Decay of massive neutrinos in a strong magnetic field

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The stimulating effect of a strong magnetic field $B \gg m_e^2/e = 4.41\times 10^{13}$ G on the decay of a massive neutrino, $\nu_i \to \nu_j \gamma$, is analyzed. This is of interest for the problem of discovering relict neutrinos formed during the Big Bang. The two-dimensional formalism developed earlier by the same author is used to examine the contribution of one-loop and multiloop diagrams to the amplitude in the standard model. The probability of decay in the neutrino-energy range $(eB)^{1/2} \gg E_\nu$ is found to be many orders of magnitude higher than that in a vacuum. In the case of ultrarelativistic neutrinos, the "decay chain" can end even in the fairly short existence of such fields, so that neutrinos now in the Universe are predominantly in their lowest mass state.

1. INTRODUCTION

We know that if the mass states of a neutrino are nondegenerate, the neutrino can decay via the following channels $(m_i > m_j)$:

$$\nu_i \to \nu_j \gamma, \tag{1a}$$

$$\nu_i \to \nu_j \gamma\gamma, \tag{1b}$$

where the states ν_i are related to the states ν_l of lepton generations via a unitary transformation,

$$\nu_i = \sum_l U_{il} \nu_l . \tag{2}$$

Massive primordial neutrinos produced in the Big Bang are believed[1,2] to have originated largely via the (1a) channel. In this connection, a theoretical study of these processes would obviously be interesting, with other astrophysical considerations also coming into the picture. Petkov[3] and Goldman and Stephenson[4] calculated the probability of process (1a), while Ghosh[5] calculated the probability of process (1b). Bilenky and Petcov[6] analyzed the structure of the amplitudes on the basis of general principles and studied the invariant form factors in specific modifications of electroweak models. They found that the amplitude of the $\nu_i \to \nu_j \gamma$ decay in the standard model contains a factor

$$\sum_l U_{il} U_{jl}^* f(\tilde{m}_l), \quad \tilde{m}_l = \frac{m_l}{m_W}, \tag{3}$$

where m_l are the masses of charged leptons. Since the expansion of $f(\tilde{m}_l)$ takes the form

$$f(\tilde{m}_l) = -\frac{3}{2} + \frac{3}{4}\tilde{m}_l^2 + \cdots, \tag{4}$$

the contribution of the first term on the right-hand side of (4) vanishes owing to the unitarity of the mixing matrix U, and the probability is proportional to the suppression factor $(m_\sigma/m_W)^4$, where m_σ is the mass of the heaviest charged lepton (the GIM mechanism[7]). For the three known lepton generations, $\sigma = \tau$, and the factor is small $(\sim 10^{-3-4})$. Various ways of overcoming this difculty have been proposed: a) the existence of a fourth generation of leptons with a large mass $\tilde{m}_\sigma$ has been considered;[8] b) the existence of a fourth generation of neutrinos (lacking charged leptons) with two singlet states $\nu_{\sigma L}$ and $\nu_{\sigma R}$ has beem suggested;[8] c) possible modifications of the standard model with an enhanced contribution of process (1b), whose amplitude does not contain the suppression factor noted above, have been examined;[9] d) the stimulating effect of strong external electromagnetic fields on the amplitude of process (1a), whose existence in collapsed states (neutron stars) is widely accepted, has been taken into account.[10]

The first three variants are somewhat far-fetched from the standpoint of the available experimental data, but the fourth approach is more attractive.

Note that the decay of the massless neutrino $\nu \to \nu\gamma$ in the Fermi framework in a crossed field was first examined in Ref. 11 and in a constant uniform magnetic field in Ref. 12. Due to four-momentum conservation in such field configurations, the phase volume of the decay of a massless particle into two massless particles is indeterminate, and to obtain an actual result it was necessary to ascribe to the photon an imaginary bare mass. The physical origin of this bare mass lies in the contribution of self-energy diagrams to the external photon line in the external field,[13] and the mass was set to zero at the end of the calculations.

