Coulomb deexcitation of muonic hydrogen in collisions with atoms of hydrogen isotopes

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The asymptotic theory of nonadiabatic transitions is used to treat Coulomb deexcitation of muonic hydrogen in hydrogen, including the effect of electron shielding of the charge of the target nucleus. The rates are calculated for an isotopically pure target and for a mixture of hydrogen isotopes. For a mixture of isotopes the rates of direct and inverse charge exchange with deexcitation are also calculated. © *1995 American Institute of Physics.*

1. As is well known, the process of deexcitation in mesonic deuterium in a deuterium—tritium mixture is of great interest in connection with muonic catalysis of nuclear fusion, since the efficiency of the mesonic catalysis cycle, i.e., the number of catalysis events per muon, is determined by the occupation q_{1s} of the ground state of mesonic deuterium.[1] The only studies of the sequence are characterized by a small energy release (of order a few electron volts) mainly determined by chemical reactions, including molecular dissociation in the target. The dissociation rates depend weakly on the collision energy. In the later stages first Coulomb deexcitation and then the external Auger effect become important, and for the lower states, radiative transitions. Besides deexcitation processes one should also take into account capture of a muon from a light isotope to a heavier one in a mixture of hydrogen isotopes, as well as capture from mesonic hydrogen to helium, which is inevitably present in a mixture as a result of the natural decay of tritium and fusion reactions. It is clear that the capture processes influence the magnitude of q_{1s} and hence the efficiency of catalysis. In order to calculate q_{1s} it is necessary to know the rates of the deexcitation and capture processes; these, generally speaking, depend on the collision energy.

Thus, one problem which arises in calculating q_{1s} is knowing the collision energy. It is well known[2] that the energy of a mesonic atom immediately after formation is about 1–1.5 eV. Subsequently in the process of deexcitation the mesonic atoms slow down as a result of elastic collisions. On the other hand, some processes (Coulomb deexcitation and the related processes of charge exchange with deexcitation) can accelerate the mesonic atom[3] due to the absence of a third light particle which would carry away most of the energy produced ($\sim 0.1/n^2$ in mesonic atomic units). Mesonic atoms with energies up to $\simeq$70 eV have been observed experimentally.[4] In a number of treatments[5,6] the occupation q_{1s} of the ground state of mesonic deuterium was calculated as a function of the tritium concentration C_t under the assumption that the occupation $\tilde{n}$ of a certain excited state is equal to unity and the energy of the mesonic atom in this state is given. At high concentrations $C_t \sim 0.5-1$ the calculated values of q_{1s} deviated from those in the experiment.[7] Generally speaking, q_{1s} should be calculated including not only deexcitation and charge-exchange processes but also elastic scattering, which is responsible for thermalizing the mesonic atoms.

2. The effective interaction potential of excited mesonic hydrogen in terms of the parabolic quantum numbers (n,n_1,n_2,m) due to a nucleus with charge Z is determined by the linear Stark effect

$$u(R) \simeq \frac{3}{2}\frac{Zn(n_1-n_2)}{R^2}, \tag{1}$$

where R is the internucleus separation. The condition for applicability of the quasiclassical approximation here is

$$\frac{1}{2\pi}\frac{d\lambda}{dR} \simeq \frac{1}{\sqrt{3MZn|n_1-n_2|}} \ll 1, \tag{2}$$

where $M=M_Z M_H/(M_Z+M_H)$ is the reduced mass of the nuclei and M_H and M_Z are the masses of the individual nuclei. We have used mesonic atomic units (m.a.e., $\hbar=m=e=1$, $m^{-1}=m_\mu^{-1}+M_H^{-1}$, where m_μ is the muon mass).

It is clear that this condition is satisfied for $n_1 \neq n_2$. Thus, we can regard the mesonic atom as moving along a classical trajectory with impact parameter ρ. Below we use the asymptotic (for particles colliding with velocity $v \to 0$) theory of nonadiabatic transitions,[8] according to which the transition probability is completely determined by the analyticity properties of the terms corresponding to the initial and final states of the system in the complex plane of the internuclear separation R. Hence the probability is large mainly close to the singularity (branch point) of the terms and can be written in the form of an exponential determined by the Massey parameter

$$\delta(\rho) = \left|\operatorname{Im}\int_C p(R)dR\right| = \left|\operatorname{Im}\int_{\operatorname{Re}R_c}^{R_c}\frac{\Delta u(R)dR}{v_R}\right|, \tag{3}$$

where $p(R) = \sqrt{2M[\varepsilon - u(R) - \varepsilon\rho^2/R^2)]}$ is the radial momentum, $\varepsilon = Mv^2/2$ is the relative energy of the colliding particles, C is the integration contour, which begins and ends on the real axis and encloses the complex branch point R_c, and $v_R = [p_1(R)+p_2(R)]/(2M)$ is the average radial velocity. For a double pass through the quasi-intersection region the transition probability is equal to

$$w(\rho) = 2\,e^{-2\delta}[1-e^{-2\delta}], \tag{4}$$

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TABLE I. Comparison of the exact branch points R_T and the approximate branch points $\tilde{R}_T$ of the T series for the g terms.

