Near-field effect and spatial distribution of spontaneous photons near a surface

O. N. Gadomskiĭ and K. V. Krutitskiĭ

Department of Theoretical Physics, Ul'yanovsk Branch of the M. V. Lomonosov Moscow State University, 432700 Ul'yanovsk, Russia

(Submitted 22 February 1994)
Zh. Eksp. Teor. Fiz. **106**, 936–955 (October 1994)

The solution of the quantum-optics boundary-value problem is found for the spontaneous emission of an excited atom near the surface of a semi-infinite dielectric medium. In addition to the energies of the atom and the lifetime of the excited state, the spatial distribution of the spontaneous photon field at different observation points is calculated. A prediction is made regarding the effect of the near field, which develops at distances from the surface small in comparison with the wavelength of the spontaneous photons; it has a substantial effect on all phase–amplitude properties of the spontaneous photon field. A technique is proposed for solving the boundary-value problem, based on the system of integral field equations together with the Heisenberg constitutive equations. An absorption theorem for quantum optics is proved, from which the quantum-mechanical amplitudes of the refracted and reflected photons are obtained. © *1994 American Institute of Physics.*

1. INTRODUCTION

In recent years considerable attention has been focused on the solution of various boundary problems in the resonant optics of surfaces and thin films, where semiclassical theory is employed. There is special interest in boundary problems of nonlinear resonant optics when light interacts in a time-dependent fashion with a surface, when the surface is irradiated by short, spatially and temporally separated high-intensity pulses. Some understanding of such processes[1] has been achieved, and the solution of the corresponding quantum-optical boundary problems is of considerable interest in view of the experience that has been gained and the special properties of quantized fields near a surface.

The interest in quantum-optical boundary problems is also attributable to the appearance of new objects of investigation, such as quantum wires on the surface of porous silicon,[2] quantum points, etc. Taking into account the considerable complexity of these objects, in this paper we still restrict ourselves to the case of the spontaneous decay of a single atom near the surface of a partially reflecting dielectric medium. We focus special attention on the behavior of spontaneous decay in the near-field region at a distance from the surface that is smaller than the photon wavelength, where, in our opinion, the near-field effect is manifested most completely. The essential physics of this effect will be described below.

Spontaneous emission from an atom near a surface, in microcavities, and in waveguides was considered in Refs. 3–13, where various aspects of this phenomenon were investigated theoretically. Let us turn our attention to some results in Ref. 3, which, in our opinion, are of the greatest interest for examining the problem of an atom interacting with a single surface of a dielectric. The quantum problem of calculating the reflection coefficient of spontaneous photons from the surface of a dielectric was formulated on a solid foundation in Ref. 3. According to the conditions of the problem posed in Ref. 3, an excited atom near a surface has a transition frequency ω_0, and the semi-infinite dielectric medium consists of N atoms per unit volume in the ground state with a transition frequency $\omega \approx \omega_0$. All the atoms in this problem have the same transition dipole moment d_0. The problem in Ref. 3 is solved by the Weisskopf–Wigner method for the probability amplitudes determining the states of the atoms (ground or excited) and the photon numbers. According to the meaning of the equations solved for the probability amplitudes, when the photons are normally incident on the surface of the dielectric, this one-dimensional boundary problem takes into account the interaction between the atom and the medium by means of a common radiation field. Cook and Milonni[3] obtained the following expression for the amplitude of the quantum transition of the atom outside the medium (d is the distance from the atom to the surface):

$$b(t)=\exp\{-K_0(1+Re^{2ik_0d})t\}, \tag{1.1}$$

where $K_0=\pi d_0\omega_0/\hbar Ac$, A is the effective surface area, $k_0=\omega_0/c$, and $R=-(n_0-1)/(n_0+1)$ is the reflection coefficient corresponding to the Fresnel formula for light incident normally on the surface of a medium with a refractive index n_0. The lifetime $\{K_0[1+R\cos(2k_0d)]\}^{-1}$ of the excited state of the atom and the frequency shift $K_0R\sin(2k_0d)$ of the atom can be found from Eq. (1.1).

The next important result in Ref. 3 is the derivation of a quantum analog of the Ewald–Oseen extinction theorem, which is well known in classical optics.[14] The condition under which the normally incident field is extinguished in the medium by the part of the field of dipoles which varies as $\exp(ikz)$, where k is the wave number of the incident field, were obtained. The amplitude of this field is determined by the Fresnel refractive index for normal incidence.

