Ultrashort dynamical chaos of biexcitons

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The emergence of ultrashort dynamical chaos of biexcitons in semiconductors upon two-photon laser excitation has been studied theoretically. It has been shown that the regular oscillations in the system become chaotic as the effective dipole moment of the two-photon excitation of a biexciton from the ground state increases. The dynamical stochasticity of coherent quasiparticles is caused by breakdown of the integrals of the motion of the system. Features of a scenario for the development of local instability in systems with $n \leqslant 2$ and $n > 2$ degrees of freedom have been studied in a numerical experiment. © *1995 American Institute of Physics.*

1. INTRODUCTION

The discovery of dynamical chaos has become one of the most important and striking achievements of laser physics in recent times. Numerous reviews, articles, and monographs have been devoted to this subject.[1–5] Dynamical chaos is a clear-cut example of the self-organization of diverse nonlinear systems, and processes involving the onset of deterministic chaos have a universal character. It can be realized in both dissipative and Hamiltonian dynamical systems.

A special place among these phenomena is occupied by the problem of optical dynamical chaos, especially in the excitonic region of the spectrum of a semiconductor as it relates to substantial nonlinearity.[6,7] The urgency of the investigation of this phenomenon is dictated primarily by the need to study the formation of fundamentally new temporal structures in semiconductors and insulators, to predict new physical phenomena in them and create new optoelectronic devices with their aid, to apply these phenomena to optical information processing, and to create a new generation of optical computers.

The theory of dynamical chaos and the formation of strange attractors in a system of coherent (in the Bogolyubov sense) excitons and biexcitons in condensed media with consideration of dissipative processes was first formulated in Refs. 8–14. It was shown, in particular, that the dynamical evolution of coherent excitons and biexcitons is described by a generalized system of Lorentz equations in a four-dimensional phase space, and that the dynamics of coherent excitons and photons can be described with consideration of the exciton–exciton interaction by a generalized system of Keldysh equations.

On the other hand, the investigation of ultrashort dynamical chaos (without consideration of dissipative processes) on the long-wavelength fundamental absorption edge of a crystal is still in its initial stages. The possibility, in principle, of the appearance of ultrashort dynamical chaos in a system of coherent excitons, photons, and biexcitons in the region of the M luminescence band of a semiconductor was predicted in Refs. 15 and 16. It was shown that stochastic instability appears at certain values of the exciton-biexciton conversion constant and the initial concentration of quasiparticles in the system owing to breakdown of the integrals of the motion.

The present research was devoted to theoretically studying a new cooperative nonlinear phenomenon, viz., the ultrashort dynamical chaos of coherent biexcitons in superconductors. Biexcitons, which were predicted by Moskalenko[17] and Lampert,[18] are widely used to interpret new absorption and luminescence bands in semiconductors. The investigation of biexcitons has essentially become a separate specialty in the physics of the condensed state. The most convincing experimental evidence of the existence of biexcitons is based on observations of two-photon excitation from the ground state of a crystal (CuCl, CuBr, etc.).[19,20] In addition, Hanamura[21] was the first to show that the two-photon excitation of biexcitons from the ground state of a crystal is characterized by an enormous oscillator strength. For this reason, the two-photon excitation of biexcitons has become widely used in experimental investigations of biexcitonic states. The two-photon absorption band has a narrow δ-shaped form. The direct generation of biexcitons owing to the immense two-photon absorption of light in a CuCl crystal was first observed by Gale and Mysyrowicz.[19] The optical bistability accompanying two-photon biexciton generation was predicted in Refs. 22–24, and it was observed experimentally in Refs. 24 and 25.

We studied the nonlinear two-photon dynamics of coherent biexcitons created by an ultrashort laser pulse during a time shorter than the relaxation times of biexcitons, which are of the order of 10^{-10}–10^{-11} s in semiconductors. Owing to the development of methods for generating and shaping light pulses with a duration as short as 10^{-15} s, the restrictions on the characteristic times of biexciton dynamics have been removed, and the experimental observation of such coherent phenomena as the propagation of solitons and the linear and nonlinear nutation of excitons and biexcitons is possible.[26,27] In those studies, the temporal dynamics of coherent excitons and biexcitons, in particular, were explored. The theory of nonlinear nutation formulated in those studies was based on a resonance approximation. However, as was noted in Ref. 28, "the neglect of nonresonance terms in the Hamiltonian has been so poorly justified, due to the formal difficulties involved, that it has become an article of faith." As was shown in Ref. 28, consideration of the nonresonance

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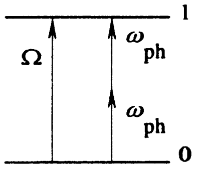

FIG. 1. Energy scheme for the two-photon generation of a biexciton from the ground state: 0—ground state of the crystal; 1—biexcitonic energy level; ω_{ph}—frequency of the light; Ω—frequency of the biexcitonic transition.

terms results in destruction of the bound states of a system of two-level atoms interacting with a radiation field and in the appearance of a new phenomenon, dynamical chaos in Hamiltonian systems. It will be shown below that consideration of the antiresonance terms in the interaction Hamiltonian in the case of two-photon excitation of biexcitons from the ground state of a crystal, in contrast to the case of nonlinear nutation,[26] fundamentally alters the nonlinear dynamics of biexcitons and, under certain conditions, results in the onset of dynamical chaos of biexcitons.

