A self-consistent theory of the metal–insulator transition in disordered systems

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We generalize the self-consistent theory of electron localization in disordered systems to allow for the electron–electron interaction. We suggest and compare several kinds of self-consistency that take into account the lowest orders of the perturbation theory in the interaction. Depending on the scheme, we arrive either at a continuous metal-insulator transition or at a transition with the lowest possible metallic conductivity. In the scheme with a continuous transition we calculate the frequency dependence of the generalized diffusion coefficient in both the metal phase and the insulator. Finally, we calculate the renormalization of the density of states caused by the interaction, which shows that in the transition from metal to insulator an effective gap is formed. © *1995 American Institute of Physics.*

1. INTRODUCTION

The disorder-induced metal–insulator transition is one of the most fundamental problems of the theory of condensed state.[1,2] The greatest difficulties lie in the problem of consistently taking into account the electron—electron interaction, whose important role has been evident for a fairly long time in the case of weakly disordered metals.[3] In recent years an entirely new avenue of research has appeared in which this problem is studied within the renormalization-group approach,[1,2,4] which generalizes the ordinary scaling localization theory.[1] Notwithstanding the indisputable successes of this approach, the problem is still far from solved, especially in the prediction of specific experimentally verifiable patterns of behavior of physical quantities. Generally the renormalization-group (scaling) approach is suited only for analyzing the immediate vicinity of the transition,[2] while experiments usually deal with variations in physical properties over a wide range of parameters controlling the transition. For one thing, we know of no papers in which the renormalization-group approach was successful in describing the properties of the insulator phase.

For a fairly long time theories of the metal–insulator transition that exclude the electron–electron interaction, in addition to the scaling approach,[1] have used what is known as the self-consistent localization theory,[5–7] which represents an effective interpolation scheme allowing the main physical characteristics to be calculated over a broad range of variation of the parameters, from a weakly disordered metal to an Anderson insulator. This theory also reproduces the basic results of the scaling approach.[5,6] Although no rigorous diagrammatic substantiation of the self-consistent justification exists, quantitatively the theory agrees with the results of exact numerical modeling of the Anderson transition.[8,9] The first attempts at allowing for the electron–electron interaction in the self-consistent localization theory were done in Refs. 10 and 11 (see also Ref. 6).

In this paper we construct a self-consistent theory of the metal–insulator transition that allows for the effects of electron–electron interaction in the first order of perturbation theory. In contrast to Refs. 10 and 11, we allow for the effect of the interaction on the generalized diffusion coefficient, the main quantity that must be determined self-consistently. Unfortunately, there is a certain ambiguity in our approach, and several self-consistency schemes can be suggested that lead to different physical results. Which scheme to choose must apparently depend on which agrees best with the experiment. For one thing, the choice will determine whether one can obtain a continuous metal–insulator transition or a transition with the lowest possible metallic conductivity. As is known, present-day experiments favor the continuous transition pattern.[1,2] Within this scheme we calculate the frequency dependence of the generalized diffusion coefficient and the renormalization of the one-particle (tunneling) density of states caused by the interaction in the transition from metal to insulator.

2. THE INTERACTION CORRECTIONS TO CONDUCTIVITY NEAR THE ANDERSON TRANSITION POINT

We introduce the two-particle Green's function of an electron in the random field of impurities:[5]

$$\Phi^{RA}_{\mathbf{pp}'}(\mathbf{q},\omega,\varepsilon)=\langle G^R(\mathbf{p}_+,\mathbf{p}'_+,\varepsilon+\omega)G^A(\mathbf{p}_-,\mathbf{p}'_-,\varepsilon)\rangle, \qquad (1)$$

where $\mathbf{p}_\pm=\mathbf{p}\pm\mathbf{q}/2$, and $\langle\cdots\rangle$ stands for averaging over the random potential. Next we introduce two quantities corresponding to the Green's function: the total vertex $\Gamma^{RA}_{\mathbf{pp}'}(\mathbf{q},\omega)$ and the "triangular" vertex $\gamma^{RA}(\mathbf{p},\mathbf{q},\omega)$ (Fig. 1).

In the self-consistent localization theory the total and triangular vertices have the following form:[6]

$$\Gamma^{RA}_{\mathbf{pp}'}(\mathbf{q},\omega)=\frac{2\gamma\rho U^2}{-i\omega+D(\omega)q^2}, \qquad (2)$$

$$\gamma^{RA}(\mathbf{p},\mathbf{q},\omega)=\frac{2\gamma}{-i\omega+D(\omega)q^2}, \qquad (3)$$

where $\gamma=1/2\tau=\pi\rho U^2N_0(0)$ is the Born damping, ρ is the impurity concentration, U is the impurity potential, $N_0(0)$ is the density of states of independent electrons on the Fermi level, and $D(\omega)$ is the generalized diffusion coefficient.

The form of the respective vertices corresponds to that obtained in the ladder approximation with the ordinary

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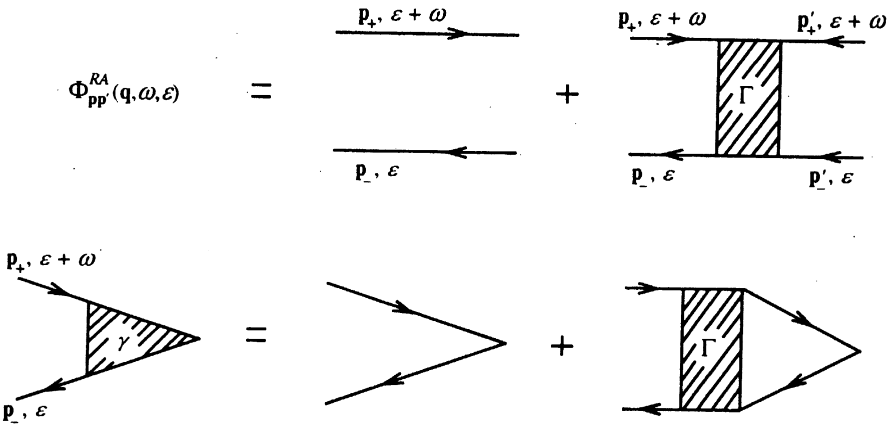

FIG. 1. The two-particle Green's function $\Phi^{RA}_{pp'}(\mathbf{q},\omega,\varepsilon)$ and the corresponding vertices $\gamma^{RA}(\mathbf{p},\mathbf{q},\omega)$ and $\Gamma^{RA}_{pp'}(\mathbf{q},\omega)$.

Drude diffusion coefficient D_0 replaced by the frequency-dependent generalized diffusion coefficient $D(\omega)$ defined by the following equation:[5,7]

$$\frac{D_0}{D(\omega)}=1+\frac{1}{\pi N_0(0)}\sum_{|\mathbf{q}|<k_0}\frac{1}{-i\omega+D(\omega)q^2}, \tag{4}$$

where $k_0=\min\{l^{-1},k_{\rm F}\}$ is the cutoff momentum, l is the mean free path, and $k_{\rm F}$ is the Fermi momentum.

In a three-dimensional system the generalized diffusion coefficient in the self-consistent localization theory has the following form

$$\frac{D_s(\omega)}{D_0}=\begin{cases}(3\pi\lambda/2)(\omega_c/E_{\rm F})^{1/3}=\alpha, & \omega\ll\omega_c,\ \alpha>0\\ \text{(metal)}, & \\ (3\pi\lambda/2)(-i\omega/E_{\rm F})^{1/3}, & \omega\gg\omega_c\\ \text{(metal and insulator)}, & \\ (3\pi\lambda/2)(\omega_c/E_{\rm F})^{-2/3}(-i\omega/E_{\rm F}), & \omega\ll\omega_c,\ \alpha<0\\ \text{(insulator)}, & \end{cases} \tag{5}$$

where $\lambda=\gamma/\pi E_{\rm F}$ is the dimensionless order parameter, $E_{\rm F}$ is the Fermi energy, $\omega_c=(2|\alpha|/3\pi\lambda)E_{\rm F}$ is the characteristic frequency, $\alpha=1-3\lambda x_0$ is a parameter characterizing the transition from metal to insulator, and $x_0=k_0/k_{\rm F}$ is the cutoff parameter.

