Axion synchrotron emission: a new bound on the axion-electron coupling constant

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We have calculated the $e \to e + a$ synchrotron contribution Q_a to the axion luminosity of a magnetized, highly degenerate relativistic electron gas. By comparing Q_a expected under neutron star conditions with the known luminosity accounted for by neutrino synchrotron emission ($e \to e + \nu + \bar{\nu}$), we have obtained a new and more stringent bound on the axion electron coupling constant, $g_{ae} \lesssim 5 \cdot 10^{-14}$. © *1995 American Institute of Physics.*

1. The axion is a pseudo-Goldstone boson invoked to address the *a priori* strong breaking of *CP* invariance in the standard model. It results from the spontaneous breaking of the global $U(1)_{PQ}$ symmetry introduced by Peccei and Quinn.[1] In the standard axion model,[2] it is assumed that the energy scale v_a for *PQ* symmetry breaking is of the same order as the electroweak scale $v_w = (\sqrt{2} G_F)^{-1/2} \approx 250$ GeV, where G_F is the Fermi constant. That model has already been invalidated by experiment,[3] a fact responsible for the appearance of various invisible axion models, in which $v_a \gg v_w$ (these have been surveyed in Ref. 4).

The interaction of axions a and fermions f [in units with $\hbar = c = 1$ and a metric with signature $(+---)$] is given by the Lagrangian Eq. (1)

$$\mathscr{L}_{af} = \frac{g_{af}}{2m_f} (\bar{\psi}_f \gamma^\mu \gamma^5 \psi_f) \partial_\mu a, \tag{1}$$

with dimensionless coupling constant

$$g_{af} = \frac{c_f m_f}{v_a}, \tag{2}$$

and where m_f is the fermion mass, c_f is a numerical coefficient that depends on the specific model,[4] and $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$ is the usual Dirac matrix. It can be shown (see, e.g., Ref. 4) that to first order in a/v_a, the Lagrangian (1) is equivalent to the pseudoscalar interaction Lagrangian

$$\tilde{\mathscr{L}}_{a_f} = -i g_{af} (\bar{\psi}_f \gamma^5 \psi_f) a. \tag{3}$$

In view of the smallness of $g_{af} (\sim 1/v_a)$, axion effects may well be most noticeable under astrophysical conditions—high matter densities, high temperature, and strong external magnetic fields (in neutron stars). Raffelt[4] has surveyed various axion production processes and astrophysical methods of placing bounds on axion model parameters. Thus far, an analysis of the bremsstrahlung emission of axions in electron scattering [due to the interaction (3)] by nuclei $(e + (Z,A) \to (Z,A) + e + a)$ in red giants and white dwarfs has yielded[5]

$$g_{ae} < 2 \cdot 10^{-13} \tag{4}$$

as a bound on the axion–electron coupling constant. Note that the bound quoted by Raffelt[4] is weaker: $g_{ae} \lesssim 3 \cdot 10^{-13}$.

In this paper, we examine the emission of axions by relativistic electrons in a magnetic field ($e \to e + a$), which by a natural analogy with the corresponding electromagnetic process ($e \to e + \gamma$)[6] we call axion synchrotron emission (ASE). We calculate the contribution of ASE to the axion luminosity Q_a of a highly degenerate magnetized electron gas under neutron star conditions.[7] By comparing Q_a with the neutrino luminosity Q_ν stemming from neutrino synchrotron emission (NSE: $e \to e + \nu + \bar{\nu}$) in neutron stars as calculated by Kaminker et al.,[8] we place a new bound on g_{ae}.