2. FORMULATION OF THE PROBLEM. DYNAMICAL EQUATIONS

Let us examine the ultrashort nonlinear dynamics of coherent biexcitons in the case of two-photon excitation from the ground state of a crystal. Since this process is characterized by an enormous oscillator strength and since the excitation pulse is assumed to be ultrashort, we shall henceforth disregard the biexciton relaxation processes. This requires a pulse duration that is shorter than the biexciton relaxation time. The estimates given in Ref. 29 show that the biexciton relaxation times due to biexciton-biexciton collisions and the interaction of biexcitons with acoustic phonons at a concentration of biexcitons equal to 10^{16} cm^{-3} are equal to $\sim 5\times 10^{-11}$ and $\sim 5\times 10^{-10}$ s, respectively. On the other hand, the spectral width of the ultrashort pulses may result in the appearance of biexciton recombination processes with the formation of free excitons and photons. To eliminate these processes, the uncertainty in the photon energy must be less than $I_m/2$, and the pulse duration τ must be greater than $2\hbar/I_m$, where I_m is the binding energy of a biexciton. Evaluations show that the pulse duration for a CuCl crystal has a lower limit: $\tau > 10^{-14}$ s. Thus, short-lived pulses in the pico- and subpicosecond ranges are needed to observe the ultrashort dynamic chaos of biexcitons upon two-photon excitation.

Let us examine the temporal evolution of a spatially homogeneous system of coherent biexcitons and photons in a semiconductor. It is assumed that the quasiparticles are coherent in the Bogolyubov sense, i.e., they have identical wave vectors, polarizations, and phases, and their amplitudes are macroscopically large. Figure 1 presents the energy-level scheme of the phenomenon under study.

The basis of the ultrashort dynamical chaos appearing upon two-photon excitation of a biexciton from the ground state of a crystal is the Hamiltonian of biexcitons and photons with a wave vector **k**

$$H = H_0 + H_{\text{int}}, \quad H_0 = \omega_{ph} c_{\mathbf{k}}^{+} c_{\mathbf{k}} + \Omega b_{2\mathbf{k}}^{+} b_{2\mathbf{k}},$$

$$H_{\text{int}} = iM_{\mathbf{k}} (c_{\mathbf{k}}^{+} + c_{-\mathbf{k}})(c_{\mathbf{k}}^{+} + c_{-\mathbf{k}}) b_{2\mathbf{k}} + \text{h.c.}, \tag{1}$$

where $b_{2\mathbf{k}}^{+}(b_{2\mathbf{k}})$ and $c_{\mathbf{k}}^{+}(c_{\mathbf{k}})$ are the creation (annihilation) operators of a biexciton and a photon, respectively, Ω is the energy of formation of a biexciton, ω_{ph} is the energy of a photon, $M_{\mathbf{k}}$ is the matrix element defined in Ref. 29 for the two-photon transition from the ground state of the crystal to the biexcitonic state. Here and in the following we set $\hbar = 1$. The interaction of light with biexcitons has both a resonant part and an antiresonant part. The resonance approximation leads to the coherent nutation of biexcitations studied in Refs. 26 and 29. It will be shown below that consideration of the antiresonance terms of the Hamiltonian results in breakdown of the integrals of the motion of the system and the appearance of ultrashort dynamical chaos of biexcitons.

We transform to action–angle variables:

$$c_{\mathbf{k}} = \sqrt{f} \exp(-i\varphi_{ph} + ikx); \quad b_{2\mathbf{k}} = \sqrt{N} \exp(-i\varphi_b + ikx), \tag{2}$$

where $f = c^{+}c$, φ_{ph}, $N = b^{+}b$, and φ_b are the numbers and phases of the photons and biexcitons, respectively. We shall henceforth omit the indices of the wave vectors. With this notation the Hamiltonian of the system takes on the form

$$H = \Omega N + \omega_{ph} f + 2M\sqrt{N} f [\sin(\varphi_b - 2\varphi_{ph}) + \sin(\varphi_b + 2\varphi_{ph}) + 2 \sin \varphi_b]. \tag{3}$$

With consideration of (3), the temporal evolution of the coherent photons and biexcitons is described by the system of equations

$$\frac{dN}{dt} = -\frac{\partial H}{\partial \varphi_b} = -2Mf\sqrt{N}[\cos(\varphi_b - 2\varphi_{ph}) + \cos(\varphi_b + 2\varphi_{ph}) + 2 \cos \varphi_b],$$

$$\frac{d\varphi_b}{dt} = \frac{\partial H}{\partial N} = \Omega + M \frac{f}{\sqrt{N}} [\sin(\varphi_b - 2\varphi_{ph}) + \sin(\varphi_b + 2\varphi_{ph}) + 2 \sin \varphi_b],$$

$$\frac{df}{dt} = -\frac{\partial H}{\partial \varphi_{ph}} = 4Mf\sqrt{N}[\cos(\varphi_b - 2\varphi_{ph}) - \cos(\varphi_b + 2\varphi_{ph})],$$

$$\frac{d\varphi_{ph}}{dt} = \frac{\partial H}{\partial f} = \omega_{ph} + 2M\sqrt{N}[\sin(\varphi_b - 2\varphi_{ph}) + \sin(\varphi_b + 2\varphi_{ph}) + 2 \sin \varphi_b]. \tag{4}$$

