Noise of thermally scattered laser light in nonlinear waveguides

Ze Cheng

Department of Chemical Physics, Weizmann Institute of Science, Rehovot 76100, Israel
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We develop a quantum theory of laser light noise in nonlinear waveguides with thermal dielectric-function fluctuations. The laser light noise is a manifestation of the interplay of the quantum noise, the radiation damping and the thermal noise. Quadrature squeezing of laser light in nonlinear waveguides shows up only at a short distance and below a threshold intensity. Increasing the incident intensity brings a transition from a squeezed state to a nonclassical state. © *1995 American Institute of Physics.*

The wave nature of light leads to the dispersion of laser pulses in fibers, and the particle nature of light gives rise to the quantum noise of laser light. Thermal scattering from fibers produces the energy loss and thermal noise of laser light. The desire to go beyond the intrinsic limits has largely driven the development of optical communications theory. Hasegawa and Tappert proposed a theory of optical solitons in nonlinear fibers.[1] The optical solitons occur owing to the cancellation of the group velocity dispersion by the Kerr nonlinearity and so propagate without distortion. The squeezed state of light in nonlinear fibers has been an active theoretical subject and has been observed in a propagation distance.[2] The squeezed state has less quantum noise in one quadrature than a coherent state, and squeezing arises due to self-phase-modulation. The author has established the photonic superguiding theory in waveguides of a self-defocusing nonlinearity.[3] In the superguiding state Rayleigh and Brillouin scattering are overcome by the virtual Raman scattering, so that the photons propagate without thermal scattering.

The present optical fibers are almost exclusively made from materials with a self-focusing nonlinearity. Thermal scattering exists inevitably in these fibers. Thermal scattering originates from thermal fluctuations in the dielectric function of fibers. The scattering loss limits the propagation distance of laser light in fibers. The thermal amplitude noise degrades the signal. The thermal phase noise tends to erase the coherence of laser light and therefore is most dangerous. To generate squeezed light in nonlinear fibers, first one must know how large the noise of thermally scattered laser light is and what factors influence squeezing. This paper is devoted to the theoretical study of these problems.

We consider a cylindrical dielectric waveguide, whose core is occupied by an isotropic dispersive crystal with a Kerr nonlinearity. The z axis coincides with the waveguide axis of symmetry. A linearly polarized coherent light field is normally incident on the end face $z=0$ of the core at the time $t=0$. The high-frequency dielectric constant of the core is larger than that of the cladding, so that the incident field excites a guided wave field in the waveguide. The object under study is a coupled system consisting of a crystal in a volume V and the field in the crystal. The central frequency of incidence is assumed to be well below the electronic transition frequencies but well above the optical phonon frequencies. With this assumption, the electron and multiphonon absorption of a photon can be omitted. The Hamiltonian of the coupled system reads

$$H=:\int d\mathbf{r}\left[\frac{1}{2\mu_0}\mathbf{B}^2+\frac{1}{2}(\mathbf{ED})+\frac{3}{4}\epsilon_0\chi^{(3)}\mathbf{E}^4\right]:+H_C, \quad (1)$$

where : : denotes normal ordering, $\chi^{(3)}$ is the third-order nonlinear susceptibility, and H_C represents the crystal Hamiltonian. The electric displacement $\mathbf{D}$ noninstantaneously depends on the electric field $\mathbf{E}$ in the form

$$\mathbf{D}(\mathbf{r},t)=\epsilon_0\int d\tau\ \epsilon(\mathbf{r},t,\tau)\mathbf{E}(\mathbf{r},\tau). \quad (2)$$

There are fluctuations in the dielectric function $\varepsilon(\mathbf{r},t,\tau)$, which are activated thermally or optically. When the incident intensity of light is below the threshold of stimulated Brillouin scattering, the crystal deviates slightly from equilibrium and thus the dielectric function has only thermal fluctuations. Near the equilibrium state of the crystal, the dielectric function can be expanded as

$$\varepsilon(\mathbf{r},t,\tau)=\varepsilon(t-\tau)+\delta(t-\tau)\delta\epsilon(\mathbf{r},t). \quad (3)$$

The dielectric function $\varepsilon(t-\tau)$ in the equilibrium state is position-independent and invariant under time translation, and its Fourier transformation $\epsilon(\omega)$ represents the material dispersion. Here $\delta\epsilon(\mathbf{r},t)$ is the fluctuation in the dielectric function due to thermal variations in the temperature and thermal vibrations in the crystal, and the delta function $\delta(t-\tau)$ exhibits the instantaneous relation between the electric displacement and the fluctuation. Thermal variations in the temperature give rise to elastic Rayleigh scattering, and thermal vibrations in the crystal lead to spontaneous Brillouin and Raman scattering.