2. The amplitude for ASE follows directly from the Lagrangian (3):

$$S_{fi} = \frac{g_{ae}}{\sqrt{2\omega V}} \int d^4x \, \bar{\psi}_{n'}(x) \gamma^5 \psi_n(x) e^{ikx}, \tag{5}$$

where $k^\mu = (\omega, \mathbf{k})$ is the four-momentum of the emitted axion, which can be assumed massless ($k^2 = 0$) under the adopted high-energy conditions (see below); ψ_n and $\psi_{n'}$ are the exact initial and final electron wave functions in the constant magnetic field $\mathbf{H} \| \hat{z}$ (which can be found in explicit form in Ref. 6); V is the normalization volume. Making use of (5), we can find the ASE probability in exactly the same way as for conventional synchrotron emission.[6] We then apply the result to the present case, with large initial and final transverse (relative to $\mathbf{H}$) electron momenta ($p_\perp \gg m$, $p'_\perp \gg m$) and magnetic fields $H \ll H_0 = m^2/e = 4.41 \cdot 10^{13}$ G, for which the initial and final (printed) electron states are semiclassical (quantization of the transverse motion can be neglected):

$$\frac{dw}{du} = \frac{g_{ae}^2}{8\pi^{3/2}} \frac{m^2}{\varepsilon} \frac{u^2}{(1+u)^3} \frac{1}{x} [-\Phi'(x)]. \tag{6}$$

This expression yields the spectral probability distribution for the process in terms of the invariant

$$u = \frac{\chi}{\chi'} - 1 = \frac{p_\perp}{p'_\perp} - 1, \tag{7}$$

where

$$\chi = \frac{e}{m^3} [-(F_{\mu\nu} p^\nu)^2]^{1/2} = \frac{p_\perp}{m} \frac{H}{H_0}, \tag{8}$$

$F_{\mu\nu}$ is the external magnetic field tensor, $p^\mu = (\varepsilon, \mathbf{p})$ is the electron four-momentum (the transverse component of the momentum $p_\perp$, the longitudinal component p_z, and the energy $\varepsilon = \sqrt{(m^2 + p_\perp^2 + p_z^2)}$ are all conserved in the magnetic

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field), χ is obtained from (8) via the substitution $p \to p'$ and $\Phi'(x) = d\Phi(x)/dx$ is the derivative of the Airy function (cf. Ref. 9, for example):

$$\Phi(x) = \pi^{-1/2} \int_0^\infty dt \cos(xt + t^3/3) \tag{9}$$

with argument

$$x = (u/\chi)^{2/3}, \tag{10}$$

and $0 \leqslant u \leqslant \infty$ (to relativistic accuracy: $p_\perp/m \gg 1$). The invariant conditions for the semiclassical Eqs. (6) to apply are[10,11]

$$\chi \gg f, \quad f = \frac{e}{m^2} \left| \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right|^{1/2} = \frac{H}{H_0} \ll 1. \tag{11}$$

The longitudinal momentum p_z is then arbitrary (see Ref. 11, p. 49), a fact that we make use of in Eq. (19). Note that (6) can be derived from Eq. (3) of Ref. 12 for the muon decay probability ($\mu \to e + \varphi$) in a magnetic field, where φ (the familon) has mixed scalar-pseudoscalar coupling to fermions. All that is necessary in the indicated equation is to retain only the pseudoscalar contribution, put $m_\mu = m_e$, and average (sum) over spin states of the initial (final) lepton. The final result is then the same as (6).

3. We now calculate the luminosity Q_a—the rate of energy loss per unit volume of the electron gas due to ASI. Using the probability (6), we have

$$Q_a = \int \frac{2d^3p}{(2\pi)^3} \int_0^\infty du \frac{dw}{du} \omega n_F(\varepsilon)[1 - n_F(\varepsilon')], \tag{12}$$

where over a substantial angular range, the energy of the emitted axion is[11]

$$\omega = \frac{u\varepsilon}{1+u}, \tag{13}$$

$$n_F(\varepsilon) = \left[\exp\left(\frac{\varepsilon - \mu}{T} \right) + 1 \right]^{-1} \tag{14}$$

is the Fermi–Dirac function for the initial electron distribution at temperature T, and μ is the chemical potential; $n_F(\varepsilon')$ is the same function for the final electrons, with $\varepsilon' = \varepsilon - \omega$.