Introducing the notation

$$\psi = \varphi_b - 2\varphi_{ph}, \quad \varphi = \varphi_b + 2\varphi_{ph}, \quad \tau = tck,$$

$$\lambda = \frac{32M}{5\omega_{ph}}, \quad \Delta = \frac{\Omega - 2\omega_{ph}}{\omega_{ph}},$$

we bring Eqs. (4) into the form

$$\dot{N} = -\frac{5\lambda}{16} f\sqrt{N}[\cos \psi + \varepsilon(\cos \varphi + 2 \cos \varphi_b)],$$

$$\dot{f}=\frac{5\lambda}{8}f\sqrt{N}(\cos\psi-\varepsilon\cos\varphi),$$

$$\dot{\psi}=\Delta+\frac{5\lambda}{32}\left(\frac{f}{\sqrt{N}}-4\sqrt{N}\right)[\sin\psi+\varepsilon(\sin\varphi+2\sin\varphi_b)],$$

$$\dot{\varphi}=\Delta+4+\frac{5\lambda}{32}\left(\frac{f}{\sqrt{N}}+4\sqrt{N}\right)[\sin\psi+\varepsilon \times(\sin\varphi+2\sin\varphi_b)],$$

$$\dot{\varphi}_b=\Delta+2+\frac{5\lambda}{32}\frac{f}{\sqrt{N}}[\sin\psi+\varepsilon(\sin\varphi+2\sin\varphi_b)]. \quad (5)$$

The parameter ε was introduced in such a manner that $\varepsilon=0$ when the antiresonance terms are neglected, and that Eqs. (5) are equivalent to Eqs. (4) when $\varepsilon=1$. The dot denotes differentiation with respect to the dimensionless parameter τ. In the resonance approximation ($\varepsilon=0$), the number of particles is an integral of the motion for Eqs. (5),

$$2N+f=C, \quad (6)$$

which is determined by the number of particles at the initial moment in time.

Using the integral of the motion (6), from (5) we easily obtain

$$\dot{\psi}=\frac{\partial P}{\partial N}=\Delta+\frac{5\lambda}{32}\left(\frac{C-2N}{\sqrt{N}}-4\sqrt{N}\right)\sin\psi,$$

$$\dot{N}=-\frac{\partial P}{\partial\psi}=-\frac{5\lambda}{16}(C-2N)\sqrt{N}\cos\psi,$$

$$P=\Delta N+\frac{5\lambda}{16}(C-2N)\sqrt{N}\sin\psi, \quad (7)$$

where $P=H/\omega_{\rm ph}-C$ is an additional integral of the motion, which plays the role of the Hamiltonian in the (N,ψ) parameter space. A system of coherent biexcitons evolves differently with time, depending on the parameters of the system, the initial values of the occupation numbers, and the offset from resonance Δ.

Introducing the notation $\bar{\lambda}=\lambda\sqrt{C}$, $\bar{f}=f/C$, and $\bar{N}=N/C$, going over to the variables $(N,\dot{N})$, and henceforth dropping the bar, we obtain

$$\dot{N}^2=\left(\frac{5\lambda}{16}\right)^2(1-2N)^2N-(P-\Delta N)^2. \quad (8)$$

In the general case, the solution of Eq. (8) has the form

$$N=N_3+N_{23}\text{sn}^2\left(\sqrt{N_{13}}\,\frac{5\lambda\tau}{32}+F(\varphi_0);k\right). \quad (9)$$

Here F is an elliptic integral of the first kind, $k=\sqrt{N_{23}/N_{13}}$ is the modulus of the elliptic function, $\varphi_0=\arcsin\sqrt{N_{03}/N_{23}}$, N_0 is the initial number of biexcitons per unit volume, $N_{ij}=N_i-N_j$, and $N_3<N_2<N_1$ are the roots of the equation

$$\left(\frac{5\lambda}{16}\right)^2(1-2N)^2N-(P-\Delta N)^2=0. \quad (10)$$

In the case of $P=\Delta/2$, the number of coherent biexcitons as a function of time is given by

$$N=\left(\frac{8\Delta}{5\lambda}\right)^2+\left[\frac{1}{2}-\left(\frac{8\Delta}{5\lambda}\right)^2\right]\tanh^2\left[\frac{5\lambda\tau}{32}\sqrt{\frac{1}{2}-\left(\frac{8\Delta}{5\lambda}\right)^2}+\log\left(\tan\frac{\varphi_0}{2}+\frac{\pi}{4}\right)\right], \quad (11)$$

i.e., all the photons are converted into biexcitons, and the evolution of the system is thereby completed.