In order to quantize the nonlinear guided-wave field, it is necessary to introduce a suitable orthonormal basis in the field space. In the Coulomb gauge, we choose the set of local plane-wave modes as the basis.[4] The wave vector $\mathbf{k}$ of a local plane-wave mode is separated into $\mathbf{k}=\mathbf{K}+\mathbf{Q}$, where $\mathbf{K}$ and $\mathbf{Q}$ are the components parallel to and transverse to the z axis. The axial wave vector $\mathbf{K}$ is real everywhere. $\mathbf{Q}$ is real in the core but imaginary in the cladding. The local plane waves in the cladding are evanescent waves and diffraction effects are eliminated. The waveguide satisfies the weak-guidance approximation, so that the local plane waves in the core are paraxial waves with $Q \ll K$. The axial wave number K is a function of the incident frequency ω, and $K(\omega)$ represents the waveguide dispersion. The group velocity dispersion consists of the material dispersion and the waveguide dispersion. In the rotating-wave approximation, the nonlinear term in the Hamiltonian (1) is quantized in the chosen basis as

$$H_{nl}=\sum_{\mathbf{KQ}\sigma}\sum_{\mathbf{k}'\sigma_1\sigma_2} \times\frac{9\chi^{(3)}\hbar^2\omega_{\mathbf{K}+\mathbf{Q}}}{16V\epsilon_0\epsilon(\omega_{\mathbf{K}+\mathbf{Q}})}\left[\frac{\omega_{\mathbf{k}_++\mathbf{k}'}\omega_{\mathbf{k}_--\mathbf{k}'}}{\epsilon(\omega_{\mathbf{k}_++\mathbf{k}'})\epsilon(\omega_{\mathbf{k}_--\mathbf{k}'})}\right]^{1/2} \times\mathbf{e}_{\sigma_1}(\mathbf{k}_++\mathbf{k}')\mathbf{e}_{\sigma_2}(\mathbf{k}_--\mathbf{k}') \times(a^\dagger_{\mathbf{k}_+,\sigma}a^\dagger_{\mathbf{k}_-,\sigma}a_{\mathbf{k}_++\mathbf{k}',\sigma_1}a_{\mathbf{k}_--\mathbf{k}',\sigma_2}+\text{H.c.}), \tag{4}$$

where $\mathbf{k}_\pm=\mathbf{K}\pm\mathbf{Q}$, $\sigma=1,2$ characterizes two polarization directions, and $\mathbf{e}_\sigma(\mathbf{k})$ is the polarization unit vector. The operator $a^\dagger_{\mathbf{k}\sigma}$ creates a photon with wave vector $\mathbf{k}$ and polarization σ, and $\omega_\mathbf{k}$ is the frequency of this photon. Let $\delta\epsilon(\mathbf{q},t)$ denotes the spatial Fourier transform of $\delta\epsilon(\mathbf{r},t)$. Consequently, the interaction term in the Hamiltonian (1) reduces to

$$H_I=\sum_{\substack{\mathbf{KQ}\\ \sigma\sigma'}}\int d\mathbf{q}\,\frac{\hbar}{4}\left[\frac{\omega_{\mathbf{K}+\mathbf{Q}}\omega_{\mathbf{K}+\mathbf{Q}+\mathbf{q}}}{\epsilon(\omega_{\mathbf{K}+\mathbf{Q}})\epsilon(\omega_{\mathbf{K}+\mathbf{Q}+\mathbf{q}})}\right]^{1/2} \times\mathbf{e}_\sigma(\mathbf{K}+\mathbf{Q})\mathbf{e}_{\sigma'}(\mathbf{K}+\mathbf{Q}+\mathbf{q}) \times[a^\dagger_{\mathbf{K}+\mathbf{Q}+\mathbf{q},\sigma'}a_{\mathbf{K}+\mathbf{Q},\sigma}\delta\epsilon(\mathbf{q},t)+\text{H.c.}]. \tag{5}$$

We require that the nonlinear Hamiltonian (4) describes the quantum optical effect and the interaction Hamiltonian (5) reflects the thermal scattering effect.

