## Polarization diagnostics of the electron distribution function in an electric field

V. P. Demkin

V. V. Kuĭbyshev Tomsk State University, 634010 Tomsk, Russia (Submitted 29 August 1994; resubmitted 28 November 1995) Zh. Éksp. Teor. Fiz. 107, 1517–1525 (May 1995)

A method is suggested for determining the electron distribution function in an electric field based on the polarization method of measuring the radiation emitted by a plasma. The initial form of the distribution function is fixed analytically with allowance for the properties of high-voltage discharges and depends on a number of parameters. Equations for calculating the parameters are derived, relating the polarization moments to the moments of the electron distribution function up to the second order inclusive. First-order perturbation theory in the field strength is used to calculate the polarization moments. The method makes it possible to use the measurements of the linear degree of polarization of the radiation from the lines with an appreciable Stark effect to obtain a more complete picture of the electron distribution function than the ordinary method of polarization spectroscopy of unperturbed states provides. © 1995 American Institute of Physics.

Polarization spectroscopy is an effective method for studying the anisotropic properties of collisional and radiative processes in a plasma. Kazantsev and Subbotenko<sup>1</sup> have shown that the polarization moments  $\rho_q^{(k)}$  in spectral lines are related to excitation of the atomic states and are proportional to the moments  $I_q^{(k)}$  of the exciting light or the moments  $f_q^{(k)}$  of the distribution function when the excitation is due to electrons. For instance, when electrons are the exciting agent,

$$\langle \rho_q^{(k)} \rangle = (-1)^q \sqrt{\frac{4\pi}{2k+1}} \int dv \ v^2 \rho_0^{(k)}(v) f_{-q}^{(k)}(v), \qquad (1)$$

where  $\langle \rho_q^{(k)} \rangle$  is the polarization tensor in the laboratory coordinate system (determined from measurements),  $\rho_0^{(k)}(v)$ (known as the dynamical tensor) is the polarization tensor in the collisional system of coordinates, and  $f_{-q}^{(k)}(v)$  is the *k*th moment of the distribution function.

Equation (1) serves as a basis for applying the methods of polarization diagnostics in finding the moments of the distribution function.

Expansion of the distribution function in orthogonal polynomials is one method of solving the Boltzmann equation.<sup>2</sup> Usually the two-term approximation is used in the expansion of the distribution function. As an analysis of experimental and theoretical studies shows,<sup>3</sup> this approximation usually yields incorrect results and cannot be used when the gradients of the external fields or the plasma parameters are large. Using higher-order terms in the expansion of the electron distribution function leads to considerable computational difficulty.

As is known, the linear polarization of radiation is determined by the polarization moment  $\rho^{(2)}$ . Measuring the value of this tensor, we can determine the corresponding moment of the distribution function<sup>4</sup> via Eq. (1). In this case, however, the moment  $\rho^{(1)}$  remains unknown and, hence, so does the moment  $f^{(1)}$ , which is the leading term in the moment expansion of the distribution function. Determining  $f^{(1)}$  requires measuring the degree of circular and linear polarization of light.<sup>5</sup>

From Refs. 4 and 5 it follows that diagnostics of the dipole radiation emitted by isolated atoms makes it possible to determine tensors whose rank is no higher than two. This limits the order of the moments that can be found in the distribution function. The limitation can be lifted by considering not only dipole transitions but also transitions of higher multiplicity.<sup>6</sup> It is difficult, however, to observe such transitions because the optical signals are weak. Furthermore, in a highly ionized plasma such transitions are difficult to observe against the background of Stark dipole-forbidden transitions and transitions of mixed multiplicity.

As shown in Ref. 7, in an electric field the probabilities of transitions corresponding to the interference of the electric dipole and quadrupole moments have an order of smallness equal to  $ka_0$  with respect to the probabilities of dipole transitions, in contrast to the probabilities of quadrupole transitions, which have an order of smallness equal to  $(ka_0)^2$ , where k is the wave vector and  $a_0$  is the Bohr radius. In the optical range,  $ka_0$  is about  $10^{-3}$ .

