Heat transport inhibition in a weakly collisional plasma

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It is shown that the reason for the inhibition of electron heat transport in a weakly collisional plasma relative to the collisionless Knudsen limit is the long-range Coulomb interaction, the effect of which is that there are always slow charged particles for which collisions are important. It is shown that heating is due to slow collisional particles and that the energy absorbed in the inverse bremsstrahlung effect is transported by collisionless particles.

1. Experiments on the interaction of high-power laser radiation with a plasma, performed within the framework of the Controlled Inertial–uclear Fusion Program in various countries, have led to the conclusion that electron heat transport in a weakly collisional plasma does not correspond to the usual Knudsen limit but rather is substantially suppressed.[1–3] For the electron heat flux density, the formula

$$q=fn_e\kappa_B T_e v_{T_e}, \tag{1.1}$$

has come into use, where n_e is the electron number density, κ_B the Boltzmann constant, T_e the electron temperature, and $v_{T_e}=(\kappa_B T_e/m_e)^{1/2}$ is the electron thermal velocity. In conventional collisionless Knudsen transport in a gas, $f\sim 1$ (Ref. 4). This is in contrast with the experimental evidence for a much smaller heat transfer inhibition coefficient f, for which values from 0.1 to 0.03 have been quoted.[1–3]

Real physical experiments have been accompanied by numerical experiments that have led to a widespread belief that the reason for the relatively low value of the heat transfer inhibition coefficient is to be found in the theory of pairwise Coulomb interactions. It should be noted here that to account for heat transport inhibition in a plasma with ion–acoustic turbulence is a simple problem. To some extent, for a turbulent plasma the problem has been solved.[5] It is for this reason that in the present publication we omit possible turbulent effects completely and concentrate on the theory of a collisional plasma. It must be said that in the absence of turbulence, for a weakly collisional plasma with its practically collisionless transport the inhibition of the heat transport as compared to the Knudsen case has thus far appeared paradoxical.

In the present report we elucidate the essence of this phenomenon. It should be noted that collisionless heat transport is, by definition, nonlocal. On the other hand, there are a number of recent publications on nonlocal heat transport in a fully ionized weakly collisional plasma heated by high-frequency electromagnetic radiation.[6–8] The results therein are presented in a heat-transport inhibition form that differs from (1.1) and appears as a nonlocal integral relation between the heat flux density and the temperature gradient. For the Fourier transform of the effective electron thermal conductivity,

$$\kappa_{\text{eff}}(\mathbf{k})=\frac{\kappa_{SH}}{1+(\alpha|\mathbf{k}|\lambda_e)^\beta}. \tag{1.2}$$

Here α and β are certain numerical quantities, $\kappa_{SH}=(128/3\pi)n_e\kappa_B v_{T_e} l_{ei}$ is the electron thermal conductivity of a highly collisional, completely ionized plasma, $l_{ei}=v_{T_e}/\nu_{ei}$ is the mean free path of electrons with respect to their collisions with the ions. For the electron–ion collision frequency we have

$$\nu_{ei}=\frac{4(2\pi)^{1/2}e^2e_i^2n_i\Lambda}{3m_e^2v_{T_e}^3}, \tag{1.3}$$

where e is the electronic charge, $e_i=Z|e|$ the ion charge, n_i is the ion number density, and Λ the Coulomb logarithm. Finally,

$$\lambda_e=(2^{1/2}l_{ei}Z/3\pi^{1/2}(Z+1)^{1/2}). \tag{1.4}$$

In Refs. 7–10, Eq. (1.2) comes from numerical electron transport simulation. The values of the exponent β thus obtained are 4/3, 1.44, and 1.148. A somewhat weak point of the numerical simulation work is the difficulty in interpreting the results.

