Local flattening of the Fermi surface and magnetoacoustic oscillations in metals

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The effect of local flattening of the Fermi surface of a metal on the geometrical oscillations of the velocity and absorption of ultrasonic waves is investigated theoretically. It is shown that the existence of local flattening of the Fermi surface can lead to a large enhancement of the oscillations. It is predicted that the amplitude of the oscillations depends on the direction of the external magnetic field. © *1995 American Institute of Physics.*

Magnetoacoustic geometrical oscillations of ultrasonic absorption, which were first observed by Bömmel[1] and interpreted by Pippard,[2] are examples of the best-studied oscillation phenomena in metals. Pippard geometrical oscillations are manifested in the propagation of sound perpendicular to a moderately strong external magnetic field **H**, in which the effective electron collision frequency $1/\tau$ is much lower than the electron cyclotron frequency Ω, and the characteristic diameter $2R$ of a cyclotron orbit is much greater than the wavelength of the sound wave. The appearance of such oscillations is associated with the periodic reappearance of the most favorable conditions for absorption of acoustic energy by electrons as the magnitude of the magnetic field changes. The period of the oscillations is determined by the largest cross-sectional diameter of the Fermi surface in a plane perpendicular to **H**. The electrons which move parallel to the wavefront and are associated with this plane absorb energy from a sound wave.

According to the theory of geometrical oscillations,[2–4] in a simple metal with a closed Fermi surface the relative number of such electrons is very large, making the amplitude of geometrical oscillations of the absorption coefficient and sound velocity small compared to their basic values. In the presence of certain singularities in the geometry of the Fermi surface, however, the number of electrons participating effectively in absorption can be much larger, which should result in a significant amplification of the oscillations. For example, it is shown in Ref. 5 that because the dispersion law for charge carriers in a quasi-two-dimensional conductor is highly anisotropic, the number of electrons participating in magnetoacoustic oscillations becomes comparable to the total number of electrons at the Fermi surface. As a result, in a conductor of the type considered, the amplitude of the geometrical oscillations of ultrasonic absorption is much larger, and the form of the oscillations is much different from the case of a quasi-isotropic metal.

A change in the number of electrons participating in the absorption of acoustic energy under the conditions of a Pippard geometrical resonance may also be associated with the existence of local flattening of the Fermi surface at the points corresponding to the stationary points of a cyclotron orbit. The objective of the present work is to analyze the effect of local flattening of the Fermi surface on the geometrical oscillations of absorption and the velocity of sound.

We assume that among the cavities of the closed Fermi surface there is present a biconvex lens, whose symmetry axis is also the x-axis of the coordinate system chosen. We write the dispersion relation for the electrons associated with the lens in the form

$$E(\mathbf{p})=\frac{p_1^2}{2m_1}\left(\frac{p_y^2+p_z^2}{p_1^2}\right)^l+\frac{p_2^2}{2m_2}\left(\frac{p_x}{p_y}\right)^2. \qquad (1)$$

Here, p_1 is the radius of the lens and p_2 is the half-thickness of the lens at the center. For $l=1$ the dispersion relation (1) corresponds to an ellipsoidal Fermi surface, for which m_1 and m_2 are the principal values of the effective-mass tensor. If the parameter l characterizing the shape of the lens assumes values greater than unity, then the Gaussian curvature of the surface vanishes at the points $(\pm p_2;\ 0;0)$, which coincide with the vertices of the lens. Because of the axial symmetry of the lens, the curvature of both principal sections vanishes at these points, i.e., the vertices are points where the surface of the lens is flattened. The lens will be flatter near its vertices, the greater the value of l.