Further treatment requires a quantum statistical description of photons in the core of the waveguide. In Eq. (4), the two photons created by the operators $a^\dagger_{\mathbf{k}_\pm,\sigma}$ have the same frequency $\omega_{\mathbf{K}+\mathbf{Q}}$ and the same polarization σ. Such photon pairs have perfect time coherence. The quantum optical effect originates from the quantum time coherence of the photons. The operator product $a_{\mathbf{k}_+,\sigma}a_{\mathbf{k}_-,\sigma}$ just reflects the quantum time coherence of the photons and therefore it needs to be handled by the quantum statistical theory. However, the two photons created by the operators $a^\dagger_{\mathbf{k}_++\mathbf{k}',\sigma_1}$ and $a^\dagger_{\mathbf{k}_--\mathbf{k}',\sigma_2}$ have unequal frequencies and polarizations. Such photon pairs are statistically independent and have no temporal coherence. The operator product $a_{\mathbf{k}_++\mathbf{k}',\sigma_1}a_{\mathbf{k}_--\mathbf{k}',\sigma_2}$ represents the incoherent correlation of photons and therefore has classical behavior. The operator product $a_{\mathbf{k}_++\mathbf{k}',\sigma_1}a_{\mathbf{k}_--\mathbf{k}',\sigma_2}$ is most naturally replaced by the product $\alpha_{\mathbf{k}_++\mathbf{k}',\sigma_1}\alpha_{\mathbf{k}_--\mathbf{k}',\sigma_2}$ of coherent state variables. In Eq. (4) we introduce the parameter $I_{\mathbf{K}+\mathbf{Q}}$ by

$$I_{\mathbf{K}+\mathbf{Q}}e^{-2i\omega_{\mathbf{K}+\mathbf{Q}}t} =\frac{\hbar c\epsilon^{1/2}(\omega_{\mathbf{K}+\mathbf{Q}})}{V}\sum_{\mathbf{k}'\sigma_1\sigma_2}\left[\frac{\omega_{\mathbf{k}_++\mathbf{k}'}\omega_{\mathbf{k}_--\mathbf{k}'}}{\epsilon(\omega_{\mathbf{k}_++\mathbf{k}'})\epsilon(\omega_{\mathbf{k}_--\mathbf{k}'})}\right]^{1/2} \times[\mathbf{e}_{\sigma_1}(\mathbf{k}_++\mathbf{k}')\mathbf{e}_{\sigma_2}(\mathbf{k}_--\mathbf{k}')] \times\alpha_{\mathbf{k}_++\mathbf{k}',\sigma_1}(t)\alpha_{\mathbf{k}_--\mathbf{k}',\sigma_2}(t). \tag{6}$$

$I_{\mathbf{K}+\mathbf{Q}}$ can be regarded as the complex intensity of the incident field at frequency $\omega_{\mathbf{K}+\mathbf{Q}}$. The nonlinear Hamiltonian (4) is therefore written in the form

$$H_{nl}=\sum_{\mathbf{KQ}\sigma}\hbar W_{\mathbf{K}+\mathbf{Q}}(I_{\mathbf{K}+\mathbf{Q}}e^{-2i\omega_{\mathbf{K}+\mathbf{Q}}t}a^\dagger_{\mathbf{K}+\mathbf{Q},\sigma}a^\dagger_{\mathbf{K}-\mathbf{Q},\sigma}+\text{H.c.}), \tag{7}$$

where $W_{\mathbf{K}+\mathbf{Q}}=9\chi^{(3)}\omega_{\mathbf{K}+\mathbf{Q}}/16c\epsilon_0\epsilon^{3/2}(\omega_{\mathbf{K}+\mathbf{Q}})$. Under the weak-guidance approximation, the two operators $a^\dagger_{\mathbf{K}\pm\mathbf{Q},\sigma}$ in Eq. (7) must be regarded as equivalent.

