# Excitation and propagation of beams of plasma waves emitted by distributed sources in a magnetized plasma

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(Submitted 28 July 1994)
Zh. Eksp. Teor. Fiz. **107**, 431–440 (February 1995)

A class of self-similar solutions is found to the simplified equation for the electric field in the linear approximation corresponding to resonant excitation of a beam of plasma waves by a monochromatic source of external current, together with the evolution of this beam as it propagates along the resonant characteristic. The class of external current (charge) spatial distributions is found corresponding to these self-similar solutions. The fundamental behavior is determined for a beam of plasma waves reflecting from a plasma–metal boundary in the presence of an arbitrarily directed magnetic field. © *1995 American Institute of Physics.*

At the present time the problem of creating artificial plasma structures with controllable parameters in an external source field is of considerable importance, as is the effect of such structures on the radiation from antennas in the plasma. The principal reason for this is the wide use of various antennas and probes in space and laboratory experiments. A series of rocket experiments with a radio transmitter, launched in the low atmosphere,[1] revealed new possibilities for actively affecting the ionospheric and magnetospheric plasma parameters by exciting a plasma-wave discharge in the electromagnetic field of the antenna, together with the effect of this artificial plasma on a number of geophysical processes. Gorubyatnikov *et al.*[2] described the results of laboratory experimental studies on the field profile and radiation efficiency of a source when quasisteady plasma structures are formed in the near field of an antenna. On the other hand, the details of the electrodynamic phenomena that occur in a plasma-source system are of general physical interest, since similar effects can occur in other physical systems. In particular, resonant wave beams for internal waves in a stratified fluid,[3] excited by harmonic sources, have a structure similar to beams of plasma waves.

As is well known,[4] the field of an external current oscillating with frequency $\omega$ which corresponds to excitation of eigenmodes of the medium, located in a uniform magnetized plasma, has a resonant structure (the field is localized near the surfaces of the plasma resonance), due to the intense excitation of quasielectrostatic waves. Under resonant excitation conditions the characteristic size of a source operating in the resonant frequency range should be small compared with the electromagnetic wavelength ($L \ll \lambda$), which allows the near field of the emitter to be described in the quasisteady approximation.[4,5] Under uniform plasma conditions the field of this source is localized on the resonant characteristics (the surfaces of the plasma resonance) passing through the region occupied by the external currents. The integral representation of the potential of the electric field in a uniform magnetized plasma takes the form[6]

$$\varphi(\mathbf{r})=\frac{1}{2\pi^2\varepsilon_1}\int_{-\infty}^{+\infty}\frac{\rho_k(\mathbf{k})e^{i\mathbf{kr}}}{D(\omega,\mathbf{k})}\,d\mathbf{k}. \tag{1}$$

Here $\rho_k(\mathbf{k})$ is the spectral density of the external charge, corresponding to the distribution of external currents on the source. We will assume that the size $L$ of the source is much greater than the parameters $v_{\mathrm{Te}}/\omega$, $v_{\mathrm{Te}}/\omega_{\mathrm{pl}}$, $v_{\mathrm{Te}}/\omega_{\mathrm{He}}$, which characterize the effect of the electron thermal motion (here $v_{\mathrm{Te}}$ is the thermal velocity, $\omega_{\mathrm{pl}}$ is the plasma frequency, and $\omega_{\mathrm{He}}$ is the gyrofrequency). In this case under resonant conditions ($L \ll \lambda$) in the dispersion relation we can disregard the effect of spatial dispersion and, taking into account the effect of weak collisions and the electromagnetic corrections, write $D(\omega,\mathbf{k})$ in the following form (Ref. 5):[1)]

$$D(\omega,\mathbf{k})=\chi^2-\frac{k_z^2}{\mu^2}+i\left(\chi^2\frac{\delta\varepsilon_1}{\varepsilon_1}+k_z^2\frac{\delta\varepsilon_3}{\varepsilon_3}\right)+\frac{\omega^2}{c^2}\left[\frac{\varepsilon_1}{\mu^2}+\frac{g^2}{\varepsilon_1(1+\mu^2)}\right]. \tag{2}$$