The probabilities of forbidden dipole transitions are proportional to  $|C_I(M)|^2$ , where  $C_I(M)$  is the coefficient in the expansion of the wave function of a Stark state,  $|M\rangle$ , in the basis states  $|JM\rangle$  of an isolated atom; the value of this coefficient depends on the polarizability  $\alpha$  of the atomic state and the field strength F. Since the induced dipole moment of an atom is proportional to  $\alpha F$ , it is obvious that as  $\alpha$  grows, the strong-field effect, where the intensity of dipole-forbidden transition becomes commensurate with the intensity of an allowed transition, can be observed at lower field strengths. Calculations<sup>8</sup> show, for instance, that for the states of the helium atom with n=5 such an effect is achieved for F  $\approx 10 \text{ kV/cm}$ while with n=4 this occurs for F  $\approx 60 \text{ kV/cm}.$ 

Thus, the presence in the atomic spectrum of transitions appreciablly affected by the electric field makes it possible to determine the polarization tensors over a broader range of ranks and with high precision in detecting the optical signals. In Ref. 9 it was shown that in an electric field the intensity of radiation involving allowed dipole transitions depends on both diagonal and off-diagonal (in the angular momentum J) tensors  $\rho_0^{(k)}(J,J')$ . Generally the value of k may vary from  $|J'-J| \le k \le J' + J$ .

The degree of linear polarization of a dipole-allowed line in an electric field is determined by the diagonal tensors  $\rho_q^{(k)}(J,J)$  for even k (k=0,2) and off-diagonal tensors  $\rho_0^{(k)} \times (J,J')$  for odd k.

In first-order perturbation theory,<sup>9–11</sup> the relation for the linear degree of polarization of radiation involving a dipoleallowed transition and emitted perpendicular to the electric field vector  $\mathbf{F}$  has the form

$$P = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}} = P_0 + P_f, \qquad (2)$$

where  $P_0$  has the same appearance as in Ref. 5 and is determined by the anisotropic polarization processes of the states of an isolated atom,

$$P_0 = 3 \begin{cases} J \ J \ 2 \\ 1 \ 1 \ J_0 \end{cases} \frac{\rho_0^{(2)}(J,J')}{G} , \qquad (3)$$

and  $P_f$  is the field's contribution, which is given by

$$P_{f} = \sqrt{6} \sum_{k=1}^{J'+J} \sqrt{2k+1} \begin{cases} J \ J \ 2 \\ 1 \ J \ J_{0} \end{cases} \sum_{J'} \rho_{0}^{(k)}(J,J') \xi_{J'}(F) \\ \times \sum_{M} \begin{pmatrix} J' \ J \ 1 \\ M \ -M \ 0 \end{pmatrix} \begin{pmatrix} J' \ J \ k \\ M \ -M \ 0 \end{pmatrix} \begin{pmatrix} J \ J \ 2 \\ M \ -M \ 0 \end{pmatrix} \middle/ G ,$$
(4)

with

$$G = -2\sqrt{10} \begin{cases} J \ J \ 0 \\ 1 \ 1 \ J_0 \end{cases} \rho_0^{(0)}(J,J) + \begin{cases} J \ J \ 2 \\ 1 \ 1 \ J_0 \end{cases} \rho_0^{(2)}(J,J),$$
(5)

and  $\xi_{J'}(F)$  the reduced coefficient in the expansion of the wave functions of Stark states in the states of an isolated atom,

$$\xi_{J'}(F) = \frac{\langle J' \| eFz \| J \rangle}{E_J - E_{J'}}$$

According to its definition, the off-diagonal tensor is calculated in first-order perturbation theory by the formula

$$\rho_{0}^{(k)}(J',J) = \sum_{M} \sqrt{(2k+1)} \xi_{J'}(F)$$

$$\times \begin{pmatrix} J' & 1 & J \\ -M' & 0 & M \end{pmatrix} \begin{pmatrix} J' & k & J \\ -M' & 0 & M \end{pmatrix} \rho_{M'M}.$$
(6)

In the case of electron pumping of Stark states, the presence of odd-rank off-diagonal tensors in Eq. (4) makes it possible to determine the moments of the electron distribution function, including the odd-order moments.