If we ignore the localization contributions, the first-order corrections in the Coulomb-interaction to the conductivity are represented by the diagrams depicted in Fig. 2 (see Ref. 12). Altshuler *et al.*[12] have shown that the total contribution of the diagrams (a), (b), and (c) is zero and that the correction to the conductivity is determined only by the diagrams (d) and (e). Here we ignore the so-called Hartree contributions to the conductivity,[3,12] which is valid in the limit $2k_{\rm F}/\kappa\gg1$, where κ is the reciprocal Debye screening radius. Strictly speaking, this is true for systems with a low electron number density. Such systems are most interesting from the standpoint of metal–insulator transition experiments. In view of the well-known divergence of the screening radius at the Anderson transition point,[6] this approximation, apparently, improves as the system approaches the transition point. This is a reservation to be kept in mind in interpreting the "point" interaction model employed here.

Using the form of the impurity vertices (2) and (3), we arrive at the following expression for the correction to conductivity (cf. Ref. 12):

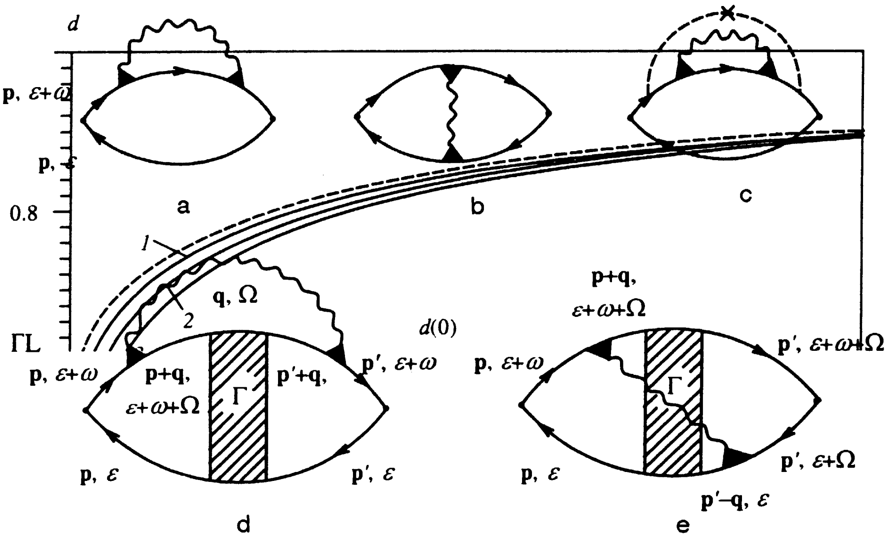

FIG. 2. First-order interaction corrections to conductivity.

$$\delta\sigma(\omega)=\frac{32i}{\pi d}\,e^2N_0(0)D_0^2\pi d\int_\omega^\infty\frac{d\Omega}{2\pi}\int\frac{d^dq}{)2\pi)^d}$$

$$\times\frac{V(q\omega)q^2}{[-i(\Omega+\omega)+D(\Omega+\omega)q^2]^2[-i\Omega+D(\Omega)q^2]^2}. \qquad (6)$$

Here e is the electron charge, d is the dimensionality of the system, and $V(q\Omega)$ is the interaction. For simplicity we consider chiefly the point interaction $V(q\Omega)=V_0$. In the case of dynamically screened Coulomb interaction and small q and Ω we have:[3]

$$V(q\Omega)=\frac{1}{2N_0(0)}\,\frac{-i\Omega+D(\Omega)q^2}{D(\Omega)q^2}. \qquad (7)$$

For the point-interaction correction to the diffusion coefficient we have

$$\frac{\delta D_C(\omega)}{D_0}=\frac{\delta\sigma(\omega)}{2e^2N_0(0)D_0}=\frac{8i}{\pi d}\,\frac{\mu D_0}{\pi N_0(0)}\int_\omega^\infty d\Omega\int\frac{d^dq}{(2\pi)^d}$$

$$\times\frac{q^2}{[-i(\Omega+\omega)+D(\Omega+\omega)q^2][-i\Omega+D(\Omega)q^2]^2}, \qquad (8)$$

where $\mu=V_0N_0(0)$. Employing in (8) the diffusion coefficient obtained in the self-consistent localization theory (5), we have for the three-dimensional system

$$\frac{\sigma D_{Cs}(\omega)}{D_0}=-\frac{2\mu}{3\pi\lambda}$$

$$\times\begin{cases}[(3\sqrt{3}+2\sqrt{2})/2\pi](E_F/\omega_c)^{1/3}, & \omega\ll\omega_c,\ \alpha>0\\ \quad\text{(metal)}, & \\ (3/\pi)(E_F/-i\omega)^{1/3} & \omega\gg\omega_c\\ \quad\text{(metal and insulator)}, & \\ (1/\pi)(\omega_c/E_F)^{5/3}(E_F/-i\omega)^2, & \omega\ll\omega_c,\ \alpha<0\\ \quad\text{(insulator)}. & \end{cases} \qquad (9)$$

Clearly, the correction to the diffusion coefficient (or the conductivity) diverges as the system approaches the Anderson transition point from the metal phase ($\omega_c\to0$), while in the insulator phase a divergence emerges as $\omega\to0$.

If instead of the point interaction we take the dynamically screened Coulomb interaction of type (7), the correction to the diffusion coefficient assumes the form

$$\frac{\delta D_C(\omega)}{D_0}=\frac{4i\lambda}{\pi}\,\frac{D_0^2}{k_F}\int_\omega^\infty d\Omega\int_0^{k_0}$$

$$\times\frac{dq\,q^{d-1}}{[-i(\Omega+\omega)+D(\Omega+\omega)q^2][-i\Omega+D(\Omega)q^2]D(\Omega)}. \qquad (10)$$

If we take the diffusion coefficient in the form (5) obtained in the self-consistent localization theory, we arrive at the expression (9) for the correction with $\mu=1/3$. Using the dynamically screened Coulomb interaction instead of the point interaction results only in μ being of order unity. Hence in what follows we chiefly consider the point interaction.

Thus, near the Anderson transition point the correction to the diffusion coefficient considerably exceeds the diffusion coefficient obtained from the ordinary self-consistent localization theory. This requires building a new self-consistency scheme that includes the electron–electron interaction from the start.

3. SELF-CONSISTENCY SCHEMES

First we recall the scheme used in the ordinary self-consistent theory without Coulomb interaction.[5–7]

When disorder is weak, the conductivity is determined by the Drude diffusion coefficient D_0. Summing the series of "maximally crossing" diagrams leads to the following localization correction to the diffusion coefficient:[5]

$$\frac{\delta D(\omega)}{D_0}=-\frac{1}{\pi N_0(0)}\sum_{|\mathbf{q}|<k_0}\frac{1}{-i\omega+D_0q^2}. \qquad (11)$$

We introduce what is known as the relaxation kernel $M(\omega)$, which is related to the generalized diffusion coefficient through the following formula:[5]

$$M(\omega)=\frac{2iE_F}{dm}\,\frac{1}{D(\omega)}. \qquad (12)$$

For one thing, the Drude relaxation kernel is

$$M_0=\frac{2iE_F}{dm}\,\frac{1}{D_0}=2i\gamma.$$

The correction to the relaxation kernel can be directly expressed in terms of the correction to the diffusion coefficient as follows:

$$\delta M(\omega)=-\frac{2iE_F}{dm}\,\frac{\delta D(\omega)}{D(\omega)}=-\frac{M(\omega)}{D(\omega)}\,\delta D(\omega). \qquad (13)$$

Let us take a Drude metal as the zeroth approximation. Then

$$\delta M(\omega)=-\frac{M_0}{D_0}\,\delta D(\omega). \qquad (14)$$

Replacing the Drude diffusion coefficient D_0 by the generalized diffusion coefficient $D(\omega)$ in the diffusion pole of (11) and substituting the result into Eq. (14), we arrive at the main equation of the self-consistent localization theory:

$$M(\omega)=M_0+\delta M(\omega)$$

$$=M_0\left(1+\frac{1}{\pi N_0(0)}\sum_{|\mathbf{q}|<k_0}\frac{1}{-i\omega+D(\omega)q^2}\right), \qquad (15)$$

which coincides with (5) if we allow for the fact that $M(\omega)/M_0=D_0/D(\omega)$.