Here $(x,y,z)$ is the Cartesian coordinate system, where the $z$ axis is directed parallel to the external magnetic field; $\varepsilon_1$ and $\varepsilon_3$ are the diagonal components of the cold-plasma dielectric tensor; $g=-\varepsilon_{xy}=\varepsilon_{yz}$ are the off-diagonal elements of this tensor, and we have written $\chi^2=k_x^2+k_y^2$, $\mu^2=\varepsilon_1/\varepsilon_3$,

$$\delta\varepsilon_1=\frac{\nu_e}{\omega}\frac{\omega_{\mathrm{pl}}^2(\omega^2+\omega_{\mathrm{He}}^2)}{(\omega^2-\omega_{\mathrm{He}}^2)^2},\qquad \delta\varepsilon_3=\frac{\nu_e}{\omega}\frac{\omega_{\mathrm{pl}}^2}{\omega^2},$$

where $\nu_e$ is the electron collision frequency. Using the dispersion relation (2) and integrating over the two components of the wave vector, taking into account the resonant behavior of the field, we can find the structure of the electric field near the resonant cone ($\xi \simeq 0$), as follows:[5]

$$\varphi(\xi,\tau)=\frac{\sqrt{\mu}\exp\left(-i\frac{\pi}{4}\right)}{\varepsilon_1\sqrt{2\pi\tau}}\int_0^{\infty}\frac{1}{\sqrt{k_\xi}}\rho_k(0,0,k_\xi)\times\exp\left[i\left(k_\xi\xi+\frac{\tau}{4\lambda^2k_\xi}\right)-sk_\xi\tau\right]dk_\xi, \tag{3}$$

where

$$\lambda=\frac{c}{\omega}\left[\frac{2\varepsilon_1}{\mu}+\frac{2g^2\mu}{\varepsilon_1(1+\mu^2)}\right]^{-1/2}$$

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is the electromagnetic wavelength, $s$ is a coefficient which takes into account the collisional dissipation, $(\xi,\tau)$ is the coordinate system with $\tau$ axis directed along the surface of the resonant cone and $\xi$ axis transverse to it, and the spectral density is

$$\rho_k(0,0,k_\xi)=\int_{-\infty}^{\infty}\rho_{\text{eff}}(\xi')\exp(-ik_\xi\xi')d\xi'$$

containing the Fourier components with transverse components of the wave vector only. This is related to the fact that under resonant conditions the phase velocity of a packet of plasma waves is directed perpendicular to the surface of the resonant cone.

Equation (3) was derived using the diffraction effects due to the electromagnetic correction and the collisional dissipation. From (3) we see that when collisions and diffraction effects are neglected the field transported along a resonant characteristic ($\xi\simeq 0$) will have a beam resonant structure. A characteristic feature of such resonant beams of plasma waves is evident from the formula (21) for the potential of a dipole source field, given below. The corresponding expression for the potential of a quasisteady electric field in terms of the effective space charge distribution $\rho_{\text{eff}}(\xi')$ under resonant conditions takes the form

$$\tilde{\varphi}=\sqrt{\pi}\varphi_c\int_{-\infty}^{\infty}\rho_{\text{eff}}(\xi')\times\frac{\exp\left\{-\sqrt{-\frac{i\tau}{\lambda^2}[s\tau-i(\xi-\xi')]}\right\}}{\sqrt{s\tau-i(\xi-\xi')}}d\xi',$$

$$\tilde{\varphi}=\varphi\sqrt{\tau},\quad \varphi_c=\frac{\sqrt{\mu}e^{-i\pi/4}}{\sqrt{2\pi\varepsilon_1}}. \tag{4}$$

Differentiating (3) with respect to $\xi$ and $\tau$ and comparing the resulting expressions, we can pass from an integral representation of the potential to a differential representation. Since the function $\tilde{\varphi}$ is complex the desired representation is a quasihyperbolic partial differential equation:

$$\frac{\partial^2\tilde{\varphi}}{\partial\tau\partial\xi}-is\frac{\partial^2\tilde{\varphi}}{\partial\xi^2}+\frac{1}{4\lambda^2}\tilde{\varphi}=0. \tag{5}$$