Note that the off-diagonal tensors are proportional to the field strength. They can be viewed as an additional parameter in the set of polarization tensors, with the result that we can consider the problem of determining all independent parameters simultaneously. In Refs. 9,12, and 13 the helium atom is used as an example for theoretical and experimental justification of a method for determining the electric field strength in a plasma.

Similarly, Eqs. (1)-(5), and (6) can be used to determine the moments of the distribution function for electrons in an electric field on the basis of measurements of the polarization characteristics of the radiation emitted by the plasma.

Let us consider this method. Suppose that the field strength is such that the atomic states considered here experience an appreciable Stark effect, with the result that the contribution of an off-diagonal tensor to the radiative intensity can be detected experimentally. On the other hand, the field is weak, so it is possible to use first-order perturbation theory in describing the atomic states.

Finding the distribution function from measurements of the polarization moments is an inverse problem in polarization spectroscopy. To determine the parameters—the polarization tensors—in a meaningful manner, one must perform a sufficient number of independent measurements, which requires knowing a sufficient number of diagnostic lines starting at the same level J.

When there is no field, the number of independent components of the tensors  $\rho_q^{(k)}(J,J)$  equals the number of independent components of elements of the density matrix  $\rho_{M'M}$ , which is equal to (2J+1)(2J+1). Turning on the field does not change the number of Stark states and, hence, the number of independent components of the density matrix. As noted earlier, the appearance of an additional parameter F increases the number of dimensions if the distribution function and the field strength are determined simultaneously. However, the field distribution in the plasma can be found by solving a simplified problem of polarization diagnostics.<sup>13</sup> Allowing for the axial symmetry of the processes and the lack of coherence in populating the Stark states, we can reduce the number of parameters to 2J+1(see Ref. 3); this quantity determines the minimum number of independent measurements and hence the necessary number of transitions.

In contrast to the isolated case, in an electric field the number of transitions increases because of the appearance of dipole-forbidden transitions from the same level J whose intensity is determined via the diagonal tensors.<sup>9</sup> The increase in the number of diagnostic lines serves as a favorable factor in solving the inverse problem in the polarization diagnostics of plasma.

Equations (5) and (6) determine the polarization of the radiation emitted from the set of Stark transitions belonging to the level J. Determining the polarization tensors in this case requires measuring the intensity integrated over the line profile.

The explanation for such an approach is that usually in a gas discharge plasma at medium pressures, the broadening of spectral lines amounts to several angstroms<sup>14</sup>; on the other hand, even in high-voltage discharges; the electric fields do not exceed several tens of kilovolts per centimeter. As a result, the Stark splitting of the spectral lines does not exceed their halfwidth, and the Stark components of a line cannot be resolved spectroscopically. This determines the choice of

method of diagnostics that uses integrated spectral line profiles. Such an approach facilitates the solution of the diagnostics problem but at the same restricts the choice of isolated spectral lines.

Spectropolarimetric measurements serve as the basis for this method. The measurement accuracy and the correspondence of the optical signals to electron excitation processes determine the accuracy of the calculated electron distribution function.

The accuracy of the measured line intensities and their degrees of polarization in the plasma depends on many factors.

The relative contribution of the field to the variation in the degree of polarization, as Eqs. (4) and (6) imply, is proportional to  $|C_{J'}(M)|^2$ , which, depending on the magnitude of the field strength, amounts to  $\leq 10^{-1}$ . The same quantity determines the relative variation of the intensity of a dipoleallowed transition in the field and, accordingly, the relative intensity of dipole-forbidden transitions. The experimental studies described in Ref. 13 show that signals of such magnitude are confidently detected in photon counting mode, and statistical averaging of the measurements makes it possible to calculate the degree of polarization with an error of less than 1%.