The analytic approach of Ref. 11 (see also Ref. 12) led to Eq. (1.2) with the parameters $\alpha=21.1$ and $\beta=10/7$, not much different from the numerical simulation results. On the other hand, Ref. 13 shows that the nonlocal thermal conductivity (1.2) leads to a heat transport inhibition coefficient of the form (1.1). This suggests that the paradoxical situation in regard to the heat transport inhibition may be resolved analytically.

It should be emphasized that Ref. 11 brought to light the previously unexamined role of cold electrons, with velocities much lower than the thermal velocity v_{T_e}, in the formation of an electron density perturbation by an electromagnetic field. In fact, one can infer from the quasihydrodynamic approach of Ref. 14 that the same cold electrons are responsible for most of the plasma temperature enhancement. The kinetic treatment below shows that this is indeed the case. This reveals the arbitrariness of the notion of temperature heating in a weakly collisional plasma. On the other hand, it is shown that heat transport in such a plasma is governed by the thermal collisionless electrons. The distinction we establish below between collisionless and collisional electrons originates from the feature of the long-range Coulomb interaction that there always exist slow particles for which collisions are of importance, even though most of the particle distribution may be considered collisionless. This is one of the qualitative differences between a gas of charged particles and an ordinary gas. This accounts for the difference between transport in a weakly collisionless plasma and in a conventional Knudsen gas. At the same time, our treatment enables one to speak of the relative character (as opposed to the notions of the conventional Knudsen kinetics) of the effective electron thermal conductivity (1.2) and, according to Ref. 13, also of the heat transport inhibition coefficient. Namely, the effective thermal conductivity of a weakly collisional plasma, Eq. (1.2), corresponds to the heating of cold (subthermal) collisional electrons and to the transfer of energy by thermal collisionless electrons.

2. To consider processes in a completely ionized plasma heated by electromagnetic radiation we shall assume that the plasma is in a high-frequency field whose electric strength is

$$\mathscr{E}(\mathbf{r},t)=1/2\mathbf{E}(\mathbf{r},t)\exp(-i\omega_0 t)+\text{c.c.} \tag{2.1}$$

We will assume that

$$\omega_0 \gg \nu_{ei}. \tag{2.2}$$

Also, we assume that

$$\omega_0 \gg (v_{T_e}/L_E), \tag{2.3}$$

where L_E is the scale of the spatial variation of the field amplitude $\mathbf{E}(\mathbf{r},t)$ which, by assumption, varies little with time over the period $2\pi/\omega_0$. Then, following Ref. 15, we divide the electron distribution function into a rapidly varying part with period $2\pi/\omega_0$, and one slowly varying over this period, and then proceed in the standard way to obtain an equation for the slow distribution function f, which involves terms quadratic in the strength of the high-frequency electric field (cf. Ref. 16). In accordance with the approach of Refs. 11 and 16, let

$$f=f_M+\delta f(\mathbf{v})\exp(i\mathbf{kr}), \tag{2.4}$$

where f_{M} is a Maxwellian distribution. Equation (2.4) restricts us to steady-state processes. We will also use the notation

$$E_iE_j^* \to \exp(i\mathbf{kr})E_iE_j^*, \tag{2.5}$$

$$\delta\varphi \to \exp(i\mathbf{kr})\delta\varphi. \tag{2.6}$$

This, in accordance with Ref. 11, enables the electron distribution perturbation f to be represented in the form

$$\delta f=f_M\left[\left(\frac{v^2}{3v_{T_e}^2}-2\right)I-\frac{e\delta\varphi}{\kappa_B T_e}\right]+\frac{1}{2}\Xi f_M+\delta f_c, \tag{2.7}$$

where

$$I=\frac{e^2|\mathbf{E}|^2}{4m_e^2\omega_0^2 v_{T_e}^2}, \tag{2.8}$$

$$\Xi=\frac{e^2(v_iv_j-\frac{1}{3}\delta_{ij}v^2)}{4m_e^2\omega_0^2 v_{T_e}^4}\left(E_iE_j^*+E_i^*E_j-\frac{2}{3}\delta_{ij}\left|\mathbf{E}\right|^2\right), \tag{2.9}$$

and the function δf_c obeys

$$i\mathbf{kv}\delta f_c-J_{ee}[\delta f_c]-J_{ei}[\delta f_c]=Y_0+Y_a. \tag{2.10}$$