In Eq. (5), we must differentiate the scattering process associated with $\mathbf{q}=0$ from that associated with $\mathbf{q}\neq 0$. In the $\mathbf{q}=0$ scattering process, the number of core photons remains unchanged but their phases are changed. The crystal in this case is characterized by a c-number $\delta\epsilon(t)=\delta\epsilon(\mathbf{q}=0,\ t)$. In the $\mathbf{q}\neq 0$ scattering process, a core photon is annihilated by the operator $a_{\mathbf{K}+\mathbf{Q},\sigma}$ and a thermal photon is created by the operator $a^\dagger_{\mathbf{K}+\mathbf{Q}+\mathbf{q},\sigma'}$. The thermal photon escapes out of the core with a certain probability, which accounts for the radiation damping. The thermal photons represent the amplitude noise in the guided wave field, and of course they can be described classically. When the operators of thermal photons are replaced by the coherent state variables, the radiation damping corresponds to the mechanism in which a core photon can lose energy by creating a crystal quantum. The crystal in this case is characterized by a q-number $\delta\epsilon(\mathbf{q},t) = iu(\mathbf{q})[b^\dagger_\mathbf{q}(t) - b_{-\mathbf{q}}(t)]$, where the operator $b^\dagger_\mathbf{q}$ creates a phonon with wave vector $\mathbf{q}$. It is necessary to point out that the phonon here is an energy quantum in the collective propagation of temperature fluctuations and crystal vibrations. Now the phonons represent the amplitude noise in the guided wave field. If we introduce the coupling parameter $f_{\mathbf{K}+\mathbf{Q},\sigma}(\mathbf{q})$ by

$$f_{\mathbf{K}+\mathbf{Q},\sigma}(\mathbf{q})e^{-i\omega_{\mathbf{K}+\mathbf{Q}}t} =\frac{1}{4}\left[\frac{\omega_{\mathbf{K}+\mathbf{Q}}\omega_{\mathbf{K}+\mathbf{Q}+\mathbf{q}}}{\epsilon(\omega_{\mathbf{K}+\mathbf{Q}})\epsilon(\omega_{\mathbf{K}+\mathbf{Q}+\mathbf{q}})}\right]^{1/2}u(-\mathbf{q}) \times\sum_{\sigma'}[\mathbf{e}_\sigma(\mathbf{K}+\mathbf{Q})\mathbf{e}_{\sigma'}(\mathbf{K}+\mathbf{Q}+\mathbf{q})]\alpha_{\mathbf{K}+\mathbf{Q}+\mathbf{q},\sigma'}(t), \tag{8}$$

then the interaction Hamiltonian (5) assumes the form

$$H_I=\sum_{\mathbf{KQ}\sigma}\int' d\mathbf{q}\, i\hbar[f^*_{\mathbf{K}+\mathbf{Q},\sigma}(\mathbf{q})e^{i\omega_{\mathbf{K}+\mathbf{Q}}t}a_{\mathbf{K}+\mathbf{Q},\sigma}b^\dagger_\mathbf{q}-\text{H.c.}] +\sum_{\mathbf{KQ}\sigma}\hbar g_{\mathbf{K}+\mathbf{Q}}a^\dagger_{\mathbf{K}+\mathbf{Q},\sigma}a_{\mathbf{K}+\mathbf{Q},\sigma}\delta\epsilon(t), \tag{9}$$

where $g_{\mathbf{K}+\mathbf{Q}}=\omega_{\mathbf{K}+\mathbf{Q}}(2\pi)^3/2\epsilon(\omega_{\mathbf{K}+\mathbf{Q}})V$. The prime on the integral symbol means that the lower limit of q is $+0$. At this point the quantized Hamiltonian of the coupled system is obtained as

$$H=\sum_{\mathbf{KQ}\sigma}\hbar\omega_{\mathbf{K}+\mathbf{Q}}a^\dagger_{\mathbf{K}+\mathbf{Q},\sigma}a_{\mathbf{K}+\mathbf{Q},\sigma}$$

$$+\int' d\mathbf{q}\, \hbar\omega(\mathbf{q}) b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + H_{nl} + H_I, \qquad (10)$$

where $\omega(\mathbf{q})$ is the phonon frequency.

We only need to investigate the noise in a local plane-wave mode. The Hamiltonian (10) is the starting point in our derivation of the quantum Langevin equations for this mode. The mode index $(\mathbf{K}+\mathbf{Q},\sigma)$ on the relevant quantities is dropped in the following. For the weak interaction, one can make the first Markov approximation:[5] $f(\mathbf{q})=\sqrt{\gamma/2\pi D(\mathbf{q})}$, where γ is independent of wave vector $\mathbf{q}$ and $D(\mathbf{q})$ is the density of phononic states. This approximation yields the Heisenberg equation of motion for the photon operators

$$\frac{da}{dt} = -i\omega a - 2iWIe^{-2i\omega t}a^\dagger - \frac{|\gamma|}{2}a - \sqrt{\gamma}e^{-i\omega t}b(t)$$
$$-ig\delta\epsilon(t)a. \qquad (11)$$