We note that aperiodic motion is possible when ε is nonzero, and that periodic oscillations are also possible when $\varepsilon=0$. These classes of solutions were omitted in our previous reports.

3. BREAKDOWN OF INTEGRALS OF THE MOTION AND STOCHASTIZATION OF PHASE TRAJECTORIES

When $P=\Delta/2$, the phase trajectory is a separatrix and corresponds to an aperiodic oscillation regime. In this case, even an infinitesimal perturbation causes significant destruction of the phase trajectories near the separatrix with resultant stochastization of the system of photons and biexcitons. Chaotic fluctuations appear owing to breakdown of the integrals of the motion, this breakdown being due to the nonresonance terms in the Hamiltonian, which act as perturbations. The variation in the integral of the motion P under the action of a perturbation, taking the canonical variables (N,ψ) into consideration, is described by

$$\dot{P}=\frac{\partial P}{\partial N}\dot{N}+\frac{\partial P}{\partial\psi}\dot{\psi}+\frac{\partial P}{\partial C}\dot{C}. \quad (12)$$

According to (5) and (7), the perturbed equations have the form

$$\dot{\psi}=\frac{\partial P}{\partial N}+\delta\dot{\psi},\quad \dot{N}=-\frac{\partial P}{\partial\psi}+\delta\dot{N},$$

$$\delta\dot{N}=-\frac{5\lambda}{16}(1-2N)\sqrt{N}(\cos\varphi+2\cos\varphi_b),$$

$$\delta\dot{\varphi}=\frac{5\lambda}{32}\left(\frac{1-2N}{\sqrt{N}}-4\sqrt{N}\right)(\sin\varphi+2\sin\varphi_b). \quad (13)$$

Plugging (13) into (12), we obtain

$$\dot{P}\approx\left\{\left(\frac{5\lambda}{8}\right)^2(1-2N)(1-6N)[\sin(\varphi-\psi)+\sin(\varphi_b-\psi)]+\frac{5\lambda\Delta}{8}\sqrt{N}(1-2N)\times(\cos\varphi+2\cos\varphi_b)\right\}\Big/\left[1+\frac{P-\Delta N}{1-2N}\right]. \quad (14)$$

We are interested in the motion of the system near the separatrix, i.e., when $P\to\Delta/2$. In this case the characteristic roots are given by

$$N_{1,2}=\frac{1}{2}+\frac{(2P-\Delta)\left(\Delta\pm\frac{5\lambda}{8\sqrt{2}}\right)}{\left(\frac{5\lambda}{8}\right)^2-2\Delta^2},\quad N_3=\frac{4P^2}{\left(\frac{5\lambda}{8}\right)^2+8\Delta P}. \tag{15}$$

$N(\tau)$ varies from almost 0 to 1/2, where its period tends to infinity at a turning point of the hyperbolic type. When $\lambda\ll 1$, the denominator in (14) is of order unity, while $\sqrt{N}(1-2N)\sim P/\lambda$ and $(1-2N)(1-6N)\sim 1$ far from the singular point $N=1/2$ over a small time interval equal to the period of the small oscillations π/ω_0. In the vicinity of the singular point we have $\sqrt{N}(1-2N)\sim P/\lambda$ and $(1-2N)(1-6N)\sim P/\lambda$ over the large time interval $2\pi/\omega(P)$, where $\omega(P)$ is the nonlinear frequency. In this approximation the equations of motion have the form

$$\dot{P}=\left(\frac{5\lambda}{8}\right)^2 A(\tau)(\sin\theta_1+\sin\theta_2),$$

$$\theta_1=4+O(\lambda),\quad \theta_2=2+O(\lambda). \tag{16}$$

Here $A(\tau)$ is a periodic function with period $2\pi/\omega(P)$, a height ~ 1, and a width $2\pi/\omega_0$, where $\omega_0=\omega_{ph}\lambda/4$ at $P\to\Delta/2$ is the frequency of small oscillations of the system of quasiparticles. Equation (16) directly yields the discrete-time map

$$P_{m+1}=P_m+\bar{\Delta}P,$$

$$\theta_{i,m+1}=\theta_{i,m}+\frac{4\pi}{\omega(P_{m+1})}=\theta_{i,m}+\frac{4\pi}{\omega(P_m)}-\frac{4\pi}{\omega^2(P_m)}\frac{d\omega(P_m)}{dP_m},$$

$$\bar{\Delta}P\approx\left(\frac{5\lambda}{8}\right)^2\int A(\tau)(\sin\theta_1+\sin\theta_2)d\tau,\quad i=1,2. \tag{17}$$

The character of the solution of Eqs. (17) is determined by K (Refs. 1 and 28)

$$K=\frac{4\pi}{\omega^2(P)}\left|\frac{d\omega(P)}{dP}\right|. \tag{18}$$

When $K<1$, the system performs quasiperiodic oscillations, and when $K>1$, the motion of the system becomes stochastic during the phase decorrelation time

$$R(\tau)=\left\langle \exp\left(\sum_{i=1,2}[\theta_i(\tau_1)-\theta_i(\tau_1+\tau)]\right)\right\rangle \sim\exp\left(-\frac{2\tau}{\tau_c}\right), \tag{19}$$

where $\tau_c=1/\omega(P)\ln K$.