Thus, Cook and Milonni[3] obtained some important results by solving the quantum-optical boundary problem; however, as will be shown in our work, the solution of this problem was far from complete. We shall show that some specific features of the field of spontaneous photons near the surface of a dielectric were not taken into account in Ref. 3. For example, the near-field effect, which should be mani-

 1063-7761/94/100513-11$10.00

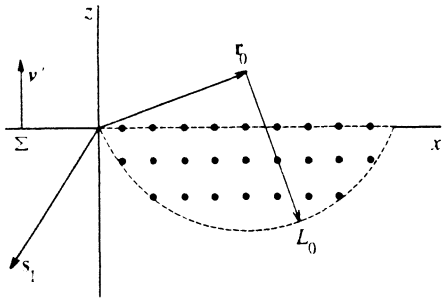

FIG. 1. Diagram of the vector configuration in the boundary problem. Here Σ is the surface $z=0$, $\mathbf{r}_0$ is the radius vector of the site of the resonant atom, $\mathbf{s}_1$ is a unit vector in the direction of photon emission, and L_0 is the radius of the Lorentzian sphere.

fested in the near-field region at a distance on the order of a wavelength from the surface, was totally disregarded. Thus, in this paper we propose a solution to the boundary problem in Ref. 3, in which the amplitude and phase characteristics of the field of spontaneous photons at any observation point can be calculated.

2. EQUATIONS FOR PHOTONIC OPERATORS

Let us consider spontaneous emission from a two-level atom at a point $\mathbf{r}_0$ near a surface Σ (Fig. 1). We may assume here that the resonant frequency ω_0 in the spectrum of the atom depends on the coordinate $\mathbf{r}_0$ due to the electrostatic interaction with the medium, during which the electrostatic and retarded interactions distort the wave functions of the states of the atom and its energy spectrum in a specific manner. Such an interaction is radiationless, is caused by virtual photon exchange, and can be described with the aid of the generalized Breit operator for atomic electrons at arbitrary distances from one another.[15]

In this paper we are interested in the radiative interaction of an atom with a surface, assuming that the influence of the nonradiating interaction is known to us. We show in this paper that two mechanisms for the influence of a surface on the spontaneous decay of the excited state of an atom can be distinguished. An example of a radiative interaction between two hydrogenic atoms is provided by a third-order effect[15] with the Feynman diagram shown in Fig. 2. We portray such an interaction in the following manner. Let the initial state of two resonant atoms correspond to the excited state of atom 1 (the observer atom) with the wave function φ_+ and the energy $E_+=\hbar\omega_+$ and to the unexcited state of atom 2 with the wave function φ_- and the energy $E_-=\hbar\omega_-$. The interaction between these atoms through the field of virtual photons is described by the generalized Breit operator,[15] in which the terms corresponding to quantum transitions of a specific type can identified. In this paper we take into account only the electric dipole transitions in the spectrum of the interacting atoms. As a result of this interaction atom 2 may end up in a certain intermediate state l_+ with a positive energy E_{l_+} and then return to the original state E_-. During this process atom 1 emits one real photon upon transition to E_-. We previ-

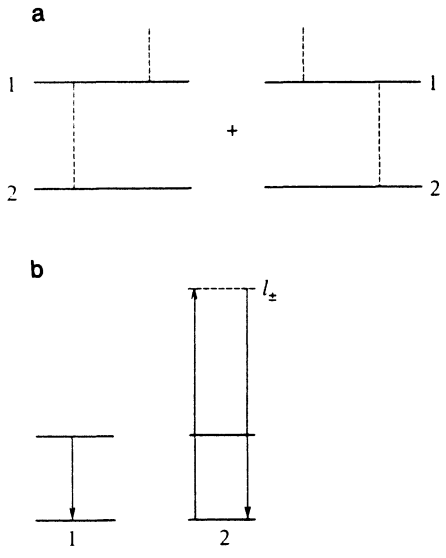

FIG. 2. a) Feynman diagram of the radial interaction between atoms 1 and 2 with the emission (absorption) of a real photon. b) Diagram of the quantum transitions in the spectrum of two resonant atoms interacting by means of a field of virtual photons. The electron and positron polarizing fields at the site of atom 1 differ in accordance with the sign of the frequency of the intermediate state $l_\pm$.

ously showed[1] that such an interaction between atoms is equivalent to the induction of a polarizing field with a vector potential $\mathbf{A}^{(e)}$ at the site of observer atom 1:

$$\mathbf{A}^{(e)}(\mathbf{r}_0,t)=\sum_{\mathbf{k}_\lambda}\ \mathrm{curl}\ \mathrm{curl}\ \mathbf{e}_{\mathbf{k}_\lambda}\alpha_2\sqrt{\frac{2\pi\hbar c^2}{V_R\omega_k}}\frac{1}{R_{02}}c_{\mathbf{k}_\lambda}\times\exp\left\{i\left[\mathbf{k}\mathbf{r}_2-\omega_k\left(t-\frac{R_{02}}{c}\right)\right]\right\}+\mathrm{h.c.}\qquad(2.1)$$

