Anisotropy in the hopping conductivity of quasi-one-dimensional systems

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It is shown that one of the characteristic features of hopping conductivity in quasi-one-dimensional systems is the special form of the percolation paths. For the case of weak crosslinking of the one-dimensional filaments, this path consists of one-dimensional fragments located along the filaments and joined to one another by sparse transverse bonds. As a result, for the transverse conductivity the carrier paths determined by the optimum sequence of hops may extend beyond the limits of the critical subnet of bonds. This causes the exponential factor in the temperature dependence of the conductivity to become strongly anisotropic. One predicted consequence of the peculiar form of the percolation paths in these systems is that the anisotropy of the conductivity increases with decreasing temperature.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

Experimental studies of disordered quasi-one-dimensional systems, e.g., salts of TCNQ,[1,2] conducting conjugated polymers,[3] including those with oriented chains,[4] and MOS field-effect transistors with narrow channels[5] or with planar systems of narrow channels,[6] indicate that it is not uncommon for the conductivity of such systems to be due to phonon-assisted hopping. Particular indications of this are the Mott type of temperature dependence of the conductivity, a temperature-dependent thermoelectric power, and an increase in the conductivity with frequency. The basic features observed in the kinetic phenomena exhibited by systems of this kind can successfully be described within the framework of a model with static disorder, in which local centers with random energies are randomly scattered along one-dimensional filaments, although in some cases a modification of the model is required (inclusion of fluctuations of the conductivity in mesoscopic systems,[5] polaron effects, and the effects of the Coulomb interaction[7,8]). It is likely that this model of a quasi-one-dimensional system can also be used to describe hopping conductivity in materials with filamentary structures such as porous silicon.

There are several different approaches to the theory of hopping conductivity in disordered quasi-one-dimensional systems. An approach based on percolation theory was developed in Ref. 9 (in the same reference, a review is given of earlier work on this question). As is well known, the percolation approach reduces the problem of calculating the conductivity of the system to the corresponding problem of bonds in percolation theory; the conductivity of this system is determined by the critical value of the transition rate that corresponds to the appearance of an infinite cluster of linked bonds in the system. It is known that for three-dimensional systems with a uniform distribution of local centers in space, transitions with rates lower than the critical rate have no effect on the basic exponential factor that multiplies the conductivity, but rather determine only the pre-exponential factor (see Refs. 10–12). In Ref. 9, the percolation approach was extended to quasi-one-dimensional systems in which the local centers are randomly scattered along the one-dimensional filaments, which are regularly located in space (or on a plane). Within the framework of this approach it was found that, as in three-dimensional systems, the conductivity is essentially determined by the critical value of the transition rate, while transitions at rates lower than critical can affect only the pre-exponential factor of the conductivity. In fact, this implies that the basic exponential dependences of the conductivity on concentration and temperature should be one and the same for the conductivities along the filaments and in the transverse direction. However, the authors of Ref. 13 carried out an independent optimization of the transverse paths, and found that there was a certain range of temperatures for which the exponential temperature dependence of the conductivity along the filaments differs from the corresponding dependence for the transverse conductivity. Unlike quasi-two-dimensional systems where the exponential anisotropy can be related to the presence of two percolation thresholds,[14] in quasi-one-dimensional systems with periodically distributed filaments there exists a single percolation threshold. Accordingly, the contradiction mentioned above suggests that the assertion that the conductivity is determined by a critical subnet of bonds may turn out to be incorrect for systems of this type.

In this paper we will discuss the relation between hopping conductivity in quasi-one-dimensional systems and the critical value of the transition rate corresponding to the threshold for the appearance of an infinite cluster. We will discuss the standard model with static disorder commonly used to describe hopping conductivity in quasi-one-dimensional systems. We will assume that local centers (nodes) with random energies are randomly scattered along the one-dimensional filaments, which are regularly distributed in space. As is customary, we choose transition rates between localized states i and j in the form

$$\Gamma_{ij}=W_{ji}f_i(1-f_j)=\Gamma_0\exp(-\eta_{ij}), \tag{1}$$

where W_{ji} is the transition probability, f_i is the equilibrium probability for filling of the state i, Γ_0 is a pre-exponential factor that usually is taken to be constant,