Here $|\gamma|$ is the damping constant of radiation, $b(t)$ is the amplitude fluctuation operator defined by

$$b(t) = \frac{1}{\sqrt{2\pi}}\int' d\mathbf{q}\, \frac{1}{\sqrt{D(\mathbf{q})}} e^{-i\omega(\mathbf{q})(t-t_0)} b_{\mathbf{q}}(t_0), \qquad (12)$$

$\delta\epsilon(t)$ produces the phase fluctuations in the guided wave field. These three quantities characterize the thermal scattering effect of core photons by the crystal. The parameter $\eta=2iWI$ characterizes the quantum optical effect. If there were no thermal scattering terms in Eq. (11), Eq. (11) would generate an ideal quadrature squeezing of core photons with a squeeze parameter given by $r=|2iWIt|$. Ideal squeezing is independent of the incident light intensity. In the presence of thermal scattering, the squeezing effect is dependent on the incident light intensity. The thermal scattering effect always tends to destroy the quantum optical effect, and so there is competition between the two effects. Equation (11) can be called a quantum Langevin equation only after the statistical properties of the random variables $b(t)$ and $\delta\epsilon(t)$ are given. $b(t)$ and $\delta\epsilon(t)$ are statistically independent. $b(t)$ is supposed to have the property of quantum white noise, i.e., $\langle b^\dagger(t)b(t')\rangle = \bar{N}\delta(t-t')$, where $\langle ...\rangle$ denotes the ensemble average for the crystal. The average $\langle \delta\epsilon(t)\delta\epsilon(t')\rangle$ is stationary but not delta-correlated.

In order to solve Eq. (11), we change to a rotating frame with $a=e^{-i\omega t}a_r$. The photon operators $a_r^\dagger(t)$ and $a_r(t)$ in the rotating frame possess the Fourier transform variables $a_r^\dagger(-\Omega)$ and $a_r(\Omega)$. In frequency space Eq. (11) supports the formal solution

$$a_r(\Omega) = -\frac{1}{(|\gamma|/2-i\Omega)^2-|\eta|^2}\left[\left(\frac{|\gamma|}{2}-i\Omega\right)\sqrt{\gamma}b(\Omega)\right.$$
$$\left. -\eta\sqrt{\gamma^*}b^\dagger(-\Omega)\right]$$
$$-\frac{ig}{|\gamma|/2-i\Omega}\int d\Omega_1 \delta\epsilon(\Omega_1)a_r(\Omega-\Omega_1), \qquad (13)$$

where $b(\Omega)$ and $\delta\epsilon(\Omega)$ are the Fourier transformations of $b(t)$ and $\delta\epsilon(t)$, respectively. In the last equation, the first term represents the amplitude fluctuations in the guided wave field, and the second term originates from the phase fluctuations in the guided wave field. Squeezing can also be interpreted as the reduction of the quantum noise in the field amplitude to below the coherent state level. Therefore, the first term is connected with the squeezing effect but the second term is not. We need to compute the covariances $\langle a_r(\Omega),a_r(\Omega')\rangle$ and $\langle a_r^\dagger(-\Omega),a_r(\Omega')\rangle$ from the solution (13). In calculating, one has to make the decoupling approximation. The decoupling approximation is based on the following idea: in a spontaneous scattering process, the phase of the guided wave field can respond only slowly to a change in the dielectric function configuration. The phase fluctuations in the guided wave field diffuse in an average background of the dielectric function. Using the decoupling approximation, we can write

$$\langle \delta\epsilon(\Omega_1)\delta\epsilon(\Omega_2)a_r(\Omega-\Omega_1)a_r(\Omega'-\Omega_2)\rangle$$
$$=\langle \delta\epsilon(\Omega_1)\delta\epsilon(\Omega_2)\rangle\langle a_r(\Omega-\Omega_1)a_r(\Omega'-\Omega_2)\rangle, \qquad (14)$$

where $\langle\delta\epsilon(\Omega_1)\delta\epsilon(\Omega_2)\rangle = S(\Omega_1)\delta(\Omega_1+\Omega_2)$ and $S(\Omega_1)$ is the spectrum of the random variable $\delta\epsilon(t)$. When the incident intensity of light exceeds the threshold of stimulated Brillouin scattering, the decoupling approximation is invalid. In a stimulated scattering process, the phase of the guided wave field responds adiabatically to a change in the dielectric function configuration. Therefore we can observe phase jumps or phase waves in the guided wave field.[6]