Here the differentiation is carried out with respect to the coordinates of the observation point $\mathbf{r}_0$; $c_{\mathbf{k}_\lambda}$ is the annihilation operator of a photon of mode $\mathbf{k}_\lambda$ in the interaction representation; $R_{02}=|\mathbf{r}_0-\mathbf{r}_2|$, where $\mathbf{r}_2$ is the radius vector of the site of atom 2; $\mathbf{e}_{\mathbf{k}_\lambda}$ is the polarization unit vector of a photon of mode $\mathbf{k}_\lambda$ with frequency ω_k and the wave vector $\mathbf{k}$; V_R is the quantization volume of the electromagnetic field; $\lambda=1,2$ is the index of the polarization of the photon; α_2 is the polarizability of atom 2, calculated in first-order nonrelativistic perturbation theory using the wave functions $\varphi_\pm$ and the Pauli operator $H=-(e/mc)\mathbf{p}_2\mathbf{A}_0(\mathbf{r}_2)$, where e and m are the charge and mass of the electron; $\mathbf{p}_2$ is the momentum operator of the electron of atom 2; and $\mathbf{A}_0(\mathbf{r}_2)$ is the vector potential operator of the free electromagnetic field at $\mathbf{r}_2$.

The vector potential (2.1) represents a so-called electron polarizing field, since it takes into account only the electronic states in the spectrum of the interacting atoms, including the intermediate state E_{l_+}. We make several remarks, which greatly generalize Eq. (2.1). 1. The wavelength of an optical photon is considerably greater than the dimensions of an atom, so that, considering the pairwise interactions of the electrons of atoms 1 and 2, we can obtain a formula for the electron polarizing field at $\mathbf{r}_0$ which is created by a more complicated atom with an arbitrary number of electrons. In

this case in Eq. (2.1) α_2 is the polarizability of the many-electron atom 2. 2. The third-order effect in Fig. 2 can occur when the energy conservation have $\hbar(\omega_+-\omega_--\omega_k)=0$ is satisfied. Therefore, the field (2.1) can also be formed by a nonresonant atom with the corresponding polarizability $\alpha_{NR}(\mathbf{r}_2)$. 3. If the field at the observation point is created by a set of atoms, rather than by a single atom, the field (2.1) must be summed over the polarizing fields formed by all the surrounding atoms. Taking into account that the characteristic wavelength $(2\pi c/\omega_0)[\omega_0=(E_+-E_-)/\hbar]$ greatly exceeds the dimensions of the atoms and the interatomic distances in the medium, we can make the transition to a continuous distribution of surrounding atoms with a certain density N. On the other hand, as follows from Refs. 1 and 15, the characteristic wavelength $2\pi c/\omega_0$ and the interatomic distance l_0 define spatial scales for the interaction of the atoms, which make it possible to separate the electrostatic and retarded parts of the generalized Breit interaction. Therefore, in the near-field region, where the distances between the interacting atoms are at most comparable to the wavelength $\lambda_0=2\pi c/\omega_0$, a representation of a discrete-continuous medium, in which the nearby atoms are distributed discretely around the observer atom, must be employed. At the same time, atoms located outside of a certain sphere of radius $L_0(l_0\ll L_0\ll 2\pi c/\omega_0)$ are distributed continuously. Thus, in all the ensuing equations in which integration over the radius vectors of the surrounding atoms will be encountered, we also assume the presence of summation over the sites of the discretely distributed atoms within a sphere of radius L_0 (Fig. 1). The existence of two spatial scales λ_0 and l_0 for calculating the field in the vicinity of the observation point gives rise to the near-field effect. We turn our attention to the important fact that the discretely distributed dipoles within a closed sphere of radius L_0 in the vicinity of the observation point create a field equal to zero at the observation point only in the case of certain types of distributions (for example, a cubic distribution) and when only the electrostatic polarizing field is taken into account. It can be demonstrated by direct calculations of the polarizing fields that within a sphere of radius L_0 the retarded field is nonzero for any type of distribution. However, the near-field effect should be manifested most completely when the sphere of radius L_0 is truncated, as occurs when the observation point approaches the surface of the optical medium. We shall investigate just this manifestation of the near-field effect in the boundary problem under consideration.

Thus, the Hamiltonian of an atom at the point $\mathbf{r}_0$ near the surface Σ of a certain dielectric medium may be described in the following manner:

$$H=\hbar\omega_0 r_3+2\omega_0 d_0\frac{1}{c}\mathbf{u}_d\mathbf{A}r_2+\sum_{\mathbf{k}_\lambda}\hbar\omega_k c^+_{\mathbf{k}_\lambda}c_{\mathbf{k}_\lambda}, \tag{2.2}$$

where r_2 and r_3 are components of the effective spin operator of the two-level atom, the ground-state of the atom equals $-\hbar\omega_0/2$, the vector potential operator is