Now we add the Coulomb interaction to the system. The first-order corrections due to this interaction are represented by the diagrams (d) and (e) in Fig. 2. Unfortunately, we cannot suggest an unambiguous self-consistency scheme that allows for corrections introduced by the electron–electron interaction. Several alternative forms of such a scheme can be considered, and a choice is made on the basis of additional ideas of a qualitative nature.

Variant I. We act in the same way as we would in the ordinary self-consistent localization theory. We take a Drude metal as the zeroth approximation and examine the localization and Coulomb corrections on an equal basis, carrying out the self-consistency procedure in terms of the relaxation kernel, which here has the form

$$M(\omega)=M_0+\delta M(\omega), \tag{16}$$

where

$$\delta M(\omega)=\delta M_l(\omega)+\delta M_C(\omega)$$
$$=-\frac{M_0}{D_0}[\delta D_l(\omega)+\delta D_C(\omega)].$$

Here the localization correction to the diffusion coefficient $D_l(\omega)$ is determined by (11) and the Coulomb correction $D_C(\omega)$ by (8). The self-consistency procedure is reduced to replacing D_0 by the generalized diffusion coefficient at all diffusion poles. As a result we arrive at the following integral equation for finding the generalized diffusion coefficient:

$$\frac{D_0}{D(\omega)}=1+\frac{1}{\pi N_0(0)}\int\frac{d^dq}{(2\pi)^d}\frac{1}{-i\omega+D(\omega)q^2}$$
$$-\frac{8i}{\pi d}\frac{\mu D_0}{\pi N_0(0)}\int_\omega^\infty d\Omega\int\frac{d^dq}{(2\pi)^d}$$
$$\times\frac{q^2}{[-i(\Omega+\omega)+D(\Omega+\omega)q^2][-i\Omega+D(\Omega)q^2]^2}\,. \tag{17}$$

Variant II. For the zeroth approximation we take an "impure" metal, which is described by the ordinary self-consistent localization theory, and add the weak Coulomb interaction. We again perform the self-consistency procedure by examining the corrections to the relaxation kernel.

The ordinary self-consistent theory is described by Eq. (15). To the right-hand side of this equation we add the Coulomb correction to the relaxation kernel, $\delta M_C(\omega)$. Here this correction has the form

$$\delta M_C(\omega)=-\frac{M_s(\omega)}{D_s(\omega)}\,\delta D_C(\omega), \tag{18}$$

where $M_s(\omega)$ and $D_s(\omega)$ are the relaxation kernel and diffusion coefficient of the ordinary self-consistent localization theory [see Eq. (5)]. In this case the equation for finding the diffusion coefficient assumes the form

$$\frac{D_0}{D(\omega)}=1+\frac{1}{\pi N_0(0)}\int\frac{d^dq}{(2\pi)^d[-i\omega+D(\omega)q^2]}$$
$$-\left(\frac{D_0}{D_s(\omega)}\right)^2\frac{8i\mu D_0}{\pi^2 dN_0(0)}\int_\omega^\infty d\Omega\int\frac{d^dq}{(2\pi)^d}$$
$$\times\frac{q^2}{[-i(\Omega+\omega)+D(\Omega+\omega)q^2][-i\Omega+D(\Omega)q^2]^2}\,. \tag{19}$$

Variant III. In contrast to variant II, we also calculate in a self-consistent manner the Coulomb correction to the relaxation kernel, $\delta M_C(\omega)$, which is added to the right-hand side of the self-consistency equation (15):

$$\delta M_C(\omega)=-\frac{M(\omega)}{D(\omega)}\,\delta D_C(\omega)\,. \tag{20}$$

The equation for finding the diffusion coefficient in this case assumes the form

$$\frac{D_0}{D(\omega)}=1+\frac{1}{\pi N_0(0)}\int\frac{d^dq}{(2\pi)^d}\frac{1}{-i\omega+D(\omega)q^2}$$
$$-\left[\frac{D_0}{D(\omega)}\right]^2\frac{8i}{\pi d}\frac{\mu D_0}{\pi N_0(0)}\int_\omega^\infty d\Omega\int\frac{d^dq}{(2\pi)^d}$$
$$\times\frac{q^2}{[-i(\Omega^+\omega)+D(\Omega+\omega)q^2][-i\Omega+D(\Omega)q^2]^2}\,. \tag{21}$$

Variant IV. We assume once more that the ordinary self-consistent localization theory is the zeroth approximation. However, the self-consistency procedure now consists in examining the corrections to the diffusion coefficient rather than to the relaxation kernel.

The self-consistency equation (4) can be written as

$$\frac{D(\omega)}{D_0}=\left[1+\frac{1}{\pi N_0(0)}\int\frac{d^dq}{(2\pi)^d}\frac{1}{-i\omega+D(\omega)q^2}\right]^{-1}. \tag{22}$$

If to the right-hand side we add the Coulomb correction to the diffusion coefficient, Eq. (8), we arrive at the following equation for finding the diffusion coefficient:

$$\frac{D(\omega)}{D_0}=\left[1+\frac{1}{\pi N_0(0)}\int\frac{d^dq}{(2\pi)^d}\frac{1}{-i\omega+D(\omega)q^2}\right]^{-1}$$
$$+\frac{8i}{\pi d}\frac{\mu D_0}{\pi N_0(0)}\int_\omega^\infty d\Omega\int\frac{d^dq}{(2\pi)^d}$$
$$\times\frac{q^2}{[-i(\Omega+\omega)+D(\Omega+\omega)q^2][-i\Omega+D(\Omega)q^2]^2}\,. \tag{23}$$

The choice between the alternative variants I–IV can hardly be done on the basis of general considerations. Hence we turn to an analysis of the qualitative implications of these equations.

4. QUALITATIVE ANALYSIS OF THE SELF-CONSISTENCY VARIANTS IN THE METAL PHASE

From now on we examine only three-dimensional systems. In the self-consistent localization theory the diffusion coefficient is determined by Eq. (5). This readily yields the relaxation kernel:

$$\frac{M_s(\omega)}{M_0}=\frac{D_0}{D_s(\omega)}\,. \tag{24}$$

Let us examine the electron–electron interaction correction to the relaxation kernel. The correction is given by the following expression:

$$\frac{\delta M_0(\omega)}{M_0}=-\frac{\delta D_C(\omega)}{D_0}\,. \tag{25}$$

Here for the zeroth approximation we have taken a "Drude" metal.

The Coulomb correction to the diffusion coefficient in the ordinary self-consistent localization theory is given by (9). Substituting this expression into (25), we arrive at the coresponding correction to the relaxation kernel, $\delta M_{0s}(\omega)$.

If we compare $\delta M_{0s}(\omega)$ with $M_s(\omega)$, we easily see that in the metal phase the following qualitative estimate holds in the entire frequency range:

$$\delta M_{0s}(\omega)\approx c\mu M_s(\omega), \tag{26}$$

where c is a numerical factor of order unity.

As detailed analysis shows, such an estimate in the metal phase is retained for a self-consistent relaxation kernel that allows for the Coulomb interaction in all self-consistency schemes. The point is that in all schemes in the metal phase the frequency behavior of the diffusion coefficient resembles the ordinary self-consistent localization theory: constant at low frequencies and $D(\omega)\sim(-i\omega)^{1/3}$ at high frequencies. Hence instead of (26) we can write

$$\delta M_0(\omega)\approx c\mu M(\omega). \tag{27}$$

Using this expression (27), we examine the qualitative results to which the different self-consistency schemes in the metal phase at $\omega=0$ lead in the various suggested self-consistency schemes.

Variant I. We examine the metal phase, so that at $\omega=0$ we have $D(\omega=0)=D\neq 0$. Multiplying both sides of Eq. (17) by M_0 we get

$$M=M_0+\frac{M_0}{\pi N_0(0)}\int\frac{d^dq}{(2\pi)^d}\frac{1}{Dq^2}+\delta M_0(0).$$

Employing (27), we obtain

$$M=M_0+3\lambda x_0M+c\mu M.$$

This yields $M=M_0/(1-3\lambda x_0-c\mu)$, or

$$\frac{D}{D_0}=1-3\lambda x_0-c\mu=\alpha-\alpha^*, \tag{28}$$

where $\alpha^*=c\mu$.