Electrons from neighborhoods of the vertices of the lens will participate in Pippard's magnetoacoustic oscillations if the magnetic field and the direction of propagation of the sound are perpendicular to the axis of the lens. Accordingly, it is assumed that **H** is parallel to the z-axis, and a sound wave with wave vector **q** and frequency ω propagates along the y-axis of the coordinate system fixed in the lens. An expression for the wave vector of a sound wave can be written down on the basis of the equations of the theory of propagation of ultrasound in metals:[6,7]

$$q=\omega/s+\Delta q. \qquad (2)$$

Here, s is the sound speed in the absence of the magnetic field and Δq is a dynamic correction which arises as a result of the interaction with electrons and is linear if the amplitude of the acoustic wave is small. For longitudinal sound Δq is determined mainly by the deformation interaction with the electrons:

$$\frac{\Delta q}{q}=-\frac{\omega}{2\rho_m s^2}\frac{1}{2\pi^2\hbar^3}\int dp_z m_\perp \times\sum_n\frac{U_{-n}(p_z,-q)U_n(p_z,q)}{\omega+i/\tau-n\Omega}, \qquad (3)$$

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where ρ_m is the mass density of the metal, $m_\perp(p_z)$ is the cyclotron mass, $U_n(p_z,q)$ is the Fourier component in the expansion in the azimuthal angle which fixes the position of the electron on the cyclotron orbit:

$$U_n(p_z,q)=\frac{1}{2\pi}\int_0^{2\pi} U(p_z,\Phi,q)e^{in\Phi}d\Phi, \quad (4)$$

$$U(p_z,\Phi,q)=U(p_z,\Phi)\exp\left[-\frac{iq}{\Omega}\int_0^{\Phi} v_y(p_z,\Phi')d\Phi'\right]$$
$$\equiv\left[\Lambda_{yy}(p_z,\Phi)-\frac{\langle\Lambda_{yy}\rangle-N}{\langle 1\rangle}\right]$$
$$\times\exp\left[-\frac{iq}{\Omega}\int_0^{\Phi} v_y(p_z,\Phi')d\Phi'\right]. \quad (5)$$

Here, $\Lambda_{yy}(p_z,\Phi)$ and $v_y(p_z,\Phi)$ are the corresponding components of the deformation-potential tensor and the electron velocity; N is the electron concentration; and, the symbol $\langle ...\rangle$ denotes averaging over the Fermi surface. For a multiply connected Fermi surface, in Eq. (3), in addition to the integration over p_z, a summation must be performed over all cavities of the Fermi surface. The quantities $U_n(p_z,q)$ are calculated for each cavity separately.

In the region considered, where $qR\gg 1$, the neighborhoods of the stationary points on the cyclotron orbits make the main contribution to the integrals over Φ in the expressions (4) for $U_n(p_z,q)$. Evaluating, accordingly, the integrals (4) by the stationary-phase method, the following asymptotic expressions are obtained for the quantities $U_{\pm n}(p_z,\pm q)$:

$$U_{\pm n}(p_z,\pm q)=\frac{1}{\pi}\, U_0(p_z)\exp\left[\mp iqR(p_z)\pm i\pi\,\frac{n}{2}\right]$$
$$\times\left\{\cos\left[qR(p_z)-\pi\,\frac{n}{2}\right]V(p_z)\right.$$
$$\left.-\sin\left[qR(p_z)-\pi\,\frac{n}{2}\right]W(p_z)\right\}, \quad (6)$$

where $U_0(p_z)\equiv U(p_z,\Phi_1)\equiv U(p_z,\Phi_2)$; $2R(p_z)$ is the diameter of a cyclotron orbit of the electrons in the lens in the direction of propagation of the sound wave; Φ_1 and Φ_2 are the values of the angle Φ which correspond to the stationary points on the cyclotron orbit; and, $\Phi_2-\Phi_1=\pi$.