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$$\eta_{ij}=\frac{2r_{ij}}{a}+\frac{\varepsilon_{ij}}{kT}, \tag{2}$$

r_{ij} is the distance between local centers, a is the localization radius of the state, and ε_{ij} is a known function of the state energies ε_i, ε_j. For one-phonon transitions,

$$\varepsilon_{ij}=\tfrac{1}{2}\left[|\varepsilon_i-\mu|+|\varepsilon_j-\mu|+|\varepsilon_i-\varepsilon_j|\right],$$

where μ is the Fermi level. An analogous expression was obtained for multiphonon transition rates in Ref. 15.

In general, the transition rates can be anisotropic. For a three-dimensional system with a uniform spatial distribution of hopping centers, the anisotropy of the transition rates does not lead to anisotropy of the conductivity exponential.[11] Accordingly, here we will limit our discussion to isotropic transition rates (1) for simplicity.

As is well known, calculating hopping conductivity with hopping probabilities given by Eq. (1) is equivalent to finding the conductivity of a network of Miller–Abrahams resistors:

$$R_{ij}=\frac{kT}{e^2}\,\Gamma_{ij}^{-1}=R_0\exp(\eta_{ij})$$

(see Refs. 11 and 15). For a three-dimensional system with a random distribution of centers in space, the exponentially wide spectrum of resistances of the net allows us to reduce the problem to one of bonds on random nodes, if we define a bond for any preset value of η by the condition $\eta_{ij}<\eta$. In this case, we obtain for the conductivity the expression $\sigma=\sigma_0\exp(-\eta_{cr})$, where σ_0 is a pre-exponential factor and η_{cr} is the threshold value corresponding to the appearance of an infinite cluster of linked bonds.

For a quasi-one-dimensional system, a critical subnet of the infinite cluster need not always determine the conductivity of the corresponding quasi-one-dimensional disordered system. This may be due to the presence of special features in the structure of the infinite cluster, e.g., a spatially nonuniform distribution of local centers. Features of this kind were discussed in Ref. 16; however, it was also shown in that paper that when the percolation is r-type, for which the spread in energy is negligible, they cannot give rise to anisotropy of the conductivity exponential.

In what follows, we will show that for quasi-one-dimensional systems under conditions of $r-\varepsilon$ percolation and weak linking between filaments, a situation is possible that cannot happen for the problem of r percolation. In this case, there is a certain range of temperatures for which the standard relation between the exponential factor of the hopping conductivity and the critical value of the transition rate may be violated for the transverse conductivity, and the conductivity exponential can become anisotropic even in cases where only one percolation threshold exists.

In the next section we will discuss the peculiarities of hopping conductivity in quasi-one-dimensional systems that follow from a spatially nonuniform distribution of local centers. In Section 3 we present a more detailed analysis of the basic characteristics of the critical subnet when the number of transverse bonds is small, while Sec. 4 deals with peculiarities of the conductivity anisotropy that follow from the special features in the structure of the critical subnet described above.

2. CERTAIN FEATURES OF HOPPING CONDUCTIVITY IN QUASI-ONE-DIMENSIONAL SYSTEMS

When the distances d between neighboring filaments are large enough, we should expect the longitudinal hopping conductivity of the system to be determined by hops between centers along the filaments, and to leading order we can find the conductivity by treating the filaments as insulating. For an insulating filament, the average number of centers whose transition rates Γ_{ij} away from the given center i of the filament in a given direction exceed $\Gamma=\Gamma_0\exp(-\eta)$ (i.e., for which $\eta_{ij}<\eta$), equals $(\eta/\eta_0)^2$, where $\eta_0=\sqrt{2/\rho akT}$ and ρ is the density of states per unit length of the filament (in the case we are interested in, $\eta_0\gg 1$). Accordingly, the probability density for a filament having no centers whose transition rates from a certain prespecified center of the same filament exceed Γ is given by $P(\eta)=\exp[-(\eta/\eta_0)^2]$. Since there is no way for a carrier to circumvent the blocked positions, we can estimate the resistance of the filament by using the Miller–Abrahams procedure, i.e., by assuming that the path of a carrier consists of hops whose probability at each step is maximal.[17] Accordingly, for the average resistance corresponding to an individual hop we have

$$\bar{R}=R_0\int_0^\infty \exp(\eta)P(\eta)\,\frac{2\eta}{\eta_0^2}\,d\eta. \tag{3}$$

