Infrared free electron laser with down-shifted frequency

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An unconventional design is described for a free electron laser with optical pumping, in which the pump frequency is Doppler-shifted "downward" with low gain in the Compton regime. It is shown that the conditions imposed on the quality of the electron beam in this design are substantially less severe than in conventional free electron laser designs with upshifted frequency. Free electron lasers with downshifted frequency are promising for applications in the IR wavelength range. © *1994 American Institute of Physics.*

The creation of high-power sources of tunable coherent radiation in the IR range of the spectrum is a pressing problem in laser physics. The use of radiation in this wavelength range is attractive for carrying out a number of fundamental and applied investigations in the physics of the solid state, high-temperature superconductors, and biophysics. Free electron lasers (FELs) are regarded as a promising IR source: the frequency range in which FELs emit is not restricted by the discrete energetic structure of atoms or molecules, and the radiation wavelength can be tuned over a wide range.

Conventional designs for FELs are based on induced scattering by a relativistic electron beam (REB) of an oppositely directed electromagnetic pump wave. Because of the "upward" Doppler shift in frequency the amplification of the signal occurs at wavelength ($\gamma_\parallel \gg 1$)

$$\lambda_s \approx \lambda_i / 4\gamma_\parallel^2 \tag{1}$$

where $\gamma_\parallel = (1-\beta_z^2)^{-1/2}$ is the "longitudinal" relativistic factor of the electrons; here $c\beta_z$ is the electron velocity in the direction of the beam and λ_i is the wavelength of the pump radiation (in the case of a static wiggler with period λ_w we have the correspondence $\lambda_i \approx 2\lambda_w$). In FEL technology magnetostatic wigglers with period $\lambda_w \sim 2-10$ cm are usually employed, so that in order to produce lasing in the IR spectral range REB sources with relatively high (tens of MeV) electron energy are needed (rf and induction linacs or microtrons[1,2]). As the energy increases, however, the gain of a FEL rapidly decreases ($\sim \gamma^{-3}$). The resulting increase in the length of the wiggler and beam current imposes severe restrictions on the quality of the REB. In order to overcome these difficulties a number of promising designs for submillimeter and IR FELs have been proposed and are currently being implemented, based on the micro-undulator technique;[3–9] the two-section FEL should also be noted.[10]

In the present work a fundamentally different scheme is reported, which makes use of a "downward" Doppler shift in the pump frequency. In a FEL with downshifted frequency[11] a high-power electromagnetic pump wave propagates in the same direction as the REB. The amplified signal then moves in the opposite direction and its wavelength is determined by the relation which is inverse to (1) ($\gamma_\parallel \gg 1$):

$$\lambda_s \approx 4\gamma_\parallel^2 \lambda_i \,. \tag{2}$$

This scheme is based on a certain symmetry in the processes of induced absorption and emission when a pump wave is scattered by an electron beam. Relations (1) and (2) are consequences of the resonance between the REB and the combination wave of the ponderomotive potential formed by the signal and pump waves. The high-frequency component is amplified when the electron velocity is slightly above resonance. In the opposite case, when the electrons lag slightly behind the combination wave, in contrast to the previous case the high-frequency component of the field is absorbed and the low-frequency component is amplified. The energy of the high-frequency wave is thus redistributed between the beam electrons and the low-frequency signal.

FELs with downshifted frequency are promising for applications in the IR wavelength range: if we assume that the pump is a high-power laser with radiation at $\lambda_i = 1.06\ \mu$m (a neodymium laser) and $\gamma \sim 5$ then we have $\lambda_s \sim 100\ \mu$m. Thus, it is possible to cover the entire IR spectral range by using relatively low-energy high-current accelerators. A further advantage of this design is the absence of a static wiggler; at the same time, existing laser systems have sufficient power to provide effective pumping.

The main properties of lasing in FELs with downshifted frequency can be clearly exhibited in the low-gain Compton regime, where simple analytical solutions are possible. In particular, in what follows we will derive the gain, the saturation intensity, and the efficiency of the device. We will show that the conditions imposed on the quality of the electron beam are considerably less severe for this device than for conventional designs with upshifted frequency.