Upper term (n, n_1, n_2, m)	R_T	$\tilde{R}_T$	Upper term (n, n_1, n_2, m)	R_T	$\tilde{R}_T$
2,0,1,0	4.75; 4.14	4.6; 4.2	6,0,5,0	126.1; 28.0	126; 28
			6,0,4,1	111.34; 26.95	111; 27
3,0,2,0	20.12; 9.45	20.0; 9.47	6,0,3,2	95.84; 25.80	96.2; 25.7
3,0,1,1	13.44; 8.54	13.36; 8.52	6,0,2,3	79.07; 24.62	81.1; 24.4
			6,0,1,4	59.85; 23.48	66.2; 22.7
4,0,3,0	45.50; 15.31	45.4; 15.3	6,1,4,0	97.51; 25.75	97.4; 25.8
4,0,2,1	36.18; 14.34	36.1; 14.3	6,1,3,1	83.01; 24.47	82.5; 24.5
4,0,1,2	25.65; 13.25	26.6; 13.1	6,1,2,2	67.42; 23.11	67.5; 23.0
4,1,2,0	27.74; 13.10	27.6; 13.2	6,2,3,0	69.62; 23.04	69.2; 23.1
5,0,4,0	80.82; 21.53	80.8; 21.6	7,0,1,5	81.65; 28.94	92.6; 27.8
5,0,3,1	68.82; 20.50	68.7; 20.5	7,1,2,3	92.33; 28.37	94.1; 28.1
5,0,2,2	55.89; 19.40	56.5; 19.3	7,2,3,1	96.90; 28.22	95.9; 28.2
5,0,1,3	41.00; 18.16	44.2; 17.8			
5,1,3,0	57.73; 19.32	57.6; 19.4			
5,1,2,1	45.89; 18.03	45.5; 18.0			

and the cross section of the process is equal to

$$\sigma=\pi\int_0^{\rho_{\max}^2} w(\rho)d\rho^2. \tag{5}$$

The maximum impact parameter is determined by the requirement that $p(R)$ be real on the actual trajectory, i.e., for $R\geqslant \mathrm{Re}\,R_c$. The reaction rate scaled by the density $N_0=4.25\cdot 10^{22}$ cm^{-3} of liquid hydrogen is equal to

$$\lambda=N_0\sigma v, \tag{6}$$

where v is the relative velocity of the colliding particles.

According to Solov'ev,[8] the HμH system has two sets of branch points: the first, the S series, is associated with the restructuring of the potential of the quasiradial equation for the problem of two Coulomb centers at small values of R and the corresponding alteration of the muon wave function from a wave function of the bound atom to a molecular wave function. The branch points of the S series are unimportant for the processes in question, since for energies of interest to us (up to 50 eV) they are well below the Coulomb barrier.

The second set of branch points is the T series, associated with terms having quantum numbers (n,n_1,n_2,m) and

TABLE II. Coulomb deexcitation rates λ in units of s^{-1}, normalized to the density of liquid hydrogen.

ε, eV	$p\mu+p$	$d\mu+d$	$t\mu+t$	$p\mu+p$	$d\mu+d$	$t\mu+t$
	$n=3$			$n=4$		
0.04	$2.11\cdot 10^9$	$9.66\cdot 10^7$	$9.79\cdot 10^6$	$2.65\cdot 10^{10}$	$2.48\cdot 10^9$	$4.27\cdot 10^8$
0.1	$1.69\cdot 10^9$	$7.39\cdot 10^7$	$7.25\cdot 10^6$	$2.49\cdot 10^{10}$	$2.22\cdot 10^9$	$3.73\cdot 10^8$
0.5	$8.30\cdot 10^8$	$3.50\cdot 10^7$	$3.38\cdot 10^6$	$1.58\cdot 10^{10}$	$1.31\cdot 10^9$	$2.11\cdot 10^8$
1	$5.96\cdot 10^8$	$2.51\cdot 10^7$	$2.43\cdot 10^6$	$1.18\cdot 10^{10}$	$9.69\cdot 10^8$	$1.55\cdot 10^8$
5	$2.99\cdot 10^8$	$1.28\cdot 10^7$	$1.26\cdot 10^6$	$6.40\cdot 10^9$	$5.37\cdot 10^8$	$8.81\cdot 10^7$
10	$2.42\cdot 10^8$	$1.06\cdot 10^7$	$1.06\cdot 10^6$	$5.59\cdot 10^9$	$4.85\cdot 10^8$	$8.21\cdot 10^7$
	$n=5$			$n=6$		
0.04	$9.54\cdot 10^{10}$	$1.34\cdot 10^{10}$	$3.16\cdot 10^9$	$2.00\cdot 10^{11}$	$3.57\cdot 10^{10}$	$1.01\cdot 10^{10}$
0.1	$9.82\cdot 10^{10}$	$1.33\cdot 10^{10}$	$3.07\cdot 10^9$	$2.16\cdot 10^{11}$	$3.75\cdot 10^{10}$	$1.04\cdot 10^{10}$
0.5	$7.59\cdot 10^{10}$	$9.57\cdot 10^9$	$2.11\cdot 10^9$	$1.86\cdot 10^{11}$	$3.03\cdot 10^{10}$	$8.07\cdot 10^9$
1	$6.11\cdot 10^{10}$	$7.51\cdot 10^9$	$1.63\cdot 10^9$	$1.59\cdot 10^{11}$	$2.53\cdot 10^{10}$	$6.65\cdot 10^9$
5	$3.80\cdot 10^{10}$	$4.75\cdot 10^9$	$1.06\cdot 10^9$	$1.21\cdot 10^{11}$	$1.95\cdot 10^{10}$	$5.30\cdot 10^9$
10	$3.66\cdot 10^{10}$	$4.80\cdot 10^9$	$1.12\cdot 10^9$	$1.32\cdot 10^{11}$	$2.26\cdot 10^{10}$	$6.49\cdot 10^9$
	$n=8$			$n=10$		
0.04	$3.95\cdot 10^{11}$	$8.72\cdot 10^{10}$	$2.89\cdot 10^{10}$	$3.93\cdot 10^{11}$	$9.04\cdot 10^{10}$	$3.06\cdot 10^{10}$
0.1	$4.62\cdot 10^{11}$	$1.00\cdot 10^{11}$	$3.27\cdot 10^{10}$	$4.66\cdot 10^{11}$	$1.07\cdot 10^{11}$	$3.61\cdot 10^{10}$
0.5	$4.75\cdot 10^{11}$	$9.74\cdot 10^{10}$	$3.06\cdot 10^{10}$	$5.07\cdot 10^{11}$	$1.11\cdot 10^{11}$	$3.64\cdot 10^{10}$
1	$4.39\cdot 10^{11}$	$8.84\cdot 10^{10}$	$2.76\cdot 10^{10}$	$5.43\cdot 10^{11}$	$1.19\cdot 10^{11}$	$3.95\cdot 10^{10}$
5	$4.99\cdot 10^{11}$	$1.09\cdot 10^{11}$	$3.69\cdot 10^{10}$	$1.34\cdot 10^{12}$	$3.44\cdot 10^{11}$	$1.34\cdot 10^{11}$
10	$7.37\cdot 10^{11}$	$1.76\cdot 10^{11}$	$6.52\cdot 10^{10}$	$2.70\cdot 10^{12}$	$7.94\cdot 10^{11}$	$3.45\cdot 10^{11}$