Recall that on an infinite filament we will always encounter blocking segments corresponding to transition rates smaller than any *a priori* given value. Hence, an infinite upper bound here reflects the possible existence of arbitrarily small transition rates on certain segments of the filament. However, since the contribution of these segments to the resistance is small, the primary contribution to the integral comes from the vicinity of $\eta\approx\eta_0^2/2$, in which the expression under the integral sign has a sharp maximum. Accordingly, for an infinitely long insulating filament we have

$$\bar{R}=R_0\exp\left(\frac{\eta_0^2}{4}\right)=R_0\exp\left(\frac{1}{2a\rho kT}\right), \tag{4}$$

i.e., the temperature dependence of the conductivity is activated. It is not difficult to estimate the average length of a hop along the filament as well:

$$r_0=\left\langle\int dr_{ij}\int d\varepsilon_j\rho P(\eta_{ij})\right\rangle_{\varepsilon_i}\approx a\eta_0,$$

where $\langle\ldots\rangle_{\varepsilon_i}$ denotes an average over the initial states (we have omitted a numerical factor of order unity here).

The average number of hops confined to a filament of finite length L is of order $L/a\eta_0$, while the fraction of bonds with transition rates no greater than $\Gamma=\Gamma_0\exp(-\eta)$ is

$$\int_\eta^\infty P(\eta)\,\frac{2\eta}{\eta_0^2}\,d\eta=\exp\left[-\left(\frac{\eta}{\eta_0}\right)^2\right].$$

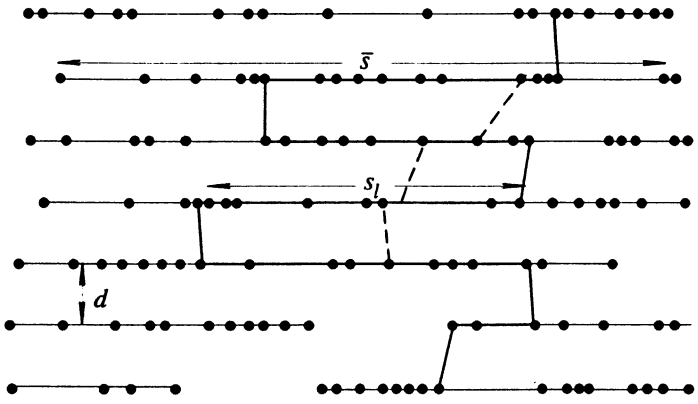

FIG. 1. Structure of a critical subnet (schematic). The solid lines show bonds between centers; $\bar{s}$ is the average length of a longitudinal chain of linked bonds, s_1 is the average length of a longitudinal macrobond. The heavy lines indicate a transverse path consisting of bonds of the critical subnet; the dashed lines show its straightened transverse transitions.

Accordingly, for a filament of length L, the number of hops whose rates do not exceed $\Gamma_0\exp(-\eta_0^2/4)$ equals $(L/a\eta_0)\exp(-\eta_0^2/4)$. Only when

$$\frac{L}{a\eta_0}\exp\left(-\frac{\eta_0^2}{4}\right)\gg 1 \tag{5}$$

does the average resistance corresponding to a hop have the value $\bar{R}$ of Eq. (3); in the opposite case (even when $L/a\eta_0\gg 1$) the resistances of those filaments that are primarily determined by minimal transition rates can fluctuate strongly.[17]

In a strictly one-dimensional system, an electron is unable to avoid blocking segments corresponding to small transition rates; however, in a quasi-one-dimensional system it is possible to do this via transitions to a neighboring filament. Inclusion of these lateral transitions (which make the paths three-dimensional) leads to finiteness of the lengths of nonbranching segments of the paths and to finiteness of the upper limit of the integral (3) due to the new possibility of avoiding blocking segments. The upper limit corresponds to the maximum value of the quantity η at infinity for the optimal trajectory (i.e., the one that traverses the entire system); we may estimate it by using the percolation bond problem.