Thus, in the presence of the weak Coulomb interaction the Anderson transition is retained, the transition index remains the same, $\nu=1$ (i.e., the diffusion coefficient vanishes linearly in the disorder parameter $\alpha-\alpha^*$), and a shift of the transition point to the region with weaker disorder, $\alpha=\alpha^*$, is observed. The dependence of the diffusion coefficient on disorder is depicted in Fig. 3a.

Note that the analysis of Eq. (17) with a frequency-independent generalized diffusion coefficient yields a transition with the lowest possible metallic conductivity.

Variant II. Multiplying both sides of Eq. (19) by M_0 and employing (27), we get

$$M=M_0+3\lambda x_0M+\frac{1}{\alpha^2}c\mu M.$$

This yields $M=M_0/(\alpha-c\mu/\alpha^2)$, or

$$\frac{D}{D_0}=\alpha-\frac{c\mu}{\alpha^2}. \tag{29}$$

In this case the Anderson transition is retained, the transition index remains the same, $\nu=1$, and a shift of the transition point to the region with weaker disorder, $\alpha=(c\mu)^{1/3}$, is observed. The dependence of the diffusion coefficient on disorder is depicted in Fig. 3b.

Variant III. Multiplying both sides of Eq. (21) by M_0 and using (27), we get

$$M=M_0+3\lambda x_0M+\left(\frac{M}{M_0}\right)^2c\mu M,$$

or

$$\alpha\frac{M}{M_0}=1+c\mu\left(\frac{M}{M_0}\right)^3.$$

Going over to the diffusion coefficient, we get

$$\alpha=\frac{D}{D_0}+\frac{c\mu}{(D/D_0)^2}. \tag{30}$$

In this case we have the lowest possible metallic conductivity:

$$\frac{D_{\min}}{D_0}=(2c\mu)^{1/3}\quad\text{for }\alpha=\alpha^*=\frac{3}{2}(2c\mu)^{1/3}.$$

The dependence of the diffusion coefficient on disorder is depicted in Fig. 3c.

Variant IV. Combining (25) with (27), we arrive at the following expression for the diffusion coefficient:

$$\frac{\delta D_C(\omega)}{D_0}=-\frac{\delta M_0(\omega)}{M_0}=-\frac{c\mu M(\omega)}{M_0}=-c\mu\frac{D_0}{D(\omega)}. \tag{31}$$

Substituting (31) into Eq. (23) yields

$$\frac{D}{D_0}=\frac{1}{1+(1-\alpha)D_0/D}-c\mu\frac{D_0}{D},$$

or

$$\alpha=\frac{c\mu}{(D/D_0)^2+c\mu}+\frac{D}{D_0}. \tag{32}$$

We again get the lowest possible metallic conductivity:

$$\frac{D_{\min}}{D_0}\approx(2c\mu)^{1/3}\quad\text{for }\alpha=\alpha^*\approx\frac{3}{2}(2c\mu)^{1/3}.$$

The dependence of the diffusion coefficient on disorder is depicted in Fig. 3d.

Qualitatively the results of the self-consistency schemes III and IV qualitatively coincide, and they lead to minimum metallic conductivity, which clearly does not agree with most of the experimental data in this area.[1,2] The schemes I and II lead to similar qualitative behavior of the diffusion coefficient; the differences lie only in estimates of the critical disorder determining the transition point. Naturally, in our approximations the electron–electron interaction strengthens the tendency toward a metal–insulator transition, which occurs at a lower disorder than without interaction.

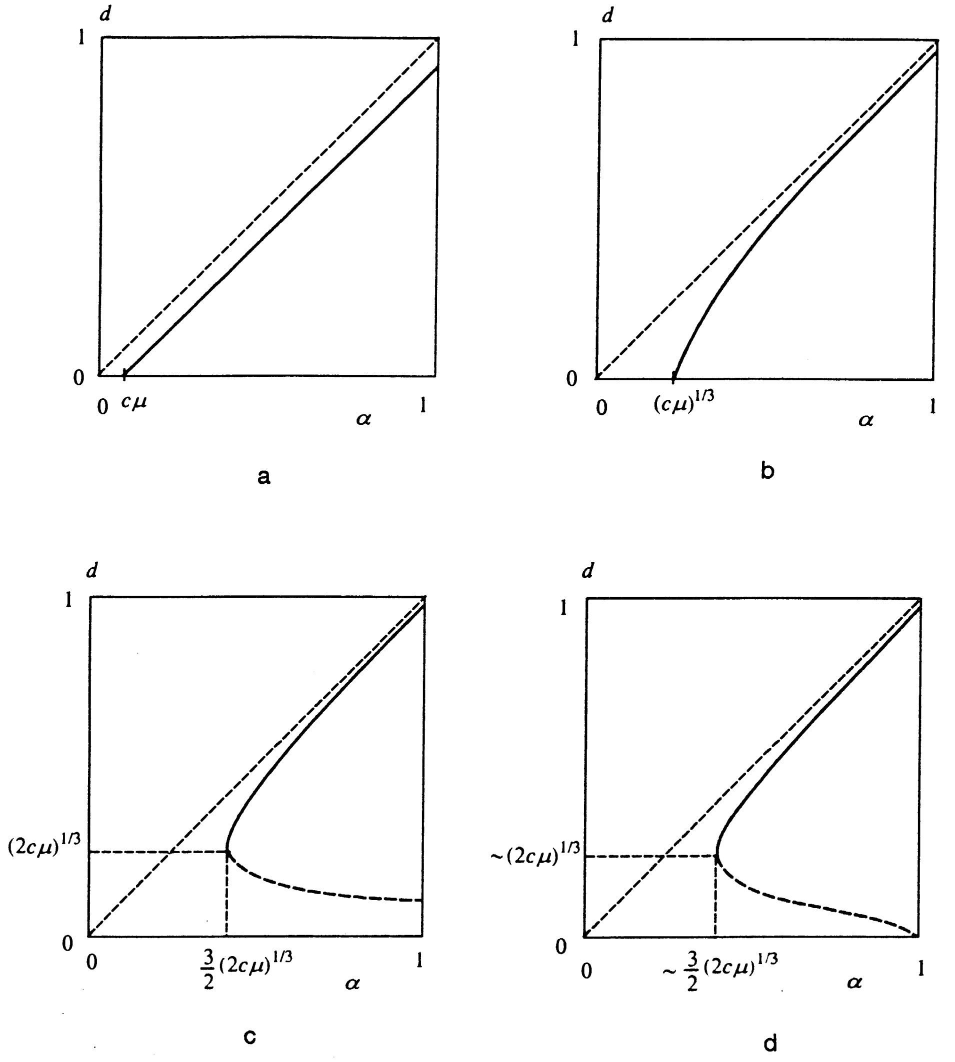

FIG. 3. The disorder-dependence of the static generalized diffusion coefficient ($d=D(0)/D_0$) for different self-consistency schemes: (a), variant I; (b), variant II; (c), variant III; and (d), variant IV.

In what follows we focus on the first self-consistency scheme, in which the Coulomb and localization contributions are considered on an equal basis.