Here

$$Y_0=-\nu_{ei}I(2\pi)^{1/2}v_{T_e}^3\frac{\partial}{\partial\mathbf{v}}\left(\frac{\mathbf{v}}{v^3}f_M\right), \tag{2.11}$$

$$Y_a=\nu_{ei}\Xi\left(3-\frac{v^2}{2v_{T_e}^2}\right)\frac{3\pi^{1/2}v_{T_e}^5}{2^{1/2}v^5}, \tag{2.12}$$

and J_{ee} and J_{ei} are the electron–electron and electron–ion collision integrals. In the electron–ion collision integral we neglect small terms of the order of the electron-to-ion mass ratio:

$$J_{ei}[\delta f]=\nu(v)\frac{\partial}{\partial v_k}\left[(v^2\delta_{rs}-v_rv_s)\frac{\partial\delta f}{\partial v_s}\right]. \tag{2.13}$$

Here

$$\nu(v)=3(\pi/8)^{1/2}\nu_{ei}(v_{T_e}/v)^3. \tag{2.14}$$

It is to be noted that when δf_c is neglected, Eq. (2.7) yields for the electron density

$$\delta n_e^{(0)}=-n_e\left[I+\frac{e\delta\varphi}{\kappa_B T_e}\right]. \tag{2.15}$$

This corresponds to a change in the electron density due to the electric potential (Boltzmann distribution) and to the Miller force potential (ponderomotive force). Now, defining the change in the electron kinetic energy according to

$$\delta\frac{3}{2}n_e\kappa_B T_e=\int d\mathbf{v}\frac{m_ev^2}{2}\delta f, \tag{2.16}$$

we obtain, neglecting δf_c,

$$\frac{3}{2}n_e\kappa_B\delta T_e^{(0)}=\frac{1}{2}n_e\kappa_B T_e I=n_e\frac{e^2|\mathbf{E}|^2}{8\omega_0^2 m_e}. \tag{2.17}$$

The last expression is the period-averaged energy of electronic oscillations in the electric field, so it can only tentatively be linked to temperature change.

From here on, we assume a weakly collisional plasma, with

$$kl_{ei}\equiv kv_{T_e}/\nu_{ei}\gg 1. \tag{2.18}$$

We will, however, distinguish between electrons with sufficiently high velocities and slow electrons, the two groups being affected in a qualitatively different manner by collisions. This is due to the fact that there are long-range Coulomb forces acting between plasma particles and that these lead to the effective frequency (2.14), which proves to be large for sufficiently slow particles. The same velocity dependence is characteristic of electron–electron collisions.

3. In the weakly collisional plasma of interest here, when the inequality (2.19) holds, collisions prove to be of no significance over most of the electron-velocity phase space. As far as Eq. (2.10) is concerned, this means that for electron velocities satisfying

$$kv\gg\nu(v) \tag{3.1}$$

or, equivalently for velocities which are not too low,

$$v > v_{\min} = v_{T_e} \frac{1}{(kl_{ei})^{1/4}}, \tag{3.2}$$

the collision integrals can be omitted. Then for the velocities (3.2), Eq. (2.10) becomes simply

$$i\mathbf{kv}\delta f_c = Y_0 + Y_a. \tag{3.3}$$

The solution has the form

$$\delta f_{c,T}(\mathbf{v}) = -i(Y_0 + Y_a)\left(\frac{P}{\mathbf{kv}} + i\pi\delta(\mathbf{kv})\right), \tag{3.4}$$

where P denotes the Cauchy principal value and the δ function arises from Landau's rule for avoiding singularities. Treating the singularity in this way can be directly justified by including electron collisions as a small effect and letting ν/kv go to zero.