The solution must satisfy boundary conditions of the form

$$\varphi(\pm\infty,\tau)=0,\quad \varphi(\xi,\tau_0)=\varphi_0(\xi). \tag{6}$$

There exists a self-similar variable $\eta$ by means of which Eq. (5) can be reduced to an ordinary differential equation:

$$\eta=\tau(\xi+\xi_0+is\tau). \tag{7}$$

Here $\xi_0$ is a specified complex scale. We represent the potential of the field as follows:

$$\tilde{\varphi}=\frac{\tau^m}{\lambda^m}\Phi(\eta), \tag{8}$$

where $\Phi(\eta)$ is some function and $m$ is a number. Substituting (8) in (5) we find an ordinary differential equation for the function $\Phi(\eta)$:

$$\eta\Phi''+(m+1)\Phi'+\frac{1}{4\lambda^2}\Phi=0. \tag{9}$$

Equation (9) is a particular form of the Bessel equation.[7] Its solution takes the form

$$\tilde{\varphi}=C_1\frac{\tau^m}{\lambda^m}(\sqrt{\eta})^{-m}Z_m\left(\sqrt{\frac{\eta}{\lambda^2}}\right)+C_2\frac{\tau^{-m}}{\lambda^{-m}}(\sqrt{\eta})^mZ_{-m}\left(\sqrt{\frac{\eta}{\lambda^2}}\right), \tag{10}$$

where $Z_m(\sqrt{\eta/\lambda^2})$ is a cylindrical function.

It is necessary to make clear to what class of charge distributions a given self-similar solution corresponds. Writing $\eta^*=\eta/\lambda^2$ and equating (10) and (4) we find

$$\frac{\tau^m}{\lambda^{2m}}\Phi_0[\sqrt{\eta^*}]^{-m}Z_m(\sqrt{\eta^*})\equiv\sqrt{\frac{-\tau}{\lambda^2}}(\sqrt{\pi i}\varphi_c)\int_{-\infty}^{\infty}\tau_{\text{eff}}(\xi')\times\frac{\exp\left[-\sqrt{\frac{\tau}{\lambda^2}(\xi'+\xi_0)-\eta^*}\right]}{\sqrt{\frac{\tau}{\lambda^2}(\xi'+\xi_0)-\eta^*}}d\xi'. \tag{11}$$

From this we see that the potential on the left-hand side depends only on $\eta^*$, so the function $\rho_{\text{eff}}$ should look like $\rho_{\text{eff}}=\rho_{\text{eff}}(\xi'+\xi_0)$. Transforming according to

$$y=\frac{\tau}{\lambda^2}(\xi'+\xi_0),$$

we have

$$\left(\frac{\tau}{\lambda^2}\right)^{m+1/2}(\sqrt{\eta^*})^{-m}\Phi_0Z_m(\sqrt{\eta^*})\equiv(\sqrt{-\eta i}\varphi_c)\int_{-\infty+i\,\mathrm{Im}\,\xi_0\tau/\lambda^2}^{\infty+i\,\mathrm{Im}\,\xi_0\tau/\lambda^2}\rho_{\text{eff}}\left(\frac{\lambda^2}{\tau}y\right)\frac{\exp(-\sqrt{y-\eta^*})}{\sqrt{y-\eta^*}}dy.$$

We assume that

$$\rho_{\text{eff}}\left(\frac{\lambda^2}{\tau}y\right)=\left(\frac{\tau}{\lambda^2}\right)^{\nu}\rho_{\text{eff}}(y),\quad \rho_{\text{eff}}(y)=\frac{1}{y^\nu}, \tag{13}$$

where $\nu$ is a number, the degree of the homogeneous function $\rho_{\text{eff}}(y)$. Then relation (12) is transformed as follows:

$$\left(\frac{\tau}{\lambda^2}\right)^{m+1/2-\nu}(\sqrt{\eta^*})^{-m}\tilde{\Phi}_0Z_m(\sqrt{\eta^*})\equiv\int_{-\infty+i\,\mathrm{Im}\,\xi_0\tau/\lambda^2}^{\infty+i\,\mathrm{Im}\,\xi_0\tau/\lambda^2}\frac{1}{y^\nu}\frac{\exp(-\sqrt{y-\eta^*})}{\sqrt{y-\eta^*}}dy, \tag{14}$$

$$\tilde{\Phi}_0=\frac{\Phi_0}{\sqrt{-\pi i\varphi_c}}.$$

We must evaluate the integral on the right-hand side of (14) for both whole and fractional values of the parameter $\nu$. The integration is performed in the complex plane; the point