5. THE FREQUENCY DEPENDENCE OF THE DIFFUSION COEFFICIENT

In the first self-consistency scheme I the diffusion coefficient is determined by the integral equation (17). In this equation we go over to dimensionless Matsubara frequencies, $-i\omega/D_0k_0^2\rightarrow\omega$ and $-i\Omega/D_0k_0^2\rightarrow\Omega$, and the dimensionless diffusion coefficient $d(\omega=D(\omega)/D_0)$. In terms of these variables the integral equation (17) assumes the form

$$\frac{1}{d(\omega)}=1+\frac{d\lambda x_0^{d-2}}{d(\omega)}\int_0^1\frac{dy\,y^{d-1}}{y^2+\omega/d(\omega)}+\frac{8}{\pi}\mu\lambda x_0^{d-2}\int_\omega^\infty\frac{d\Omega}{d(\omega+\Omega)d^2(\Omega)}\times\int_0^1\frac{dy\,y^{d+1}}{[y^2+\omega+\Omega/d(\omega+\Omega)][y^2+\Omega/d(\Omega)]^2}. \tag{33}$$

We consider only the three-dimensional case in what follows. The diffusion coefficient of the ordinary self-consistent localization theory [Eq. (5)] in these variables has the form

$$d(\omega)=\begin{cases}\alpha=1-3\lambda x_0, & \omega\ll\omega_c,\ \alpha>0\\ \text{(metal)}, & \\ (3\pi\lambda x_0/2)^{2/3}\omega^{1/3}, & \omega\gg\omega_c\\ \text{(metal and insulator)}, & \\ (3\pi\lambda x_0/2)^2/\alpha^2\omega=(\xi k_0)^2\omega, & \omega\ll\omega_c,\ \alpha<0\\ \text{(insulator)}, & \end{cases} \tag{34}$$

where $\omega_c=|\alpha|^3/(3\pi\lambda x_0/2)^2$, and ξ is the localization radius.

We introduce the function $K(\omega)=\omega/d(\omega)$ and analyze Eq. (33), assuming that $K(\omega)$, $K(\Omega)$, and $K(\omega+\Omega)\ll 1$. Expanding the right-hand side of Eq. (33) in a power series in these small parameters, we get

$$\frac{\alpha}{d(\omega)}=1-\frac{3\pi\lambda x_0}{2d(\omega)}\sqrt{K^{1/2}(\omega)}+2\mu\lambda x_0\int_\omega^\infty\frac{d\Omega}{d(\omega+\Omega)d^2(\Omega)}\times\frac{\sqrt{K(\Omega)}+2\sqrt{K(\omega+\Omega)}}{[\sqrt{K(\Omega)}+\sqrt{K(\omega+\Omega)}]^2}. \tag{35}$$

Let us examine the metal phase. We look for the diffusion coefficient $d(\omega)$ in the form

$$d(\omega)=\begin{cases} d, & \omega\ll\omega_c, \\ d(\omega/\omega_c)^{1/3}, & \omega\gg\omega_c. \end{cases} \tag{36}$$

Substituting (36) into (35), we find d and ω_c. This yields the following expression for the diffusion coefficient:

$$d(\omega)=\begin{cases} \alpha-\alpha^*, & \omega\ll\omega_c, \\ (3\pi\lambda x_0/2)^{2/3}\omega^{1/3}, & \omega\gg\omega_c, \end{cases} \tag{37}$$

where $\omega_c=|\alpha-\alpha^*|^3(3\pi\lambda x_0/2)^{-2}$, $\alpha^*=c\mu$, and $c\approx 0.89$.

Thus, in the metal phase we have found confirmation of our qualitative approach (Sec. 4). The Anderson transition is retained, the transition index remains the same, $\nu=1$, and the transition has shifted to the region of somewhat lower disorder, $\alpha=\alpha^*=c\mu$. Qualitatively the frequency behavior of the diffusion coefficient in the metal phase coincides with that in the ordinary self-consistent localization theory [Eq. (34)]. In the high-frequency range, $\omega\gg\omega_c$, the behavior of the diffusion coefficient undergoes no change when interaction is added to the system.

Now let us examine the insulator phase. In the high-frequency range, $\omega\gg\omega_c$, the diffusion coefficient is clearly frequency-dependent: $d(\omega)\propto\omega^{1/3}$. Assume that in the low-frequency range the diffusion coefficient is a power function of frequency:

$$d(\omega)=\begin{cases} d(\omega/\omega_c)^{\delta}, & \omega\ll\omega_c, \\ d(\omega/\omega_c)^{1/3}, & \omega\gg\omega_c, \end{cases} \tag{38}$$

where δ is an unknown exponent.

Substituting (38) into (35) and examining the case where $\alpha<0$ (the insulator phase in the ordinary self-consistent localization theory) and $|\alpha|\gg\alpha^*$, we get

$$d(\omega)=\begin{cases} \dfrac{(3\pi\lambda x_0/2)^2}{\alpha^2}\omega=(\xi k_0)^2\omega, & \omega^*\ll\omega\ll\omega_c, \\ (3\pi\lambda x_0/2)^{2/3}\omega^{1/3}, & \omega\gg\omega_c, \end{cases} \tag{39}$$

where $\omega_c=|\alpha|^3(3\pi\lambda x_0/2)^2$, and we have introduced a new characteristic frequency determined by the interaction, $\omega^*\approx 0.1\mu\alpha^2/(3\pi\lambda x_0/2)^2=0.1\mu(\xi k_0)^2$. Note that $\omega^*\to 0$ as the system moves closer to the transition point when $\xi\to\infty$.

Thus, in the insulator phase quite far from the transition point, where $\alpha<0$ and $|\alpha|\gg\alpha^*$ for $\omega\gg\omega^*$, the diffusion coefficient remains the same as in the ordinary self-consistent localization theory, i.e., at low frequencies it is linear in frequency and at high frequencies it is proportional to $\omega^{1/3}$.

Analysis of Eq. (35) shows unfortunately that for frequencies ω much less than ω^* it is impossible to select a power dependence for $d(\omega)$, i.e., the diffusion coefficient in the insulator phase cannot, apparently, be represented in the form $d(\omega)=d(\omega^*/\omega_c)(\omega/\omega^*)^{\delta}$, where δ is an unknown exponent. This fact makes it impossible to study Eq. (35) analytically in the $\omega\ll\omega^*$ range in the insulator phase.

Now let us examine the behavior of the system in the insulator phase near the transition point, when we have $\alpha-\alpha^*<0$ but $\alpha>0$, i.e., when in the absence of impurities the system would be in the metal phase.

We assume once more that the frequency dependence of the diffusion coefficient for $\omega\ll\omega_c$ is a power function, i.e., the diffusion coefficient is given by Eq. (38).

Substituting (38) into (35) yields $\delta=1/3$. As a result for the diffusion coefficient we have

$$d(\omega)=\begin{cases} (4.2\mu\lambda x_0/\alpha)^{2/3}\omega^{1/3}, & \omega\ll\omega_c, \\ (3\pi\lambda x_0/2)^{2/3}\omega^{1/3}, & \omega\gg\omega_c, \end{cases} \tag{40}$$

where $\omega_c=|\alpha-\alpha^*|^3/(3\pi\lambda x_0/2)^2$. Naturally, under exact solution the coefficient of $\omega^{1/3}$ is a smooth function of frequency and ensures a smooth matching in the region $\omega\sim\omega_c$.

Thus, in the insulator phase in close proximity to the transition point, where the system without interaction would still be a metal, the diffusion coefficient behaves in the entire frequency range like $\omega^{1/3}$, but at low frequencies the coefficient of $\omega^{1/3}$ differs from the one in the ordinary self-consistent theory and depends on the Coulomb interaction. Note that if for the interaction we use the dynamically screened Coulomb interaction (7) for which $\mu\sim 1$, the range of applicability of (40) broadens considerably because $\alpha^*\sim\mu\sim 1$.

We have studied the integral equation (33) numerically in a broad frequency range for the metal phase (Fig. 4) and the dielectric phase (Fig. 5). The numerical results for the frequency behavior of the diffusion coefficient are in good agreement with the results obtained analytically.

In the high-frequency range, the frequency behavior of the diffusion coefficient for both the metal phase and the insulator phase differ little from that in ordinary self-consistent localization theory.

In the low-frequency range, $d(\omega)$ for the metal phase is observed to decrease as the strength of interaction grows. The disorder-dependence of the static generalized diffusion coefficient at $\mu=0.24$ is practically linear (see the inset in Fig. 4). The metal–insulator transition here is observed at $\alpha=\alpha^*=c\mu$, where $c\approx 0.5$, which agrees rather well with our analytical results.

In the low-frequency range ($\omega\ll\omega^*$), $d(\omega)$ for the insulator phase deviates rather strongly from its behavior in the ordinary self-consistent localization theory. The behavior of the diffusion coefficient in this range is, apparently, nonanalytic in frequency and tends to form a gap at $\omega\ll\omega^*$.