We restrict ourselves to the case of collinear interaction geometry, so that the problem reduces to one dimension. Neglecting the effect of the self-fields of the electron beam (the Compton regime) and harmonic generation effects, we can represent the potential of the electromagnetic field as a superposition of the pump (i) and signal (s) waves:

$$\mathbf{A} = \mathbf{A}_i + \mathbf{A}_s , \tag{3}$$

where in accordance with the assumed geometry the vector potentials will be taken in the form of plane waves with circular polarization:

$$\begin{aligned} \mathbf{A}_i(z,t) &= \mathbf{e}_- A_i(z) \exp\{i(\omega_i t - k_i z)\} + \text{c.c.} \\ \mathbf{A}_s(z,t) &= \mathbf{e}_- A_s(z) \exp\{i(\omega_s t + k_s z)\} + \text{c.c.} \end{aligned} \tag{4}$$

($\mathbf{e}_{\pm}=\mathbf{e}_x \pm i\mathbf{e}_y$). A uniform relativistic electron beam propagates in the positive z direction (together with the pump wave A_i) with velocity $c\beta_z$. In the low-gain regime the amplitude of the signal is a slowly varying function $(k_s A_s)^{-1} dA_s/dz \ll 1$. The intensity of the pump wave needed to achieve an effective interaction should be relatively high, so that we can neglect its depletion in the course of the amplification, $A_i(z) \simeq \text{const}$.

We will take the pulse lengths τ_b of the beam current, τ_i of the pump, and τ_s of the amplified signal to be large in comparison with the time of flight of an electron through the interaction region L:

$$\tau_b, \tau_i, \tau_s \gg L/\beta_z c, \tag{5}$$

so that the problem becomes quasistationary.

With these approximations the electron dynamics is described by a one-dimensional kinetic equation[12]

$$\frac{\partial f}{\partial t} + \frac{p_z}{m\gamma} \frac{\partial f}{\partial z} = \frac{e^2}{m\gamma} \frac{\partial}{\partial z} \left(\frac{A^2}{2} \right) \frac{\partial f}{\partial p_z}. \tag{6}$$

Here e and m are the electron charge and rest mass, $f(z, p_z, t)$ is the factored beam electron distribution function, and $p_z = m\gamma\beta_z c$ is the dynamical electron longitudinal momentum (in real light fields the mass shift is negligible, $\delta m/m \sim e^2 A^2/m^2 c^4 \ll 1$). The boundary condition for Eq. (6) is given in the form of the steady momentum distribution at the input to the interaction region ($z=0$):

$$f(z=0, p_z, t) = F(p_z). \tag{7}$$

We will use the resonance approximation, retaining in (6) only the terms corresponding to electrons synchronized with the combination wave of the ponderomotive potential:

$$\beta_z = \omega/ck, \tag{8}$$

where $\omega = \omega_i - \omega_s$ and $k = k_i + k_s$ are the frequency and wave number respectively of the combination wave.

The kinetic equation (6) is to be solved together with the wave equation describing the evolution of the low-frequency signal. In the weak-signal and low-gain approximation the solution to the problem is completely analogous to the case of a conventional scheme with upshifted frequency.[12] To first order in perturbation theory we have for the power amplification of the signal

$$\alpha = \frac{4\pi^2 r_0^2 m n_e L}{k_i k_s c} I_i \int_{-\infty}^{\infty} dp_z F(p_z) \frac{d}{dp_z} \left\{ \frac{1}{\gamma p_z} \left(\frac{\sin \eta}{\eta} \right)^2 \right\}. \tag{9}$$

Here r_0 is the electron classical radius, I_i is the pump radiation intensity, n_e is the electron density in the beam, and we have written $\eta = \mu L/2$; $\mu = \omega/c\beta_z - k$ is the parameter which specifies the detuning from resonance. The formal analogy between (9) and the corresponding result of Ref. 12 for schemes with upshifted frequency is due to the symmetry in the induced processes of energy exchange between the pump and signal waves mentioned above.

By virtue of (9) the absorption and emission lines are below the resonance $\mu \approx 0$, which, as is easily seen, determines the relation (2) for the wavelengths of the pump and the low-frequency signal.

Maximum gain is achieved in the regime in which homogeneous line-broadening dominates, and in one pass amounts to

$$G = \alpha L \approx 8\pi^2 \frac{r_0^2 n_e L^3}{mc^2 \gamma^3} \frac{I_i}{\omega_i} \frac{d}{d\eta} \left(\frac{\sin \eta}{\eta} \right)^2. \tag{10}$$

To reach this regime we must satisfy a condition on the electron beam quality, since it is just the electron momentum distribution which determines the magnitude of the inhomogeneous broadening. We can easily find the necessary conditions by analyzing the integrand in (9). Accordingly, the initial spread of the beam electrons, $\Delta\gamma$ in energy and $\langle \Delta\theta^2 \rangle$ in angle (related to the beam emittance), must satisfy the inequality

$$\langle \Delta\theta^2 \rangle/2, \Delta\gamma/\gamma < 4\gamma^2/2N, \tag{11}$$

where $N = L/\lambda_i$ is the effective number of periods of the dynamic wiggler formed by the laser pump wave.

It is clear that for a fixed value of N and a beam of a specified quality, the criterion (11) can be satisfied only above some value of γ. The possibilities for raising the electron energy, however, are limited by the condition that the length of the interaction region should be substantially greater than the wavelength of the generated signal, $L \gg \lambda_s$. As a result, the right-hand side of Eq. (11) is small compared to unity.