It follows from the standard theory of hopping conductivity[11] that because of the low density of the infinite cluster in the immediate vicinity of threshold, the conductivity is determined by the infinite cluster that arises for values of $\eta-\eta_{cr}$ of order unity, which we call the critical subnet. In this case, for a three-dimensional system with a random distribution of centers in space and a hopping probability given by Eq. (1), the pre-exponential factor can be written in the form

$$\sigma_0=\frac{Ae^2}{kTL_0},$$

where A is a number of order unity and L_0 is the correlation length of the critical subnet.

As in three-dimensional systems, for the quasi-one-dimensional system under discussion we define a bond for any chosen value of η by the condition $\eta_{ij}<\eta$. Let η_{cr} be the percolation threshold corresponding to the appearance of an infinite cluster of linked bonds in the system. In the presence of this infinite cluster (i.e., for $\eta>\eta_{cr}$), the quantities η_{ij} along any infinite path lying in the cluster do not exceed η. For such paths the upper limit in (3) equals η.

For quasi-one-dimensional systems we can use the one-strand net model proposed for three-dimensional systems. Here we assume that we can identify a skeleton net in the critical subnet for which the segments between neighboring nodes (macrobonds) are one-strand (see Ref. 11). Actually, for large d the distinctive features of the structure of the critical subnet are associated with the existence of long longitudinal unbranched chains of bonds whose characteristics are analogous to those of a one-dimensional filament of the corresponding length. The characteristic length of unbranched fragments of these chains (longitudinal macrobonds) is defined to be the average distance between lateral macrobonds, which consist of simple bonds (crosslinks) that join centers located on neighboring filaments. For large distances d between the filaments, these longitudinal macrobonds can be extremely long (see Fig. 1).

The distinctive features of the structure of the critical subnet mentioned above can have a very strong influence on the characteristics of the optimal sequence of hops in directions along and across the filaments, and accordingly on the longitudinal and transverse conductivity of the system. Indeed, if the optimal path lies in the critical subnet, the longitudinal displacement of a carrier and the length of the path are quantities of order unity, whereas the ratio of the transverse displacement to the length of the path is of order d/L_1, where L_1 is the characteristic length of a longitudinal macrobond. For large L_1/d, it may turn out that the optimum sequence of hops in directions perpendicular to the filaments corresponds to straightened paths that include transitions with small rates which do not belong to the critical subnet. This implies that the standard percolation approach to hopping conductivity will be inapplicable. As a result, not only the magnitudes but also the exponential factors of the longitudinal and transverse conductivities (i.e., their exponential temperature and concentration dependences) can turn out to be different.

3. CRITICAL SUBNET OF AN INFINITE CLUSTER FOR QUASI-ONE-DIMENSIONAL SYSTEMS

Let us discuss the structure of the critical subnet in more detail for a quasi-one-dimensional system under conditions

where the concentration of bonds between centers on different chains is small. For a given η, the average number of bonds in longitudinal chains of linked bonds located on the same filament is

$$\bar{s}=\sum_{s=1}^{\infty} s(1-P_\eta)^s P_\eta=\exp[(\eta/\eta_0)^2]-1 \qquad (6)$$

(for $\eta \gg \eta_0$ we have $\bar{s} \gg 1$). The average length of unbranched segments of this chain (macrobonds) is determined by the number of transverse bonds between centers of different filaments. We call the average number of transverse bonds connecting a given center with centers on one of the neighboring filaments $\nu_\perp$. Then the average number of nodes lying on a filament between neighboring crosslinks in the critical subnet is $s_1 \approx \nu_\perp^{-1}$.