$$\mathbf{A}(\mathbf{r}_0,t)=\sum_{\mathbf{k}_\lambda} g_{k_\lambda}\mathbf{e}_{\mathbf{k}_\lambda}[c_{\mathbf{k}_\lambda}(0)\exp\{i(\mathbf{kr}_0-\omega_k t)\} + c^+_{\mathbf{k}_\lambda}(0)\exp\{-i(\mathbf{kr}_0-\omega_k t)\}]+\mathbf{A}^{(e)}(\mathbf{r}_0,t),$$

$$g_{k_\lambda}=\sqrt{\frac{2\pi\hbar c^2}{V_R\omega_k}}, \tag{2.3}$$

i.e., equals the sum of the vector potential operators of the free field and the electron polarizing field at the observation point $\mathbf{r}_0$, and $\mathbf{d}_0/d_0=\mathbf{u}_d$, where d_0 is the matrix element of the electric dipole moment operator of the atom calculated with the wave functions of the atom $\varphi_\pm$. If the atom is found near Σ or on Σ itself, the fundamental frequency ω_0 and the transition dipole moment d_0 are functions of the relative distance z (Fig. 1). In view of the remarks just made, we write the vector potential of the polarizing field in the form

$$\mathbf{A}^{(e)}(\mathbf{r}_0,t)=\sum_{\mathbf{k}_\lambda} g_{k_\lambda}c_{\mathbf{k}_\lambda}(0)\,\mathrm{curl}\,\mathrm{curl}\int_V\frac{1}{R_0}\mathbf{e}_{\mathbf{k}_\lambda}\alpha N \times\exp\{i[\mathbf{Kr}'-\omega_k(t-R_0/c)]\}dV', \tag{2.4}$$

where the integration is carried out over the entire volume of the medium bounded by the surface Σ, $\mathbf{r}'$ is a point within the medium or on its surface, $R_0=|\mathbf{r}_0-\mathbf{r}'|$, and the value of αN is determined by the refractive index of the medium n_0 according to the Lorentz-Lorenz equation

$$\frac{4\pi}{3}N\alpha=\frac{n_0^2-1}{n_0^2+2}. \tag{2.4a}$$

The curl curl operator in (2.4) was moved outside of the integral sign, since the observation point is located outside the medium.

A. Positron polarizing field

We also take into account the intermediate states with a negative frequency $(1/\hbar)E_{l_-}=-(mc^2/\hbar)+\varepsilon_{l_-}$ in the spectrum of the interacting atoms, which naturally appear in the third-order effect in Fig. 2. Such interactions were considered by Drake[17] for two electrons in a helium-like atom and in our studies[1,15] for two atomic electrons belonging to different hydrogenic atoms. These investigations included derivation of the Hamiltonian of the interaction of two electrons with an external field of the type $H_2\sim\mathbf{p}_i\mathbf{A}_j/r_{ij}$, where r_{ij} is the distance between the electrons, $\mathbf{p}_i$ is the momentum operator of the ith electron, and $\mathbf{A}_j$ is the operator of the vector potential at the site of the jth electron. H_2 can be obtained by two methods. The first method is based on the replacement of the electron momentum $\mathbf{p}_j\to\mathbf{p}_j-(e/c)\mathbf{A}_j$ in the Darwin Lagrangian,[18] followed by a transition to operators. The other method for obtaining H_2 is based on successive treatment of the third-order effect in Fig. 2 followed by a transition to nonrelativistic quantum mechanics. When the internuclear distance between two hydrogenic atoms equals zero, H_2 (Refs. 15 and 18) becomes the corresponding Drake operator.[17]

According to Refs. 1 and 15, for a medium bounded by the surface Σ we obtain the following vector potential operator

$$\mathbf{A}^{(p)}(\mathbf{r}_0,t)=-\frac{e^2}{mc^2}\sum_{\mathbf{k}_\lambda}\int_V Ng_{k_\lambda}\frac{1}{R_0}[\mathbf{e}_{\mathbf{k}_\lambda}-\mathbf{n}_0(\mathbf{e}_{\mathbf{k}_\lambda}\mathbf{n}_0)]$$
$$\times c_{\mathbf{k}_\lambda}(0)\exp\{i[\mathbf{Kr}'-\omega_k$$
$$\times(t-R_0/c)]\}dV'+\text{h.c.}, \qquad (2.5)$$

where $\mathbf{n}_0=\mathbf{R}_0/R_0$. We call this field the positron polarizing field or, simply, the positron polarization of the medium. As a result, at the observation point $\mathbf{r}_0$ we have the field $\mathbf{A}(\mathbf{r}_0,t)$ (2.3), to which field (2.5) must also be added. We shall see below that the resultant field at the site of the atom $\mathbf{r}_0$ consists of four components, where, apart from the components just cited, there is a self-field due to the reciprocal influence of the photons on the spontaneous emission process.