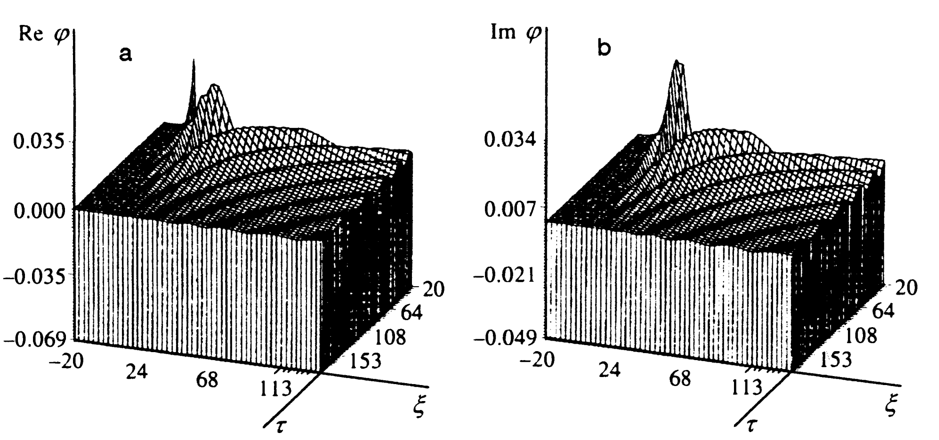


FIG. 1. Resonant behavior of the real (a) and imaginary (b) parts of the potential of the quasisteady electric field of a dipole source.

$y=0$ is a singular point. For integral values of $\nu$ it is a pole of order $\nu$ and for fractional $\nu$ it is a branch point. Finally, after integration the right-hand side for fractional $\nu$ becomes

$$\left(\frac{\tau}{\lambda^2}\right)^{m+1/2-\nu} (\sqrt{\eta^*})^{-m}\tilde{\Phi}_0 Z_m(\sqrt{\eta^*}) \equiv \frac{2^{5/2-\nu}\Gamma(1/2)}{\Gamma(\nu)} (\sqrt{\eta^*})^{1/2-\nu} K_{\nu-1/2}(\sqrt{-\eta^*}), \tag{15a}$$

for the whole $\nu$

$$\left(\frac{\tau}{\lambda^2}\right)^{m+1/2-\nu} (\sqrt{\eta^*})^{-m}\tilde{\Phi}_0 Z_m(\sqrt{\eta^*}) \equiv \frac{-2\pi i}{(\nu-1)!} \lim_{y\to 0} \frac{d^{\nu-1}}{dy^{\nu-1}} \frac{\exp(-\sqrt{y-\eta^*})}{\sqrt{y-\eta^*}}, \tag{15b}$$

where $K_m(\sqrt{-\eta^*})$ is the modified Bessel function of the second kind. When (15) holds identically we must choose the cylindrical function on the left-hand side properly, taking into account the boundary conditions at infinity and $m+1/2-\nu\equiv 0$. In this case the function $Z_m(\sqrt{\eta^*})$ is a modified Bessel function of the second kind $K_m(\sqrt{-\eta^*})$ with half-integral index when $\nu$ is a whole number and with arbitrary index for fractional values of $\nu$ (Refs. 8 and 9).

Thus, the above self-similar solution of the reduced equation for the potential of a distributed source corresponds to a class of algebraic distributions of external charge of the form (13), or in the original variables,

$$\rho_{\text{eff}}(\xi'+\xi_0)=\frac{\rho_0}{(\xi'+\xi_0)^{\nu}}. \tag{16}$$

The parameter $\nu$ may take on either an integral or fractional value; $\xi_0$ is a complex number. For example, for the dipole free-charge distribution (20) we have $\xi_0=iL$. The complex nature of the scale $\xi_0$ is a factor which determines the wavelike behavior of the field under resonant conditions. Fields produced by such sources for $\nu=1,2$ are described in Refs. 5 and 10. The spectral density of algebraic distributions from the class (16) take the form

$$\rho_k(k_\xi)=\frac{2\pi(-i)^{\nu}\rho_0}{\Gamma(\nu)} k_\xi^{\nu-1} \exp(i\xi_0 k_\xi), \tag{17}$$

where $k_\xi>0$ and $\operatorname{Im}\xi_0>0$. Then for the self-similar solution of (3) we find

$$\varphi(\xi,\tau)=\frac{\varphi_0 2\pi(-1)^{m+1/2}\rho_0}{\Gamma(m+1/2)\sqrt{\tau}} \int_0^\infty k_\xi^{m-1} \times\exp\left[ik_\xi(\xi+\xi_0+is\tau)+i\frac{\tau}{4\lambda^2}\frac{1}{k_\xi}\right]dk_\xi. \tag{18}$$

