Suppression of photocurrent shot noise in a feedback loop

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(Submitted 14 March 1994; resubmitted 27 July 1994)
Zh. Eksp. Teor. Fiz. **106**, 1278–1285 (November 1994)

We employ the theory of continuous quantum measurements to examine the detection of an electromagnetic field when feedback exists between the detector and the source of the field. We obtain the spectrum of photocurrent fluctuations for both open-loop and closed-loop detectors. We demonstrate that even though the noise has been reduced in the latter, the field in the feedback loop is not nonclassical. © *1994 American Institute of Physics.*

Suppressing fluctuations in an optical beam below the shot-noise limit is one of the central problems of contemporary quantum optics, due in particular to the possibility of markedly enhancing the signal-to-noise ratio in optoelectronic circuits in the presence of sources of such radiation.[1,2] It is not difficult to show that such light does not yield to a description in classical terms, and this is one of the distinguishing features of a squeezed state of the electromagnetic field.

Indeed, it is well known[3] that photodetector shot noise can be explained by the impulsive nature of the photocurrent, which relates to the fact that atomic ionization produced by the photoelectric effect is a discrete process. Describing a random current in the form

$$i(t)=e\sum_k \delta(t-t_k), \tag{1}$$

where e is the elementary charge and the t_j ($j=1,2,\ldots$) are random arrival times of photoelectrons, we obtain[4]

$$\langle i(t)\rangle = eh_1(t), \tag{2}$$

$$\langle \delta i(t_0)\delta i(t)\rangle = e^2h_1(t)\delta(t-t_0)+e^2g_2(t_0,t), \tag{3}$$

where

$$g_2(t_0,t)=h_2(t_0,t)-h_1(t_0)h_1(t). \tag{4}$$

Here $h_l(t_1,\ldots,t_l)$ denotes the so-called coincidence probability density,[5] which is the probability density for photocounts being produced at times $t_1,\ldots,t_l$ (with counts possibly occurring at other times as well).

The set of functions $h_l(t_1,\ldots,t_l)$, $l=1,2,\ldots$ completely characterizes the random point process of photoelectron production, and thereby the statistics of the photocurrent. The relationship between these functions and the statistical properties of the incident radiation was first derived by Glauber.[6] In the P representation,[6,7] which expands $\hat{E}^{(+)}(\mathbf{r},t)$ (coherent states) in eigenstates of the free field operator $\hat{E}^{(+)}(\mathbf{r},t)|\mathscr{E}^{(+)}(\mathbf{r},t)\rangle=\mathscr{E}^{(+)}(\mathbf{r},t)|\mathscr{E}^{(+)}(\mathbf{r},t)\rangle$ the Glauber relation takes the form

$$h_l(t_1,\ldots,t_l)=\lambda^l\langle I(t_1)\ldots I(t_l)\rangle_P, \tag{5}$$

where λ is the detector efficiency. Averaging denoted by $\langle\ldots\rangle_P$ in Eq. (5) is performed over P-functions, which are positive definite for classical fields, but can be negative or singular for pure quantum fields. The random quantity $I(t)$ signifies the field intensity at the surface of the detector D:

$$I(t)=\frac{c}{4\pi}\int_D \mathscr{E}^{(-)}(\mathbf{r},t)\mathscr{E}^{(+)}(\mathbf{r},t)d^2\mathbf{r}. \tag{6}$$

For a coherent field, $\langle I(t_1)I(t_2)\rangle_P=\langle I(t_1)\rangle_P\langle I(t_2)\rangle_P$, and the photocounts have Poisson statistics: $h_2(t_1,t_2)=h_1(t_1)h_1(t_2)$. For an incoherent field, we have in general

$$(\delta i^2)_\omega\equiv\int_{-\infty}^{+\infty}\langle\delta i(t)\delta i(t+\tau)\rangle e^{-i\omega\tau}d\tau=e\langle i\rangle + e^2\lambda^2S_{II}(\omega), \tag{7}$$

where the first term corresponds to shot noise, and

$$S_{II}(\omega)=\int_{-\infty}^{+\infty}\langle\delta I(t)\delta I(t+\tau)\rangle_P e^{-i\omega\tau}d\tau \tag{8}$$

is the intensity spectrum of the incident radiation. According to the Wiener–Khinchin theorem,[8] this spectrum is nonnegative at all frequencies for any classical random process $I(t)$. Consequently, a photocurrent noise spectrum can only drop below the shot noise limit when the light being detected is in a pure quantum state. The best known forms of such nonclassical states are fields with sub-Poissonian statistics for the photon numbers (at measurement times much greater than the coherence time), for which $S_{II}(0)<0$, and fields with anticorrelated photons, for which $\int_0^{+\infty}S_{II}(\omega)d\omega<0$. Obviously, however, the most general manifestation of nonclassical behavior that can be directly detected is a photocurrent spectral intensity (at some frequency) below the shot noise level.