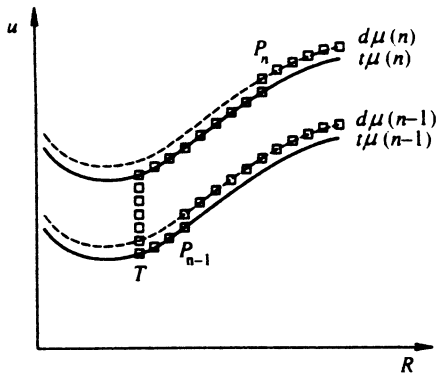

FIG. 1. Term configuration and T and P branch points for a $d-t$ mixture. The squares indicate the path of a transition needed to bring about the Coulomb deexcitation reaction (7).

$(n+1,n_1,n_2+1,m)$. It determines the rate at which a term reaches the peak of the barrier in the quasi-angle equation of the two-center problem. One must distinguish between symmetric (g) states and antisymmetric (u) states. Both types have a T series. The real part R_T of the branch point is the same for both g and u states, but $\mathrm{Im}\,R_T(u)\simeq 2\,\mathrm{Im}\,R_T(g)$. For this reason the probability of a transition between u terms is much smaller due to the large values of the Massey parameter.

For $H\mu H'$ systems with $H\neq H'$ a P series of branch points appears, associated with the exchange interaction. The difference in the masses of the H and H′ isotopes is equivalent to a difference in the nuclear charges,[8] so that the heavy isotope H′ corresponds to a larger charge, equal to $1+\Delta Z$. The difference in the charges is $\Delta Z\simeq(\mu_2-\mu_1)/2$, where μ_1 and μ_2 are the reduced masses of the $H\mu$ and $H'\mu$ atoms, respectively.

The branch points of the P series form an infinite sequence of equivalent points having approximately identical values of $\mathrm{Re}\,R_P$. All these points combine the specified initial term (n,n_1,n_2,m) with the corresponding final term, which has the same quantum numbers but a different muon localization.

For the pairs $p\mu d$, $p\mu t$ and $d\mu t$ we have $\mathrm{Re}\,R_P>\mathrm{Re}\,R_T$. The coordinates of the T points, in contrast to the points of the P series, are essentially independent of the isotopic composition of the mixture.

The branch points of the P series are responsible for the process of quasiresonant charge exchange of excited mesonic hydrogen in mixtures of hydrogen isotopes.[6] These transitions are characterized by a small Massey parameter, due to the smallness of the resonant defect Δu, which for the isotopic mixtures treated is equal to $(0.008-0.03)/n^2$ m.a.e.

3. The Coulomb excitation process

$$(H\mu)^*_n+H'\rightarrow(H\mu)^*_{n-1}+H' \qquad (7)$$

(where $H, H'\equiv p, d, t$) was treated previously in a number of papers (see Refs. 9 and 10 and the works cited in Ref. 9). The use of crude approximation for the coordinates of the branch points and also the use of an asymptotic expansion of the terms in calculating the Massey parameter should be regarded as a shortcoming in previous calculations of the Coulomb deexcitation rate.[9] As is well known,[8] the location of the T point limits the applicability of the asymptotic expansion for the lower term. For the upper term the asymptotic expansion is invalid at these separations.

TABLE IIIa. Coulomb deexcitation rates λ in units of s^{-1}, normalized to the density of liquid hydrogen, in mixtures of hydrogen isotopes. The initial term is of the form eZ_1.