In the standard model and in the Feynman gauge, the set of diagrams in Fig. 1 corresponds to the decay of a massive Dirac neutrino with the nonphysical scalar boson $\varphi^\pm$, the $W^\pm$-boson, and the charged leptons $l = e, \mu, \tau$. The amplitude of the decay $\nu_i \to \nu_J \gamma$ converges when all contributions are taken into account, but the situation changes when an external field is turned on, since this changes the propagators of the internal lines. More precisely, the field-dependent part of the amplitude corresponding to the diagram (a) with the W-boson is convergent "in and of itself" and provides the leading field contribution, since the field amplitudes corresponding to the other diagrams are suppressed by the propagators and form factors at reasonable field values $eF/m_W^2 \ll 1$. This also means that for neutrino energies $E_\nu \ll m_W$ (in the same reference frame), the propagator of the W boson can be taken in the contact form. Vasilevskaya,

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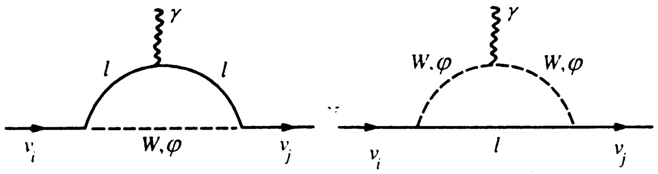

FIG. 1.

Gvozdev, and Mikheev[10] did their calculations for a crossed field, in which case both field invariants vanish. As is known,[14] in the ultrarelativistic case, when the discreteness of levels plays no role, this transition yields correct results even for fairly smooth fields of arbitrary configuration. For instance, in a magnetic field with induction $B \gg B_e = m_e^2/e = 4.41 \times 10^{13}$ G (such fields occur with high probability in collapsed objects and at the Big Bang), the crossed-field method is valid in the case under investigation only at neutrino energies in the range $(Be)^{1/2} \gg E_\nu \gg m_e$, which corresponds to the contribution of the lightest charged particle of the vacuum of the model (the electron). It is obvious, however, that in order to solve the foregoing astrophysical problems, including the missing-mass problem, it would be interesting to calculate the probability over the entire range $(Be)^{1/2} \gg E_\nu$, including the case with nonrelativistic neutrinos.

Earlier, in the QED setting, a two-dimensional covariant method of calculating Feynman diagrams in the subspace (0, 3) in a strong constant and uniform magnetic field was developed (the 3 axis is directed along the field).[15] In particular, it was found (see, e.g., Refs. 13 and 16) that QED perturbation theory in α (and accordingly, the loop expansion in electroweak models) works only up to field strengths $\lesssim 10^{18}$ G, while in higher fields the perturbation series must be summed[17–20] as in the doubly logarithmic asymptotic energy limit in ordinary QED. Note also that many researchers (see, e.g., Ref. 21) ignore these results and use the loop expansion in fields $\sim 10^{18}$ G or higher, which generally makes no sense (in particular, it is unlikely that a tachyonic mode[22] is present in the W boson spectrum, since in ultrahigh fields $\sim 10^{24}$ G one must sum all self-energy diagrams). Disappearance of the zero-charge problem in summing the diagrams in the two-dimensional asymptotics can be pointed to as an example.[18]

Thus, within the range of applicability of the perturbation method ($B \ll 10^{18}$ G) we can ignore the effect of the magnetic field on the propagators of μ and τ in the loop because of the large mass of these particles, which results in the leading field contribution at $l=e$ in diagram (a). Since the electron contribution in the range is special, it is clear that there can be no GIM-type suppression mechanism. With fields $B \gtrsim 10^{18}$ G one must generally sum the loop contributions with allowance for the "two-dimensionalization" of the charged-lepton propagators in the corresponding diagrams.

In this paper, the amplitude and probability of the $\nu_i \to \nu_j \gamma$ decay process in a strong magnetic field are calculated over the entire range of neutrino energy, $(Be)^{1/2} \gg E_\nu$, and the field asymptotics and energy asymptotics are discussed in connection with the astrophysical aspects mentioned earlier.

2. THE AMPLITUDE AND A GENERAL EXPRESSION FOR THE PROBABILITY

In the conditions formulated in Sec. 1, the propagation of an electron in a magnetic field takes the form

$$S_e(x,y) = \frac{\gamma}{2\pi} f(x_\perp, y_\perp) S_e(x-y), \quad \gamma = |eB|, \tag{5}$$

$$f(x_\perp, y_\perp) = \exp\left\{ -\frac{i\gamma}{2}(x_1+y_1)(x_2-y_2) - \frac{\gamma}{4}[(x_1-y_1)^2 + (x_2-y_2)^2] \right\}, \tag{5a}$$

$$S_e(z) = \frac{1}{(2\pi)^2} \int d^2p \, \frac{\check{p}+m_e}{p^2-m_e^2} e^{-i(pz)}, \tag{5b}$$