From Eq. (3.4), the contribution of electrons with moderate velocities (3.2) to the electron density perturbation and to the mean electron kinetic energy due to the high-frequency field are of order

$$\delta n_{e,T} \sim n_e I \frac{1}{kl_{ei}}, \quad \delta T_{e,T} \sim T_e I \frac{1}{kl_{ei}}. \tag{3.5}$$

This is kl_{ei} times less than the corresponding values (2.15) and (2.16). Here and in what follows, k denotes the absolute value of the vector $\mathbf{k}$. One therefore concludes that the increase in the energy of thermal electrons is small compared to their oscillatory energy due to the permanently acting force from the externally applied electromagnetic field.

Subsequent discussion relies on the implication of Eq. (3.4) for the nonequilibrium electron heat flux. Thus, for the divergence of the energy flux density of thermal electrons we obtain

$$i\mathbf{k}\mathbf{q}^{(0)} = \frac{1}{2} m_e \int d\mathbf{v} i(\mathbf{kv}) v^2 \delta f_{c,T}(\mathbf{v}) = 2\nu_{ei} n_e \kappa_B T_e I$$
$$= n_e \frac{e^2 |\mathbf{E}|^2}{2 m_e \omega_0^2} \nu_{ei}. \tag{3.6}$$

The error in this expression is due to the low velocities, and is governed by the parameter

$$\frac{v_{\min}^2}{v_{T_e}^2} \sim \frac{1}{(kl_{ei})^{1/2}}. \tag{3.7}$$

Neglecting small terms of this order, the expression above for the divergence of the energy density transferred by electrons is just the energy absorbed by the plasma per unit time owing to the inverse bremsstrahlung effect. In fact, Eq. (3.6) follows directly from the electron kinetic equation for steady-state processes and for the exact electron distribution function [see, e.g., Eq. (5) of Ref. 14]. This enables an important assertion to be made, that heat transport in a weakly collisional plasma occurs via collisionless electrons, whose nonequilibrium distribution is described by Eq. (3.4).

Noting that the temperature increase in the thermal electrons is given to order of magnitude by Eq. (3.5), one easily obtains from Eq. (3.6) the following electron heat flux expression:

$$q \sim n_e \kappa_B v_{T_e} \delta T_{e,T}. \tag{3.8}$$

This is a natural result for the collisionless Knudsen-type transport process in which thermal electrons transfer the energy increase they acquire. We emphasize that this result does not involve any significant inhibition of the heat transport.

4. We next turn our attention to the role of slow, or cold, electrons, whose velocities will be assumed to obey the inequality

$$kv < \nu(v) \tag{4.1}$$

or

$$v < v_{\max} < v_{T_e} \frac{1}{(kl_{ei})^{1/4}}. \tag{4.2}$$

Note that the $v_{\max}$ of this section equals, to order of magnitude, $v_{\min}$ of the preceding section, which is natural as being consistent with the distinction between the collisional and collisionless regions of velocity phase space. In fact, for cold electrons with velocities (4.2), collisions dictate the form of their velocity distribution.

Equation (2.10), with inequality (2.18) for slow electrons, was solved in Ref. 11 under the assumption of high ionization

$$Z = |e_i/e| \gg 1, \tag{4.3}$$

which we also will employ. Then the perturbation of the cold electron distribution has the form

$$\delta f_c = \delta f_0 \left\{1 - \frac{i\mathbf{kv}}{2\nu(v)}\right\} + \frac{1}{6\nu(v)} Y_a, \tag{4.4}$$

where the symmetric part (in velocities δf_0) of the perturbation of the electron distribution is determined by the equations[11]

$$\delta f_0 = \frac{9\pi}{8} \frac{I}{k^2 l_{ei}^2} f_M(v) \Phi\left(\frac{v^2}{2 v_{T_e}^2}\right), \tag{4.5}$$