B. Transition to the Heisenberg representation

We pass from the operators F in the representation of the interaction to the operators $\tilde{F}$ in the Heisenberg representation:

$$\tilde{F}=\exp\left[\frac{i}{\hbar}(H_1+H_2)t\right]F\exp\left[-\frac{i}{\hbar}(H_1+H_2)t\right], \qquad (2.6)$$

where the Hamiltonian of the interaction with the photon field

$$H_1+H_2=2\omega_0 d_0\frac{1}{c}\mathbf{u}_d(\mathbf{A}+\mathbf{A}^{(p)})r_2 \qquad (2.7)$$

includes the electron (2.4) and positron (2.5) polarizing fields, which depend on the observation point $\mathbf{r}_0$. The photonic operators in the Hamiltonian (2.7) correspond to the free field and are defined by the following relations:

$$c_{\mathbf{k}_\lambda}(\mathbf{r},t)=c_{\mathbf{k}_\lambda}(0)\exp\{i(\mathbf{kr}-\omega_k t)\},$$
$$c^+_{\mathbf{k}_\lambda}(\mathbf{r},t)=c^+_{\mathbf{k}_\lambda}(0)\exp\{-i(\mathbf{kr}-\omega_k t)\}, \qquad (2.8)$$

where $c_{\mathbf{k}_\lambda}(0)$ and $c^+_{\mathbf{k}_\lambda}(0)$ are independent of space and time. This permits moving the operators $c_{\mathbf{k}_\lambda}(0)$ and $c^+_{\mathbf{k}_\lambda}(0)$ outside of the integral sign and the curl curl operator in Eqs. (2.4) and (2.5) and writing the Hamiltonian (2.7) as

$$H_1+H_2=2\frac{\omega_0}{c}\sum_{\mathbf{k}_\lambda}d^{\text{eff}}_{\mathbf{k}_\lambda}A^{(0)}_{\mathbf{k}_\lambda}r_2+\text{h.c.}, \qquad (2.9)$$

where the effective transition dipole moment has the form

$$d^{\text{eff}}_{\mathbf{k}_\lambda}=d_0\mathbf{u}_d(\mathbf{I}^{(1)}_{\mathbf{k}_\lambda}+\mathbf{I}^{(2)}_{\mathbf{k}_\lambda}+\mathbf{e}_{\mathbf{k}_\lambda}\exp(i\mathbf{kr}_0)), \qquad (2.10)$$

$$\mathbf{I}^{(1)}_{\mathbf{k}_\lambda}(\mathbf{r}_0)=\text{curl curl}\int_V\mathbf{e}_{\mathbf{k}_\lambda}\frac{N\alpha}{R_0}\exp\{i(\mathbf{Kr}'$$
$$+\omega_k R_0/c)\}dV',$$

$$\mathbf{I}^{(2)}_{\mathbf{k}_\lambda}(\mathbf{r}_0)=-\frac{e^2}{mc^2}\int_V\frac{N}{R_0}[\mathbf{e}_{\mathbf{k}_\lambda}-\mathbf{n}_0(\mathbf{e}_{\mathbf{k}_\lambda}\mathbf{n}_0)]\exp\{i(\mathbf{Kr}'$$
$$+\omega_k R_0/c)\}dV', \qquad (2.10a)$$

and the photonic operator $A^{(0)}_{\mathbf{k}_\lambda}$ equals

$$A^{(0)}_{\mathbf{k}_\lambda}=g_{k_\lambda}c_{\mathbf{k}_\lambda}(0)\exp(-i\omega_k t). \qquad (2.11)$$

The Heisenberg equations for the photonic operators $\tilde{c}_{\mathbf{k}_\lambda}$ and $\tilde{c}^+_{\mathbf{k}_\lambda}$ are found with the aid of the Hamiltonian (2.9), in which the replacements $r_2\to\tilde{r}_2$, $c_{\mathbf{k}_\lambda}\to\tilde{c}_{\mathbf{k}_\lambda}$, and $c^+_{\mathbf{k}_\lambda}\to\tilde{c}^+_{\mathbf{k}_\lambda}$ should be made. Using the known transposition relations for atomic and photonic operators at coinciding moments in time, we obtain the following equation at the observation point $\mathbf{r}=\mathbf{r}_0$:

$$\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r}_0,t)=\tilde{c}^{\text{v}}_{\mathbf{k}_\lambda}(\mathbf{r}_0,t)-\frac{i}{\hbar}2\frac{\omega_0}{c}(d^{\text{eff}}_{\mathbf{k}_\lambda})^*g_{k_\lambda}\int\tilde{r}_2(t',r_0)$$
$$\times G_{k_\lambda}(t-t')dt', \qquad (2.12)$$

where ω_0,d_0, and the effective dipole moment $d^{\text{eff}}_{\mathbf{k}_\lambda}$ are assigned at the observation point $\mathbf{r}_0$, G_{k_λ} is the retarded Green's function