where the scalar products and contractions in (5b) (and in what follows, unless otherwise stated) are two-dimensional in the subspace (0, 3), and

$$p^2 = p_0^2 - p_3^2, \quad \check{p} = \tilde{\gamma}_0 p_0 - \tilde{\gamma}_3 p_3, \tag{6}$$

with $\tilde{\gamma}_\alpha$ being 2×2 matrices in the two-dimensional subspace (0, 3). The representation (5b) is valid if

$$\gamma \gg |(p^2 - m_e^2)_{\text{eff}}|, \tag{7}$$

a condition that is met in a strong field $B \gg B_e$ if the integrals over the loop momenta on the electron mass converge, which corresponds to a situation in which the ground Landau level provides the dominant contribution to the virtual states.

Below, we make use of the following properties of the matrices $\tilde{\gamma}_\alpha$ and the matrix $\tilde{\gamma}^5 = \tilde{\gamma}^0 \tilde{\gamma}^3$, which plays the role of γ^5 in the two-dimensional case:

$$\begin{gathered}
\frac{1}{2}\operatorname{Tr}(\tilde{\gamma}_\alpha \tilde{\gamma}_\beta) = \tilde{g}_{\alpha\beta}, \\
\frac{1}{2}\operatorname{Tr}(\tilde{\gamma}_\alpha \tilde{\gamma}_\beta \tilde{\gamma}_\rho \tilde{\gamma}_\sigma) = \tilde{g}_{\alpha\beta}\tilde{g}_{\rho\sigma} + \tilde{g}_{\alpha\sigma}\tilde{g}_{\beta\rho} - \tilde{g}_{\alpha\rho}\tilde{g}_{\beta\sigma}, \\
\tilde{\gamma}_\alpha \check{p} \tilde{\gamma}^\alpha = 0, \\
\frac{1}{2}\operatorname{Tr}(\tilde{\gamma}^5 \tilde{\gamma}_\alpha \tilde{\gamma}_\beta) = \tilde{e}_{\alpha\beta}, \\
\frac{1}{2}\operatorname{Tr}(\tilde{\gamma}^5 \tilde{\gamma}_\alpha \tilde{\gamma}_\beta \tilde{\gamma}_\rho \tilde{\gamma}_\sigma) = \tilde{g}_{\alpha\beta}\tilde{e}_{\rho\sigma} + \tilde{g}_{\rho\sigma}\tilde{e}_{\alpha\beta}, \\
\tilde{e}_{\alpha\beta} p^\beta p_\rho = \tilde{e}_{\rho\beta} p^\beta p_\alpha - p^2 \tilde{e}_{\rho\alpha},
\end{gathered} \tag{8}$$

and so on. Here $\tilde{g}_{\alpha\beta}$ and $\tilde{e}_{\alpha\beta}$ are the metric and totally antisymmetric tensors in (0, 3):

$$\tilde{g}_{03} = \tilde{g}_{30} = \tilde{e}_{00} = \tilde{e}_{33} = 0,$$

$$\tilde{g}_{00} = -\tilde{g}_{33} = \tilde{e}_{30} = -\tilde{e}_{03} = 1.$$

Note that when the contact approximation is employed for the propagator of the W boson, the two phase factors that look like (5a) in the resulting "electron loop" cancel out and, hence, the four-momentum in the process $\nu_i \to \nu_j \gamma$ is conserved.

This leads to the following expression for the matrix element (defined in the ordinary manner) that describes the contribution of the leading diagram considered here:

$$M_e = \frac{eG\gamma U_{ei}U_{ej}^*}{2(2\pi)^{5/2}} [\bar{u}_\nu(k_j)\gamma^\mu(1+\gamma^5)u_\nu(k_i)]e^\sigma I_{\sigma\mu}(q), \tag{9}$$

where e^σ and q are the photon polarization vector and momentum, the quantities in the neutrino brackets are obviously four-dimensional, and the two-dimensional tensor $I(q)$ takes the form

$$I_{\sigma\mu} = -2i \int d^2k \mathrm{Tr} \times \left[\tilde{\gamma}_\sigma \frac{\check{k}+m_e}{k^2-m_e^2} \tilde{\gamma}_\mu (1+\tilde{\gamma}^5) \frac{\check{k}+\check{q}+m_e}{(k+q)^2-m_e^2} \right]. \tag{10}$$