The change in P in one transformation step is

$$\max\bar{\Delta}P=40\lambda\pi\exp\left(-\frac{64\alpha\pi}{5\lambda}\right), \tag{20}$$

where the constant $\alpha\sim 1$. The nonlinear frequency is given by

$$\omega(P)=5\pi\lambda\Bigg/32\sqrt{2}\ln\left[\frac{\frac{5\lambda}{8}-2\Delta^2}{2\sqrt{2}P-\sqrt{2}\Delta}\right]. \tag{21}$$

Substituting (20) and (21) into (18), we obtain

$$K=1024\sqrt{2}\lambda\pi\exp\left(-\frac{64\alpha\pi}{5\lambda}\right). \tag{22}$$

The boundary of the stochastic layer is determined from the condition $K(P_0,H)\sim 1$

$$P_0=\frac{\Delta}{2}+1024\sqrt{2}\pi\exp\left(-\frac{64\alpha\pi}{5\lambda}\right). \tag{23}$$

The decay constant of the nutational motion in the stochastic layer equals

$$\gamma_s=5\pi\lambda\ln K\Bigg/32\sqrt{2}\ln\left[\frac{\frac{5\lambda}{8}-2\Delta^2}{2\sqrt{2}P-\sqrt{2}\Delta}\right]. \tag{24}$$

4. COMPUTER EXPERIMENT

In the general case, Eqs. (5) have one integral of the motion (the energy of the system), and the region of motion in the phase space is a three-dimensional hypersurface defined by (1) in the four-dimensional phase space. When the antiresonance terms are neglected, the additional integral of the motion P appears. The temporal evolution of coherent quasiparticles in this case takes the form of either nonlinear periodic oscillations described by (9) or aperiodic oscillations defined by (11). Figure 2 presents the temporal variation of the number of coherent biexcitons, the number of photons, and the phase, as well as a projection of the phase trajectory onto the phase/number-of-biexcitons plane. As the separatrix is approached, the oscillation period increases, and it becomes infinite at the separatrix.

Consideration of the nonresonance terms results in breakdown of the integral of the motion P. When $\lambda<1$, the motion has a quasiperiodic character. Figure 3 presents the temporal evolution of the number of coherent biexcitons, the number of photons, and the phase, as well as projections of the phase trajectories onto the number-of-photons/number-of-biexcitons plane, the phase/number-of-biexcitons plane, and the phase/number-of-photons plane, and a three-dimensional phase portrait in the space of the number of photons, the phase, and the number of biexcitons in the perturbed case. As is seen from the figure, the system moves on a perturbed torus with a nonlinear frequency $\omega(P)$ and amplitudes modulated by the perturbation.

As λ increases, the diameter of the torus increases, and when $\lambda\lesssim 1$, it overlaps itself. Figure 4 presents the time-dependent number of coherent biexcitons, the number of photons, and the resonance phase, as well as the corresponding projections of the phase trajectories for $\lambda=0.96$.

As can be seen from the figure, the motion becomes more complex, the number of harmonics in the oscillation spectrum increases, and the amplitude of the oscillations increases. As λ increases, both the frequency of the resonant interaction and the frequency of the nonresonant interaction

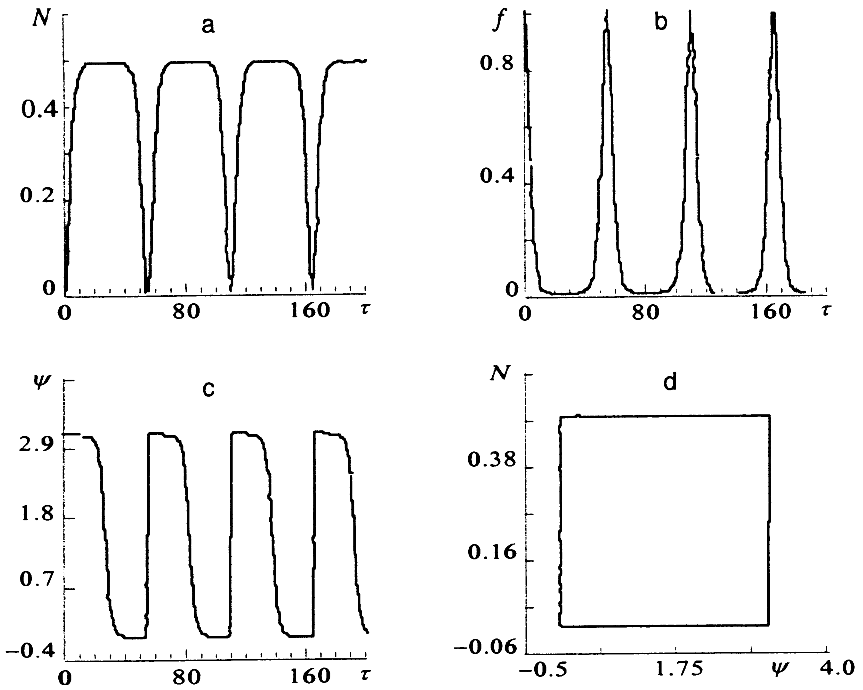

FIG. 2. Temporal evolution of the number of coherent biexcitons (a), the number of photons (b), the coherent phase (c), and projection of the phase trajectory onto the phase/number-of-biexcitons plane (d) for $P=10^{-6}$, $C=1.001$, and $\lambda=1$.

