=J\frac{\lambda^2(d\omega_L/dz)}{\kappa\omega_p\tau_1}. \tag{11}$$

The rate of change of the precession frequency ω_p is found by multiplying both sides of Eq. (11) by $\nabla\omega_L$. The values of the coefficient J increase rapidly with the number of rotations of $\boldsymbol{\sigma}$ within the wall. For a π-wall we have $J_1\simeq 2.35$, for a 3π-wall we have $J_3\simeq 18.1$, and for a 5π-wall we have $J_5\simeq 37.1$.

4. OSCILLATIONS OF THE PRECESSING STRUCTURE

In order to find the frequencies of the oscillations we shall follow the standard procedure and consider a small perturbation of the precessing structure, i.e., we set $\boldsymbol{\sigma}=\boldsymbol{\sigma}^0+\boldsymbol{\psi}$ and $\mathbf{j}_i=\mathbf{j}_i^0+\mathbf{g}_i$, where $\boldsymbol{\psi}$ and $\mathbf{g}_i$ are small. Next, we substitute $\boldsymbol{\sigma}$ and $\mathbf{j}_i$ into Eqs. (1), (2), and confine ourselves to terms linear in $\boldsymbol{\psi}$ and $\mathbf{g}_i$ in these equations. In the coordinate frame rotating with frequency ω_p we obtain for $\boldsymbol{\psi}$ and $\mathbf{g}_i$ the following system of equations:

$$\frac{\partial\boldsymbol{\psi}}{\partial t}+\frac{\partial\mathbf{g}_i}{\partial x_i}=[\boldsymbol{\psi}(\boldsymbol{\omega}_L-\boldsymbol{\omega}_p)], \tag{12}$$

$$\frac{\partial\mathbf{g}_i}{\partial t}+u^2\frac{\partial\boldsymbol{\psi}}{\partial x_i}=[\mathbf{g}_i(\boldsymbol{\omega}_L-\boldsymbol{\omega}_p+\kappa\boldsymbol{\sigma}^0)]+\kappa[\mathbf{j}_i^0\boldsymbol{\psi}]-\frac{\mathbf{g}_i}{\tau_1}. \tag{13}$$

To separate the perturbations that are longitudinal with respect to $\vec{\sigma}_0$ from the transverse perturbations we rewrite Eqs. (12), (13) in a coordinate system $(\hat{\xi},\hat{\eta},\hat{\zeta})$ such that the $\hat{\zeta}$ axis is along the direction of $\boldsymbol{\sigma}_0$ at each point, the $\hat{\xi}$ axis is along the direction of $\mathbf{j}_i^0$, and the direction of $\hat{\eta}$ is chosen so that $(\hat{\xi},\hat{\eta},\hat{\zeta})$ form a right-handed basis. Since, with change of z, this basis rotates about $\hat{\xi}$, when we go over to the new coordinate system the space derivatives must be transformed in accordance with the rule

$$\frac{\partial\mathbf{a}}{\partial x_l}=-\delta_{l3}\frac{d\theta}{dz}[\hat{\xi}\mathbf{a}]+\left(\frac{\partial\mathbf{a}}{\partial x_l}\right)', \tag{14}$$

where $\mathbf{a}$ is any of the indicated vectors, and $(\partial\mathbf{a}/\partial x_l)'$ denotes a derivative of the projections of the vector $\mathbf{a}$ onto the axes $(\hat{\xi},\hat{\eta},\hat{\zeta})$. Taking into account the rule (14), we obtain the following equations for the longitudinal components ψ^ζ, g_l^ζ and circularly polarized transverse components $\psi^\pm=\psi^\xi\pm i\psi^\eta$, $g_l^\pm=g_l^\xi\pm ig_l^\eta$ of $\boldsymbol{\psi}$ and $\mathbf{g}_l$:

$$\frac{\partial\psi^\zeta}{\partial t}+\frac{\partial g_l^\zeta}{\partial x_l}=g_3^\eta\frac{d\theta}{dz}-\psi^\xi\Delta\omega\sin\theta, \tag{15}$$

$$\frac{\partial g_l^\zeta}{\partial t}+u^2\frac{\partial\psi^\zeta}{\partial x_l}=-g_l^\xi\Delta\omega\sin\theta-\frac{g_l^\zeta}{\tau_1}, \tag{16}$$

$$\frac{\partial\psi^+}{\partial t}+\frac{\partial g_l^+}{\partial x_l}+i\psi^+\Delta\omega\cos\theta=\psi^\zeta\Delta\omega\sin\theta-ig_3^\zeta\frac{d\theta}{dz}, \tag{17}$$

$$\frac{\partial g_l^+}{\partial t}+\frac{g_l^+}{\tau_1}+u^2\frac{\partial\psi^+}{\partial x_l}=-ig_l^+(\Delta\omega\cos\theta+\kappa\sigma_0)+g_l^\zeta\Delta\omega\sin\theta, \tag{18}$$