In view of the property (8), the logarithmically divergent contribution to (10) cancels out, and the condition (7) for the applicability of representation (5) with $B \gg B_e$ is met. Calculations yield the following gauge-invariant form of the tensor:

$$I_{\sigma\mu} = A(q^2)(\tilde{e}_{\sigma\alpha} q^\alpha q_\mu + \tilde{g}_{\sigma\mu} q^2 - q_\sigma q_\mu),$$
$$q^\sigma I_{\sigma\mu} = 0, \tag{11}$$

where $A(q^2)$, a scalar function in (0, 3), is determined by the following expressions for different values of q^2 (note that $q^2 = q_1^2 + q_2^2 \neq 0$ for a real photon owing to the two-dimensional nature of the contractions):

(a) $0 \leqslant q^2 < 4m_e^2$:

$$A = -\frac{8\pi}{q^2} \left[1 - \frac{4}{\sqrt{\tilde{q}^2(4-\tilde{q}^2)}} \tan^{-1} \left(\frac{\tilde{q}^2}{\sqrt{\tilde{q}^2(4-\tilde{q}^2)}} \right) \right], \tag{11a}$$

$$\tilde{q}^2 = \frac{q^2}{m_e^2},$$

(b) $q^2 > 4m_e^2$:

$$A = \frac{8\pi}{q^2} \left(1 - \frac{2\xi \ln\xi}{1-\xi^2} \right) - i \frac{16\pi^2}{q^2} \frac{\xi}{1-\xi^2}, \tag{11b}$$

$$\tilde{q}^2 = \frac{(1+\xi)^2}{\xi}, \quad 0 < \xi < 1.$$

In view of the invariance of $k_0 W_e$ (with k_0 the 4-momentum of the initial neutrino and W_e the decay probability per unit time) under boosts along the 3 axis, we can set k_3 to zero with no loss of generality. Then by summing, averaging over polarizations, and performing standard transformations we arrive at the following expression for the differential probability with respect to the photon variables:

$$\frac{k_0\, dW_e}{d\varphi\, dq_\perp} = w_e q_\perp^3 \frac{|A(q_\perp^2)|^2}{\sqrt{R}} [-2k_\perp q_\perp^3 \cos\varphi + (4k_\perp^2 \cos^2\varphi - 2k_0^2 - \Delta^2) q_\perp^2 + 4k_\perp \Delta^2 q_\perp \cos\varphi + \Delta^4], \tag{12}$$

$$R = -4(k_0^2 - k_\perp^2 \cos^2\varphi) q_\perp^2 + 4\Delta^2 k_\perp q_\perp \cos\varphi + \Delta^4,$$
$$\Delta^2 = m_i^2 - m_j^2, \tag{12a}$$

$$0 \leqslant \varphi \leqslant 2\pi, \quad 0 \leqslant q_\perp \leqslant \frac{\Delta^2}{2(k_0 - k_\perp \cos\varphi)}, \tag{12b}$$

$$w_e = \frac{\alpha G^2 \gamma^2 |U_{ei}U_{ej}^*|^2}{2(2\pi)^7}, \tag{12c}$$

where $k_\perp$ and $q_\perp$ are the components of the initial neutrino and the photon momentum perpendicular to the field, and φ is the azimuthal angle of the vector $\mathbf{q}$ reckoned from $k_\perp$. In the general case where $k_3 \neq 0$, one must put $k_0^2 \to k^2$ in the final result.

At this point, we note that the probability is proportional to the square of the magnetic field strength. A linear dependence of the amplitude on the field strength is a general property of diagrams with an electron loop with an even number of vector and/or pseudovector vertices in the two-dimensional formalism used here, for values of B much larger than B_e (see Refs. 12 and 23).

There is no way to integrate (12) using the exact form factor $A(q_\perp^2)$ of Eq. (11). This requires that we examine certain special cases.

3. HIGH-ENERGY AND THE LOW-ENERGY ASYMPTOTIC BEHAVIOR OF THE DECAY PROBABILITY

The parameter in the expansion of the form factor $A(q_\perp^2)$ is $q_\perp / m_e^2$, so that in what follows we discuss the asymptotic behavior in the neutrino energy.

a)

$$\frac{1}{2} \left(1 - \frac{m_j^2}{m_i^2} \right) (k_0 + k_\perp) \ll m_e. \tag{13}$$