Beginning in 1977,[9] various schemes for generating nonclassical fields were proposed and evaluated, but in most, the power carried by nonclassical light and the extent to which the statistics deviated from Poisson noise were extremely small.[10,11] It was for this reason that the experiments carried out by Yamamoto and his colleagues[12] attracted so much attention: they exhibited a five-fold suppression of shot noise over the frequency range from 0 to 15 MHz (the width of a laser mode), using amplitude modulation of a single-mode laser in a feedback loop (Fig. 1; for similar schemes, see also Refs. 13 and 14). One interesting aspect of these experiments

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was that the noise level of light diverted from the feedback loop by a beamsplitter (sampling detector D2) was above the shot noise level at all frequencies—in other words, it did not behave nonclassically. This gave rise to a number of theories on detection in a feedback loop,[12,14–16] which indicate that the presence of feedback significantly affects the field-splitting process. It should be pointed out that all of these theories are either implicitly or explicitly based on Eq. (5), and therefore imply that nonclassical light is generated in a feedback loop.

In the present paper, we propose another description of experiments that deal with detection in a feedback loop, based on the rapidly developing theory of continuous quantum measurements.[5,17,18] The need for reconsideration results from the fact that the Glauber approach holds only for fields whose sources are not correlated with the atomic states in the detector (cf. Ref. 19), so that Eq. (5) does not apply to detection in a feedback loop.

The theory of continuous photodetection, apart from the functions $h_l(t_1,\ldots,t_l)$, $l=1,2,\ldots$, operates on a set of "elementary probability densities" $p^{(n)}_{[0,t)}(t_1,\ldots,t_n)$, $n=0,1,2,\ldots$, each of which is the probability that exactly n counts are detected at times $t_1,\ldots,t_n$ in the half-open interval $[0,t)$, and that no counts are detected at any other time. These functions also completely determine the photocount statistics over the interval $[0,t)$, and are related to the functions $h_l(t_1,\ldots,t_l)$ (see, e.g., Ref. 4):

$$p^{(n)}_{[0,t)}(t_1,\ldots,t_n)=\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\times\int_0^t\ldots\int_0^t h_{l+n}\times(t_1,\ldots,t_n,t_1',\ldots,t_l')dt_1'\ldots dt_l', \tag{9}$$

$$h_l(t_1,\ldots,t_l)=\sum_{l=0}^{\infty}\frac{1}{n!}\int_0^t\ldots\int_0^t p^{(n+l)}_{[0,t)}\times(t_1,\ldots,t_l,t_1',\ldots,t_n')dt_1'\ldots dt_n'. \tag{10}$$

In what follows, we make use of the fact that for a wide range of modulation schemes based on a linear interaction between the current and field (the electrooptic modulator employed in Ref. 14, or the slow modulation—compared to the lifetime of the upper lasing level—of the laser pump rate above threshold, as used in Ref. 12), the coherent state $|\alpha\rangle$ is transformed by the modulation into the coherent state $|\alpha'\rangle$ with a different amplitude.[6] In the process, the average in (5) reduces to an average over the initial intensity I_0 (prior to modulation), the distribution of which is unrestricted—it can be positive definite (classical) or not (essentially quantum mechanical). Since practical interest is mainly concentrated on the transformation of coherent radiation into nonclassical radiation, we will assume that the initial state is coherent, and that I_0 is constant.

For the detection of a nonsteady coherent field with intensity $I(t)$, Eqs. (5) and (9) yield the elementary probability densities

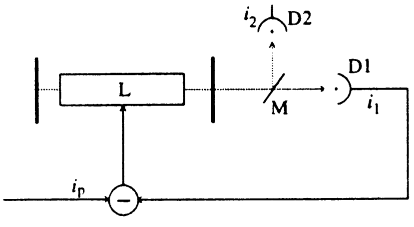

FIG. 1. Optical layout used in Ref. 12. L—laser; D1 and D2—detectors; M—beamsplitting mirror. The photocurrent from detector D2 modulates the laser pump current i_p via the feedback loop.

$$P^{(n)}_{[0,t)}(t_1,\ldots,t_n)=\lambda^n I(t_1)\ldots I(t_n)\exp\left\{-\lambda\int_0^t I(\tau)d\tau\right\}. \tag{11}$$