ε, eV	$p\mu+d$	$p\mu+t$	$d\mu+t$	$p\mu+d$	$p\mu+t$	$d\mu+t$
	$n=3$			$n=4$		
0.04	$8.25\cdot10^5$	$2.75\cdot10^5$	$1.36\cdot10^5$	$3.16\cdot10^7$	$1.23\cdot10^7$	$1.09\cdot10^7$
0.1	$1.30\cdot10^6$	$4.34\cdot10^5$	$2.13\cdot10^5$	$4.97\cdot10^7$	$1.95\cdot10^7$	$1.70\cdot10^7$
0.5	$2.87\cdot10^6$	$9.64\cdot10^5$	$4.51\cdot10^5$	$1.08\cdot10^8$	$4.31\cdot10^7$	$3.45\cdot10^7$
1	$4.00\cdot10^6$	$1.35\cdot10^6$	$5.98\cdot10^5$	$1.49\cdot10^8$	$6.02\cdot10^7$	$4.40\cdot10^7$
5	$8.12\cdot10^6$	$2.91\cdot10^6$	$9.48\cdot10^5$	$2.87\cdot10^8$	$1.27\cdot10^8$	$6.25\cdot10^7$
10	$1.07\cdot10^7$	$4.03\cdot10^6$	$1.08\cdot10^6$	$3.76\cdot10^8$	$1.77\cdot10^8$	$7.18\cdot10^7$
	$n=5$			$n=6$		
0.04	$2.64\cdot10^8$	$1.02\cdot10^8$	$1.36\cdot10^8$	$1.07\cdot10^9$	$4.11\cdot10^8$	$6.69\cdot10^8$
0.1	$4.16\cdot10^8$	$1.61\cdot10^8$	$2.10\cdot10^8$	$1.68\cdot10^9$	$6.48\cdot10^8$	$1.02\cdot10^9$
0.5	$8.97\cdot10^8$	$3.54\cdot10^8$	$4.08\cdot10^8$	$3.59\cdot10^9$	$1.43\cdot10^9$	$1.93\cdot10^9$
1	$1.22\cdot10^9$	$4.93\cdot10^8$	$5.04\cdot10^8$	$4.87\cdot10^9$	$1.99\cdot10^9$	$2.35\cdot10^9$
5	$2.32\cdot10^9$	$1.09\cdot10^9$	$6.88\cdot10^8$	$9.64\cdot10^9$	$4.44\cdot10^9$	$3.37\cdot10^9$
10	$3.16\cdot10^9$	$1.59\cdot10^9$	$8.42\cdot10^8$	$1.42\cdot10^{10}$	$7.32\cdot10^9$	$4.60\cdot10^9$
	$n=8$			$n=10$		
0.04	$3.86\cdot10^9$	$1.88\cdot10^9$	$3.03\cdot10^9$	$6.78\cdot10^9$	$4.09\cdot10^9$	$3.62\cdot10^9$
0.1	$6.10\cdot10^9$	$2.98\cdot10^9$	$4.60\cdot10^9$	$1.09\cdot10^{10}$	$6.58\cdot10^9$	$5.53\cdot10^9$
0.5	$1.35\cdot10^{10}$	$6.83\cdot10^9$	$8.45\cdot10^9$	$2.51\cdot10^{10}$	$1.65\cdot10^{10}$	$1.11\cdot10^{10}$
1	$1.95\cdot10^{10}$	$1.00\cdot10^{10}$	$1.04\cdot10^{10}$	$4.03\cdot10^{10}$	$2.68\cdot10^{10}$	$1.59\cdot10^{10}$
5	$5.42\cdot10^{10}$	$2.94\cdot10^{10}$	$2.14\cdot10^{10}$	$1.83\cdot10^{11}$	$1.19\cdot10^{11}$	$7.42\cdot10^{10}$
10	$1.05\cdot10^{11}$	$6.17\cdot10^{10}$	$4.03\cdot10^{10}$	$5.03\cdot10^{11}$	$3.27\cdot10^{11}$	$2.01\cdot10^{11}$

TABLE III b. Coulomb deexcitation rates λ in units of s^{-1}, normalized to the density of liquid hydrogen, in mixtures of hydrogen isotopes. The initial term is of the form eZ_2.