increase. Near the separatrix, however, the resonant frequency decreases sharply. When $\lambda\sim1$, the energy of the perturbing interaction is of the order of the energy of the resonant interaction. The system of coherent quasiparticles performs quasiperiodic oscillations with the frequency of the antiresonant interaction, modulated by the resonant interaction.

Figure 5 presents the temporal evolution of the number of coherent biexcitons, the number of photons, and the phase, as well as projections of the phase trajectories for $\lambda=1.056$. At this value of the parameter, the number of harmonics increases sharply, and the system performs increasingly complex oscillations, which attest to the onset of stochastic motion. When λ increases further, the motion of the system becomes completely stochastic (Fig. 6).

Figure 7 presents the development of local instability at various values of λ. The distance between the two close trajectories is given by

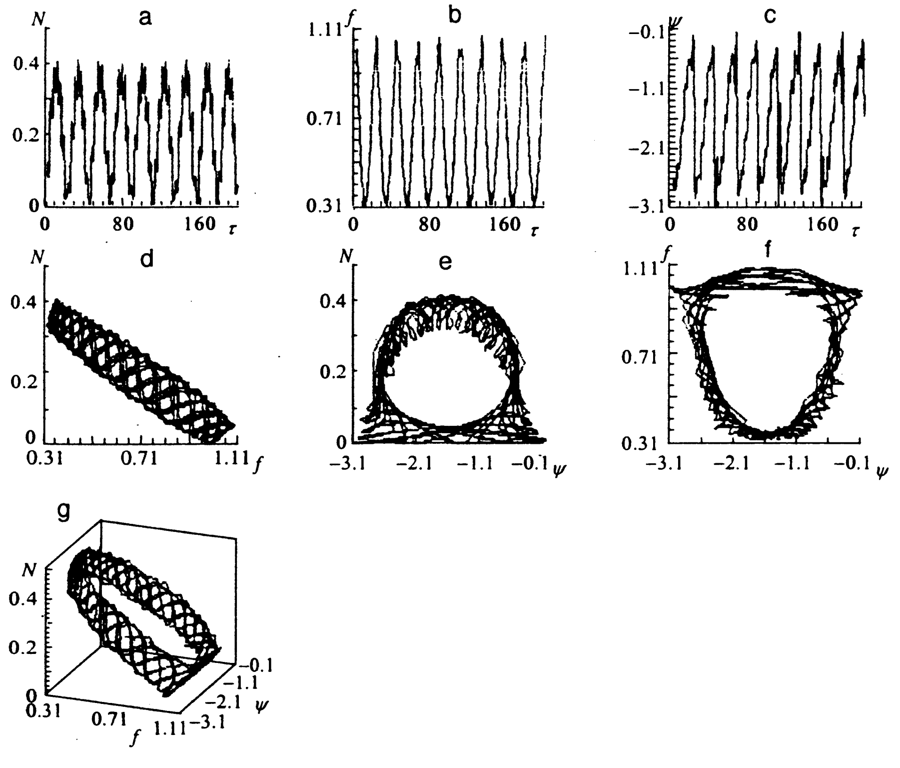

FIG. 3. Temporal evolution of the number of coherent biexcitons (a), the number of photons (b), and the coherent phase (c), projections of the phase trajectory onto the number-of-photons/number-of-biexcitons plane (d), the phase/number-of-biexcitons plane (e), and the phase/number-of-photons plane (f), and phase portrait of the evolution of the system in the space of the number of photons, the phase, and the number of biexcitons (g) for $N_0=10^{-8}$, $f_0=1$, $\varphi_{b0}=\varphi_{ph0}=\pi/2$, and $\lambda=0.64$.