$$\Phi(x) = \tilde{\Phi}(\xi)$$
$$= -\frac{4}{\pi}\left(\frac{2}{7}\right)^{1/7} \Gamma\left(\frac{6}{7}\right) \sin\frac{\pi}{7} N^{8/7} \frac{1}{\xi^{1/4}} K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)$$
$$+ \frac{4}{7} N^{6/7} \frac{1}{\xi^{1/4}} \left\{ I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right) \right.$$
$$\times \int_\xi^\infty \frac{dy}{y^{1/4}} K_{1/7}\left(\frac{4}{7} y^{7/4}\right)$$
$$\left. + K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right) \int_0^\xi \frac{dy}{y^{1/4}} I_{1/7}\left(\frac{4}{7} y^{7/4}\right) \right\}, \tag{4.6}$$

where $\xi = XN^{2/7}$, $I_{1/7}$ and $K_{1/7}$ are the Bessel functions of imaginary argument, Γ is the Euler Γ function, and finally

$$N = \frac{4\sqrt{2\pi}}{9} Zk^2 l_{ei}^2. \tag{4.7}$$

The solution (4.6) is for the asymptotic limit $N \gg 1$. It is worthwhile to note a useful asymptotic representation

$$\Phi(X)=\frac{1}{X^3}=\frac{N^{6/7}}{\xi^3}, \tag{4.8}$$

which also obtains for small values of X, when $N \gg NX^{7/2} \gg 1$, that is, when $\xi \gg 1$. For the variable ξ, the inequality (4.2) has the form

$$\xi < Z^{2/7}(kl_{ei})^{1/14}. \tag{4.9}$$

The right-hand side of Eq. (4.9) is large compared to unity.

In Ref. 11 it is shown that the distribution (4.4) leads to the following electron density perturbation:

$$\delta n_e=\delta n_{e,c}=-n_e I\,\frac{1.73\ Z^{5/7}}{(kl_{ei})^{4/7}}\ . \tag{4.10}$$

The main contribution to the formation of such a perturbation is given by that region of velocity phase space in which $\xi \sim 1$, or equivalently

$$v \sim v_{T_e} N^{-1/7}. \tag{4.11}$$

These velocity values lie in the region (4.2), and it is this fact which ensures the asymptotic validity of Eqs. (4.4)–(4.6).

The electron density perturbation (4.10) due to the cold electrons exceeds that due to the ponderomotive force (2.15) if

$$1.73\ Z^{5/7} > (kl_{ei})^{4/7} \gg 1. \tag{4.12}$$

It is under these conditions that a correct description of the cold electrons leads to qualitative changes in the theory of the filamentation of laser radiation in a plasma (as discussed in Ref. 11) and in the theory of the stimulated Mandelstam–Brillouin scattering (as discussed in Ref. 17).

On the other hand, it is to be noted that the density perturbation (4.10), due to the cold collisional electrons, is always large compared to the electron density perturbation (3.5) due to the thermal collisionless electrons. We will show now that the same conclusion about thermal energy perturbation follows from the distribution (4.4)–(4.6). To do so, we find the perturbation

$$\begin{aligned}\delta(\tfrac{3}{2}\, n_e\kappa_B T_e)_c &= \tfrac{3}{2}\,\kappa_B T_e \delta n_{e,c} + \tfrac{3}{2}\, n_e \kappa_B \delta T_{e,c} \\ &= \tfrac{1}{2}\, m_e \int_{v<v_{\max}} d\mathbf{v} v^2 \delta f_c \\ &= \frac{9\pi^{1/2} n_e \kappa_B T_e I}{4k^2 l_{ei}^2} \int_0^{X_{\max}} dX X^{3/2}\Phi(X), \end{aligned} \tag{4.13}$$

where $\xi_{\max}$ is defined by Eq. (4.2). It is readily seen that

$$\delta\left(\frac{3}{2}\, n_e\kappa_B T_e\right)_c \sim n_e \kappa_B T_e \frac{IZ^{2/3}}{(kl_{ei})^{8/7}} \ll \kappa_B T_e \delta n_{e,c}\,. \tag{4.14}$$