$$G_{k_\lambda}(t-t')=\begin{cases}0, & t<t',\\ \lim\limits_{\varepsilon\to0}\exp\{-i(\omega_k-i\varepsilon)(t-t')\}, & t>t',\end{cases} \qquad (2.13)$$

and the photonic operator

$$c^{\text{v}}_{\mathbf{k}_\lambda}(\mathbf{r}_0,t)=c_{\mathbf{k}_\lambda}(0)\exp\{i(\mathbf{kr}_0-\omega_k t)\}$$

corresponds to the free field of the photons. The second term on the right-hand side of Eq. (2.12) may be interpreted as the self-field of the photons, which is distorted in a definite manner by the surface in comparison to the self-field of a free atom.[16] Thus, the transition to the Heisenberg representation (2.6) significantly alters the coordinate dependence of the photonic operators, especially in the near-field region, according to the coordinate dependence of the quantities defined by (2.10a), as well as ω_0 and d_0 at the observation point $\mathbf{r}_0$.

C. Extinction theorem in quantum optics

Equation (2.12) is defined at the observation point $\mathbf{r}_0$. Let us find operator equations for calculating the field at an arbitrary observation point $\mathbf{r}\neq\mathbf{r}_0$, using the concepts of electron and positron polarization of the medium. The operator of the electric field strength at $\mathbf{r}$ is written in the following manner:

$$\tilde{\mathbf{E}}(\mathbf{r},t)=\tilde{\mathbf{E}}_{\text{v}}(\mathbf{r},t)+\text{curl curl}\frac{\tilde{\mathbf{d}}(\mathbf{r}_0,t-R_0/c)}{R_0}+\tilde{\mathbf{E}}^{(e)}+\tilde{\mathbf{E}}^{(p)}, \qquad (2.14)$$

where $\tilde{\mathbf{d}}$ is the dipole moment operator of the atom, $\tilde{\mathbf{E}}_{\text{v}}$ is the electric field strength corresponding to the field of photons in free space,

$$\tilde{\mathbf{E}}^{(e)}(\mathbf{r},t)=\int_V\text{curl curl}\frac{N\alpha}{R}\tilde{\mathbf{E}}(\mathbf{r}',t-R/c)dV', \qquad (2.14a)$$

$$\tilde{\mathbf{E}}^{(p)}(\mathbf{r},t)=-\frac{e^2}{mc^2}\int_V\frac{N}{R}[\tilde{\mathbf{E}}-\mathbf{n}(\tilde{\mathbf{E}}\mathbf{n})]_{\mathbf{r}',t-R/c}dV', \qquad (2.14b)$$

and $\mathbf{n}=\mathbf{R}/R$. We transform $\tilde{\mathbf{E}}^{(e)}$ in the following manner:

$$\tilde{\mathbf{E}}^{(e)}=\frac{i}{\sqrt{V_R}}h^{1/2}\sum_{\mathbf{k}_\lambda}\omega_k^{1/2}\exp(-i\omega_k t)$$
$$\times\int \text{curl curl}\,\mathbf{Q}_{\mathbf{k}_\lambda}\tilde{c}_{\mathbf{k}_\lambda}G_k dV'+\text{h.c.}, \qquad (2.15)$$

where

$$\mathbf{Q}_{\mathbf{k}_\lambda}(\mathbf{r}')=\mathbf{e}_{\mathbf{k}_\lambda}N\alpha\exp(i\mathbf{k}\mathbf{r}'),$$
$$G_k=\exp\left(i\frac{\omega_k}{c}R\right)/R,\quad \mathbf{Q}_{\mathbf{k}_\lambda}\tilde{c}_{k_\lambda}=\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}$$

and the photonic operator $\tilde{c}_{\mathbf{k}_\lambda}=\tilde{c}_{\mathbf{k}_\lambda}(r',t-R/c)$ in the Heisenberg representation depends on the coordinates and time in the general case. Let us investigate the properties of the photon field near Σ (Fig. 1) at various observation points, including observation points within the medium, when the emitting atom has a fixed position $\mathbf{r}=\mathbf{r}_0$. The first two terms on the right-hand side of Eq. (2.14) play the role of the initial field $\tilde{E}_{\text{in}}$ in our treatment. The local dipole moment $\tilde{\mathbf{d}}$ is then determined from Eq. (2.12).

We first consider the case in which the observation point is located within the medium ($z<0$, Fig. 1). We move the curl curl operator outside of the integral sign in (2.14):

$$\text{curl curl}\int_{\sigma_0}^{\Sigma}\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}G_k dV'=\int_{\sigma_0}^{\Sigma}\text{curl curl}\,\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}G_k dV'$$
$$+\frac{8\pi}{3}\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}(\mathbf{r},t), \qquad (2.16)$$

where the integral is taken over the volume bounded by the outer surface Σ and the inner surface σ_0 surrounding the observation point $\mathbf{r}_0$.