A rigorous examination of detection in a feedback loop in the context of the theory of continuous quantum measurements shows[20] that when an interaction exists between the field (or its sources) and the photocurrent in the detector (resulting in a dependence of $I(t)$ on the photocount times of arrival t_j), Eq. (11) is still valid, but Eq. (5) is not. Transformation to the functions $h_l(t_1,\ldots,t_l)$ via Eq. (10) then becomes nontrivial by virtue of the dependence of $I(t_n)$ on the time of arrival of previous photocounts, $t_j<t_n$. For amplitude modulation of coherent light, this dependence takes the form

$$I(\tau)=I_0+\sum_j F(\tau-t_j), \tag{12}$$

where the t_j, $j=1,2,\ldots$ are the arrival times of photocounts from $D1$ (Figs. 1, 2), and $F(\tau)$ is the response of the entire feedback loop to a δ-function current impulse at the detector [$F(\tau)=0$ at $\tau<0$, so only counts with $t_j<\tau$ contribute to (12)].

The results of measurements in the two-detector scheme of Figs. 1 and 2 can be best described by the joint probability density for the compound event that during the interval $[0,t)$, detector D1 registered exactly n counts at times $t_1,\ldots,t_n$ (event A), and detector D2 registered exactly m counts at times $t_1',\ldots,t_m'$ (event B), which is equal to the product of the probability of event A and the probability of event B under the condition that A actually occurred:

$$P^{(n,m)}_{[0,t)}(t_1,\ldots,t_n,t_1',\ldots,t_m')$$
$$=\lambda_1^n I(t_1)\ldots I(t_n)$$

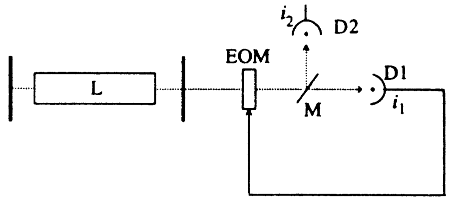

FIG. 2. Optical layout used in Ref. 14. Notation as in Fig. 1. Feedback is implemented via the electrooptic modulator (EOM).

$$\times\exp\left\{-\lambda_1\int_0^t I(\tau)d\tau\right\}\lambda_2^m I(t_1')\dots I(t_m')$$

$$\times\exp\left\{-\lambda_2\int_0^t I(\tau)d\tau\right\}. \tag{13}$$

Here λ_1 and λ_2 are the efficiencies of detectors D1 and D2 with allowance for the beamsplitter (i.e., they include a factor of ε and $1-\varepsilon$, respectively, where ε is the (intensity) transmission coefficient of beamsplitter M), and event A specifies the intensity in accordance with (12). Note that in the present approach, the transformation of the field state at the beamsplitter is of the form $|\alpha\rangle\rightarrow|\varepsilon^{1/2}\alpha\rangle$, which is equivalent to the unitary transformation of field operators in Refs. 12 and 14–16.

The mean value and the photocurrent fluctuation spectrum can be obtained for each detector by averaging (13) over the readings of the other detector and carrying out the calculations indicated in Eqs. (1)–(4) and (10).

For a detector in the feedback loop, we have

$$h_1^{\text{in}}(t)=\sum_{n=0}^{\infty}\frac{\lambda_1^{n+1}}{n!}\int_0^t dt_n\dots\int_0^t dt_1\left(I_0+\sum_{j=1}^{n}F(t-t_j)\right)I(t_n)\dots I(t_1)\exp\left\{-\lambda_1\int_0^t I(\tau)d\tau\right\}$$

$$=\lambda_1 I_0+\lambda_1\int_0^t F(t-t')h_1^{\text{in}}(t')dt'. \tag{14}$$

Similarly, for the out-of-loop detector,

$$h_1^{\text{out}}(t')=\lambda_2 I_0+\lambda_1\int_0^t F(t-t')h_1^{\text{out}}(t')dt'. \tag{15}$$

It is also straightforward to obtain integral equations corresponding to the two-time coincidence probability densities for the closed-loop detector and the out-of-loop detector, $h_2^{\text{in}}(t,t_0)$ and $h_2^{\text{out}}(t,t_0)$:

$$h_2^{\text{in}}(t,t_0)=\sum_{n=0}^{\infty}\frac{\lambda_1^{n+2}}{n!}\int_0^t dt_n\dots\int_0^t dt_1\left(I_0+F(t-t_0)+\sum_{j=1}^{n}F(t-t_j)\right)\left(I_0+\sum_{j=1}^{n}F(t_0-t_j)\right)$$

$$\times\tilde{I}(t_n)\dots\tilde{I}(t_1)\exp\left\{-\lambda_1\int_0^t\tilde{I}(\tau)d\tau\right\}$$