It is obvious that only centers with energies such that $|\varepsilon-\mu|<\varepsilon_m(1-d/r_m)$ can form transverse bonds; here, as usual, $\varepsilon_m=kT\eta$ is the maximum hopping energy, while $r_m=(a/2)\eta$ is the maximum length of a hop for a given η. As far as the situation we are interested in goes, where the number of transverse bonds is small, we may assume that $|\varepsilon-\mu|<\Delta$, where Δ is the width of the band of localized states. In this case, for the number of bonds averaged over initial energy, we find from calculations for a single center that

$$\nu_\perp=\rho\int_{-\infty}^{\infty}dx\int d\left(\frac{\varepsilon}{\varepsilon_m}\right)\int_{-\Delta}^{\Delta}d\varepsilon'\,\theta\left(1-\frac{\sqrt{x^2+d^2}}{r_m}-\frac{|\varepsilon-\mu|+|\varepsilon'-\mu|+|\varepsilon-\varepsilon'|}{2\varepsilon_m}\right)=\rho\varepsilon_m r_m F(\kappa), \qquad (7)$$

where

$$F(\kappa)=(3-2\kappa^2)\kappa-\frac{3}{2}\,1-\kappa^2\,\ln\frac{1+\kappa}{1-\kappa}, \qquad (8)$$

and $\kappa=\sqrt{1-(d/r_m)^2}$. In the range of temperatures that are not too low, for which we may neglect transitions to all filaments except neighboring ones, i.e., for $r_m<d_2$, where d_2 is the distance to the next-nearest neighbor, we have $\kappa<\sqrt{1-d^2/d_2^2}$. The leading terms of the expansion of the function $F(\kappa)$ in κ have the form

$$F(\kappa)\approx\tfrac{2}{5}\kappa^5+\tfrac{6}{35}\kappa^7. \qquad (9)$$

For $\kappa<0.5$, the error connected with using the approximate Eq. (9) does not exceed 3.5%. Retaining only the first term increases the maximum error in this region to around 15%.

For $\eta<2d/a$ there are no lateral bonds, and percolation does not take place in the system. The threshold for appearance of an infinite cluster corresponds to a concentration of lateral bonds for which the number of such bonds between longitudinal chains of conjugate bonds reaches the critical value. The threshold value for the transition rate can be found with the help of the modified bond criterion (compare with Ref. 15) $z\bar{s}\nu_\perp=\nu_{cr}$, where z is the number of filaments close to a given filament (for filaments in a plane $z=2$), while the critical concentration of lateral bonds ν_{cr} is a number of order unity. Taking into account (7), (9), and (6), we find from the condition $z\bar{s}\nu_\perp=\nu_{cr}$ the critical value of the quantity η corresponding to the appearance of an infinite cluster:

$$\eta_{cr}=\frac{2d}{a}\left\{1+\frac{1}{2}\,\frac{5\nu_{cr}}{z}\left(\frac{a}{2d}\right)^2\frac{1}{\rho akT}\exp\left[-\frac{2}{5}\left(\frac{2d}{a\eta_0}\right)^2\right]\right\}. \qquad (10)$$

Since $2d/a\,\eta_0 \gg 1$, the critical value η_{cr} differs from $2d/a$ by a quantity considerably less than unity.

Note that for values of η close to threshold, the average number of lateral bonds per center $\nu_\perp$ in this calculation is small, and the value of $\bar{s}$ is large. When

$$1 \ll s_1 \lesssim \bar{s}, \qquad (11)$$

the infinite cluster is strongly anisotropic, consisting of long longitudinal fragments with rare lateral crosslinks (see Fig. 1). As long as the average distance between lateral bonds remains large, for values of $\eta-\eta_{cr}$ of order unity the critical subnet has the same anisotropic structure as well.

As we approach the critical subnet of an infinite cluster, i.e., as the parameter η_{cr} increases to a value of order unity above the critical value η_{cr} from (10), we have $\kappa \approx \sqrt{2/\eta_{cr}}$, and the average number of lateral bonds per center (7) becomes of order

$$\nu_\perp=\frac{8}{5}\,\rho akT\sqrt{\frac{a}{d}}. \qquad (12)$$

If

$$\rho akT \ll \sqrt{d/a}, \qquad (13)$$

the number of transverse bonds between centers of the critical subnet (in the calculation for one center) is small. In this case, the critical subnet consists of long longitudinal macrobonds with infrequent transverse bonds (crosslinks). Since there is significant correlation between the occupation numbers of nodes located on macrobonds, the correlation length determined by the distance between endpoints of a macrobond is different for directions along and transverse to the filament.