The factors distorting the optical signals and leading to an error in calculating the electron distribution function are radiation capture and transition cascades. A detailed study of their effect on the shape of the optical signals can be found in Refs. 15-18. In the case of electron excitation these factors can be eliminated by selecting the diagnostic lines from highly excited states and by additional monitoring of the behavior of the optical signals as a function of the discharge parameters and gas density.

The errors introduced by the optical system are well known.<sup>17</sup> They are common to all spectropolarimetric measurements and can easily be controlled.

Thus, determining polarization moments from experimental data on the linear degree of polarization of the radiation emitted by a plasma requires in this method a careful choice of the diagnostic lines, which are isolated and which originate at levels that exhibit an appreciable Stark effect and are excited by direct electron impact.

It is convenient to calculate the electron distribution function with Eq. (1) using a parametric method of specifying the distribution function. As an example we calculate the distribution function for electrons in discharges with a beam of "escaping" electrons.

In weak electric fields, the electron distribution function is close to Maxwellian with insignificant anisotropy<sup>1,2,19</sup>:

$$f(v) = C \exp\left[\frac{(\mathbf{v} - \mathbf{u})^2}{v_0^2}\right], \qquad (7)$$

where u and  $v_0$  are the drift and most likely velocities of an electron, with  $u \ll v_0$ . The drift velocity grows with field strength, and for  $u \ge \sqrt{I}$ , where I is the ionization potential of an atom, the electron velocity distribution "acquires" a group of escaping electrons with a delta-like distribution function. As shown in Refs. 20 and 21, the presence of two groups of electrons in the distribution function is typical of

plasma-beam discharges, where the group of low-energy electrons is characterized by weak anisotropy, and the group of high-energy electrons has the properties of a beam. The fraction of high-energy electrons may be considerable, and for high-voltage pulse discharges amounts to several tens of percent.<sup>22</sup>

Allowing for these features, we write the distribution function in the form

$$f(\mathbf{v}) = C \left\{ \exp \left[ \frac{(\mathbf{v} - \mathbf{u})^2}{v_0^2} \right] + \gamma \delta(\mathbf{v} - \mathbf{v}_b) \right\}, \qquad (8)$$

where  $\gamma$  and  $\mathbf{v}_b$  are, respectively, the fraction and beam velocity of the escaping electrons. It is assumed that  $u \ll v_0$  and  $v_b \gg v_0$ . The quantities u,  $v_b$ , and  $\gamma$  in Eq. (8) are parameters to be determined. We write the distribution function of the low-energy electrons as a moment expansion:

$$\exp\left[\frac{(\mathbf{v}-\mathbf{u})^2}{v_0^2}\right] = \sum_{k,q} f_q^{(k)}(v) Y_q^{(k)}(\Omega).$$
(9)

The inverse transformation yields a formula for calculating the moments:

$$f_{q}^{(k)}(v) = \int d\Omega \exp\left[\frac{(\mathbf{v}-\mathbf{u})^{2}}{v_{0}^{2}}\right] (Y_{q}^{(k)}(\Omega))^{*}.$$
 (10)

Suppose that the z axis of the laboratory coordinate system coincides in direction with the electric field vector. Since the function (8) possesses axial symmetry with respect to the direction of the vector **F**, only the moments  $f_0^{(k)}(v)$  are non-zero. Separating out the group of high-energy electrons in Eq. (8) makes it possible to retain only a few terms in the expansion of the distribution function for the low-energy electrons. The most interesting are the moments  $f_0^{(k)}(v)$  at k=0,1,2. We find them from Eq. (10) by expanding the exponential function in series, retaining terms up to fourth order:

$$f_0^{(0)}(v) = 2\sqrt{\pi} \exp\left[-\frac{(v^2 - u^2)}{v_0^2}\right] \left(1 + \frac{\beta^2}{6} + \frac{\beta^4}{120}\right), \quad (11)$$

$$f_0^{(1)}(v) = 2\sqrt{\frac{\pi}{3}} \exp\left[-\frac{(v^2 - u^2)}{v_0^2}\right] \beta\left(1 + \frac{\beta^2}{10}\right), \quad (12)$$

$$f_0^{(2)}(v) = \frac{2}{3} \sqrt{\frac{\pi}{5}} \exp\left[-\frac{(v^2 - u^2)}{v_0^2}\right] \beta^2 \left(1 + \frac{\beta^2}{2}\right).$$
(13)

Here  $\beta = 2vu/v_0^2$ .