Thus, a FEL with downshifted frequency has an unquestionable advantage: the requirements on the REB quality here are substantially less severe than in conventional laser designs. This makes it possible to use wigglers with a considerably larger number N of periods (up to values $N \gtrsim 10^4$), and in view of the scaling $G \sim N^3$, Eq. (10) gives rise to a corresponding increase in the gain.

The saturation intensity can be estimated from the condition for complete trapping of the REB by the combination wave. Saturation occurs when the spread in electron energy $\Delta\gamma^*$ that develops in the course of generating a useful signal causes a spread in the detuning parameters on the order of the width of the amplification waveband:

$$\Delta\mu L/2 \sim \pi. \tag{12}$$

The pump wave energy is transferred as a result of induced scattering to the beam electrons and the signal wave:

$$\Delta I_i = mc^3 \Delta\gamma^* \beta_z n_e + \Delta I_s, \tag{13}$$

where $\Delta I_{i,s}$ are the changes in the pump and signal wave intensities respectively. Here, however, we should also take into account the conservation of photon number in an elementary Compton scattering event:

$$\Delta I_i/\hbar\omega_i = \Delta I_s/\hbar\omega_s. \tag{14}$$

We finally obtain for the saturation intensity of the low-frequency signal

$$I_s \approx \tfrac{1}{4} m\gamma c^3 \beta_z^3 n_e (NG)^{-1}. \tag{15}$$

An interesting feature of this FEL design should be emphasized: in the process of amplifying the signal the electron beams are accelerated, which follows from relation (13), whereas in conventional designs with upshifted frequency energy is extracted from the REB. The maximum relative increase in the electron energy is given by

$$\Delta\gamma^*/\gamma\sim\left(\frac{\omega_i}{\omega_s}-1\right)\Delta I_s/m\gamma c^3\beta_z n_e\sim\gamma^2\beta_z^2/N, \qquad (16)$$

where $\Delta I_s=GI_s$ is the increase in the signal intensity in one pass. Consequently, the efficiency of a FEL with downshifted frequency should be defined relative to the pump wave, not to the electron beam. However, it is easy to see that the maximum of the device (assuming total depletion of the pump) must be less than

$$\eta\sim(4\gamma^2-1)^{-1}. \qquad (17)$$

Let us make some estimates. Assuming that the pump consists of radiation from a neodymium laser ($\lambda_i=1.06\ \mu$m) with intensity $I_i=5\cdot10^{13}$ W/cm^2, for a REB with $\gamma=3.5$ and electron density $n_e=10^{12}$ cm^3 (which corresponds to a beam density $\sim$5 kA/cm^2) we find a gain coefficient $\sim$30% per pass through an amplification region of length $L\approx2$ cm. The wavelength of the emitted signal then lies in the IR region of the spectrum, $\lambda_s\sim50\ \mu$m, and the saturation intensity is $I_s\sim0.3$ MW/cm^2. Typical pulse lengths of high-power lasers and high-current electron beams are tens of nanoseconds; consequently, it is possible to design multipass generators with $\tau_p c/2L\sim10^2-10^3$. Assuming that the conditions for focusing the pump radiation determine the size of the ray caustic equal to the length of the interaction region $\approx$2 cm, it is easy to see that to reach the necessary pump intensity level a laser pulse energy $\sim$10 J is required with pulse lengths $\tau_p\sim10$ ns. Laser systems with such parameters are not too exotic and are reasonably compact.[13]

Thus, a FEL with downshifted frequency may offer a very promising source of high-power coherent IR, tunable over essentially the entire IR spectral range. This scheme does not require cumbersome special-purpose accelerators and is relatively compact. An additional advantage is the absence of a static wiggler, and also the small interaction region, which makes it substantially easier to implement experimentally.

The possibility of raising the gain is evidently related to increasing the pump intensity and/or the length of the interaction region. Both approaches take us outside the assumptions made in the present work (the amplification of the signal can then no longer be assumed small) and require further study. Furthermore, new physical factors come into play. In particular, under the condition

$$\omega_p L/c>\gamma^{3/2},$$

where ω_p is the beam electron plasma frequency, collective modes of the REB develop and the Raman mechanism for IR signal generation can become dominant.

In conclusion, we note that the efficiency of a FEL with downshifted frequency can be increased by phase-modulating the pulse of the pump radiation. The mechanism for this increase is completely analogous to the case of a magnetostatic wiggler with variable parameters:[1,2] for a corresponding adiabatic increase in frequency during the pump pulse the electron remains in phase with the ponderomotive potential combination wave and takes part in generating a useful signal throughout the entire interaction region.

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