ε, eV	$d\mu + p$	$t\mu + p$	$t\mu + d$	$d\mu + p$	$t\mu + p$	$t\mu + d$
	$n = 3$			$n = 4$		
0.04	$5.36 \cdot 10^8$	$3.98 \cdot 10^8$	$2.40 \cdot 10^7$	$1.08 \cdot 10^{10}$	$8.01 \cdot 10^9$	$7.88 \cdot 10^8$
0.1	$4.26 \cdot 10^8$	$3.08 \cdot 10^8$	$1.82 \cdot 10^7$	$9.81 \cdot 10^9$	$7.27 \cdot 10^9$	$7.04 \cdot 10^8$
0.5	$2.06 \cdot 10^8$	$1.47 \cdot 10^8$	$8.55 \cdot 10^6$	$5.96 \cdot 10^9$	$4.39 \cdot 10^9$	$4.11 \cdot 10^8$
1	$1.47 \cdot 10^8$	$1.06 \cdot 10^8$	$6.14 \cdot 10^6$	$4.41 \cdot 10^9$	$3.25 \cdot 10^9$	$3.02 \cdot 10^8$
5	$7.30 \cdot 10^7$	$5.29 \cdot 10^7$	$3.09 \cdot 10^6$	$2.34 \cdot 10^9$	$1.74 \cdot 10^9$	$1.64 \cdot 10^8$
10	$5.86 \cdot 10^7$	$4.30 \cdot 10^7$	$2.54 \cdot 10^6$	$2.03 \cdot 10^9$	$1.52 \cdot 10^9$	$1.47 \cdot 10^8$
	$n = 5$			$n = 6$		
0.04	$5.11 \cdot 10^{10}$	$3.12 \cdot 10^{10}$	$5.52 \cdot 10^9$	$1.15 \cdot 10^{11}$	$8.33 \cdot 10^{10}$	$1.57 \cdot 10^{10}$
0.1	$5.16 \cdot 10^{10}$	$3.27 \cdot 10^{10}$	$5.41 \cdot 10^9$	$1.27 \cdot 10^{11}$	$9.13 \cdot 10^{10}$	$1.64 \cdot 10^{10}$
0.5	$3.83 \cdot 10^{10}$	$2.95 \cdot 10^{10}$	$3.77 \cdot 10^9$	$1.06 \cdot 10^{11}$	$8.18 \cdot 10^{10}$	$1.29 \cdot 10^{10}$
1	$3.03 \cdot 10^{10}$	$2.33 \cdot 10^{10}$	$2.94 \cdot 10^9$	$8.89 \cdot 10^{10}$	$6.96 \cdot 10^{10}$	$1.07 \cdot 10^{10}$
5	$1.81 \cdot 10^{10}$	$1.39 \cdot 10^{10}$	$1.81 \cdot 10^9$	$6.38 \cdot 10^{10}$	$5.29 \cdot 10^{10}$	$7.98 \cdot 10^9$
10	$1.72 \cdot 10^{10}$	$1.32 \cdot 10^{10}$	$1.81 \cdot 10^9$	$6.86 \cdot 10^{10}$	$5.66 \cdot 10^{10}$	$9.17 \cdot 10^9$
	$n = 8$			$n = 10$		
0.04	$2.34 \cdot 10^{11}$	$1.85 \cdot 10^{11}$	$4.83 \cdot 10^{10}$	$2.49 \cdot 10^{11}$	$1.66 \cdot 10^{11}$	$6.10 \cdot 10^{10}$
0.1	$2.77 \cdot 10^{11}$	$2.19 \cdot 10^{11}$	$5.54 \cdot 10^{10}$	$2.95 \cdot 10^{11}$	$1.98 \cdot 10^{11}$	$7.26 \cdot 10^{10}$
0.5	$2.84 \cdot 10^{11}$	$2.23 \cdot 10^{11}$	$5.29 \cdot 10^{10}$	$3.19 \cdot 10^{11}$	$2.25 \cdot 10^{11}$	$7.15 \cdot 10^{10}$
1	$2.58 \cdot 10^{11}$	$2.16 \cdot 10^{11}$	$4.68 \cdot 10^{10}$	$3.30 \cdot 10^{11}$	$2.57 \cdot 10^{11}$	$7.29 \cdot 10^{10}$
5	$2.96 \cdot 10^{11}$	$2.30 \cdot 10^{11}$	$5.38 \cdot 10^{10}$	$7.99 \cdot 10^{11}$	$6.81 \cdot 10^{11}$	$1.91 \cdot 10^{11}$
10	$4.46 \cdot 10^{11}$	$3.51 \cdot 10^{11}$	$8.65 \cdot 10^{10}$	$1.64 \cdot 10^{12}$	$1.37 \cdot 10^{12}$	$4.27 \cdot 10^{11}$

Using the computers available to us, an exact calculation of the coordinates of the branch points of the T series is possible only for $n \leqslant 6$. Consequently, the branch points of higher states have been calculated using the quasiclassical approximation for the eigenvalues of the two-center problem derived in Ref. 11. To determine the coordinates of the T points the system of Eqs. (25)–(27) of Ref. 11 has been solved numerically[1)] (and the eigenvalues for the two-center problem at the branch point are determined at the same time). It is found that the approximate branch points agree well with the exact values in the range of n where a comparison is possible (Table I).

To calculate the Massey parameter we have used the quasiclassical values of the terms,[11] but not the asymptotic expansion. For the difference of the terms in this case we have used the approximation

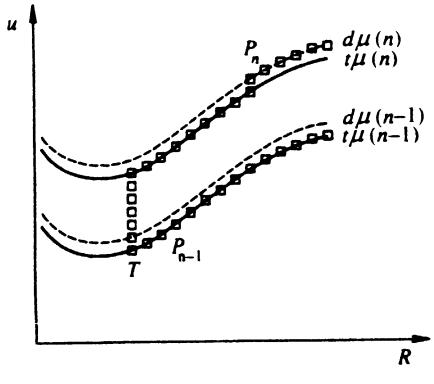

FIG. 2. Terms in a d–t mixture. The squares indicate the path of a transition needed to bring about the charge-exchange reaction with deexcitation (14).

$$\Delta u(R) \simeq \frac{\Delta u(\mathrm{Re}\, R_c)}{\mathrm{Im}\, R_c} \sqrt{(R-R_c)(R-R_c^*)}, \tag{8}$$

which is a correct description of the basic behavior of the terms close to a branch point. This yields for the Massey parameter

$$\delta(\rho) = \frac{\pi \Delta u(\mathrm{Re}\, R_c)}{4 v_c(\rho)} \mathrm{Im}\, R_c, \tag{9}$$

where $v_c(\rho)$ is the average radial velocity at the point $R = \mathrm{Re}\, R_c$.

Only attractive potentials ($\Delta\nu = n_1 - n_2 < 0$) have been treated; these make a larger contribution to the cross section, due to particle "focusing." The shielding of the target nucleus charge by atomic electrons is taken into account in the same way as in Ref. 5. The shielding potential $\tilde{u}(R)$ is calculated as

$$\tilde{u}(R) = u(R)[1 + 2\mathscr{R}(1+\mathscr{R})] \exp(-2\mathscr{R}), \tag{10}$$

where we have written $\mathscr{R} = R/a_e$, and a_e is the Bohr radius.