Thus, under these conditions the critical subnet is strongly anisotropic. In this case, the resistance R_1 of a long longitudinal macrobond, for which $s_1 \gg s_0$, where $s_0=\exp(\eta_0^2/4)$, is defined not by the maximum resistance of a bond or the quantity η_{cr}, Eq. (10), but rather by the characteristic resistance (3), i.e., $R_1 \approx s_1\bar{R}$. In view of what was said in the previous section, this resistance of the longitudinal macrobonds also determines the resistance of long chains made up of bonds of the critical subnet, independent of their orientation.

4. CONDUCTIVITY

The hopping conductivity of a system is determined by the optimum sequences of hops (paths) that correspond to maximum probability for transfer of a carrier between opposite ends of the macroscopic sample. Based on our discussion of the critical subnet in the previous section, we will now calculate the longitudinal and transverse hopping conductivities.

The critical subnet is anisotropic when inequality (11) holds, i.e., when both the average length of longitudinal chains of linked bonds $\bar{s}$ and the average length of longitudinal macrobonds s_1 are large, where $\bar{s} \gtrsim s_1$. According to (6) and (13), these inequalities bound the temperature range

$$T_1 \lesssim T \ll T_2. \tag{14}$$

Here the upper boundary $T_2 = (\rho a k)^{-1}\sqrt{d/a}$, while the lower boundary T_1 is determined by the equation $\bar{s}=s_1$, which leads to the following transcendental equation for T_1:

$$(\eta_{cr}/\eta_0)^2 \exp[(\eta_{cr}/\eta_0)^2] = \eta_{cr}^{5/2};$$

we find that $T_1 \approx (\rho a k)^{-1}(a/d)^2$ for reasonable values of the parameter η_{cr}.

We will show that the optimal paths that determine the longitudinal conductivity σ_1 consist of chains of bonds lying in the critical subnet, but that the conductivity is not always determined by the critical value η_{cr}. We first consider the low-temperature portion of region (14), $T_1 \lesssim T \ll T_3$, which corresponds to the inequalities $s_0 \ll s_1 \lesssim \bar{s}$. The first of these inequalities bounds the temperature range from above, with the bound T_3 determined by the equation $\eta_0^{-2}\exp(\eta_0^2/4) = \sqrt{\eta_{cr}}$, we find that the quantity $\rho a k T_3$ does not differ much from unity.

Using the method introduced in Ref. 11 to calculate the pre-exponential factor, to accuracy up to a factor of order unity we obtain

$$\sigma_1 = L_{c,l} L_{c,t}^{-2} R_l^{-1}, \tag{15}$$

where $L_{c,l}$ and $L_{c,t}$ are the longitudinal and transverse correlation lengths (i.e., the projections of the characteristic distances between nodes of the critical subnet onto the longitudinal and transverse directions), and R_l is the resistance of a longitudinal macrobond. Here we have taken into account the fact that for the present case of small concentrations of transverse bonds, the resistance of a longitudinal macrobond greatly exceeds the characteristic resistance of a crosslink. Since $L_{c,l} \approx s_1$, while $L_{c,t} \approx d$, we obtain for the total longitudinal resistivity of the system

$$\sigma_1 = a\,\eta_0 (d^2 \bar{R})^{-1}, \tag{16}$$

where $\bar{R}$ is given by Eq. (4). The paths under discussion that lie in the critical subnet are optimal, i.e., any departure of a path from the bounds of the critical subnet increases its length and resistance. For the range of temperatures under discussion, it is these paths that determine the longitudinal conductivity. However, their basic temperature dependence is determined not by the critical value of the transition rate, but by the quantity $\bar{R}^{-1}$, and is activated in character; the activation energy is proportional to $(\rho a)^{-1}$.