$$=\lambda_1[I_0+F(t-t_0)]h_1^{\text{in}}(t_0)+\lambda_1\int_0^t F(t-t')h_2^{\text{in}}(t',t_0)dt', \tag{16}$$

$$h_2^{\text{out}}(t,t_0)=\sum_{n=0}^{\infty}\frac{\lambda_2^2\lambda_1^n}{n!}\int_0^t dt_n\dots\int_0^t dt_1\left(I_0+\sum_{j=1}^{n}F(t-t_j)\right)\left(I_0+\sum_{j=1}^{n}F(t_0-t_j)\right)$$

$$\times I(t_n)\dots I(t_1)\exp\left\{-\lambda_1\int_0^t I(\tau)d\tau\right\}$$

$$=\lambda_2 I_0 h_1^{\text{out}}(t_0)+\lambda_1\int_{t_0}^t F(t-t')h_2^{\text{out}}(t',t_0)dt'$$

$$+\frac{\lambda_2^2}{\lambda_1}\int_0^{t_0}F(t-t')h_2^{\text{in}}(t_0,t')dt', \tag{17}$$

where

$$\tilde{I}(t)=I_0+F(t-t_0)+\sum_{j=1}^{n}F(t-t_j), \tag{18}$$

$$I(t)=I_0+\sum_{j=1}^{n}F(t-t_j), \tag{19}$$

and the integral equations have been derived, under the assumption that $t\geqslant t_0$, by expanding the first set of parentheses in the n-fold multiple integral.

We can then work our way from Eqs. (16) and (17) to the equations for the $g_2(t,t_0)$ defined by Eq. (4):

$$g_2^{\text{in}}(t,t_0)=\lambda_1 F(t-t_0)h_1^{\text{in}}(t_0)+\lambda_1\int_0^t F(t-t')g_2^{\text{in}}(t',t_0)dt', \tag{20}$$

$$g_2^{\text{out}}(t,t_0)=\lambda_1\int_{t_0}^t F(t-t')g_2^{\text{out}}(t',t_0)dt'$$

$$+\frac{\lambda_2^2}{\lambda_1}\int_0^{t_0}F(t-t')g_2^{\text{in}}(t_0,t')dt'. \tag{21}$$

To solve (20) and (21), we introduce the function

$$G(t,t_0)=\left(\frac{g_2^{\text{in}}(t,t_0)}{\lambda_1^2}-\frac{g_2^{\text{out}}(t,t_0)}{\lambda_2^2}\right)\frac{\lambda_1}{h_1^{\text{in}}(t_0)}. \tag{22}$$

We can then rewrite (20) and (21) in the form

$$G(t,t_0)=F(t-t_0)+\lambda_1\int_{t_0}^t F(t-t')G(t',t_0)dt', \tag{23}$$

$$g_2^{\text{out}}(t,t_0)=\lambda_1\int_0^t F(t-t')g_2^{\text{out}}(t',t_0)dt'$$

$$+\lambda_2\lambda_1\int_0^{t_0}F(t-t')G(t_0,t')h_1^{\text{out}}(t')dt'. \tag{24}$$

It can easily be shown by direct substitution that $g_2^{\text{out}}(t,t_0)$ can be expressed in terms of $G(t,t_0)$:

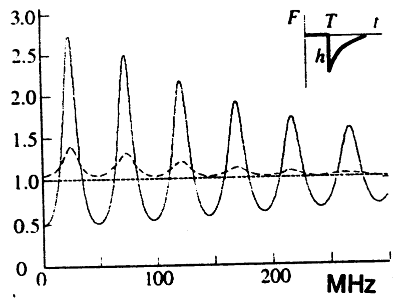

FIG. 3. The function $F(t)$ (inset) and photocurrent fluctuation spectrum from detectors D1 and D2 of Fig. 1 normalized to the shot-noise level. The plot shows $(\delta i_1^2)_\omega/e\langle i_1\rangle$ (solid curve) and $(\delta i_2^2)_\omega/e\langle i_2\rangle$ (dotted curve) for a feedback delay time $T=20$ ns, $\lambda_1 h=\lambda_2 h=4\cdot 10^8$ photons/sec, and exponential field relaxation time $\tau_c=1$ ns (photon lifetime in the resonator). The dashed line is the shot-noise level.