We adjust the incident intensity of light so that the parameter $|\eta|=|2iWI|$ is less than $|\gamma|/2$. In this case the variance of the photon operator $a_r(t)$ is calculated to be

$$\langle a_r(t),a_r(t)\rangle = \frac{1}{2}\frac{\eta(|\gamma|/2)(2\bar{N}+1)}{(|\gamma|/2)^2-|\eta|^2}$$
$$-\frac{1}{2\pi}\int d\Omega\int d\Omega'\int d\Omega_1 e^{-i(\Omega+\Omega')t}$$
$$\times\frac{g^2S(\Omega_1)}{(|\gamma|/2)^2-\Omega\Omega'-(|\gamma|/2)i(\Omega+\Omega')}$$
$$\times\langle a_r(\Omega-\Omega_1),a_r(\Omega'+\Omega_1)\rangle. \qquad (15)$$

The double integrals over Ω and Ω' in Eq. (15) are concentrated mainly on the curve $\Omega\Omega'=|\gamma|^2/4$ in the $\Omega^- - \Omega'$ plane. The integral over Ω_1 can therefore be separated from the double integrals. This is exactly what the decoupling approximation demands. Differentiating Eq. (15) with respect to t yields a rate equation for the variance $\langle a_r(t),a_r(t)\rangle$. The first term in Eq. (15) gives the variance without phase fluctuations. Obviously there are no phase fluctuations at the initial time $t=0$. This initial condition requires that $\langle a_r(\Omega),a_r(\Omega')\rangle$ be an even function of $\Omega+\Omega'$. The above analysis also applies to the covariance $\langle a_r^\dagger(t),a_r(t)\rangle$. The variance and covariance of the photon operators $a_r^\dagger(t)$ and $a_r(t)$ are then

$$\langle a_r(t),a_r(t)\rangle = \frac{1}{2}\frac{\eta(|\gamma|/2)(2\bar{N}+1)}{(|\gamma|/2)^2-|\eta|^2}e^{-\beta t},$$

$$\langle a_r^\dagger(t),a_r(t)\rangle = \frac{1}{2}\frac{(|\gamma|/2)^2 2\bar{N}+|\eta|^2}{(|\gamma|/2)^2-|\eta|^2}e^{\beta t}, \qquad (16)$$

where $\beta=2g^2\langle[\delta\epsilon(t)]^2\rangle/|\gamma|$. In the absence of phase fluctuations, the variance and covariance are stationary. Now phase fluctuations destroy this stationarity. As shown in Eq. (16), the variance decreases exponentially with time, whereas the covariance increases. The physical mechanism for this is as follows. Phase fluctuations diffuse with time. There are interference effects in the diffusion process. The interference effect for the variance is destructive but the interference effect for the covariance is constructive.

The photon operators $a_r^\dagger$ and a_r represent the non-Hermitian amplitudes of the local plane-wave mode. The Hermitian amplitudes of the local plane-wave mode must be expressed in terms of the quadrature phase operators X and Y defined by $a_r=e^{i\theta/2}(X+iY)$, where θ is the phase of η. The noise in the local plane-wave mode is therefore described by the normally ordered variances of the quadrature phase operators

$$\langle:X(t),X(t):\rangle=\frac{1}{4}\left[\frac{|\eta|(|\gamma|/2)(2\bar{N}+1)}{(|\gamma|/2)^2-|\eta|^2}e^{-\beta t}+\frac{(|\gamma|/2)^2 2\bar{N}+|\eta|^2}{(|\gamma|/2)^2-|\eta|^2}e^{\beta t}\right],$$

$$\langle:Y(t),Y(t):\rangle=-\frac{1}{4}\left[\frac{|\eta|(|\gamma|/2)(2\bar{N}+1)}{(|\gamma|/2)^2-|\eta|^2}e^{-\beta t}-\frac{(|\gamma|/2)^2 2\bar{N}+|\eta|^2}{(|\gamma|/2)^2-|\eta|^2}e^{\beta t}\right], \quad (17)$$