After integrating expression (18) with respect to $k_\xi$ we find that the potential takes the following form:

$$\varphi(\xi,\tau)=\frac{4\pi(-i)^{m+1/2}\rho_0\varphi_0}{\Gamma(m+1/2)2^m}\frac{\tau^m}{\lambda^m\sqrt{\tau}}\left[\frac{1}{\tau(\xi+\xi_0+is\tau)}\right]^{m/2} \times K_m\left(\sqrt{\frac{-\tau(\xi+\xi_0+is\tau)}{\lambda^2}}\right). \tag{19}$$

Asymptotic estimates of the integral (18) (by the method of stationary phase for $\xi>0$ and by the method of steepest descent for $\xi<0$) when the parameter $\tau$ is large ($\tau\gg\xi_0$) and $|\xi|\gg|L|$, where $\xi_0$ is the characteristic length scale of the source, show that in the "illuminated" region ($\xi>0$) the field has an oscillatory character and falls off according to a power law, while for $\xi<0$ (the "shadow" region) there are no oscillations and the field falls off exponentially. The structure of the field produced by sources from the class (16) is shown in Fig. 1. This transparent interpretation was obtained by transforming expression (18) to dimensionless form and integrating it numerically. It is clear that under these conditions the field can be wavelike. From the asymptotic form of expression (18) it follows that the surfaces of stationary phase are hyperbolas, and the dispersion law also has a hyperbolic character:

$$\tau_\xi=C\lambda^2, \quad C=\text{const},$$

$$k_\tau k_\xi=\frac{1}{4\lambda^2}.$$

From these relations it also follows that the field energy flows out from the resonant characteristic.

This class of distributions (16) can be used to investigate the reflection of beams of plasma waves from steep gradients in a magnetized plasma.

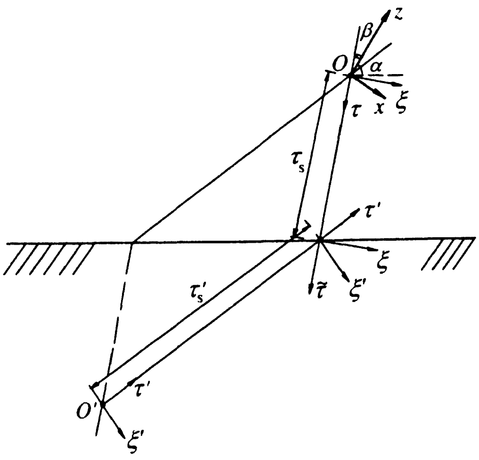


FIG. 2. Showing the reflection of beams of plasma waves from a sharp plasma–metal boundary.

Here, using distributions of the external current and the resonant field studied previously, we consider the distribution of beams of plasma waves in an inhomogeneous medium. We take a thin antenna with a dipole distribution of free charge:

$$\rho_{\mathrm{ex}}(\mathbf{r})=\frac{P}{2\pi}\left[(z-iL)^{-2}-(z+iL)^{-2}\right]\delta(x)\delta(y), \tag{20}$$

where $P$ is the dipole moment. The quasisteady potential of the source field, leaving out the spatial dispersion and collisional dissipation, takes the form[5]

$$\varphi(\xi,\tau)=\frac{iP_{\mathrm{eff}}}{\sqrt{\tau(\xi+iL)^3}}. \tag{21}$$

Here $P_{\mathrm{eff}}$ is the effective dipole moment, given by

$$P_{\mathrm{eff}}=\frac{P\sqrt{\mu}\sin\beta}{2\sqrt{2}\varepsilon_1},$$

where $\beta$ is the opening angle of the resonant cone (Fig. 2).