In the case H=H′ the main contribution to the cross section for the process comes from transitions between g terms, since we have $\mathrm{Im}\, R_T(u) \simeq 2\, \mathrm{Im}\, R_T(g)$ and the transition probability between u terms is strongly suppressed. In this case the cross section for the reaction (7) with a given value of n is calculated as the sum

$$\sigma_n = \sum_{\nu<0} \frac{n - |\Delta\nu|}{n^2} \sigma_n(\Delta\nu) \tag{11}$$

where $\sigma_n(\Delta\nu)$ is the mean value of the cross section for the g and u states:

$$\sigma_n(\Delta\nu) = \tfrac{1}{2}[\sigma_n(\Delta\nu, g) + \sigma_n(\Delta\nu, u)] \simeq \tfrac{1}{2}\sigma_n(\Delta\nu, g).$$

TABLE IV. Charge-exchange rates in units of s^{-1} with the deexcitation (14).

ε, eV	$p\mu+d$	$p\mu+t$	$d\mu+t$	$p\mu+d$	$p\mu+t$	$d\mu+t$
	$n=3$			$n=4$		
0.04	$1.22\cdot10^6$	$4.17\cdot10^5$	$1.62\cdot10^5$	$4.31\cdot10^7$	$2.02\cdot10^7$	$1.34\cdot10^7$
0.1	$1.92\cdot10^6$	$6.58\cdot10^5$	$2.54\cdot10^5$	$6.78\cdot10^7$	$3.19\cdot10^7$	$2.09\cdot10^7$
0.5	$4.23\cdot10^6$	$1.46\cdot10^6$	$5.36\cdot10^5$	$1.48\cdot10^8$	$7.04\cdot10^7$	$4.23\cdot10^7$
1	$5.89\cdot10^6$	$2.05\cdot10^6$	$7.11\cdot10^5$	$2.03\cdot10^8$	$9.81\cdot10^7$	$5.40\cdot10^7$
5	$1.19\cdot10^7$	$4.36\cdot10^6$	$1.13\cdot10^6$	$3.87\cdot10^8$	$2.04\cdot10^8$	$7.65\cdot10^7$
10	$1.57\cdot10^7$	$5.97\cdot10^6$	$1.28\cdot10^6$	$4.99\cdot10^8$	$2.80\cdot10^8$	$8.76\cdot10^7$
	$n=5$			$n=6$		
0.04	$3.15\cdot10^8$	$1.35\cdot10^8$	$1.61\cdot10^8$	$1.31\cdot10^9$	$4.77\cdot10^8$	$8.38\cdot10^8$
0.1	$4.95\cdot10^8$	$2.13\cdot10^8$	$2.49\cdot10^8$	$2.06\cdot10^9$	$7.51\cdot10^8$	$1.28\cdot10^9$
0.5	$1.07\cdot10^9$	$4.67\cdot10^8$	$4.84\cdot10^8$	$4.42\cdot10^9$	$1.65\cdot10^9$	$2.41\cdot10^9$
1	$1.45\cdot10^9$	$6.48\cdot10^8$	$5.98\cdot10^8$	$5.98\cdot10^9$	$2.30\cdot10^9$	$2.91\cdot10^9$
5	$2.71\cdot10^9$	$1.34\cdot10^9$	$8.10\cdot10^8$	$1.15\cdot10^{10}$	$5.04\cdot10^9$	$4.12\cdot10^9$
10	$3.61\cdot10^9$	$1.91\cdot10^9$	$9.82\cdot10^8$	$1.63\cdot10^{10}$	$7.73\cdot10^9$	$5.55\cdot10^9$
	$n=8$			$n=10$		
0.04	$4.33\cdot10^9$	$2.54\cdot10^9$	$3.54\cdot10^9$	$8.15\cdot10^9$	$5.19\cdot10^9$	$3.70\cdot10^9$
0.1	$6.84\cdot10^9$	$4.02\cdot10^9$	$5.37\cdot10^9$	$1.31\cdot10^{10}$	$8.37\cdot10^9$	$5.72\cdot10^9$
0.5	$1.52\cdot10^{10}$	$9.25\cdot10^9$	$9.96\cdot10^9$	$3.09\cdot10^{10}$	$2.11\cdot10^{10}$	$1.19\cdot10^{10}$
1	$2.17\cdot10^{10}$	$1.35\cdot10^{10}$	$1.23\cdot10^{10}$	$4.94\cdot10^{10}$	$3.41\cdot10^{10}$	$1.74\cdot10^{10}$
5	$5.58\cdot10^{10}$	$3.82\cdot10^{10}$	$2.46\cdot10^{10}$	$2.05\cdot10^{11}$	$1.44\cdot10^{11}$	$7.93\cdot10^{10}$
10	$1.01\cdot10^{11}$	$7.42\cdot10^{10}$	$4.47\cdot10^{10}$	$4.86\cdot10^{11}$	$3.49\cdot10^{11}$	$2.04\cdot10^{11}$

Table II displays calculated Coulomb deexcitation rates, normalized to the density of liquid hydrogen, for a symmetric system (H=H′).

For a potential $u(R)\propto-\alpha/R^2$ the square of the maximum impact parameter is $\rho_{\max}^2=\alpha/\varepsilon+R_0^2$, where R_0 is the turning point (in this case $R_0=\mathrm{Re}\,R_T$). Thus, in the absence of shielding the cross section satisfies $\sigma\propto\alpha/\varepsilon+R_0^2$ and the reaction rate λ at low energies is proportional to $1/\sqrt{\varepsilon}$. When shielding is included, $\rho_{\max}$ decreases sharply at low energies and the energetic dependence of λ is smeared out.