The functions $V(p_z)$ and $W(p_z)$, in Eq. (6) are correspondingly given by

$$V(p_z)=\int_0^{\infty}\cos[qR(p_z)Q_l(y,p_z)]dy,$$

$$W(p_z)=\int_0^{\infty}\sin[qR(p_z)Q_l(y,p_z)]dy. \quad (7)$$

Here,

$$Q_l(y,p_z)=\sum_{k=1}^{l} a_k(p_z)y^{2k}\left(\frac{m_\perp^2}{m_1m_2}\right)^k, \quad (8)$$

and all dimensionless coefficients $a_k(p_z)$, except $a_l(p_z)$, vanish at $p_z=0$, and $a_l(p_z)$ is of the order of 1 at $p_z=0$. In particular, for the case $l=2$ we obtain

$$Q_2(y,p_z)=\frac{m_\perp^2}{m_1m_2}\,a_1(p_z)y^2+\left(\frac{m_\perp^2}{m_1m_2}\right)^2 a_2(p_z)y^4, \quad (9)$$

where

$$a_1(p_z)=\frac{p_z^2}{p_1^2},\quad a_2(p_z)=\frac{1}{2}\left[1-\frac{4}{3}\frac{p_z^4}{p_1^4}\right]. \quad (10)$$

For small values of p_z, corresponding to a neighborhood at the center of the lens, for which

$$\frac{p_z^2}{p_1^2}<[qR(p_z)]^{-1}, \quad (11)$$

the leading term of the asymptotic expansion of the function $V(p_z)$ in inverse powers of qR with $l=2$ has the form

$$V(p_z)=\frac{\Gamma(1/4)}{4}\,\frac{\sqrt{m_1m_2}}{m_\perp^{ex}}\left(\frac{2}{qR_{ex}}\right)^{1/4}\cos(\pi/8). \quad (12)$$

Here, $\Gamma(x)$ is the gamma function, $m_\perp^{ex}=m_\perp(0)$, and $R_{ex}=R(0)$.

For sufficiently large values of p_z, where the inequality (11) is not satisfied, the following approximation can be used for $V(p_z)$:

$$V(p_z)=\frac{1}{2}\sqrt{\frac{\pi m_1m_2}{m_\perp^2}\,\frac{p_1^2}{p_z^2}}\;\frac{\cos(\pi/4)}{\sqrt{qR(p_z)}}. \quad (13)$$

The asymptotic expressions for $W(p_z)$ in the corresponding ranges of p_z are obtained from Eqs. (12) and (13) by replacing the cosine by a sine with the same argument.

In calculating the dynamic correction arising in the wave vector of the sound wave as result of the interaction with the electrons of the lens, the range of integration over p_z in the expression (3) must be divided into regions, with small and large values of p_z. When the integration in each region is performed, the corresponding asymptotic form must be used for the functions $V(p_z)$ and $W(p_z)$. The result is

$$\Delta q=\Delta q_1+\Delta q_2. \quad (14)$$

Here, the leading term in the expansion of the first term in inverse powers of the parameter qR can be represented in the form

$$\Delta q_1=i\gamma_0\left\{\frac{\Gamma^2(1/4)}{2\sqrt{2\pi}}\,\overline{U_0^2}(0)\coth\left[\frac{\pi}{\Omega_{ex}\tau}(1-i\omega\tau)\right]+\sqrt{qR_{ex}}\right.$$
$$\left.\times\int_1^{\sqrt{qR_{ex}}}\overline{U_0^2}(tp_1/\sqrt{qR_{ex}})\frac{\coth\left[\frac{\pi}{\Omega\tau}(1-i\omega\tau)\right]}{t^2\sqrt{(qR_{ex})^2-t^4}}\,dt\right\}, \quad (15)$$

where $\overline{U_0^2}(p_z)=m_2U_0(p_z)/(p_2\sqrt{p_1p_2})$, and $\gamma_0=m_1N\omega p_2/2\pi\rho_m m_\perp^{ex}s^2$ is of the same order of magnitude as the sound absorption coefficient in the absence of a magnetic field. For $qR\gg 1$ the integral in the expression (15) is of order $1/\sqrt{qR_{ex}}$. Therefore, both terms in Eq. (15) are of the same order of magnitude. At not too high frequencies ($\omega\tau<1$), they do not exhibit an oscillating dependence on the magnetic field and give the continuous part of the contribution of the lens electrons to the absorption and renormalization of the velocity of the ultrasonic wave.