We seek a solution of Eq. (2.14) in the form

$$\tilde{\mathbf{E}}_{\mathbf{k}_\lambda}(\mathbf{r},t)=\frac{i}{\sqrt{V_R}}h^{1/2}\omega_k^{1/2}\mathbf{e}_{\mathbf{k}_\lambda}\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r},t)\exp\{i(\mathbf{K}\mathbf{r}-\omega_k t)\}+\text{h.c.}, \qquad (2.17)$$

where $\mathbf{K}=(\omega/c)n_0\mathbf{s}_T$ is the wave vector of a photon within the medium, n_0 is a quantity which will be defined below, and $\mathbf{s}_T$ is the unit vector in the direction of propagation of the photon within the medium. We also assume that the amplitude

$$\tilde{\mathbf{E}}^{(0)}_{\mathbf{k}_\lambda}(\mathbf{r},t)=\frac{i}{\sqrt{V_R}}h^{1/2}\omega_k^{1/2}\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r},t)\mathbf{e}_{k_\lambda} \qquad (2.17a)$$

of the trial solution (2.17) is a slowly varying function of the position and time.

We rewrite the integrand in (2.16) as

$$G_k\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}=\frac{(c/\omega_k)^2}{n_0^2-1}[G_k\nabla^2\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}-\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}\nabla^2 G_k],$$

where

$$\nabla^2\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}+\left(\frac{\omega_k}{c}n_0\right)^2\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}=0, \qquad (2.18)$$

and then using Green's theorem we can write the following relation:

$$\int_{\sigma_0}^{\Sigma}\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}(\mathbf{r}',\mathbf{r},t)G_k(R)dV'=\frac{(c/\omega_k)^2}{n_0^2-1}\{\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}+4\pi\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}(\mathbf{r},t)\},$$

$$\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}=\int_\Sigma\left\{\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}\frac{\partial G_k}{\partial\nu'}-G_k\frac{\partial\mathbf{Q}_{\mathbf{k}_\lambda}}{\partial\nu'}\right\}dS', \qquad (2.19)$$

where $\partial/\partial\nu'$ denotes differentiation along an external normal to Σ (Fig. 1) and the second term on the right-hand side of the equality corresponds to the value of the surface integral over the small sphere σ_0 when the radius of the sphere tends to zero.

We transform the volume integral (2.14b) in a similar manner. We leave only the first term in the integrand in (2.14b), since the second term makes a contribution to the positron polarizing field that is approximately three times smaller. Then we introduce the notation

$$-\frac{e^2}{mc^2}N\frac{i}{\sqrt{V_R}}h^{1/2}\omega_k^{1/2}\mathbf{e}_{\mathbf{k}_\lambda}\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r}',t)\exp(i\mathbf{K}\mathbf{r}')$$
$$=\tilde{\mathbf{P}}_{\mathbf{K}_\lambda}(\mathbf{r}',t) \qquad (2.20)$$

and obtain the following approximate expression:

$$\tilde{\mathbf{E}}^{(p)}(\mathbf{r},t)=\frac{(c/\omega_k)^2}{n_0^2-1}[\mathbf{I}^{(4)}_{\mathbf{k}_\lambda}+4\pi\tilde{\mathbf{P}}_{\mathbf{k}_\lambda}(\mathbf{r},t)],$$

$$\mathbf{I}^{(4)}_{\mathbf{k}_\lambda}(\mathbf{r},t)=\int_\Sigma\left\{\tilde{\mathbf{P}}_{\mathbf{k}_\lambda}\frac{\partial G_k}{\partial\nu'}-G_k\frac{\partial\tilde{\mathbf{P}}_{\mathbf{k}_\lambda}}{\partial\nu'}\right\}dS'. \qquad (2.21)$$

Thus, after all the transformations we can distinguish two groups of terms in Eq. (2.14). The first group of terms is defined at the observation point (r,t) and forms the local equation

$$\tilde{\mathbf{E}}=-\frac{8\pi}{3}\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}+\text{curl curl}\frac{(c/\omega_k)^2}{n_0^2-1}4\pi\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}$$
$$+\frac{(c/\omega_k)^2}{n_0^2-1}4\pi\tilde{\mathbf{P}}_{\mathbf{k}_\lambda}. \qquad (2.22)$$

When the condition $\text{div}\tilde{\mathbf{Q}}_{\mathbf{k}_\lambda}=0$ and Eq. (2.18) are satisfied, from (2.22) we obtain the following formula for calculating n_0:

$$n_0^2=\frac{1+(8\pi/3)N\alpha-4\pi N(e^2/m\omega_k^2)}{1-(4\pi/3)N\alpha}. \qquad (2.23)$$