Then $q_\perp^2 / m_e^2 \ll 1$ over the entire range of integration, and in (12) we can, in accordance with (11), put

$$A(q_\perp^2) \to A(0) = \frac{4\pi}{3m_e^2}.$$

Integration with respect to $dq_\perp$, which is a lengthy procedure, yields the following angular distribution of the final photon:

$$W_e = \frac{\pi^2 w_e \Delta^{10}}{36 k_0^5 m_e^4} \int_0^{2\pi} \frac{d\varphi}{u^2} \left\{ \frac{1}{5} + 7 \left[\frac{1}{5u} - \frac{9}{4u^2} + \frac{9}{4u^3} + I v_\perp \cos\varphi \left(-\frac{3}{u^2} + \frac{9}{2u^3} \right) \right] + \frac{\Delta^2}{k_0^2} \left(\frac{1}{6u} + 7 \left[-\frac{39}{40u^2} + \frac{47}{16u^3} - \frac{33}{16u^4} + I v_\perp \cos\varphi \left(-\frac{1}{u^2} + \frac{9}{2u^3} - \frac{33}{8u^4} \right) \right] \right) \right\}, \tag{15}$$

$$I=\frac{1}{2\sqrt{u}}\left[\frac{\pi}{2}+\sin^{-1}(v_\perp\cos\varphi)\right],$$

$$u=1-v_\perp^2\cos^2\varphi,\quad v_\perp=\frac{k_\perp}{k_0}. \tag{15a}$$

As expected, when $v_\perp\to 1$, the final particles are emitted in a narrow cone in the direction of the momentum of the initial neutrino, and when $v_\perp\to 0$, the distribution becomes isotropic.

Integrating over the angle, some rearrangement yields the final result:

$$W_e=\frac{\alpha G^2|U_{ei}U_{ej}^*|^2\Delta^{10}}{180(2\pi)^4k_0^5}\left(\frac{B}{B_e}\right)^2\gamma_\perp^8\times\left[4\gamma_\perp^2-3-\frac{\Delta^2}{k_0^2}\left(\frac{10}{3}\gamma_\perp^4-4\gamma_\perp^2+1\right)\right], \tag{16}$$

where we have introduced the transverse Lorentz factor

$$\gamma_\perp=(1-v_\perp^2)^{-1/2}. \tag{16a}$$

We now write the probabilities of decay of a neutrino at rest ($v_\perp=0$) and an ultrarelativistic neutrino ($v_\perp\to 1$), both with the constraint (13):

$$W_e(v_\perp=0)=\frac{\alpha G^2|U_{ei}U_{ej}^*|^2m_i^5}{270(2\pi)^4}\left(\frac{B}{B_e}\right)^2\times\left(1-\frac{m_j^2}{m_i^2}\right)^5\left(1+\frac{m_j^2}{2m_i^2}\right), \tag{17a}$$

$$W_e(v_\perp\to 1)=\frac{\alpha G^2|U_{ei}U_{ej}^*|^2m_i^6}{270(2\pi)^4k_0}\left(\frac{B}{B_e}\right)^2\times\left(1-\frac{m_j^2}{m_i^2}\right)^5\left(1+5\frac{m_j^2}{m_i^2}\right)\gamma_\perp^6. \tag{17b}$$

These relations correspond to Eqs. (6) and (11) of Ref. 10 written in a similar approximation for a crossed field. Naturally, the two sets of equations do not coincide since the two approaches are not equivalent in the region specified by (13). The most important difference consists in the different powers of the large factor (F/B_e), where F is the field's amplitude. More precisely, for a magnetic field the probability is proportional to $(F/B_e)^2$, while for a crossed field the probability is proportional to $(F/B_e)^6$. Nevertheless, even for realistically strong astrophysical magnetic fields, the probability W_e is considerably higher than the probability W_{vac} of decay in a vacuum:

$$\frac{W_e}{W_{\text{vac}}}\sim\left(\frac{B}{B_e}\right)^2\left(\frac{m_W}{m_\tau}\right)^4.$$

In the situation opposite that specified by (13), the main contribution to the integral over $dq_\perp$ in (12) is provided by the region near the upper limit with an effective value of $\cos^2\varphi\sim 1$. Employing the asymptotic value of the form factor that follows from (11b),

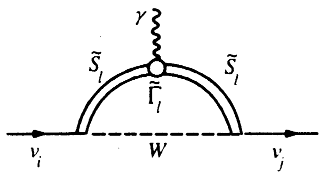

FIG. 2.

$$A(q_\perp^2)\Big|_{q_\perp^2\gg m_e^2}\simeq\frac{8\pi}{q_\perp^2}, \tag{18}$$

some simple calculations yield

$$W_e\simeq\frac{2\alpha G^2|U_{ei}U_{ej}^*|^2m_e^4m_i^2}{(2\pi)^4k_0}\left(\frac{B}{B_e}\right)^2\left(1-\frac{m_j^4}{m_i^4}\right)\gamma_\perp^2. \tag{19}$$

Equation (19) is identical to Eq. (12) of Ref. 10 apart from notation (we ascribe the difference of $\frac{1}{2}$ to errors committed by the authors), which is as it should be, as the two methods overlap in this region (see also Sec. 1).