Let us now discuss the contribution to the transverse conductivity from paths lying in the critical subnet. This contribution can be written in the form

$$\sigma_t^{net} = L_{c,l}^{-1} R_l^{-1} = a\,\eta_0 (L_{c,l}^2 \bar{R})^{-1}, \tag{17}$$

where, as before, the primary contribution to the resistance of a path passing through nodes located on adjacent filaments is determined by the longitudinal macrobonds. It is clear that the length of the chain of bonds joining nodes lying on neighboring filaments is $\beta = L_{c0}/d$ times the distance between filaments; the component of the transverse conductivity associated with transport along the critical subnet is β^2 times smaller than the longitudinal conductivity. Since $\beta \gg 1$, the ratio σ_l/σ_t^{net} turns out to be large. Note that for the r-percolation problem this ratio is of order unity, and the anisotropy of the exponential factor does not appear in the conductivity.[16] The reason for this is that for $na \gg 1$, where n is the linear density of centers, as η increases above the critical value η_{cr}, the number of transverse bonds increases so rapidly that the average number in a calculation for a single center becomes greater than unity even for $\eta - \eta_{cr} \approx 1$; as a result, despite the anisotropy of the infinite cluster in the immediate vicinity of threshold, the critical subnet is essentially isotropic. When $na \ll 1$, there is no anisotropy in the exponential factor because the resistance of the paths is determined by the transverse transitions, even when their longitudinal unbranched segments are long.[16]

In accordance with what was said in Sec. 3, in the present case of an anisotropic critical subnet it can turn out that the contribution of paths that depart the critical subnet is nonnegligible, since despite their small hopping probabilities, these paths can be appreciably shorter than the subnet paths. Let us find the optimum transverse path that is not limited to bonds that belong to the critical subnet. By including in the finite cluster those bonds that correspond to larger and larger values of η, i.e., larger than ε and larger than $\nu_\perp$, we obtain a cluster with shorter and shorter unbranched chains of bonds on the filaments. As before, we will refer to these as longitudinal macrobonds, and we will assume that the number of bonds in a macrobond s_1 is large. For fixed η, an infinite cluster appears when $z s_1 \nu_\perp = \nu_{cr1}$, consisting of longitudinal macrobonds with length of order s_1 joined by transverse bonds; as in the earlier sections, the critical number of bonds ν_{cr1} is of order unity. Taking (7) into account, we obtain for small κ

$$(32\rho k T d^2/15a) z s_1 \kappa^5 = \nu_{cr1}, \tag{18}$$

from which we have

$$\eta_s = 2d/a + (T_0/s_1 T)^{2/5}, \tag{19}$$

where $T_0 = (15\nu_{cr1}/32 z \rho a k)\sqrt{d/a}$. We note that, generally speaking, the quantity

$$y \equiv (T_0/s_1 T)^{2/5} = (d/a)\kappa^2,$$

can be large, despite the smallness of the parameter κ, since $d \gg a$.

The transverse path consists of series-connected longitudinal macrobonds and transverse bonds, for which the corresponding resistances $R_s = R_0 \exp(\eta_s)$. For large $s_1 \gg s_0$, the resistance of the path segment that joins nodes located on adjacent filaments is of order

$$R_{tot}(s) = s_1 \bar{R} + R_s. \tag{20}$$

The optimum transverse path is found by minimizing this expression with respect to s_1. As a result we find

$$R_{tot}(s_{opt}) = \bar{R}\,\frac{T_0}{T}\,\frac{y_{opt}+1}{y_{opt}^{7/2}}, \tag{21}$$

where $y_{opt}=(T_0/s_{opt}T)^{2/5}$ is determined from the equation

$$y_{opt}^{7/2}\exp(y_{opt})=\frac{5T_0\bar{R}}{2TR_{cr}},$$

while $R_{cr}=R_0\exp(2d/a)$ is the resistance corresponding to the percolation threshold. When $T_0\bar{R}/TR_{cr}\gg 1$ we have $y_{opt}\gg 1$, and the optimal path departs the critical subnet. It is clear that in this case the temperature dependence of y_{opt} and the length of the longitudinal fragment of the optimal path s_{opt} associated with it are nonexponential. Taking (21) into account, we find that the primary contribution to $R_{tot}(s_{opt})$ is given by longitudinal macrobonds, so that

$$R_{tot}(s_{opt})=s_{opt}\bar{R}=\bar{R}(T_0/T)y_{opt}^{-5/2}.$$

For the transverse conductivity determined by the optimized transverse paths, we have

$$\sigma_t=[a\eta_0 s_{opt}R_{tot}(s_{opt})]^{-1}=(a\eta_0 s_{opt}^2\bar{R})^{-1}. \quad (22)$$