To calculate the dynamical tensor  $\rho_0^{(k)}(v)$  we use a formula that relates it to the polarization tensor in the laboratory coordinate system:

$$\rho_q^{(k)}(v) = (-1)^q \sqrt{\frac{4\pi}{2k+1}} \,\rho_0^{(k)}(v)(Y_q^{(k)}(\vartheta,\varphi))^*, \quad (14)$$

where  $\vartheta$  and  $\varphi$  are the angles that specify the velocity vector of an outgoing electron. The transformation that is the inverse of (14) yields

$$\rho_0^{(k)}(v) = (-1)^q \sqrt{\frac{2k+1}{4\pi}} \int d\Omega \ \rho_q^{(k)}(\mathbf{v}) Y_q^{(k)}(\vartheta, \varphi).$$
(15)

The polarization tensor  $\rho_q^{(k)}(\mathbf{v})$  can be calculated in the following way. According to the definition of Refs. 9 and 23, in an electric field the tensor  $\rho_q^{(k)}(\mathbf{v},J',J'')$ , which is offdiagonal in J, is given by the following formula if the ground state  $J_0$  is populated isotropically:

$$\rho_{q}^{(k)}(\mathbf{v},J',J'') = \sum_{M_{0}M'M''} C_{J'}(M')C_{J''}^{*}(M'') \frac{(-1)^{J'-M'}}{2J_{0}+1}$$
$$\times \sqrt{(2k+1)} \begin{pmatrix} J' & k & J'' \\ -M' & q & M'' \end{pmatrix} f_{M'M_{0}}f_{M''M_{0}}^{*}.$$
(16)

Here  $C_{I}(M)$  are the coefficients in the expansion of the wave functions of the Stark states in the basis of the wave functions of an unperturbed atom, and  $f_{MM_0}$  is the amplitude of excitation of a Stark state by electron impact.

We write the excitation amplitude as a multipole expansion:14

$$f_{M'M_0} = \langle M' | T | M_0 \rangle$$
  
=  $\sum_{k_1 q_1} \langle M' | T_{q_1}^{(k_1)}(r) | M_0 \rangle Y_{q_1}^{(k_1)}(\vartheta, \varphi).$  (17)

Then

$$\rho_{q}^{(k)}(\mathbf{v},J',J'') = \sum_{\substack{M_{0}M'M''\\k_{1}q_{1}k_{2}q_{2}}} \frac{(-1)^{J''-M''}}{2J_{0}+1} C_{J'}(M') C_{J''}^{*}(M'') \\ \times \begin{pmatrix} J' & k & J''\\ -M' & q & M'' \end{pmatrix} \begin{pmatrix} J' & k_{1} & J_{0} \\ -M' & q_{1} & M_{0} \end{pmatrix} \\ \times \begin{pmatrix} J'' & k_{2} & J_{0} \\ -M'' & q_{2} & M_{0} \end{pmatrix} \times Y_{q_{1}}^{(k_{1})}(\vartheta,\varphi) [Y_{q_{2}}^{(k_{2})} \\ \times (\vartheta,\varphi)]^{*} \langle J' \| T^{(k_{1})} \| J_{0} \rangle \langle J'' \| T^{(k_{2})} \| J_{0} \rangle^{*}.$$
(18)