where normal ordering simplifies the formulae. The thermal phase noise accumulates with time and β is its cumulative rate. In the last equation, $\bar{N}$ characterizes the thermal amplitude noise and βt measures the thermal phase noise at time t. In the absence of the thermal noise, Eq. (17) represents the effect on the quantum noise of the radiation damping. Since the local plane-wave mode under study is arbitrary, the noise in the guided wave field is a manifestation of the interplay of the quantum noise and the radiation damping, amplitude noise and phase noise due to thermal scattering. The larger the noise, the smaller the coherence. Because of the thermal phase noise, the noise in the guided wave field increases exponentially with time, so that all coherence is erased. From many experimental facts,[2] we find that $\bar{N}\ll 1$ holds below room temperature. The thermal amplitude noise can be neglected in comparison with the quantum noise. β has a calculable expression

$$\beta=\omega^2\overline{\langle\delta\epsilon(\mathbf{r},t)\delta\epsilon(0,t)\rangle}/2\epsilon^2(\omega)|\gamma|,$$

where the overline denotes the spatial average in the volume V. $|\gamma|$ is expressed in terms of the absorption coefficient α of the waveguide as $|\gamma|=\alpha v_g$, where v_g is the group velocity of the guided wave field. When $\alpha=0.2$ dB/km and $v_g=2.1\cdot 10^8$ $\mathrm{m\cdot s^{-1}}$ $|\gamma|=0.967\cdot 10^4\ \mathrm{s^{-1}}$. Given that $\omega=1.215\cdot 10^{15}\ \mathrm{s^{-1}}$, $\epsilon(\omega)=2.25$ and $\overline{\langle\delta\epsilon(\mathbf{r},t)\delta\epsilon(0,t)\rangle}=1.20\cdot 10^{-20}$, one finds $\beta=1.81\cdot 10^5\ \mathrm{s^{-1}}$. If the propagation distance z is smaller than 100 m or the transit time t is less than $4.762\cdot 10^{-7}$ s, $\beta t\ll 1$. The thermal phase noise can be ignored in this case.

When the thermal noise can be omitted, Eq. (17) becomes

$$\langle:X,X:\rangle=\frac{1}{4}\frac{|\eta|}{|\gamma|/2-|\eta|},\quad \langle:Y,Y:\rangle=-\frac{1}{4}\frac{|\eta|}{|\gamma|/2+|\eta|}. \quad (18)$$

The condition for squeezing is that the normally ordered variance in one quadrature phase be less than zero. If there were no thermal scattering, perfect squeezing in one quadrature phase would be achieved with a normally ordered variance of $-1/4$. We know that $|\eta|$ is directly proportional to the frequency component of the incident light intensity. There is a threshold intensity at which $|\eta|=|\gamma|/2$. As shown in Eq. (18), below the threshold intensity squeezing increases with the incident light intensity. At the threshold intensity, the maximum squeezing in the Y quadrature phase is attained with the normally ordered variance of $-1/8$ while the X quadrature phase is infinitely unsqueezed. Above the threshold intensity, we find that the normally ordered variance in each quadrature phase is positive. In this case the guided wave field exhibits no squeezing, but it is in a nonclassical photon state due to the nonlinear effect. Increasing the incident light intensity brings a transition from a squeezed state to a nonclassical photon state. The squeezed state here is not a minimum uncertainty state because of the radiation damping. As the propagation distance increases, the thermal phase noise becomes nonnegligible. The thermal phase noise rapidly destroys these two nonclassical photon states by deleting phase information. When the guided wave field loses all phase information, one can show that the state of the guided wave field is a superposition of number states. The guided wave field in this superposition state has manifestly classical behavior.

In conclusion, we have developed a quantum theory of laser light noise in nonlinear waveguides with thermal dielectric-function fluctuations. A method for quantizing the nonlinear guided-wave field is presented, and a quantum statistical description of thermally scattered photons is given. We have derived the quantum Langevin equations in the photon operators, thereby finding a time-dependent analytic solution of laser light noise. The laser light noise is a manifestation of the interplay of the quantum noise and the radiation damping, amplitude noise and phase noise due to thermal scattering. Quadrature squeezing of laser light in nonlinear waveguides shows up only at a short propagation distance and below a threshold light intensity. Increasing the incident light intensity causes a transition from a squeezed state to a nonclassical photon state. Increasing the propagation distance induces the guided wave field to enter a state having manifestly classical behavior. Since our theory does not involve the propagation form of the guided wave field, the above conclusions apply to continuous-wave, pulse and soliton propagation.

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