We now consider a highly degenerate relativistic gas, with

$$T \ll \varepsilon_F = \sqrt{m^2 + p_F^2} \simeq p_F = \sqrt[3]{3\pi^2 n_e} \gg m, \tag{15}$$

where n_e is the electron number density. Equation (15) is a valid expression for the Fermi momentum when

$$p_F \gg \omega_F = eH/\varepsilon_F, \tag{16}$$

whereupon many Landau levels are occupied, and we can neglect the magnetic field strength dependence of the Fermi energy ε_F (see, e.g., Ref. 13). We assume that the magnetic field is nonquantizing,

$$\omega_F \ll T, \tag{17}$$

i.e., the electron's "transverse" energy spectrum is essentially continuous (in general, the transverse momentum is quantized:[6] $p_\perp^2 = 2eHn$, $n = 0,1,2,\ldots$). In (12), therefore, we can use the semiclassical Eq. (6) for the probability dw, and summation over the initial and final electron states can be replaced by an integration over phase space.

Subsequent calculations follow those in Ref. 8 for the neutrino luminosity resulting from neutrino synchrotron emission ($e \to e\nu\bar{\nu}$). When (15)–(17) hold, the Fermi factor $n_F(\varepsilon)[1 - n_F(\varepsilon')]$ in (12) has a sharp maximum at $p \simeq p_F$, and the principal contribution to the integral over p comes from the narrow interval $|p - p_F| \lesssim T \ll p_F$. We can therefore put $p = p_F$ everywhere in the integrand of Eq. (12) except the argument of n_F. The resulting integral can easily be evaluated after we make the substitution $p \to z = (\varepsilon - \mu)/T \simeq (p - p_F)/T$:

$$\int_0^\infty dp n_F(\varepsilon)[1 - n_F(\varepsilon')] \approx \omega[\exp(\omega/T) - 1]^{-1}. \tag{18}$$

Here we take $\mu \approx \varepsilon_F \approx p_F$, and the limits of integration over z can be set to infinity by virtue of the fact that $p_F/T \gg 1$ [see (15)]. In (18), the axion energy (13) can be set to $\omega = up_F$, since $|\varepsilon' - \varepsilon_F| \lesssim T$ in the most important range, and $u \ll 1$. Gathering all of this together, including (18) and (16), we have for the luminosity (12)

$$Q_a = \frac{g_{ae}^2 m^5}{(4\pi)^2 \pi^{3/2}} \gamma_F^3 \int_0^\pi d\theta \sin\theta \int_0^\infty du u^4 \left[\exp\frac{up_F}{T} - 1 \right.$$
$$\left. \times \left(-\frac{1}{x} \Phi'(x) \right) \right]. \tag{19}$$

Here, x is given by (10), where the parameter χ of (8) is

$$\chi = \chi_F \sin\theta, \quad \chi_F = \gamma_F H/H_0, \tag{20}$$

in which $\gamma_F = p_F/m$ and θ is the angle between **p** and **H**.

The case of most interest is

$$T \ll T_c = \gamma_F^3 \omega_F. \tag{21}$$

This inequality (21) is satisfied over a wide range of neutron star parameters n_e and T.[8] Note that to order of magnitude, T_c equals (in the adopted system of units) the frequency at which the synchrotron spectrum peaks for an individual electron with energy $\varepsilon = \varepsilon_F \simeq p_F$ (see Ref. 6).

Drawing upon (15) and (21), we can simplify the integral in (19) considerably. The main contribution to the integral comes from $u \lesssim T/p_F \ll 1$ and $\sin\theta \sim 1$, and the argument of the Airy function is

$$x \lesssim \left(\frac{T}{p_F \chi_F} \right)^{2/3} = \left(\frac{T}{T_c} \right)^{2/3} \ll 1.$$