Therefore, neglecting small terms of the order of (4.14), it can be concluded that

$$\delta T_{e,c} = T_e I\,\frac{1.73\ Z^{5/2}}{(kl_{ei})^{4/7}}\ . \tag{4.15}$$

Such an expression was previously obtained by considering hydrodynamic pressure balance.[14] Now it has been obtained directly from the electron velocity distribution. It is to be emphasized here that the expression (4.15) has nothing to do with the conventional temperature, but corresponds instead to the increase predicted by the kinetic theory of very slow (cold) electrons with velocities (4.11). However, the increase in this cold electron temperature turns out to be greater than the increase in the thermal energy of the thermal electrons, Eq. (3.5). Finally, the increase in the cold electron temperature, $\delta T_{e,c}$, turns out to exceed the mean energy of electron oscillations in an external electromagnetic field when the inequality (4.12) holds.

It remains to consider the energy flux density of the cold electron component,

$$\mathbf{q}_c = \frac{1}{2}\, m_e \int_{v<v_{\max}} d\mathbf{v} v^2 \mathbf{v}\, \delta f_c\,. \tag{4.16}$$

As with the thermal electrons, we consider the flux density divergence, for which we obtain

$$i\mathbf{k}\mathbf{q}_c = \frac{k^2 m_e}{12} \int_{v<v_{\max}} d\mathbf{v}\, \frac{v^4}{\nu(v)}\, \delta f_0 = 2\nu_{ei} n_e \kappa_B T_e I J, \tag{4.17}$$

where

$$J=\int_0^{X_{\max}} dX X^4 \Phi(X). \tag{4.17'}$$

The two terms in Eq. (4.6) yield for J the simple upper bounds $(Zk^2 l_{\rm ei}^2)^{-2/7}$ and $(kl_{\rm ei})^{-1}$, respectively. Either is small compared to unity. This confirms the earlier conclusion that the energy of the absorbed radiation is transported by collisionless thermal electrons, and that cold electrons are of no significance for this transport.

The above analysis facilitates the separate treatment of the effects due to collisionless thermal electrons and those due to collisional cold electrons. The qualitative difference in their behavior appears to be an important feature of a weakly collisional plasma as a system of particles with the Coulomb interaction.

5. The analysis above shows that the main contribution to the heating of a weakly collisional plasma, due to the inverse bremsstrahlung absorption of electromagnetic radiation, comes from the increase in the thermal energy of the cold electrons with velocities of the order of (4.11), for which electron Coulomb collisions are the major determinant of the distribution. At the same time, heat transport is mainly due to collisionless thermal electrons. We have shown that if one associates such transport with the relatively small heating of the thermal electrons themselves, no heat transport inhibition occurs and, according to Eq. (3.8), the conventional Knudsen picture of collisionless electron heat transport arises. This simple kinetic picture of electron heat transport is valid only for the thermal electron component. However, the picture looks entirely different when using the notion of an electron gas as a whole, without dividing it into a hot and a cold part. The heat transport description using the flux divergence (3.6), but with the temperature increase (4.15), enables Eq. (3.6) to be rewritten in the form

$$\operatorname{div}\mathbf{q} = 2\nu_{ei} n_e \kappa_B \delta T_e\, \frac{(kl_{ei})^{4/7}}{1.73\ Z^{5/7}}\ . \tag{5.1}$$

Here we make no distinction between $q^{(0)}$ and q, nor between $\delta T_{e,c}$ and δT_e, as this distinction was shown to be of no consequence in the discussion above. Expression (5.1) has been written earlier in Ref. 14 in the development of a phenomenological description of nonlocal heat transport. Equation (5.1) allows a representation of the form

$$\operatorname{div} \mathbf{q} = -k^2 \kappa_{\text{eff}}(\mathbf{k}) \delta T_e, \tag{5.2}$$

where the effective thermal conductivity κ_{eff} has a form corresponding to Eq. (1.2) without the 1 in the denominator, the latter being included with a view to passing to the strongly collisional limit.