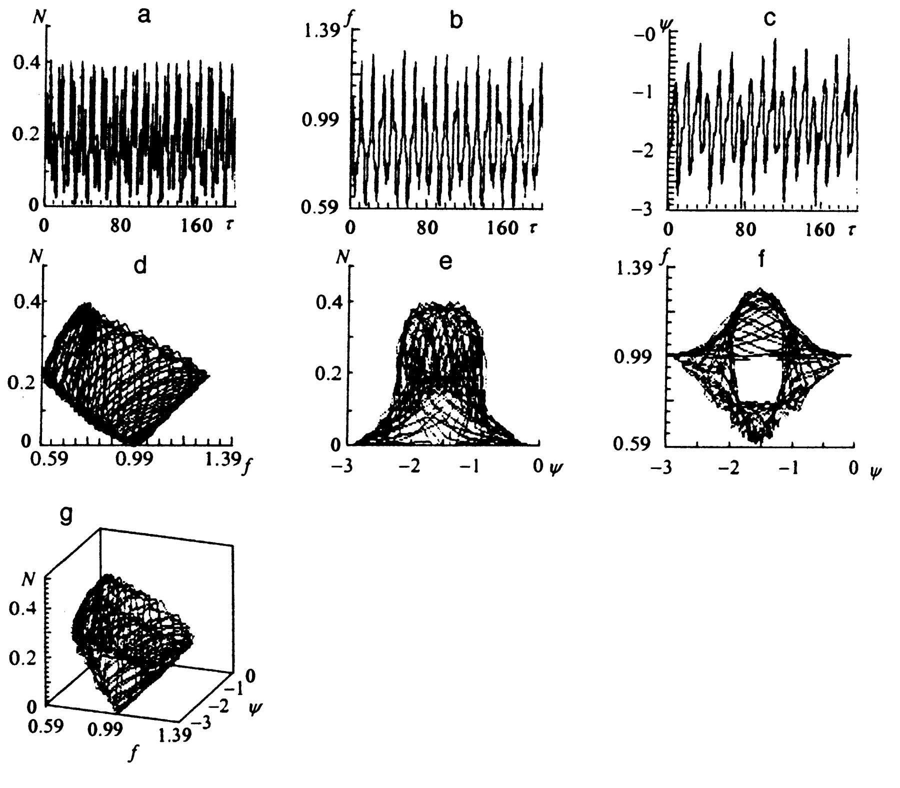

FIG. 4. Same as in Fig. 3, but for $\lambda=0.96$.

$$D=\left[(N_1-N_2)^2+(f_1-f_2)^2+\left\{\frac{\varphi_{b1}-\varphi_{b2}}{2\pi}\right\}^2+\left\{\frac{\varphi_{ph\,1}-\varphi_{ph\,2}}{2\pi}\right\}^2\right]^{1/2}.$$

We find the decay constant γ_s from the expression

$$D=D_0\exp(\gamma_s\tau).$$

In conclusion, we note that the trajectory always remains on the torus. Varying the integrals of the motion of the system, we find a family of invariant tori. Their relative arrangement in the phase space is determined by the dimensionality of the phase space of the system. When the number of de-

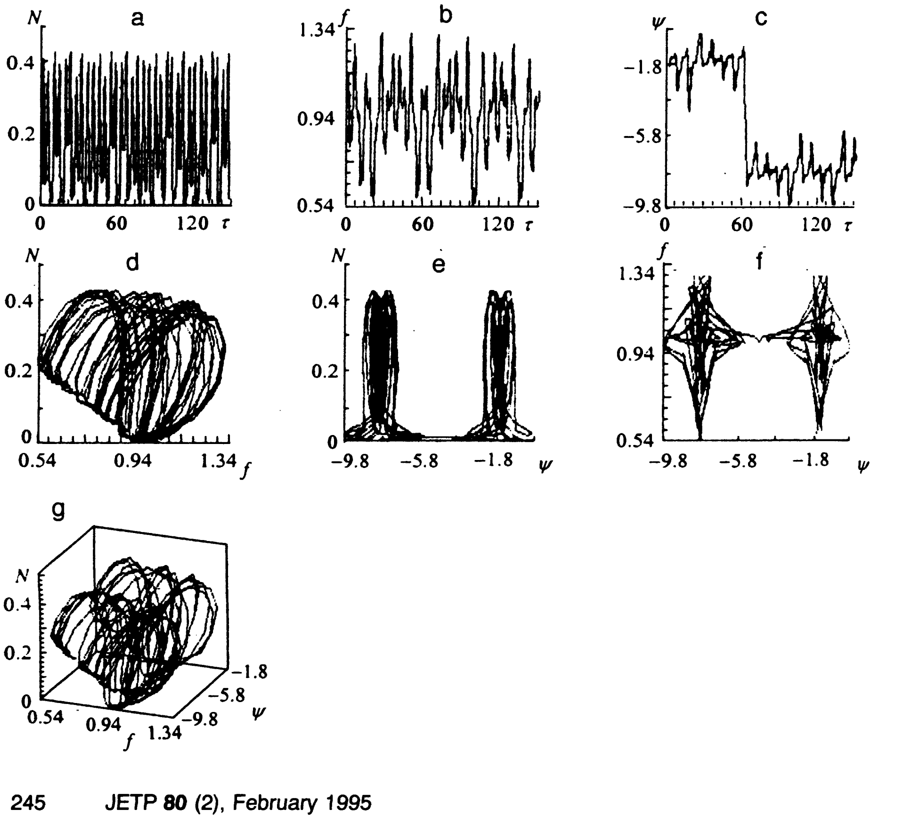

FIG. 5. Same as in Fig. 3, but for $\lambda=1.056$.