Substituting Eq.(18) into Eq. (15), we arrive at the following expression for the dynamical excitation tensor:

$$\rho_{0}^{(k)}(v,J',J'') = (-1)^{q} \frac{2k_{1}+1}{4\pi(2J_{0}+1)} \sqrt{(2k_{1}+1)(2k_{2}+1)} \\ \times \sum_{\substack{M_{0}M'M''\\k_{1}q_{1}k_{2}q_{2}}} (-1)^{J''-M''+q_{2}}C_{J'}(M')C_{J''}^{*}(M'') \\ \times \langle J' \| T^{(k_{1})} \| J_{0} \rangle \langle J'' \| T^{(k_{2})} \| J_{0} \rangle^{*} \\ \times \langle J' \| T^{(k_{1})} \| J_{0} \rangle \langle J'' \| T^{(k_{2})} \| J_{0} \rangle^{*} \\ \times \begin{pmatrix} J' & k & J''\\ -M' & q & M'' \end{pmatrix} \begin{pmatrix} J' & k_{1} & J_{0} \\ -M' & q_{1} & M_{0} \end{pmatrix} \\ \times \begin{pmatrix} J'' & k_{2} & J_{0} \\ -M'' & q_{2} & M_{0} \end{pmatrix} \begin{pmatrix} k_{2} & k & k_{1} \\ -q_{2} & q & q_{1} \end{pmatrix}.$$
(19)

To calculate the expansion coefficients  $C_{I}(M)$  we use first-order perturbation theory. Let us examine the diagonal tensor  $\rho_0^{(k)}(v,J,J)$ . In this case  $C_{J'}(M') = C_{J''}(M'')$ =  $C_I(M) \equiv 1$ . Using the rules for contracting 3j-symbols, we arrive at the following expression for the diagonal tensor:

$$\rho_{0}^{(k)}(v,J,J) = \frac{1}{4\pi(2J_{0}+1)} \sqrt{(2k_{1}+1)(2k_{2}+1)}$$

$$\times \sum_{k_{1}k_{2}} {\binom{k_{2} \ k \ k_{1}}{0 \ 0 \ 0}} {\binom{k_{2} \ k \ k_{1}}{J \ J_{0} \ J}}$$

$$\times \langle J \| T^{(k_{1})} \| J_{0} \rangle \langle J \| T^{(k_{2})} \| J_{0} \rangle^{*}.$$
(20)

Equations (4)–(6) imply that k is an even number, so  $k_1 = k_2$  in Eq. (20). The final expression is

$$\rho_{0}^{(k)}(v,J,J) = \frac{2k_{1}+1}{4\pi(2J_{0}+1)} \sum_{k} \begin{pmatrix} k_{1} & k & k_{1} \\ 0 & 0 & 0 \end{pmatrix} \begin{cases} k_{1} & k & k_{1} \\ J & J_{0} & J \end{cases}$$
$$\times |\langle J \| T^{(k_{1})} \| J_{0} \rangle|^{2}.$$
(21)

Summation with respect to  $k_1$  is from  $|J-J_0| \le k \le J + J_0$ . Similarly, for the off-diagonal tensor  $\rho_0^{(k)}(v,J',J'')$  we assume that

$$C_{J'}(M') = C_J(M) = 1,$$
  

$$C_{J''}(M'') = (-1)^{J''-M''} \begin{pmatrix} J'' & 1 & J \\ -M'' & 0 & M \end{pmatrix} \xi_{J''}(F),$$

with k=1. Doing the necessary calculations, we get

$$\rho_0^{(1)}(v,J,J'') = \frac{3}{4\pi k_1 k_2} \sqrt{(2k_1+1)(2k_2+1)} \xi_{J'}(F)$$
  
×  $S(k_1k_2) \langle J \| T^{(k_1)} \| J_0 \rangle \langle J' \| T^{(k_2)} \| J_0 \rangle^*,$  (22)

where

$$S(k_{1}k_{2}) = \sum_{MM_{0}q_{1}q_{2}} \begin{pmatrix} J' & 1 & J \\ -M & 0 & M \end{pmatrix}^{2} \begin{pmatrix} J & k_{1} & J_{0} \\ -M & q_{1} & M_{0} \end{pmatrix} \times \begin{pmatrix} J' & k_{2} & J_{0} \\ -M & q_{2} & M_{0} \end{pmatrix} \begin{pmatrix} k_{2} & 1 & k_{1} \\ -q_{2} & 0 & -q_{1} \end{pmatrix}.$$
 (23)

Comparison of Eqs. (22) and (21) shows that the tensor ratio  $\rho_0^{(1)}(v,J,J'')/\rho_0^{(2)}(v,J,J) \sim \xi_{J''}(F)$  and depends on the field strength. The quantity  $\xi_{I''}(F)$  can be estimated as the ratio  $eFa_0/\Delta\varepsilon_{II'}$ , and for levels with an appreciable Stark effect it comes to several tens of percent.