The magnetoacoustic oscillations are described by the term Δq_2, whose leading term is

$$\Delta q_2 = 1\,\gamma_0 \overline{U_0^2}(0) b \cos\left(2qR_{ex}+\frac{\pi}{4}\right) \times \frac{1}{\sinh[(\pi/\Omega_{ex}\tau)(1-i\omega\tau)]}, \tag{16}$$

where $b=\Gamma^2(1/4)/(4/\sqrt{2}\pi^2)$.

The real and imaginary parts of the dynamic correction Δq determine the renormalization of the velocity of and the energy absorption coefficient for the ultrasonic wave. As follows from Eq. (16), the leading-order approximation in the small parameter $(qR)^{-1}$ for the oscillating part of the absorption coefficient has the form

$$\gamma = \mathrm{Im}\,\Delta q_2 = \gamma_0 \overline{U_0^2}(0) b \cos\left(2qR_{ex}+\frac{\pi}{4}\right) \times \mathrm{Re}\left[\frac{1}{\sinh[(\pi/\Omega_{ex}\tau)(1-i\omega\tau)]}\right]. \tag{17}$$

The amplitude of the oscillations described by the expression (17) is of the same order of magnitude in $(qR)^{-1}$ as the nonoscillating contribution to the absorption coefficient $\mathrm{Im}\Delta q_1$. This is a direct consequence of the larger number of effective electrons as a result of the flattening at the neighborhoods of the vertices of the electron lens. In a simple metal whose Fermi surface is closed and convex everywhere, the oscillating correction to the sound absorption coefficient is small compared to the continuous part.

The expressions (15)–(17) were derived under the assumption that the parameter l characterizing the degree of flatness of the lens near its vertices is equal to 2. Therefore, even a moderate flattening of the Fermi surface can result in a significant amplification of the geometric oscillations. For $l>2$, the amplification of the oscillations will be even more pronounced. For an arbitrary value of l, the function $V(p_z)$ in a neighborhood of $p_z=0$ is described by the asymptotic expression

$$V(p_z)=\frac{1}{2l}\frac{\Gamma(1/2l)}{(qR_{ex})^{1/2l}}\sqrt{\frac{m_1 m_2}{m_\perp^{ex}(a_l(0))^{1/l}}}\cos(\pi/4l). \tag{18}$$

A similar expression can also be written for $W(p_z)$.

To calculate the contribution from the neighborhood of the stationary point $p_z=0$ to the integral over p_z in Eq. (3) with $l>2$, the asymptotic expression (18) must be used. As a result, an expression which extends the expression (17) to arbitrary values of the parameter l is obtained for the oscillating part of the sound absorption coefficient:

$$\gamma=\gamma_0\overline{U_0^2}(0) b_l (qR_{ex})^{(l-2)/2l}\cos\left(2qR_{ex}+\frac{\pi}{2l}\right) \times \mathrm{Re}\left[\frac{1}{\sinh[(\pi/\Omega_{ex}\tau)(1-i\omega\tau)]}\right]. \tag{19}$$

Here, $b_l=\Gamma^2(1/2l)/(\pi^2 l^2[a_l(0)]^{1/l})$.