It is thus apparent that electron heat transport inhibition in a weakly collisional plasma is due to the fact that the energy absorbed in a plasma is transported by hot collisionless electrons and that the heating process occurs for the cold low-velocity electrons. The fact that a hydrodynamically averaged description involves only one electron component makes heat transfer by this single component paradoxical, at first glance. However, in contrast to a simple Knudsen gas of atoms with a short-range potential, in a gas of interacting charged particles, one finds that even in the usual collisionless limit (2.18) there are always slow collisional particles obeying Eq. (4.2).

In conclusion, recall that in certain other cases as well, the application of the concept of a gas with short-range forces to plasma problems has caused difficulties in understanding the properties of a system of particles with long-range Coulomb forces.

Finally, note that the above properties of the electron component of a weakly collisional plasma are undoubtedly shared to some extent by the ion component, whose properties have yet to be explored theoretically.

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APPENDIX. ROLE OF THERMAL AND COLD ELECTRONS IN THE COMPENSATION OF THE DRAG CURRENT

Consider the divergence of the drag current density

$$\operatorname{div} \mathbf{j} = i\mathbf{kj} = ie \int d\mathbf{v}(\mathbf{kv}) \delta f_c. \tag{A1}$$

Substituting Eq. (3.4), which corresponds to thermal collisionless electrons,

$$\operatorname{div} \mathbf{j}_T = -en_e I \frac{\nu_{ei}}{2\pi} \int dv \frac{\partial}{\partial \mathbf{v}} \left(\frac{\mathbf{v}}{v^3} e^{-v^2/2v_{Te}^2} \right). \tag{A2}$$

We now discuss the above integral in a little more detail. Since

$$\frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{v}}{v^3} = -\Delta_{\mathbf{v}} \frac{1}{v} = 4\pi\delta(\mathbf{v}), \tag{A3}$$

and noting that, by the collisionless inequality (3.2), the low-velocity region must be excluded from the integration in Eq. (A2), the $\delta(\mathbf{v})$ contribution will not be taken into account. Then

$$\operatorname{div} \mathbf{j}_T = 2en_e I \nu_{ei}. \tag{A3$'$}$$

On the other hand, note that if we had included the contribution (A3) then, being of opposite sign, it would completely cancel the right-hand side of Eq. (A3).

Now let us turn to the divergence of the drag current due to cold electrons,

$$\operatorname{div} \mathbf{j}_c = \frac{ek^2}{6} \int_{v<v_{\max}} d\mathbf{v} \frac{v^2}{\nu(v)} \delta f_0 = 2en_e \nu_{ei} I J_j, \tag{A4}$$

where

$$J_j = \int_0^{X_{\max}} dX X^3 \Phi(x). \tag{A5}$$

It is readily seen that the neighborhood of the upper limit, corresponding to the asymptotic form (4.8), contributes to (A5) a quantity of order $(kl_{\text{ei}})^{-1/2}$, which is negligible to within the numerical errors. Then

$$J_j = -\frac{4}{\pi} \left(\frac{2}{7}\right)^{1/7} \Gamma\left(\frac{6}{7}\right) \sin\frac{\pi}{7} \int_0^\infty d\xi \times \xi^{11/4} K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right) = -1. \tag{A6}$$

Hence

$$\operatorname{div} \mathbf{j}_c = -2en_e I \nu_{ei}. \tag{A7}$$

Thus, the cold electron contribution cancels the thermal electron contribution (A3) completely. It can be said that the cold electron distribution (4.4)–(4.6) enables one to discuss in a sensible manner the influence of the singularity (A3) on the drag current compensation effect.

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