Relation (21) describes the complex amplitude of a beam of plasma waves propagating in the resonant direction $\tau$. Suppose that this beam is incident on a plasma–metal boundary at an angle $\alpha+\beta$, where $\alpha$ is the slant of the magnetic field. In the direction of propagation (the resonant characteristic) the transverse structure of the beam is conserved.[5] The incident beam is reflected into an adjacent resonant characteristic in the direction $\tau'$. In this quasi-two-dimensional approximation the field of the reflected beam can be calculated as the field produced by the "image" of the original source, taking into account the appropriate boundary conditions. The transformation from the coordinate system $(\xi,\tilde{\tau})$ to the system $(\xi',\tilde{\tau}')$ is as follows:

$$\tilde{\tau}'=\xi\sin 2\beta-\tilde{\tau}\cos 2\beta,$$

$$\xi'=\xi\cos 2\beta+\tilde{\tau}\sin 2\beta, \tag{22}$$

where we have written $\tilde{\tau}=\tau-\tau_s$, $\tilde{\tau}'=\tau'-\tau_s'$, $\tau_s$ is the shift in the origin of coordinates $O$, $\tau_s'$ is a shift in the origin $O'$, and the relation between them is given by

$$\tau_s'=\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}\tau_s. \tag{23}$$

Using the boundary condition in the coordinate system $(\xi,\tilde{\tau})$: $\tilde{\tau}=-\xi\,\mathrm{ctg}(\alpha+\beta)$, and some simple trigonometric formulas, we derive the relation for the transverse coordinates at the boundary:

$$\xi'+iL'=\xi\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}+iL'. \tag{24}$$

We assume that the relation between the transverse coordinates $\xi$ and $\xi'$ is as follows:

$$\xi'+iL'\equiv\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}(\xi+iL). \tag{25}$$

We assume that the structure of the reflected beam is similar to that of the incident beam, so that

$$\varphi'=\frac{iP'_{\mathrm{eff}}}{\sqrt{\tau'(\xi'+iL')^3}}. \tag{26}$$

At the resonant surface for $\tau'\gg\xi'$, $\tau\gg\xi$ on the boundary we can set $\tau'\simeq\tau_s'$, $\tau\simeq\tau_s$. Consequently, for complex amplitudes of the incident and reflected beam potentials we have

$$\varphi=\frac{iP_{\mathrm{eff}}}{\sqrt{\tau_s(\xi+iL)^3}},\quad \varphi'=\frac{iP'_{\mathrm{eff}}}{\sqrt{\tau_s'(\xi'+iL')^3}}. \tag{27}$$

From (27) it is easy to find the tangential components of the electric fields on the interface. Using (23) and (25) we obtain

$$E_\xi^t=-\frac{\partial\varphi}{\partial\xi}\sin(\alpha+\beta)=\frac{3iP_{\mathrm{eff}}\sin(\alpha+\beta)}{2\sqrt{\tau_s(\xi+iL)^3}} \tag{28a}$$

$$E_\xi'^t=-\frac{\partial\varphi'}{\partial\xi'}\sin(\alpha-\beta)=\frac{3iP'_{\mathrm{eff}}\sin(\alpha-\beta)\sin(\alpha+\beta)}{2\sqrt{\tau_s(\xi+iL)}\sin^2(\alpha-\beta)}. \tag{28b}$$

On the boundary the total tangential component of the electric field must vanish, whereupon

$$P'_{\mathrm{eff}}=-P_{\mathrm{eff}}\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}. \tag{29}$$

The reflection coefficient, which is equal to the ratio of the amplitudes of the reflected and incident waves, is

$$\Gamma=\frac{E'_\xi}{E_\xi}=-\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}. \tag{30}$$

Thus, in this problem the method of images is applicable, and the boundary conditions are satisfied assuming that the reflected beam has the same structure as the incident one and is produced by an image source with characteristic scale length

$$L'=L\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} \tag{31}$$

and with the dipole moment given by Eq. (29).

In the foregoing we derived a class of external distributions describing self-similar solutions of the original equations. This class can be used to treat the reflection of beams of plasma waves from steep gradients in a plasma. Consider a quasisteady electric field of the form (19) with amplitude $A$, produced by a source distribution from the class (16) near a metal–plasma interface, without taking into account the collisional dissipation:

$$\varphi=A\,\frac{\tau(m-1)/2}{(\xi+\xi_0)^{m/2}}\,K_m\left(\frac{\sqrt{-\tau(\xi+\xi_0)}}{\lambda}\right). \tag{32}$$

We assumed that Eq. (32) is the potential of a beam of plasma waves incident along the resonant characteristic on the interface. Using the notation and results obtained above, we find analogously the potential of the reflected beam, given by

$$\varphi'=A'\,\frac{\tau'^{(m-1)/2}}{(\xi'+\xi_0')^{m/2}}\,K_m\left(\frac{\sqrt{-\tau'(\xi'+\xi_0')}}{\lambda}\right). \tag{33}$$