At the same time, the first term in the expression (15) for the nonoscillating part of the dynamic correction Δq_1 can be replaced by

$$i\gamma_0\frac{\Gamma^2(1/2l)}{\pi l^2[a_l(0)]^{1/l}}\overline{U_0^2}(0)(qR_{ex})^{(l-2)/2l} \times \coth\left[\frac{\pi}{\Omega_{ex}\tau}(1-i\omega\tau)\right]. \tag{20}$$

Even under these conditions, the second term in the expression for Δq_1 is of the same order of magnitude. Therefore, even with greater flattening of the surface of the lens, the amplitude of the geometric oscillations of ultrasonic absorption (19) is of the same order of magnitude as the continuous part of the absorption coefficient and is much greater than (by a factor qR_{ex} for $l\gg 1$) the amplitude of the corresponding oscillations in a simple metal, whose Fermi surface has no local flattenings.

The oscillating contribution to the velocity of the ultrasonic wave is expressed in terms of the real part of the oscillating contribution to the correction Δq:

$$\frac{\Delta s}{s}=\frac{s}{2\omega}\mathrm{Re}\Delta q_2 = \frac{m_1 N}{4\pi\rho_m s}\frac{p_2}{m_\perp^{ex}}\overline{U_0^2}(0) b_l (qR_{ex})^{(l-2)/2l} \times \cos\left(2qR_{ex}+\frac{\pi}{2l}\right)\mathrm{Im}\left[\frac{1}{\sinh[(\pi/\Omega_{ex}\tau)(1-i\omega\tau)]}\right]. \tag{21}$$

The amplification produced in the magnetoacoustic oscillations as a result of the increase in the number of electrons participating in the oscillations is manifested in the oscillations of the sound speed just as in the oscillations of the ultrasonic absorption which we analyzed above.

Since the amplification, which we studied in this work, of the geometric resonances in the velocity and absorption of ultrasound is due to the local geometric characteristics of the Fermi surface, it can be observed only for a definite choice of the direction of the magnetic field with respect to the symmetry axes of the crystal lattice. When the magnetic field is tilted away from the direction for which the point of flattening of the Fermi surface falls on its section corresponding to the cyclotron orbit of the electrons participating effectively in the formation of the oscillations, the influence of this point vanishes and the amplitude of the oscillations decreases. Therefore, the amplification of geometric oscillations, just as a number of other effects which result from local geometric features of the Fermi surface of a metal,[8–11] should exhibit a pronounced dependence on the direction of the external magnetic field.

Specifically, for the model Fermi surface (1) considered here, the amplitude of the geometric oscillations of the velocity and sound absorption coefficient will depend on the angle φ between the external magnetic field and a plane perpendicular to the axis of the lens. The range of variation of the amplitude of the oscillations with increasing φ is determined by the degree of flattening of the lens near its vertex.

[1] H. E. Bömmel, Phys. Rev. **100**, 758 (1955).
[2] A. B. Pippard, Phil. Mag. **2**, 1147 (1957).
[3] V. L. Gurevich, Zh. Éksp. Teor. Fiz. **37**, 71 (1959) [Sov. Phys. JETP **37**, 51 (1960)].
[4] M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).
[5] O. V. Kirichenko and V. G. Peschanskiĭ, Fiz. Nizk. Temp. **20**, 574 (1994) [Sov. J. Low Temp. Phys. **20**, 453 (1994)].
[6] J. Mertsching, Phys. Status Solidi **37**, 465 (1970).
[7] V. M. Kontorovich, Usp. Fiz. Nauk **142**, 265 (1984) [Sov. Phys. Usp. **27**, 134 (1984)].
[8] D. Shoenberg, *Magnetic Oscillations in Metals*, Cambridge University Press, N. Y., 1984 [Russian translation, Mir, Moscow, 1986, p. 588].
[9] P. T. Coldridge, G. B. Scott, and I. M. Templeton, Can. J. Phys. **50**, 1999 (1972).
[10] N. A. Zimbovskaya, Fiz. Nizk. Temp. **19**, 1337 (1993) [Sov. J. Low Temp. Phys. **19**, 949 (1993)].
[11] N. A. Zimbovskaya, Fiz. Nizk. Temp. **20**, 441 (1994) [Sov. J. Low Temp. Phys. **20**, 350 (1994)].

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