Note that our numerical study was carried out in terms of the Matsubara frequencies, for which Eq. (33) is written. No analytic continuation of the numerical results to the range of real frequencies was done.

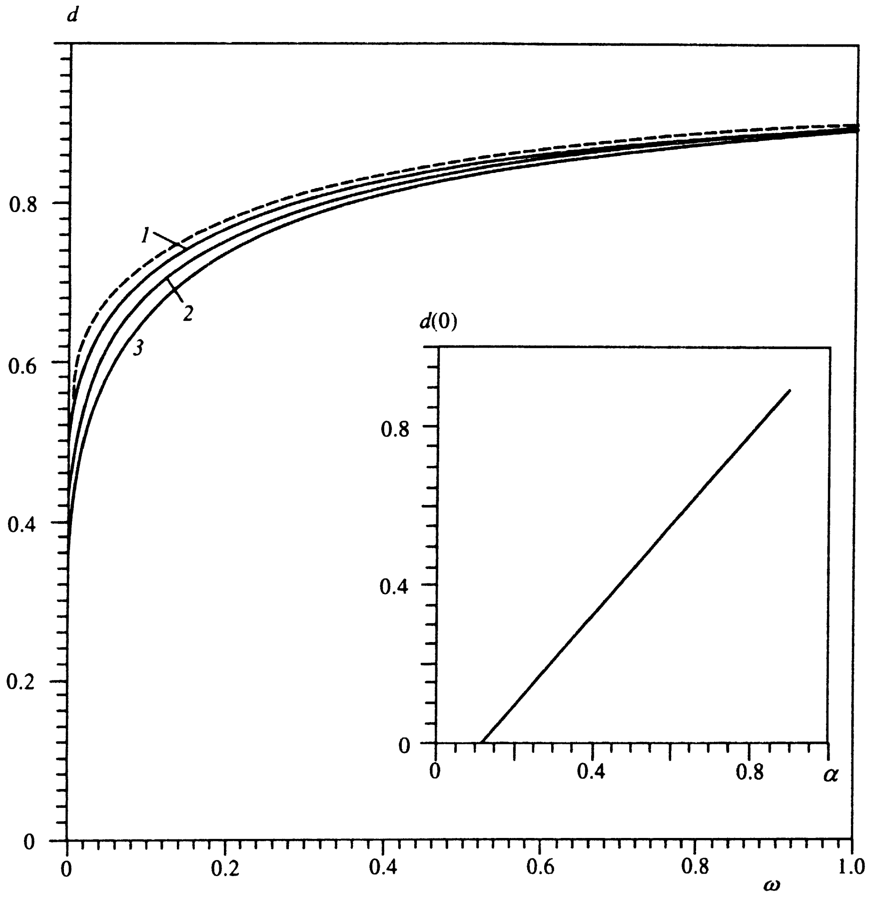

FIG. 4. The dimensionless generalized diffusion coefficient vs the dimensionless Matsubara frequency in the metal phase ($\alpha=0.5$) obtained by solving Eq. (33) numerically for different values of μ: curve *1*, 0.24; curve *2*, 0.6; curve *3*, 0.95; the dashed curve corresponds to the ordinary self-consistent localization theory with $\mu=0$. Inset: the disorder-dependence of the static diffusion coefficient ($d=D(0)/D_0$) at $\mu=0.24$.

6. THE DENSITY OF STATES NEAR THE METAL–INSULATOR TRANSITION

Let us examine the effect of electron–electron interaction on the one-electron ("tunneling") density of states, which is determined by the well-known relation

$$N(\varepsilon)=-\frac{1}{\pi}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\,\mathrm{Im}\,G^R(\mathbf{p},\varepsilon),\tag{41}$$

where $\varepsilon=E-E_F$ is the electron energy measured from the Fermi level, and $G^R(\mathbf{p},\varepsilon)$ is the retarded Green's function:

$$G^R(\mathbf{p},\varepsilon)=\frac{1}{\varepsilon-\xi_\mathbf{p}+i\gamma-\Sigma^R_{ee}(\varepsilon,\mathbf{p})}\,.\tag{42}$$

We examine the "Fock" contribution to the self-energy part (Fig. 6):

$$\Sigma^R_{ee}(\varepsilon,\mathbf{p})=i\int\frac{d\mathbf{q}}{(2\pi)^3}\int_\varepsilon^{1/\tau}\frac{d\omega}{2\pi}\,G^A_0(\mathbf{p}-\mathbf{q},\varepsilon-\omega)v(\mathbf{q})\gamma^2(\mathbf{q},\omega)$$

$$\approx i\gamma^2\mu G^A_0(\mathbf{p},\varepsilon)[f_{\varepsilon,\omega_c}+ig_{\varepsilon,\omega_c}],\tag{43}$$

where the functions f_{ε,ω_c} and g_{ε,ω_c} are determined by the following relationships:

$$f_{\varepsilon,\omega_c}=4N_0^{-1}(0)\,\mathrm{Re}\int\frac{d\mathbf{q}}{(2\pi)^3}\int_\varepsilon^{1/\tau}\frac{d\omega}{2\pi}$$

$$\times\frac{1}{[-i\omega+D_E(\omega)q^2]^2}\,,\tag{44}$$

$$g_{\varepsilon,\omega_c}=4N_0^{-1}(0)\,\mathrm{Im}\int\frac{d\mathbf{q}}{(2\pi)^3}\int_\varepsilon^{1/\tau}\frac{d\omega}{2\pi}$$

$$\times\frac{1}{[-i\omega+D_E(\omega)q^2]^2}\,,\tag{45}$$

and $G^A_0(\mathbf{p},\varepsilon)$ is the advanced Green's function without interaction. Here and in what follows $N_0(0)$ stands for the density of states at the Fermi level in the absence of interaction. Substituting the expression (43) for $\Sigma^R_{ee}(\varepsilon,\mathbf{p})$ into (42), we find, as $\varepsilon\to0$, that

$$\frac{N(\varepsilon)}{N_0(0)}\approx-\frac{1}{\pi}\int_{-\infty}^{\infty}d\xi_\mathbf{p}\,\mathrm{Im}\,G^R(\mathbf{p},\varepsilon)$$

$$=\frac{\gamma}{\pi}\int_{-\infty}^{\infty}d\xi_\mathbf{p}\,\frac{\xi_\mathbf{p}^2+\gamma^2+\mu\gamma^2g_{\varepsilon,\omega_c}}{(\xi_\mathbf{p}^2+\gamma^2+\mu\gamma^2g_{\varepsilon,\omega_c})^2+(\mu\gamma^2f_{\varepsilon,\omega_c})^2}$$

$$=\frac{1}{\sqrt{2[1+\mu g_{\varepsilon,\omega_c}+\sqrt{[(1+\mu g_{\varepsilon,\omega_c})^2+(\mu f_{\varepsilon,\omega_c})^2]}}}$$

$$\times\left(1+\frac{1+\mu g_{\varepsilon,\omega_c}}{\sqrt{(1+\mu g_{\varepsilon,\omega_c})^2+(\mu f_{\varepsilon,\omega_c})^2}}\right).\tag{46}$$

Now let us calculate the functions f_{ε,ω_c} and g_{ε,ω_c}. As noted earlier, the frequency behavior of the diffusion coefficient in the metal phase qualitatively agree with the results of the ordinary self-consistent theory (with a shifted mobility