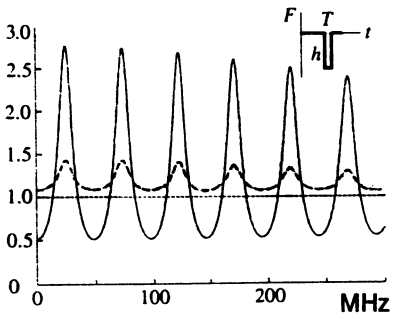

FIG. 4. Same quantities as in Fig. 3, but for the optical layout of Fig. 2. Parameter values T, $\lambda_1 h$, and $\lambda_2 h$ are the same as in Fig. 3; modulation pulse width is $\tau_m=1$ ns.

$$g_2^{\rm out}(t,t_0)=\lambda_2\lambda_1\int_0^{t_0} G(t,t')G(t_0,t')h_1^{\rm out}(t')dt'. \tag{25}$$

In the steady state, as $t\to\infty$, $t_0\to\infty$, $t-t_0=\tau$, we obtain

$$h_1^{\rm in}=\lambda_1 I_0+\lambda_1 h_1^{\rm in}\int_0^{+\infty} F(s)ds,$$

$$h_1^{\rm out}=\lambda_2 I_0+\lambda_1 h_1^{\rm out}\int_0^{+\infty} F(s)ds, \tag{26}$$

and

$$G_-(\tau)=F(\tau)+\lambda_1\int_{-\infty}^{\infty} F(\tau-s)G_-(s)ds, \tag{27}$$

$$g_2^{\rm out}(\tau)=h_1^{\rm out}\lambda_1\lambda_2\int_{-\infty}^{\infty} G_-(\tau+s)G_-(s)ds, \tag{28}$$

$$g_2^{\rm in}(\tau)=\frac{\lambda_1^2}{\lambda_2^2}\, g_2^{\rm out}(\tau)+h_1^{\rm out}\lambda_1[G_-(\tau)+G_-(-\tau)], \tag{29}$$

where

$$G_-(\tau)=\begin{cases} G(\tau), & \tau>0,\\ 0, & \tau<0.\end{cases} \tag{30}$$

From (27)–(29) and (3), we find the mean and the photocurrent fluctuation spectrum in the two channels:

$$\langle i_1\rangle=e\lambda_1 I_0\left(1-\lambda_1\int_0^{\infty} F(\tau)d\tau\right)^{-1},$$

$$(\delta i_1^2)_\omega=\frac{e\langle i_1\rangle}{|1-\lambda_1 F(\omega)|^2}, \tag{31}$$

$$\langle i_2\rangle=e\lambda_2 I_0\left(1-\lambda_1\int_0^{\infty} F(\tau)d\tau\right)^{-1},$$

$$(\delta i_2^2)_\omega=e\langle i_2\rangle\left(1+\frac{\lambda_1\lambda_2|F(\omega)|^2}{|1-\lambda_1 F(\omega)|^2}\right), \tag{32}$$

where $F(\omega)$ is the Fourier transform of $F(\tau)$.

Figure 3 shows the form of $F(\tau)$ and the corresponding photocurrent spectrum in the two channels for the layout of Fig. 1 (neglecting the selectivity of the feedback loop), and Fig. 4 shows the same functions for the layout of Fig. 2. It can be seen that at several frequencies, the photocurrent fluctuations in detector D1 clearly lie below the shot-noise level, while in detector D2 they are always above that level.

Equation (31) and (32) for the fluctuation spectra of the two detectors yield the same results as the corresponding equations in Ref. 16, suggesting that analysis of the rate equations is indeed a valid approach to such problems. The interpretation of these equations in the present theory, however, is an entirely different matter. In our approach, the field is in a time-dependent coherent state $|\alpha(t)\rangle$, which is randomly time-variable in amplitude as the result of some modulation. But of course such a field is not nonclassical, and its noise exceeds the shot-noise level (as indicated by the out-of-loop detector), while the sub-shot character of the current noise in detector D1 can be accounted for by the inapplicability of the traditional (Glauber) theory of photodetection to that detector. Therefore, even if we were to replace D1 with a nondestructive detector (as proposed in a number of papers[12,15,16]), we would not be able to obtain a nonclassical state of the electromagnetic field with the given setup.

In light of the foregoing theory, the agreement between the results obtained in the operator and c-number approaches to the description of feedback systems[15] becomes clear. The agreement results from the fact that the generated field has a classical analog, and all means over operators in Ref. 15 are entirely classically stochastic in nature.

This research was supported in part by the Belarus Foundation for Fundamental Research (grant No. F4-027) and in part by the NATO Scientific Program (grant No. LG 921174). The basic results were reported in a seminar at the International School on Quantum Optics (St. Petersburg, Repino, 1993). The authors thank all those who took part in discussions on the topic of the present paper, and in particular, A. S. Troshin, A. V. Masalov, V. N. Gorbachev, and A. I. Trubilko.

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Translated by Marc Damashek