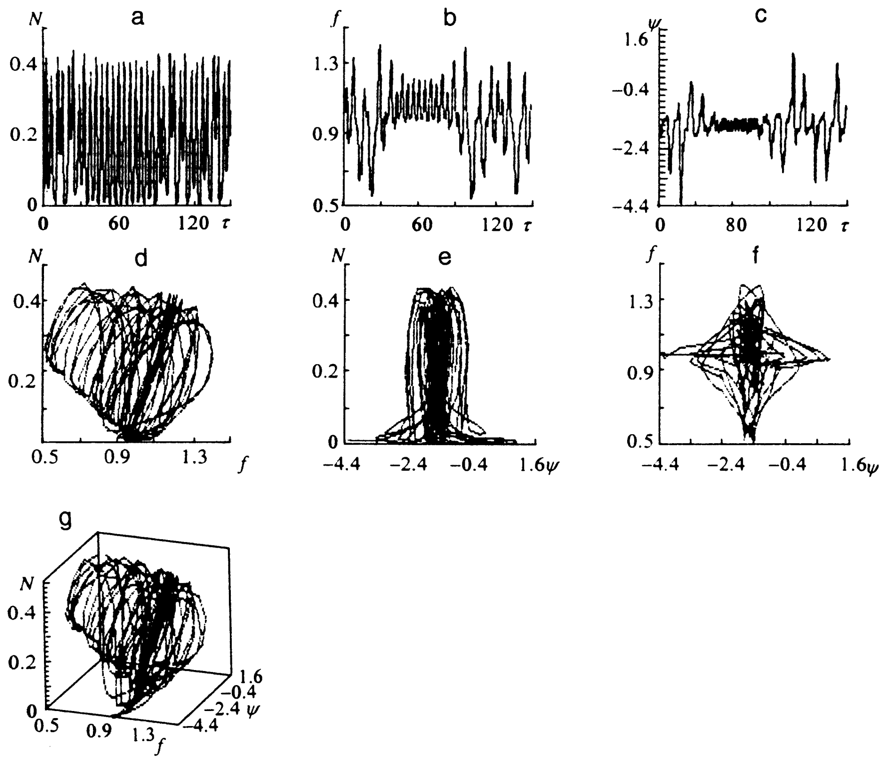

FIG. 6. Same as in Fig. 3, but for $\lambda=1.064$.

grees of freedom $n\leqslant 2$, the tori corresponding to different integrals of the motion are nested and do not intersect. In this case it can be said that they partition the space. When $n>2$, the tori fail to partition the phase space, and intersect. Therefore, the regions of breakdown may permeate the entire phase space. In this case the phenomenon known as Arnol'd diffusion[30] is observed. The excited trajectories may wander unrestrictedly far from their unperturbed values, even when the majority of the tori do not collapse.

In Ref. 16 $n=3$, and initially nearby trajectories wandered apart to a distance $D\sim 1$ after sufficiently long time intervals even when $\lambda\ll 1$. In contrast to Ref. 28, in the present problem $n=2$, and the collapsed tori are compressed between the uncollapsed tori. In this case $D\ll 1$. Only when the critical value $\lambda=1.04$ is achieved do the majority of the tori collapse and does the stochastic layer spread throughout the phase space. When $\lambda>1.04$, $D\sim 1$, and the value of γ_s increases sharply as λ increases: $\gamma_s=0.03$, 0.08, and 0.18 when $\lambda=1.056$, 1.064, and 1.12, respectively.

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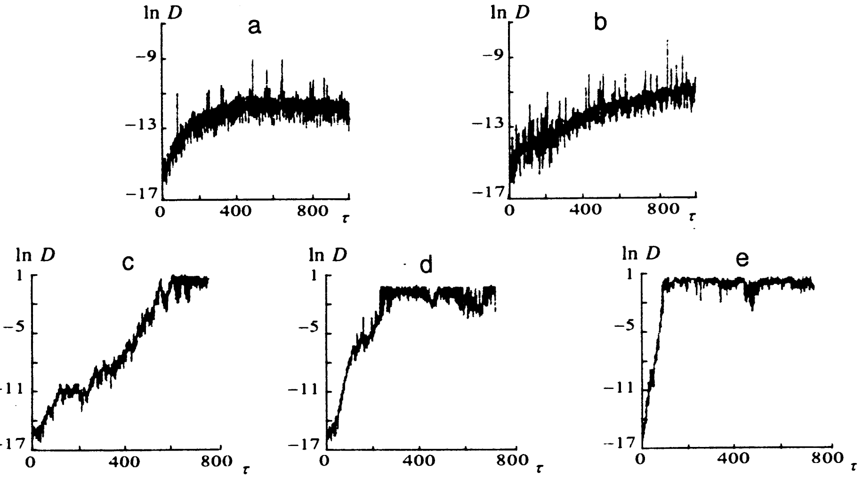

FIG. 7. Time dependence of the distance between initially nearby trajectories for $N_0=10^{-8}$, $f_0=1$, $\varphi_{b0}=\varphi_{ph0}=\pi/2$, and $\lambda=0.8$ (a), $\lambda=1.04$ (b), $\lambda=1.056$ (c), $\lambda=1.064$ (d), and $\lambda=1.12$ (e).

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