4. DECAY IN ULTRAHIGH FIELDS. SUMMATION OVER LOOPS

For fields in the range

$$B_W=\frac{m_W^2}{e}\simeq 10^{24}\text{ G}\gg B\gtrsim 10^{18}\text{ G}, \tag{20}$$

we can still ignore their effect on the W boson propagator, while the structure of the other parts of the diagram in Fig. 2 is determined by the two-dimensional vertex functions Γ_l and lepton propagators $\tilde S_l$, which are exact (in the QED sector) in α, and are leading in the field.

We start with the electron contribution $l=e$. Representing $\tilde\Gamma_e$ in the form $\tilde\Gamma_e^\alpha=\tilde\gamma^\alpha+\tilde\Lambda^\alpha$, we note that if (13) holds and the integrals over the two-dimensional momentum of the loop on m_e are convergent, the effective "field" contribution $\langle\tilde\Lambda\rangle$ is smaller than the "bare" contribution $\langle\tilde\gamma\rangle$ in accordance with a relationship that follows from Ref. 20:

$$\frac{\langle\tilde\Lambda\rangle}{\langle\tilde\gamma\rangle}\ll\frac{\alpha}{\pi}\ln\left(\frac{\pi}{\alpha}\right)\left(\frac{B}{B_e}\right)^{(\alpha/2\pi)\ln(\pi/2)}. \tag{21}$$

Since even for $B\sim B_W$ the value of the exponentiated factor is of order unity, we have

$$\frac{\langle\tilde\Lambda\rangle}{\langle\tilde\gamma\rangle}\ll 10^{-2},$$

and $\tilde\Gamma_e$ can be assumed equal to $\tilde\gamma$ to a high degree of accuracy.

In the same field range (20), the two-dimensional electron propagator takes the form[17,18]

$$\tilde S_e=\frac{\check p+m_e^*}{p^2-m_e^{*2}}, \tag{22}$$

with the effective renormalized mass

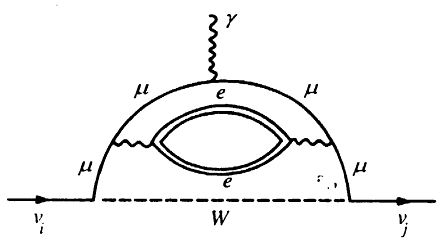

FIG. 3.

$$m_e^* = m_e\left(\frac{B}{B_e}\right)^{(\alpha/2\pi)\ln(\pi/\alpha)}. \tag{23}$$

In comparison to (21), the difference between $\tilde{S}_e$ from the "bare" two-dimensional propagator in (5b) must still be taken into account, since in the field range (20) the value of m_e^* can exceed that of m_e by some tens of percent.

Note that the weak dependence of $\tilde{S}_e$ and $\tilde{\Gamma}_e$ on the field strength is obtained only if we sum the perturbation series in α, while the subsequent terms of the series in fields $\gtrsim 10^{18}$ G yield contributions at least of the order of the preceding terms because of the actual dependence of the expansion parameter on B.

Thus, the matrix element corresponding to the electron contribution is determined everywhere in the range (20) by Eqs. (9), (10), and (14) with m_e^* substituted for m_e. Furthermore, for all values $B_e \ll B \ll 10^{24}$ G when (13) holds, we can formally invoke these equations with the indicated substitution, since for $B \lesssim 10^{18}$ G, m_e^* differs from m_e by at most several percent.