Hence,

$$\Phi'(x) \approx \Phi'(0) = -3^{1/6}\Gamma(2/3)/(2\sqrt{\pi}). \tag{22}$$

Substituting (22) and (20) into (19) and letting $u = (T/p_F)y$, we obtain

$$Q_a = \frac{3^{1/6}\Gamma(2/3)}{2^5 \pi^4} g_{ae}^2 m^5 \gamma_F^3 \chi_F^{2/3} \left(\frac{T}{p_F} \right)^{13/3}$$
$$\times \int_0^\pi d\theta \sin^{5/3}\theta \int_0^\infty dy y^{10/3} (e^y - 1)^{-1}.$$

Plugging in the expressions for the remaining tabulated integrals,[14] we finally have

$$Q_a=c_a g_{ae}^2 m^5 \gamma_F^{-2/3}\left(\frac{T}{m}\right)^{13/3}\left(\frac{H}{H_0}\right)^{2/3},$$

$$c_a=\frac{7}{2}\left(\frac{3}{4}\right)^{1/6}\frac{\Gamma^3(1/3)}{(3\pi)^4}\,\zeta\left(\frac{13}{3}\right)\approx 8.67\cdot 10^{-3}. \qquad (23)$$

4. For real applications, it is convenient to cast (23) in "astrophysical" form:

$$Q_a=1.59\cdot 10^{40} g_{ae}^2 \gamma_F^{-2/3} T_9^{13/3} H_{13}^{2/3}\ \text{erg/cm}^{-3}\cdot\text{s}, \qquad (24)$$

where $T_9=T/10^9$ K and $H_{13}=H/10^{13}$ H.

We now compare (24) with the neutrino luminosity Q_ν derived from neutrino synchrotron emission that was obtained in Ref. 8 under the same conditions:

$$Q_\nu=8.97\cdot 10^{14} T_9^5 H_{13}^2\ \text{erg/cm}^{-3}\cdot\text{s}, \qquad (25)$$

If we require that $Q_a<Q_\nu$, (24) and (25) yield

$$g_{ae}<2.38\cdot 10^{-13}\gamma_F^{1/3} T_9^{1/3} H_{13}^{2/3}. \qquad (26)$$

We now apply (26) to the shell of a neutron star,[8] which has $n_e\sim 10^{30}-10^{37}$ cm^{-3}, $T\sim 10^8-10^{10}$ K, and $H\sim 10^{12}-10^{14}$ G. We put $n_e=10^{33}$ cm^{-3}, $T=10^8$ K, and $H=10^{12}$ G. Then $\gamma_F=11.9$, $p_F=7.1\cdot 10^{10}$ K, $\omega_F=eH/p_F=1.1\cdot 10^7$ K, and $T_c=\gamma_F^3\omega_F=1.9\cdot 10^{10}$ K. Under these conditions, (15)–(17) and (21) are satisfied. Moreover, we know from astrophysics that the axion mass is extremely small:[3] 10^{-5} eV$\lesssim m_a\lesssim 10$ eV, and since $\omega\sim T\gg m_a$ [see (13) and (18)], we are justified in adopting a massless axion. As a result, (26) yields an upper bound on the coupling constant:

$$g_{ae}<5.4\cdot 10^{-14}. \qquad (27)$$

This is a much more stringent bound [see Eq. (4)] than the one found under red giant and white dwarf conditions.[5]

Taking $f=e$ and $c_e\sim 1$ in (2), (27) places a lower bound on the corresponding symmetry-breaking scale:

$$v_a\gtrsim 10^{10}\ \text{GeV}, \qquad (28)$$

which is consistent with recent observations of SN 1987A.[3]

Thus, under neutron star conditions with $g_{ae}\gtrsim 5\cdot 10^{-14}$, axion synchrotron energy losses ($e\to ea$) are comparable to neutrino synchrotron losses ($e\to e\nu\bar{\nu}$), which have been shown[8] to be competitive with other neutrino loss mechanisms. In closing, we note that by multiplying the right-hand side of (27) by $1/\sqrt{10}$, we can estimate the value of the coupling constant at which the axion luminosity is at least an order of magnitude less than the neutrino luminosity: $g_{ae}<1.7\cdot 10^{-14}$.

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