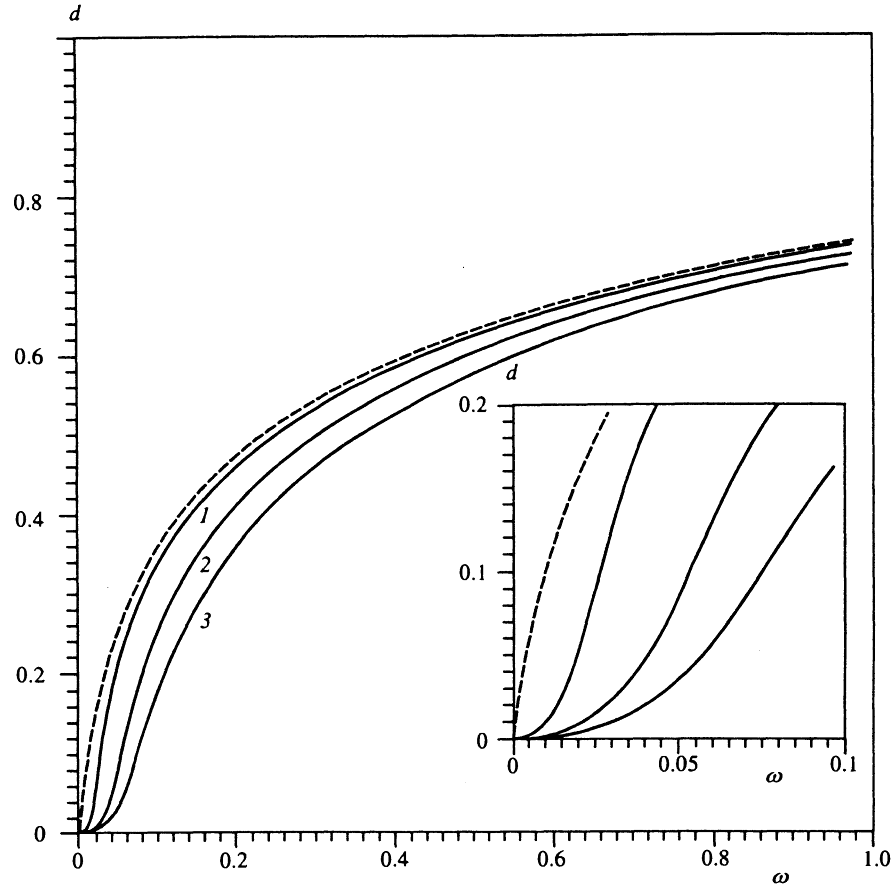

FIG. 5. The dimensionless generalized diffusion coefficient vs the dimensionless Matsubara frequency in the insulator phase ($\alpha=-0.5$) obtained by solving Eq. (33) numerically for different values of μ: curve *1*, 0.12; curve *2*, 0.6; curve *3*, 1.2; the dashed curve corresponds to the ordinary self-consistent localization theory with $\mu=0$.

threshold). In the insulator phase within a narrow frequency range $\omega \ll \omega^*$ the frequency dependence of $D(\omega)$ is not known. However, if we bear in mind that near the transition point $\omega^* \rightarrow 0$, it is reasonable to use for the generalized diffusion coefficient $D(\omega)$ the expression (5), which follows from the self-consistent localization theory, assuming that the role of interaction is reduced to a simple shift of the transition point.

In the metal phase, as $\varepsilon \rightarrow 0$,

$$f_{\varepsilon,\omega_c} = \sqrt{\frac{27}{2}} \left(\frac{\gamma}{E_F}\right)^2 \left(1 - \frac{\varepsilon}{\omega_c}\right) + \frac{27}{\pi}\left(\frac{\gamma}{E_F}\right)^3 \left[\left(\frac{\omega_c}{1/\tau}\right)^{1/3} - 1\right], \tag{47}$$

$$g_{\varepsilon,\omega_c} = \sqrt{\frac{27}{2}} \left(\frac{\gamma}{E_F}\right)^2 \left(1 - \sqrt{\frac{\varepsilon}{\omega_c}}\right) + \frac{\sqrt{27}}{2}\left(\frac{\gamma}{E_F}\right)^2 \ln \frac{1/\tau}{\omega_c}. \tag{48}$$

In the $\omega_c < \varepsilon < 1/\tau$ range the functions f_{ε,ω_c} and g_{ε,ω_c} are reduced to

$$f_{\varepsilon,\omega_c} = \frac{27}{\pi}\left(\frac{\gamma}{E_F}\right)^3 \left[\left(\frac{\varepsilon}{1/\tau}\right)^{1/3} - 1\right], \tag{49}$$

$$g_{\varepsilon,\omega_c} = \sqrt{\frac{27}{2}} \left(\frac{\gamma}{E_F}\right)^2 \ln \frac{1/\tau}{\varepsilon}. \tag{50}$$

Using Eqs. (47)–(50), we can analyze the expression (46) for the density of states $N(\varepsilon)$. In the event of fairly weak disorder, when f_{ε,ω_c} and $g_{\varepsilon,\omega_c} \ll 1$, for $\mu \ll 1$ we can restrict ourselves to the term linear in μ. The result is the correction found in Refs. 10 and 11:

$$\frac{N(\varepsilon)}{N_0(0)} \approx 1 - \frac{\mu}{2} g_{\varepsilon,\omega_c}. \tag{51}$$

As the system moves toward the mobility threshold ($\gamma \sim E_F$ and $\omega_c \rightarrow 0$), the function g_{ε,ω_c} becomes logarithmically divergent [$g_{\varepsilon,\omega_c} \propto \ln(1/\tau\omega_c)$] and the correction (51) proves to be insufficient. The density of states must be calculated by the complete formula (46). Note that neither in Ref. 10 nor in Ref. 11 was the frequency dependence of the diffusion coefficient taken into account. This frequency dependence, as we will shortly see, plays an important role in the movement of the system toward the mobility threshold. As $\omega_c \rightarrow 0$, f_{ε,ω_c} in Eq. (46) can be neglected in comparison to the divergent function g_{ε,ω_c}. We have

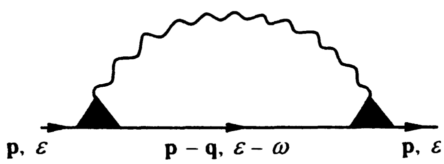

FIG. 6. The "Fock" contribution of the interaction to the self-energy part.

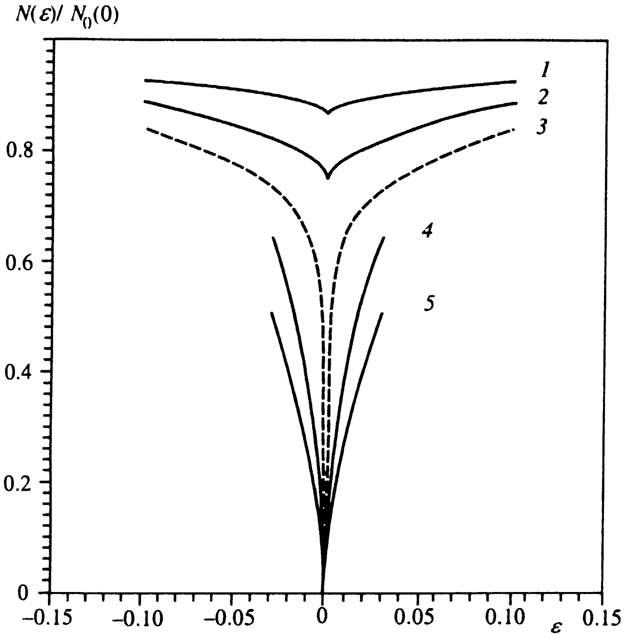

FIG. 7. The evolution of the density of states in the metal–insulator transition for a system with interaction, $\mu=0.2$. The conductivity band is infinitely wide. The curves are given for different values of the disorder parameter λ/λ_c: curve *1*, 0.5; curve *2*, 0.7; curve *3*, 1.0; curve *4*, 1.4; and curve *5*, 1.8.

$$\frac{N(\varepsilon)}{N_0(0)}\approx\frac{1}{\sqrt{1+\mu g_{\varepsilon,\omega_c}}}$$
$$\approx 1-\mu\sqrt{\frac{27}{8}}\left(\frac{\gamma}{E_\mathrm{F}}\right)^2\left(1-\sqrt{\frac{\varepsilon}{\omega_c}}+\frac{1}{\sqrt{2}}\ln\frac{1/\tau}{\omega_c}\right). \tag{52}$$