We average the derived tensors over the electron distribution function (8) in accordance with Eq. (1). The result is determined by the velocity dependence of the dynamical tensors.

For the beam component, integration yields

$$\langle \rho_0^{(k)} \rangle = \sqrt{\frac{4\pi}{2k+1}} \, \rho_0^{(k)}(v_b).$$
 (24)

For the low-energy electrons it is generally impossible to integrate analytically because of the complex velocity dependence of the cross section. However, for optically allowed transitions a good approximation for the cross section of atomic excitation by electron impact is  $\sigma \propto v^{-2} \ln v^2$ , where the electron velocity is expressed in units of the threshold velocity  $v_c = \sqrt{2 \Delta \varepsilon / m}$ , with  $\Delta \varepsilon$  the excitation energy.<sup>14</sup> Then, with allowance for Eqs. (11)-(13), Eq. (1) acquires integrals of the form

$$I_n = \int \exp(-\alpha v^2) \ln(v^2) v^n dv, \qquad (25)$$

where  $\alpha = (v/v_0)^2$ , and n = 0, 1, 2, 3, 4. Obviously,

$$I_2 = -\frac{\partial I_0}{\partial \alpha}$$
,  $I_4 = \frac{\partial^2 I_0}{\partial \alpha^2}$ ,  $I_3 = -\frac{\partial I_1}{\partial \alpha}$ 

and for the integrals  $I_0$  and  $I_1$  we easily obtain<sup>24</sup>

$$I_0 = \frac{1}{2\alpha} \frac{\partial}{\partial \nu} \left\{ \alpha^{1-\nu} \Gamma(\alpha, \nu) \right\}_{\nu=1/2},$$
(26)

$$I_1 = \frac{\alpha u}{v_c} \frac{\partial}{\partial \nu} \left\{ \alpha^{1-\nu} \Gamma(\alpha, \nu) \right\}_{\nu=1}, \qquad (27)$$

where  $\Gamma(\alpha, \nu)$  is the incomplete gamma function.

Analysis of Eqs. (12), (13), (21), and (22) shows that the average value  $\langle \rho_0^{(2)} \rangle \sim u^2$  is determined by the dynamics of the collision processes and the field dependence of their cross sections. The value  $\langle \rho_0^{(1)} \rangle \sim \xi_{J''}(F)u$  is determined not only by the drift velocity of the incident electron but also by the polarization of states of the target atom in the field. The degree of polarization depends on  $\xi_{J''}(F)$ , and this determines the choice of atomic states with an appreciable field effect.

The drift velocity u is equal to  $\mu F$ , where  $\mu$  is the electron mobility<sup>25</sup>; this implies that both  $\langle \rho_0^{(1)} \rangle$  and  $\langle \rho_0^{(2)} \rangle$  are proportional to the square of the field strength. If we allow for the fact that  $\langle \rho_0^{(1)} \rangle$  and  $\langle \rho_0^{(2)} \rangle$  are calculated using measurements of the polarization characteristics of the radiation, the numerical accuracy is determined solely by the initial choice of states being diagnosed.

To find the parameters u,  $v_b$ , and  $\gamma$  of the distribution function (8), we need only determine the values of three tensors,  $\rho^{(0)}$ ,  $\rho^{(1)}$ , and  $\rho^{(2)}$ , from measurements of the linear degree of polarization and Eqs. (3) and (4). Doing the measurements over a large number of spectral lines for dipoleallowed and dipole-forbidden transitions, we can optimize these parameters.

Thus, the present method makes it possible to use measurements of the degree of linear polarization of lines with an appreciable Stark effect to obtain a more complete picture of the electron distribution function than that provided by the usual method of polarization spectroscopy of unperturbed states.

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