The tangential components of the electric fields of the incident and reflected beams on the boundary take the form

$$E^{\mathrm{t}}=-A\,\frac{\tau_s^{(m-1)/2}}{(\xi+\xi_0)^m}\left[\frac{\sqrt{-\tau_s}}{2\lambda}\,K_m'(\xi+\xi_0)^{(m-1)/2}-\frac{m}{2}\,K_m(\xi+\xi_0)^{(m-2)/2}\right]\sin(\alpha+\beta),$$

$$E'^{\mathrm{t}}=-A'\,\frac{\tau_c^{(m-1)/2}}{(\xi+\xi_0)^m}\left[\frac{\sqrt{-\tau_s}}{2\lambda}\,K_m'(\xi+\xi_0)^{(m-1)/2}-\frac{m}{2}\,K_m(\xi+\xi_0)^{(m-2)/2}\right]\sin(\alpha-\beta)\times\left[\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}\right]^{m+1/2}. \tag{34}$$

By equating to zero the tangential component of the field on the boundary we find a relation between the amplitudes of the incident and reflected beams:

$$A'=-A\left[\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}\right]^{m-1/2}. \tag{35}$$

For typical beam scale sizes we have

$$\xi_0'=\xi_0\,\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}. \tag{36}$$

In this case the reflection coefficient is the same as that given by Eq. (30). Equation (35) holds for the amplitudes of arbitrary beams of plasma waves emitted by sources from the class (16), described by the self-similar solutions derived above. It is not hard to see that the distribution (20) belongs to this class. In this case we have $\nu=2$, $m=3/2$, and Eq. (35) goes over to (29).

In order to explain the qualitative picture of the structure of the field we can turn to Ref. 2. In Ref. 2 a model problem was treated, involving the reflection of a quasistatic wave from a sharp oblique boundary. When electrostatic waves are reflected and refracted the wavelength changes. The angle $\alpha$ (the angle of the slope in the boundary with respect to the magnetic field) is important. As $\alpha$ approaches $\beta$, the angle of the slope in the resonant characteristics, the amplitude of the reflected wave field grows resonantly, the reflection coefficient formally diverges, and the reflected wavelength approaches zero. For $\alpha=\beta$ the group velocity of the reflected wave is parallel to the boundary, and the wave vector $\mathbf{k}$ is orthogonal to it (the reflected wavelength then vanishes). All wave fronts of the reflected wave lie in the plane of the boundary, i.e., the energy of the reflected wave is concentrated in an infinitesimal layer. For $\nu_e\simeq\omega$ (where $\nu_e$ is the electron collision frequency and $\omega$ is the frequency of the radiation field) in the quasisteady state, which is formed by a source in a magnetized plasma, the reflecting boundary has a slope close to the critical value.[2]

Thus, the reduction in the wavelength in connection with reflection at angles close to the critical value allows narrow spatial distributions of the field of the quasistatic waves to develop with length scale comparable to the thickness of the boundary of the heated region. In passing through the critical angle the direction of the group velocity of the reflected wave changes. For $\alpha>\beta$ the energy flux in the reflected wave is directed parallel to the resonant characteristic toward the plane on which the source is located, while for $\alpha<\beta$ it is in the opposite direction ("forward" and "backward" reflection). This means essentially that, by varying the gradient in temperature or density one can create artificial plasma irregularities of different sizes. For backward reflection a plasma density gradient develops which is localized near the source; for forward reflection and the subsequent geometric focusing of the beam of plasma waves a density variation can develop which is highly elongated parallel to the magnetic field (a plasma channel). These cases have been observed in laboratory experiments.[1,2,10]

In conclusion we note that the foregoing analysis of the resonant behavior of beams of plasma waves and of the evolution of these beams in propagating along resonant characteristics in a magnetized plasma, carried out using the simplified equation (5), is fundamental for the study of the role of nonlinear effects in similar problems.

Note also that the problem regarding the reflection of electrostatic waves propagating in a magnetized plasma is similar to the problem of the reflection of internal gravitational waves propagating in a stratified fluid.

This work was performed with financial support from the Russian Fund for Fundamental Studies (project No. 93-02-848).

[1)]When the source is sufficiently large, small-scale effects due to the thermal motion are averaged due to interference effects.

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Translated by David L. Book