In the range $B_W \gg B \gg B_\mu = (m_\mu^2/e) = 1.88\times 10^{18}$ G, for the contribution of $l=\mu$ we can accurately use the one-loop approximation with "two-dimensional" propagators of μ (Fig. 1a). Multiloop diagrams of the type depicted in Fig. 3 may compete here because of the smaller electron mass and the fact that the perturbation theory is inapplicable to the "electron" sector in this interval. The contribution of diagrams of this type is small, however. Indeed, the following propagator corresponds to the "heavy" photon line:[19]

$$\tilde{\mathscr{D}}_{\alpha\beta} = \frac{4\pi\tilde{g}_{\alpha\beta}}{\kappa^2 - \kappa_\perp^2 - \mathscr{P}_e(\kappa^2,\kappa_\perp^2)}, \tag{24}$$

where κ and $\kappa_\perp$ are the two-dimensional and transverse momenta of an internal photon line, and $\mathscr{P}_e(\kappa^2,\kappa_\perp^2)$ the photon polarization operator, exact in α in the electron sector and leading in the field strength:

$$\mathscr{P}_e(\kappa^2,\kappa_\perp^2) = \frac{4}{\pi}\alpha\gamma\left(\frac{1}{2} + \frac{\eta\ln\eta}{1-\eta^2}\right)\exp\left(-\frac{\kappa_\perp^2}{2\gamma}\right),$$

$$-\frac{\kappa^2}{m_e^{*2}} = \frac{(1-\eta)^2}{\eta}, \quad \kappa^2<0, \quad 0<\eta\leq 1. \tag{25}$$

If (13) holds in this section, the integral with respect to κ^2 converges to m_μ^2, and, bearing in mind that $m_\mu^2 \gg m_e^{*2}$, in Eq. (24) we can approximately replace the expression (25) for $\eta\to 0$ with

$$\mathscr{P}_e(\kappa^2,\kappa_\perp^2) \simeq \frac{2}{\pi}\alpha\gamma\exp\left\{-\frac{\kappa_\perp^2}{2\gamma}\right\}. \tag{26}$$

The convergence of the integral over $\kappa_\perp^2$ to γ implies that to an accuracy of 10^{-2}, we can neglect $\mathscr{P}_e(\kappa^2,\kappa_\perp^2)$ in (24). But then the photon lines in Fig. 3 will be "bare," and such diagrams can be discarded in the muonic sector, so that the one-loop approximation suffices.

Since in the applicability range of the two-dimensional approximation for the muonic sector, $B \gg B_\mu$, but the corresponding amplitude is proportional to $1/m_\mu^2$ [see Eq. (14) where e is replaced by μ], and the analysis conducted in this section suggests that the "electron contribution" is proportional to $1/m_e^{*2}$, the electron sector dominates. On the other hand, since even for $B \lesssim B_\mu$ the diagram with $l=\mu$ is suppressed by the propagator factors, it is obvious that over the entire range $B_W \gg B \gg B_e$, we can to a high degree of accuracy ignore the muonic and, all the more so, the tauonic contribution.

5. CONCLUSION

Thus, with initial-neutrino energies satisfying condition (13), the probability of decay in the field-strength interval $B_W \gg B \gg B_e$ is thus specified by Eqs. (15), (16), (17a), and (17b) with $m_e \to m_e^*$ as given by (23). In the interval

$$m_\mu \gtrsim \sqrt{\gamma} \gg \frac{1}{2}\left(1 - \frac{m_j^2}{m_i^2}\right)(k_0 + k_\perp) \gg m_e \tag{27}$$

the probability coincides with the result of calculations for a crossed field and is given by Eq. (19). In all cases considered here it exceeds by many orders of magnitude the corresponding expression for the decay in a vacuum because of the absence of the suppressing GIM mechanism, and because of the presence of a field factor $(B/B_e)^2 \gg 1$ and a relativistic factor $\gamma_\perp^{(2-6)} \gg 1$.

In the narrow neutrino energy range

$$m_W \gg \sqrt{\gamma} \gg \frac{1}{2}\left(1 - \frac{m_j^2}{m_i^2}\right)(k_0 + k_\perp) \gg m_\tau, \tag{28}$$

and still retaining the contact and two-dimensional approximation, the matrix elements M_e, M_μ, and M_τ are independent of the masses m_e^* [see Eqs. (9), (11), and (18)]; because of the unitarity of the mixing matrix, the leading contribution in the field is suppressed.