Thus, in the case of weak disorder, near the Fermi level the well-known square-root dependence of the density of states appears.[3] As the system moves toward the mobility threshold $(\omega_c\to 0)$, the minimum deepens owing to the divergent term $\ln(1/\tau\omega_c)$, and at the mobility threshold $(\omega_c=0)$ the density of states at the Fermi level vanishes. Note that the square-root dependence exists only in the $0<\varepsilon<\omega_c$ range, which narrows as the system moves toward the mobility threshold. In the $\omega_c<\varepsilon<1/\tau$ range we have

$$\frac{N(\varepsilon)}{N_0(0)}\approx\left[1+\mu\frac{\sqrt{27}}{2}\left(\frac{\gamma}{E_\mathrm{F}}\right)^2\ln\frac{1/\tau}{\varepsilon}\right]^{-1/2}, \tag{53}$$

while at the mobility threshold $(\gamma\sim E_\mathrm{F})$ we have

$$\frac{N(\varepsilon)}{N_0(0)}\approx\left(1+\mu\frac{\sqrt{27}}{2}\ln\frac{1/\tau}{\varepsilon}\right)^{-1/2}. \tag{54}$$

Figure 7 (curves *1*–*3*) depicts the results of numerical calculations of the functions f_{ε,ω_c} and g_{ε,ω_c} and the density of states by Eq. (46), which demonstrate the formation and growth of a pseudogap in the density of states. The diagrams are represented for different values of λ/λ_c, where λ_c is the critical order parameter at which the metal–insulator transition takes place. The dashed curve shows the behavior of the density of states at the mobility threshold $(\lambda/\lambda_c=1)$.

In the insulator phase the functions g_{ε,ω_c} and f_{ε,ω_c} for $\varepsilon\to 0$ have the form

$$f_{\varepsilon,\omega_c}=\frac{\sqrt{27}}{2}\left(\frac{\gamma}{E_\mathrm{F}}\right)^2\left(1-\frac{\omega_c}{\varepsilon}\right)+\frac{27}{\pi}\left(\frac{\gamma}{E_\mathrm{F}}\right)^3\left[\left(\frac{\omega_c}{1/\tau}\right)^{1/3}-1\right], \tag{55}$$

$$g_{\varepsilon,\omega_c}=\frac{\sqrt{27}}{2}\left(\frac{\gamma}{E_\mathrm{F}}\right)^2\ln\frac{1/\tau}{\omega_c}. \tag{56}$$

If in (46) g_{ε,ω_c} is ignored in comparison to the function f_{ε,ω_c}, which becomes divergent as $\varepsilon\to 0$, we obtain

$$\frac{N(\varepsilon)}{N_0(0)}\approx 4\sqrt{\frac{\varepsilon}{27\mu\omega_c}}. \tag{57}$$

Thus, in the insulator phase, as $\varepsilon\to 0$, the square-root dependence of the density of states remains, broadening as the disorder grows. In the $\omega_c<\varepsilon<1/\tau$ range the dependence specified by Eq. (53) remains valid (Fig. 7, curves *4* and *5*).

Above we considered the case of an infinitely wide conductivity band. Substantial differences emerge if we consider a band of finite width $2B$ in the insulator phase. In the linear-spectrum model we have

$$\frac{N(\varepsilon)}{N_0(0)}=\frac{\gamma}{\pi}\int_{-B}^{B}d\xi_\mathrm{p}\,\frac{\xi_\mathrm{p}^2+\gamma^2}{(\xi_\mathrm{p}^2+\gamma^2)^2+(\mu\gamma^2 f_{\varepsilon,\omega_c})^2}. \tag{58}$$

Analysis of Eqs. (55) and (58) shows that in the range $\varepsilon\ll\mu\omega_c(\gamma_c/B)^2$ the density of states is a quadratic function of ε:

$$\frac{N(\varepsilon)}{N_0(0)}\sim\left(\frac{B}{\gamma}\right)^3\left(\frac{E_\mathrm{F}}{\gamma}\right)^4\frac{1}{\mu^2}\left(\frac{\varepsilon}{\omega_c}\right)^2, \tag{59}$$

which resembles the well-known behavior of the Éfros–Shklovskiĭ[13,14] "Coulomb gap" in the insulator phase far from the transition point. This fact may be accidental, since in the Éfros–Shklovskiĭ scheme the long-range nature of the Coulomb interaction plays a decisive role. The results of numerical calculation of $N(\varepsilon)$ are depicted in Fig. 8 (curves *5* and *6*). As numerical calculations have shown, in the metal phase with a finite-width band the situation is practically the same as with an infinitely wide band considered earlier.

The behavior of the tunneling density of states of the type represented by Fig. 8 was observed in experiments involving a great number of disordered systems near the metal–insulator transition point,[1,2] from amorphous alloys[15–17] to disordered metal-oxide simple crystals, including high-T_c superconductors.[18] We note, however, that although qualitatively the correspondence of the experimental data and this theory is complete, quantitatively there is no correspondence: the effective "pseudogap" width near the transition point is greater and is described by a power dependence rather than a logarithmic dependence. On the other hand, we find that the density of states over a broad energy interval is much lower than that observed in all the experiments. As far as we know, these equations describe for the first time the evolution of the tunneling density of states in

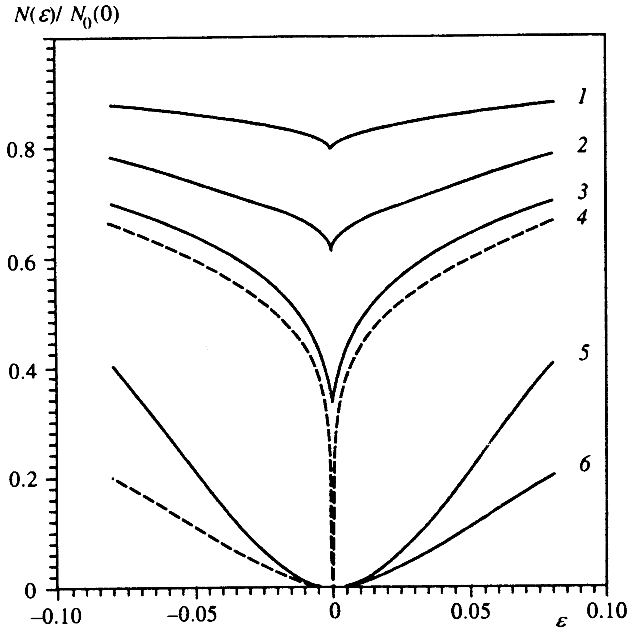

FIG. 8. The evolution of the density of states in the metal–insulator transition for a system with interaction, $\mu=0.3$. The conductivity band has a finite width $2B=4E_F$. The curves are given for different values of the disorder parameter λ/λ_c: curve *1*, 0.5; curve *2*, 0.7; curve *3*, 0.9; curve *4*, 1.0; curve *5*, 2.0; and curve *6*, 2.4.

the disorder-induced transition from the metal phase to the insulator over the entire range of parameters controlling this transition.

7. CONCLUSION

We have presented a self-consistency scheme for describing a disorder-induced metal–insulator transition with the electron–electron interaction, which is taken into account by first-order perturbation theory. Our approach assumes that the transition without interaction is described in a fairly complete manner by the self-consistent localization theory, while the interaction can be taken into account by allowing, on an equal basis, for the contributions of the corresponding diagrams in an equation for the generalized diffusion coefficient. Such an approach we find, makes it possible to describe by unified equations the transition from the metal state to the insulator and obtain results that qualitatively agree with the experimental data (the tunneling density of states). At the same time, the interaction in this approach plays a rather moderate role, and within a broad range of parameters the self-consistent localization theory provides an adequate apparatus for describing the behavior of the diffusion coefficient. For one thing, metallic conductivity (at $T=0$) vanishes linearly at the transition point, which agrees well with the experimental data on some systems[1,2] but does not permit description of a known class of disordered systems in which such behavior tends rather to express a square-root behavior in the disorder parameter.[1,2]

From a purely theoretical standpoint, even when we limit ourselves to the first order in the interaction, we must still analyze the role of the discarded "Hartree" diagrams, as well as that of the screening features in the insulator phase.[5,11] We have also completely ignored spin effects, whose important role near the metal–insulator transition points has long been known.[1,2]

Nevertheless, the suggested scheme does have certain merits, since it makes it possible to allow in a certain manner for the interaction effects in a broad range of parameters controlling the metal–insulator transition, even including the insulator phase, which no single known approach has succeeded in doing up to now.

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