where $\Delta\omega=\omega_L-\omega_p$. The equations for ψ^- and g_l^- are obtained by complex conjugation of Eqs. (17) and (18). Together with Eqs. (15)–(18) they form a linear system of homogeneous differential equations, the coefficients in which are independent of t, x, and y. The solution of this system must be sought in the form of $\psi^+(x,y,z,t)=\exp(i\mathbf{k}_\perp\boldsymbol{\rho}-i\omega t)\psi^+(z)$ with analogous expressions for the other unknown functions. Here, $\boldsymbol{\rho}$ has components (x,y), and $\mathbf{k}_\perp$ is perpendicular to the z axis. Introducing the abbreviated notation $q_\perp^+=k_xg_x^++k_yg_y^+$, we obtain for ψ^+ and $\mathbf{g}^+$ the following equations:

$$-i\omega\psi^++iq_\perp^++\frac{dg_3^+}{dz}+i\psi^+\Delta\omega\cos\theta=\psi^\zeta\Delta\omega\sin\theta-ig_3^\zeta\frac{d\theta}{dz}, \tag{19}$$

$$\left(\frac{1}{\tau_1}-i\omega\right)q_\perp^++iu^2\mathbf{k}_\perp^2\psi^+=-iq_\perp^+(\Delta\omega\cos\theta+\kappa\sigma_0)+q_\perp^\zeta\Delta\omega\sin\theta, \tag{20}$$

$$\left(\frac{1}{\tau_1}-i\omega\right)g_3^++u^2\frac{d\psi^+}{dz}=-ig_3^+(\Delta\omega\cos\theta+\kappa\sigma_0)+g_3^\zeta\Delta\omega\sin\theta. \tag{21}$$

Below, we shall be interested in the low-frequency vibrations, i.e., those for which $\omega\ll\kappa\sigma_0$. Use of this smallness, and also of the previously made assumption that $\Delta\omega$ is small in comparison with $\kappa\sigma_0$, makes it possible to solve Eqs. (20), (21) for g_3^+ and $q_\perp^+$:

$$g_3^+=i\frac{u^2}{\kappa\sigma_0}\frac{d\psi^+}{dz},\quad q_\perp^+=-\frac{u^2k_\perp^2}{\kappa\sigma_0}\psi^+. \tag{22}$$

Substituting the expressions obtained into Eq. (19), we find an equation for ψ^+. It can be written conveniently using the dimensionless coordinate $s=(z-z_0)/\lambda_w$ and the dimensionless frequency $\Omega=\omega/\lambda_w(d\omega_L/dz)$:

$$\frac{d^2\psi^+}{ds^2}+[s\cos\theta-\Omega-(k_\perp\lambda_w)^2]\psi^+=-is\sin\theta\psi^\zeta-\frac{g_3^\zeta}{(d\omega_L/dz)\lambda_w^2}\frac{d\theta}{ds}. \tag{23}$$

The equation for ψ^- is obtained from (23) by changing the sign in front of the frequency Ω and in front of i. The derivation of the equation for ψ^ζ is analogous to that of Eq. (23), and the result is

$$\frac{d^2\psi^\zeta}{ds^2}+\left[\frac{i\Omega}{\kappa\sigma_0\tau_1}-(k_\perp\lambda_w)^2\right]\psi^\zeta=-\frac{\lambda}{u^2\tau_1}\frac{d\theta}{ds}g_3^\eta+\frac{1}{\kappa\sigma_0\tau_1}s\sin\theta\psi^\xi. \tag{24}$$

Here,

$$g_3^\zeta=-\frac{u\tau_1}{\lambda_w}\left(u\frac{d\psi^\zeta}{ds}+\frac{\xi_w}{\lambda_w}s\sin\theta g_3^\xi\right). \tag{25}$$

In the right-hand sides of Eqs. (23) and (24) we have collected the terms that couple the longitudinal component and transverse components of the perturbation ψ. We note, however, that the functions $\sin\theta$ and $d\theta/ds$, which are nonzero only in the region $s\sim 1$ (or $z-z_0\sim\lambda_w$), appear as coefficients in the right-hand sides. For Eq. (23) λ_w also determines the scale over which the solution varies. For Eq. (24), at the same frequency, the combination $\lambda_w(\kappa\sigma_0\tau_1)^{1/2}\gg\lambda_w$ serves as the characteristic scale. For this reason, we can assume that the right-hand side of Eq. (24) has the form const. $\delta(s)$. To determine the constant we must integrate the right-hand side of Eq. (24) over s between infinite limits. If, now, we take into account that in Eq. (25) $\lambda_w\gg\xi_\omega$, and that the dependence $\theta(s)$ is determined by the equation

$$\frac{d^2\theta}{ds^2}=-s\sin\theta, \tag{26}$$

the integral vanishes. This implies that in the leading approximation in $(\kappa\sigma_0\tau_1)^{-1}\ll 1$ the oscillations of the transverse components of ψ do not perturb the longitudinal component, and the changes of the longitudinal and transverse components can be treated independently. For longitudinal perturbations in the indicated approximation we obtain a diffusion equation, the solutions of which are not characterized by definite frequencies. Therefore, we shall consider only transverse perturbations. Putting $\psi^\zeta=0$ and $g^\zeta=0$ in Eq. (23), we obtain the equation for ψ^+:

$$\frac{d^2\psi^+}{ds^2}+(s\cos\theta+\varepsilon)\psi^+=0, \tag{27}$$

where $\varepsilon=\Omega_++(k_\perp\lambda_w)^2$. For ψ^- we obtain an equation that differs from (27) by the replacement of Ω_+ by $-\Omega_-$. Thus, to each ε there correspond two modes of oscillations, with frequencies $\Omega_\pm=\pm[\varepsilon-(k_\perp\lambda_w)^2]$. Equation (27) is the Schrödinger equation with potential $U(s)=-s\cos\theta$; it has solutions localized on the domain wall, i.e., satisfying the conditions $\psi\to 0$ as $s\to\pm\infty$, with corresponding eigenvalues $\varepsilon=\varepsilon_n$. The first few eigenvalues ε_n were found numerically: $\varepsilon_0=0.850$, $\varepsilon_1=2.234$, $\varepsilon_2=3.207$, $\varepsilon_3=4.040$, $\varepsilon_4=4.792$. Since $U(s)$ is an even function, the solutions will be even or odd functions of s, depending on whether n is even or odd. For $s\gg 1$ we have $U\approx|s|$, and for large values of n the solutions are close to Airy functions with known eigenvalues. Even at $n=4$, the eigenvalue ε_n found as the root of the derivative of the Airy function is equal to $\varepsilon_4^A=4.820$. In dimensional units, the formula for the frequencies of the oscillations has the following form:

$$\Omega_\pm=\pm\left[\varepsilon_n\left(\frac{u^2(\nabla\omega)^2}{\kappa\sigma_0}\right)^{1/3}-\frac{1}{\kappa\sigma_0}(uk_\perp)^2\right]. \tag{28}$$

For cells that are bounded in the transverse direction, $k_\perp$ is also quantized. If the cell is a circular cylinder with base of radius R, the azimuthal dependence of the perturbations is described, as usual, by the factor $\exp(im\phi)$, and for each m there is an infinite set of numbers ν_{mj}, which are found as the roots of the equations $dJ_m(\nu)/d\nu=0$ and determine the values of $k_\perp$: $k_\perp R=\nu_{mj}$. The fundamental mode corresponds to $m=0$ and $k_\perp=0$, and the first nonzero ν corresponds to $m=1$: $\nu_{10}\approx 1.84$. For $k_\perp=0$ the frequencies of all the modes are proportional to $(d\omega_L/dz)^{2/3}$. In experiments with solutions of ^{3}He in ^{4}He (Refs. 6, 8) a periodic modulation of the induction signal has been observed, with a frequency of the same order as that calculated from Eq. (28). The dependence of the frequency on the gradient as found in Ref. 8 is close to the expected $(d\omega_L/dz)^{2/3}$. However, the available data are not sufficient for a quantitative comparison with experiment, since there is no independent way of determining the quantity σ_0. In addition, in both cases the height of the experimental chamber was comparable to the domain-wall thickness, and so it is not strictly possible to apply Eq. (28), which was obtained for a wall far from the upper and lower boundaries of the chamber.

The oscillations considered here are essentially spin waves localized on the domain wall. They are similar in many respects to the previously studied spin waves localized near a vessel wall.[3,6,5] There are, however, substantial differences between the two types of wave. In the case of the domain wall, there are, roughly speaking, four times as many modes. This happens for two reasons. First, in the case of the domain wall the boundary conditions on the perturbations admit the existence of both even and odd solutions, whereas on the vessel wall the odd solutions are excluded by the condition that the spin current vanish. Second, to each ε_n for oscillations of the domain wall there correspond two modes, with a positive and a negative frequency. With increase of n the eigenvalues ε_n for the even modes rapidly approach the corresponding values for waves at the vessel wall. We note also that waves localized on the domain wall are determined entirely by properties of the liquid itself, and do not depend on assumptions about the properties of the vessel wall.

5. CONCLUSION

The coherently precessing structure in a normal Fermi liquid has much in common with the structure that exists in the superfluid B phase of ^{3}He, and even more in common with the structure predicted for antiferromagnetic solid ^{3}He (Ref. 14). In all the cases considered, for the structure to form it is essential that there exist an interaction (BCS or exchange) that would maintain the spatial uniformity of the state, and also that there exist a mechanism that sustains anabsolute value of the magnetization that is constant or almost constant over the entire volume under investigation. The structure is thereby described by one function, e.g., the coordinate dependence of the angle θ between the spin and the magnetic field. The formal reason for the formation of the structure is the presence of an expression of the form $s\sin\theta$ in the equation determining the dependence of θ on s. The essential point is that $\sin\theta$ has only simple zeros, and this leads to alternating signs of the derivative of this function at successive zeros. Such properties are possessed by the equations describing the spin dynamics of other systems, e.g., spin-polarized hydrogen[15] or a ferromagnet;[16] in such systems coherently precessing magnetic structures can also exist. In the case of the ferromagnet the spin-dynamics equations should be solved together with the magnetostatics equations, and this can complicate the problem.

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