The isotopic dependence of the reaction rate is determined by the dependence of the Massey parameter on the reduced mass, $\delta\propto\sqrt{2M}$.

When H and H′ correspond to different isotopes, we have to treat eZ_1 and eZ_2 terms, which for $R<\mathrm{Re}\,R_P$ coincide with the u and g terms of a symmetric system (H=H′).

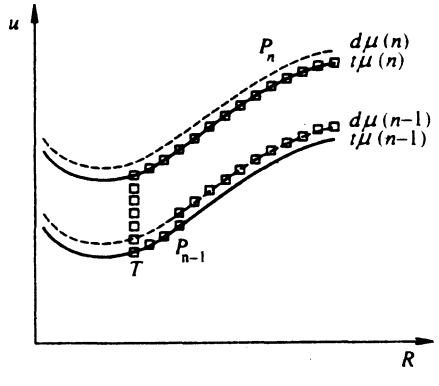

FIG. 3. Terms in a $d-t$ mixture. The squares indicate the path of a transition needed to bring about the inverse charge-exchange reaction with deexcitation (16).

In the region $R>\mathrm{Re}\,R_P$ the eZ_1 term corresponds to a muon localized at the light nucleus, and the eZ_2 term is localized at the heavy one. As in the case of a symmetric system, Coulomb deexcitation arises from the T series of branch points. If H is lighter than H′, the initial eZ_1 term corresponds to the u term. This would seem to imply that the reaction (7) is strongly suppressed in this case. This conclusion, however, is valid only for the direct transition between u terms through the T point. There exists a reaction channel, the $P-T-P$ triple transition (Fig. 1), in which there is no suppression. The transition probability in this case can be written as

$$w(\rho)=2w_T(1-w_T)w_{P1}w_{P2}, \tag{12}$$

where $w_{Pi}=(1-\tanh\delta_{Pi})/2$ and $w_T=\exp(-2\delta_T)$ are the probabilities for a single transition through the P and T point (g-type), respectively. Here the suppression due to the two additional P transitions is negligible, since in the P transition the Massey parameter is small.

If H is heavier than H′, the initial term corresponds to the g term and the probability of Coulomb deexcitation is calculated as the product of the probability of a transition between terms through the T point and the probability of remaining in the g term while passing through the P points:

$$w(\rho)=2w_T(1-w_T)(1-w_{P1})(1-w_{P2}). \tag{13}$$

The Coulomb deexcitation rates for H≠H′ are shown in Table III.

4. Charge-exchange reactions with deexcitation,

$$(\mathrm{H}\mu)_n^*+\mathrm{H}'\to(\mathrm{H}'\mu)_{n-1}^*+\mathrm{H}, \tag{14}$$

where H′ is heavier than H, can proceed as a double transition through the T and P points (see Fig. 2). The transition probability in this case is equal to

TABLE V. Inverse charge-exchange rates in units of s^{-1} with the deexcitation (16).

ε, eV	$d\mu+p$	$t\mu+p$	$t\mu+d$	$d\mu+p$	$t\mu+p$	$t\mu+d$
	$n=3$			$n=4$		
0.04	$3.63\cdot10^{8}$	$1.90\cdot10^{8}$	$2.02\cdot10^{7}$	$5.25\cdot10^{9}$	$3.35\cdot10^{9}$	$6.44\cdot10^{8}$
0.1	$2.88\cdot10^{8}$	$1.51\cdot10^{8}$	$1.53\cdot10^{7}$	$4.96\cdot10^{9}$	$3.13\cdot10^{9}$	$5.74\cdot10^{8}$
0.5	$1.39\cdot10^{8}$	$7.26\cdot10^{7}$	$7.19\cdot10^{6}$	$3.16\cdot10^{9}$	$1.95\cdot10^{9}$	$3.35\cdot10^{8}$
1	$9.94\cdot10^{7}$	$5.20\cdot10^{7}$	$5.16\cdot10^{6}$	$2.35\cdot10^{9}$	$1.44\cdot10^{9}$	$2.46\cdot10^{8}$
5	$4.93\cdot10^{7}$	$2.59\cdot10^{7}$	$2.60\cdot10^{6}$	$1.24\cdot10^{9}$	$7.70\cdot10^{8}$	$1.34\cdot10^{8}$
10	$3.97\cdot10^{7}$	$2.09\cdot10^{7}$	$2.14\cdot10^{6}$	$1.06\cdot10^{9}$	$6.71\cdot10^{8}$	$1.20\cdot10^{8}$
	$n=5$			$n=6$		
0.04	$1.99\cdot10^{10}$	$1.07\cdot10^{10}$	$3.70\cdot10^{9}$	$4.40\cdot10^{10}$	$2.46\cdot10^{10}$	$1.07\cdot10^{10}$
0.1	$2.06\cdot10^{10}$	$1.13\cdot10^{10}$	$3.69\cdot10^{9}$	$4.87\cdot10^{10}$	$2.71\cdot10^{10}$	$1.13\cdot10^{10}$
0.5	$1.61\cdot10^{10}$	$9.92\cdot10^{9}$	$2.66\cdot10^{9}$	$4.27\cdot10^{10}$	$2.45\cdot10^{10}$	$9.24\cdot10^{9}$
1	$1.29\cdot10^{10}$	$7.92\cdot10^{9}$	$2.08\cdot10^{9}$	$3.64\cdot10^{10}$	$2.09\cdot10^{10}$	$7.67\cdot10^{9}$
5	$7.69\cdot10^{9}$	$4.75\cdot10^{9}$	$1.27\cdot10^{9}$	$2.60\cdot10^{10}$	$1.58\cdot10^{10}$	$5.64\cdot10^{9}$
10	$7.31\cdot10^{9}$	$4.55\cdot10^{9}$	$1.28\cdot10^{9}$	$2.81\cdot10^{10}$	$1.73\cdot10^{10}$	$6.53\cdot10^{9}$
	$n=8$			$n=10$		
0.04	$7.52\cdot10^{10}$	$4.75\cdot10^{10}$	$2.63\cdot10^{10}$	$6.01\cdot10^{10}$	$2.75\cdot10^{10}$	$2.46\cdot10^{10}$
0.1	$8.87\cdot10^{10}$	$5.59\cdot10^{10}$	$3.01\cdot10^{10}$	$7.05\cdot10^{10}$	$3.23\cdot10^{10}$	$2.89\cdot10^{10}$
0.5	$9.08\cdot10^{10}$	$5.60\cdot10^{10}$	$2.91\cdot10^{10}$	$7.57\cdot10^{10}$	$3.59\cdot10^{10}$	$2.92\cdot10^{10}$
1	$8.36\cdot10^{10}$	$5.37\cdot10^{10}$	$2.61\cdot10^{10}$	$8.19\cdot10^{10}$	$4.31\cdot10^{10}$	$3.08\cdot10^{10}$
5	$1.00\cdot10^{11}$	$6.41\cdot10^{10}$	$3.12\cdot10^{10}$	$2.62\cdot10^{11}$	$1.67\cdot10^{11}$	$9.67\cdot10^{10}$
10	$1.61\cdot10^{11}$	$1.07\cdot10^{11}$	$5.20\cdot10^{10}$	$6.40\cdot10^{11}$	$4.27\cdot10^{11}$	$2.41\cdot10^{11}$