We see that the temperature dependence of the transverse conductivity is the same as that of the longitudinal conductivity. Like the latter, it is determined by the activated temperature dependence of the reciprocal of the mean resistance of the macrobonds (4); however, the pre-exponential factor is considerably smaller. For the ratio of longitudinal to transverse conductivity we have

$$K=\sigma_1/\sigma_t=(a\eta_0 s_{opt}/d)^2. \quad (23)$$

Thus, the anisotropy coefficient (23) is large and depends weakly (nonexponentially) on temperature.

Recall that Eq. (22) was obtained for large $s_{opt}\gg s_0$. For $s_{opt}\ll s_0$, i.e., in the temperature range

$$T_3\ll T\ll T_2, \quad (24)$$

the primary contribution to the resistance of the optimal transverse paths is given by transverse bonds, and the total resistance (21) is determined by the second term with $s=1$. Accordingly, in this case the transverse resistance has a temperature dependence of the form

$$\sigma_t=\sigma_0\exp[-(T_0/T)^{2/5}], \quad (25)$$

corresponding to the Mott law for a one-dimensional system. The difference between this result and that obtained in Ref. 13 by independent direct optimization of transverse hops is the power in the argument of the exponential function (2/5 instead of 1/2).

5. DISCUSSION OF RESULTS

Our discussion of the structure of the critical subnet in this paper shows that the conductivity of a quasi-one-dimensional system cannot always be obtained using the standard approach based on percolation theory. In fact, for a system of asymptotically long unlinked filaments, the longitudinal conductivity is finite due to the low probability of very large resistances between nodes. When the transverse transition rates are small, the calculation of an individual resistance is directly related to averaging the resistance corresponding to transitions along the chain. The average obtained (the characteristic resistance of a bond) differs in general from the critical value corresponding to the appearance of an infinite cluster.

It is for this reason that anomalously large anisotropy can appear in the conductivity. However, for systems with exponential scatter of the transition rates, in order for anisotropy of the exponential factor of the conductivity to appear it is necessary that the critical subnet be highly anisotropic. In systems with r-percolation, the rapid growth in the number of bonds near the percolation threshold causes the critical subnet to become isotropic, and prevents anisotropy in the conductivity exponential. For systems with $r-\varepsilon$ percolation, the temperature range (14) over which anisotropy is large becomes significant. This range is bounded from above by the condition of smallness of the number of transverse bonds, which increases with temperature due to the increase in the thickness of the layer of energy that contains centers for which the transitions are possible. The bound from below comes about because as the temperature decreases, the number of bonds $\bar{s}$ in the longitudinal chains of bonds decreases as well. At low temperatures the hopping length becomes large compared to the distance between filaments, the quasi-one-dimensional character of the distribution of centers becomes insignificant, and the paths effectively become three-dimensional. The change in the temperature dependence of the conductivity in this region, where the anisotropy is small, was discussed in Ref. 9 by the methods of percolation theory.

A decrease in the anisotropy connected with spatial inhomogeneity of the system as the temperature decreases due to an increase in the hopping length is a general property of hopping problems. However, in Ref. 18 Kovacik *et al.* reported anomalous behavior of the anisotropy of the hopping conductivity for single crystals of $SmBa_2Cu_3O_{6.2}$. At temperatures around 20 K, they observed a transition from anisotropic activated conductivity to anisotropic Mott conductivity with different Mott parameters in the longitudinal and transverse directions; this transition has not been explained. In connection with this, we note the anomalous temperature dependence of the conductivity anisotropy obtained for our model within a certain temperature range. As the temperature drops below a value of order $(\rho ak)^{-1}\sqrt{d/a}$, corresponding to a transition to the region of anisotropy (14) from above, the conductivity anisotropy increases. However, the applicability to this system of the quasi-one-dimensional model we have discussed here requires additional analysis, since the nature of the localized states, their parameters, and features of their spatial distribution for $SmBa_2Cu_3O_{6.2}$ have not yet been clarified.

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