Having in mind possible astrophysical applications, we give below the principal results written in various forms for the convenience of estimates:

$$W(v_\nu=0) \simeq 5.7\times 10^{-26}\ \mathrm{yr}^{-1}\left(\frac{m_i}{10\ \mathrm{eV}}\right)^5\left(\frac{B}{B_e^*}\right)^2$$

$$\times(1-x^2)^5\left(1+\frac{x^2}{2}\right)|U_{ei}U_{ej}^*|^2. \tag{29}$$

$$W(v_\nu\to 1,\ E_\nu \ll m_e) \simeq 1.2\times 10^{-10}\ \mathrm{s}^{-1}\left(\frac{E_\nu}{m_e}\right)^5\left(\frac{B}{B_e^*}\right)^2$$

$$\times(1-x^2)^5(1+5x^2)|U_{ei}U_{ej}^*|^2, \tag{30}$$

$$W(E_\nu \gg m_e) \simeq 6.6 \times 10^{-8}\ \mathrm{s}^{-1} \left(\frac{E_\nu}{m_e}\right)\left(\frac{B}{B_e^*}\right)^2 \times (1-x^4)|U_{ei}U_{ej}^*|^2, \qquad (31)$$

where we have introduced the notation $x=m_j/m_i$ and $B_e^*=m_e^{*2}/e$. As Eqs. (30) and (31) show, the time it takes a high-energy neutrino to decay in fields with an induction $B \gg B_e^*$ is quite comparable to the characteristic times of the Big Bang, when such fields are possible. This suggests that the ultrarelativistic neutrinos in the Universe are in the lowest mass state. In the nonrelativistic sector [Eq. (29)], the decay time is much longer than the most optimistic estimates of the lifetime of fields $B \gg B_e^*$, and hence, here the dominant process is the decay of a free neutrino, since the corresponding time interval is comparable to the age of the Universe.

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[1] A. de Rujula and S. Glashow, Phys. Rev. Lett. **45**, 942 (1980).
[2] F. W. Stecker, Phys. Rev. Lett. **45**, 1460 (1980).
[3] S. T. Petkov, Yad. Fiz. **25**, 641 (1977) [Sov. J. Nucl. Phys. **25**, 340 (1977)].
[4] T. Goldman and G. J. Stephenson, Phys. Rev. D **16**, 2256 (1977).
[5] R. K. Ghosh, Phys. Rev. D **29**, 493 (1984).
[6] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
[7] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
[8] P. B. Pal and L. Wolfenstein, Phys. Rev. D **25**, 766 (1982).
[9] J. Liu, Phys. Rev. D **44**, 2879 (1991).
[10] L. A. Vasilevskaya, A. A. Gvozdev, and N. V. Mikheev, Yad. Fiz. **57**, 124 (1994) [Phys. Atom. Nuclei **57**, 117 (1994)].
[11] D. V. Gal'tsov and N. S. Nikitina, Zh. Éksp. Teor. Fiz. **62**, 2008 (1972) [Sov. Phys. JETP **35**, 1047 (1972)].
[12] V. V. Skobelev, Zh. Éksp. Teor. Fiz. **71**, 1263 (1976) [Sov. Phys. JETP **44**, 660 (1976)].
[13] V. V. Skovelev, Izv. Vyssh. Uchebn. Zaved., Fiz., No. 10, 142 (1975); Yu. M. Loskutov and V. V. Skobelev, Phys. Lett. A **56**, 151 (1976).
[14] V. I. Ritus, Trudy Fiz. Inst. Akad. Nauk SSSR **111**, 3 (1979).
[15] V. V. Skobelev, Doctor Phys.–Math. Sci. Dissertation [in Russian], Moscow State University (1982).
[16] V. V. Skobelev, Zh. Éksp. Teor. Fiz. **73**, 1301 (1977) [Sov. Phys. JETP **46**, 684 (1977)].
[17] Yu. M. Loskutov and V. V. Skobelev, Teoret. Mat. Fiz. **48**, 44 (1981).
[18] Yu. M. Loskutov, B. A. Lysov, and V. V. Skobelev, Teoret. Mat. Fiz. **53**, 469 (1982).
[19] Yu. M. Loskutov and V. V. Skobelev, Vestn. Mosk. Univ., Ser. III Fiz. Astronom. **24**, 95 (1983).
[20] Yu. M. Loskutov and V. V. Skobelev, Vestn. Mosk. Univ., Ser. III Fiz. Astronom. **25**, 70 (1984).
[21] V. R. Khalilov, *Electrons in a Strong Magnetic Field* [in Russian], Energoatomizdat, Moscow (1988).
[22] Tsai Wu-Yang and A. Yildiz, Phys. Rev. D **4**, 3643 (1971).
[23] Yu. M. Loskutov and V. V. Skobelev, Phys. Lett. A **62**, 53 (1977).

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