$$w(\rho)=2w_T(1-w_T)w_{P1}(1-w_{P2}). \tag{15}$$

The charge-exchange rates with deexcitation are shown in Table IV.

Similarly, inverse charge exchange with deexcitation,

$$(H'\mu)_n^*+H\rightarrow(H\mu)_{n-1}^*+H' \tag{16}$$

can occur (Fig. 3), since the energy release associated with deexcitation $(n-1/2)/[n(n-1)]^2$, is much larger than the resonance defect. Here the transition probability is

$$w(\rho)=2w_T(1-w_T)(1-w_{p1})(1-w_{p2}). \tag{17}$$

The inverse charge-exchange rates with deexcitation are shown in Table V.

Note that reactions in which the initial state corresponds to the eZ_1 term (Tables IIIa and IV) occur through the transition to the eZ_2 term, which is associated with additional effective acceleration of the colliding particles at the transition point R_T (Figs. 1 and 2). The energy of the additional acceleration is approximately equal to $\Delta Z/n^2$. For the case in question, with an attractive potential, we have $\rho_{\max}^2\sim1+|u|/\varepsilon$. Thus, this additional acceleration causes a reduction in $\rho_{\max}$ and the cross section relative to the opposite case, when the initial state corresponds to the eZ_2 term. This is especially pronounced for small values of n and low energies, when the energy of the additional acceleration exceeds that of the colliding particles (cf. Tables IIIa and IIIb).

TABLE VI. Coulomb deexcitation rates in units of s^{-1} for the $p\pi+p$ system, normalized to the density of liquid hydrogen, and the rates λ_A for the external Auger effect, taken from Refs. 12 and 13.

ε, eV	$n=3$	$n=4$	$n=5$
0.004	$4.36\cdot10^{9}$	$3.36\cdot10^{10}$	$9.83\cdot10^{10}$
0.01	$4.73\cdot10^{9}$	$3.97\cdot10^{10}$	$1.20\cdot10^{11}$
0.04	$4.13\cdot10^{9}$	$4.38\cdot10^{10}$	$1.45\cdot10^{11}$
0.1	$3.22\cdot10^{9}$	$4.01\cdot10^{10}$	$1.46\cdot10^{11}$
0.2	$2.41\cdot10^{9}$	$3.40\cdot10^{10}$	$1.35\cdot10^{11}$
0.5	$1.54\cdot10^{9}$	$2.44\cdot10^{10}$	$1.08\cdot10^{11}$
1	$1.10\cdot10^{9}$	$1.80\cdot10^{10}$	$8.49\cdot10^{10}$
2	$7.93\cdot10^{8}$	$1.32\cdot10^{10}$	$6.58\cdot10^{10}$
5	$5.34\cdot10^{8}$	$9.26\cdot10^{9}$	$4.90\cdot10^{10}$
10	$4.18\cdot10^{8}$	$7.65\cdot10^{9}$	$4.36\cdot10^{10}$
λ_A [12]	$8.32\cdot10^{10}$	$6.66\cdot10^{11}$	$2.94\cdot10^{12}$
λ_A [13]	$8.10\cdot10^{10}$	$6.08\cdot10^{11}$	$2.25\cdot10^{12}$

In Table VI the Coulomb deexcitation rates are shown for $n=3$, 4, and 5 in the $p\pi+p$ system, along with the rates for the external Auger effect taken from Refs. 12 and 13. These rates do not depend on the collision energy if the latter is much less than the transition energy in the mesonic atom. As can be seen from the table, the Coulomb deexcitation rates are an order of magnitude smaller than the Auger transition rate. Thus, acceleration of a substantial fraction of mesonic atoms to energy $\simeq$70 eV, as observed in Ref. 4, cannot be explained by Coulomb deexcitation.

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