As $\omega_k\to\infty$, the positron polarization makes a negligibly small contribution to the refractive index of the medium, and Eq. (2.23) evidently becomes the Lorentz–Lorenz equation of classical optics. The role of positron polarization in the optical properties of the medium may become appreciable when the polarizabilities of the atoms are small and the frequencies of the photons decrease. The contribution of the positron polarization of the medium can be analyzed by calculating the ratio of the vector potentials of the electron ($\mathbf{A}^{(e)}$) and positron ($\mathbf{A}^{(p)}$) polarization. As was shown in Ref. 1, the ratio $A^{(e)}/A^{(p)}$ for two resonant atoms ($N=2$) in the two-level approximation equals $3c^2m/4\omega_0e^2$. Thus, when

we have $N=2$, $\mathbf{A}^{(e)}$ is considerably greater than $\mathbf{A}^{(p)}$ at optical frequencies. The situation may be different in the case of $N\gg 1$ when the inhomogeneous broadening of the spectral lines with an rms deviation from resonance $\Delta\omega$ is taken into account. It can easily be shown that the ratio between $A^{(e)}$ and $A^{(p)}$ in this case equals

$$A^{(e)}/A^{(p)}\approx m\omega_0^2 d_0^2/(\hbar e^2\Delta\omega).$$

If $d_0\sim 10^{-18}$ esu, $\omega_0=10^{13}$ to 10^{15} sec^{-1}, this ratio is approximately equal to $(10^9-10^{13})\cdot\Delta\omega^{-1}$. Thus, when the inhomogeneous broadening is sufficiently great, the role of positron polarization may be comparable to the electron polarization, i.e., to the field of electric dipoles in optics. Some attention should be focused on the following two points, which were ignored during the derivation of Eq. (2.23). 1. The observation point is located on Σ or in close proximity to it at a distance less than the characteristic scale of the medium. We previously[1] considered this special case in nonlinear optics. 2. The lifetime of the excited state of the atom is considerably smaller than the characteristic relaxation times of the medium. This permits the use of the stationary solutions of the material equations for the medium, and the polarizability α does not depend on time, i.e., the medium is linear and stationary in our treatment.

The second group of terms in Eq. (2.14) forms the nonlocal operator equation

$$\tilde{\mathbf{E}}^{\text{in}}_{\mathbf{k}_\lambda}+\frac{(c/\omega_k)^2}{n_0^2-1}\,[\text{curl curl }\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}+\mathbf{I}^{(4)}_{\mathbf{k}_\lambda}]=0, \tag{2.24}$$

which, according to the correspondence principle, is an expression of the extinction theorem of quantum optics.

The field at an observation point located above Σ can be calculated, if the replacement $-z\to z$ is made in the surface integrals $\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}$ and $\mathbf{I}^{(4)}_{\mathbf{k}_\lambda}$. Then the field of reflected photons has the form

$$\tilde{\mathbf{E}}^{\text{R}}_{\mathbf{k}_\lambda}(\mathbf{r},t)=\frac{(c/\omega_k)^2}{n_0^2-1}\,[\text{curl curl }\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}+\mathbf{I}^{(4)}_{\mathbf{k}_\lambda}]. \tag{2.25}$$

Thus, the boundary problem of calculating the photon field at different observation points can be solved using Eqs. (2.24), (2.25), and (2.12), if these equations are supplemented by the corresponding equations for the atomic operators.

3. EQUATIONS FOR ATOMIC OPERATORS

We use Hamiltonian (2.9) to define the atomic operators $\tilde{r}_1$, $\tilde{r}_2$, and $\tilde{r}_3$ of the effective spin of a two-level atom. The corresponding Heisenberg equations have the form

$$\dot{\tilde{r}}_1=-\omega_0\tilde{r}_2+2(\omega_0/\hbar c)d^{\text{eff}}\tilde{A}\tilde{r}_3,\quad d^{\text{eff}}\tilde{A}=\sum_{\mathbf{k}_\lambda} d^{\text{eff}}_{\mathbf{k}_\lambda}\tilde{A}_{\mathbf{k}_\lambda}, \tag{3.1a}$$

$$\dot{\tilde{r}}_2=\omega_0\tilde{r}_1, \tag{3.1b}$$

$$\dot{\tilde{r}}_3=-2(\omega_0/\hbar c)d^{\text{eff}}\tilde{A}\tilde{r}_1, \tag{3.1c}$$

where $d^{\text{eff}}_{\mathbf{k}_\lambda}$ depends on the coordinate $\mathbf{r}_0$ in accordance with the above remarks and the field $\tilde{A}_{\mathbf{k}_\lambda}$ is determined with the aid of photonic operators (2.12). Thus, we have a closed system of equations for determining the atomic and photonic characteristics at the observation point $\mathbf{r}_0$.

A. Lifetime of the excited state of an atom near the surface of a dielectric

We use Eqs. (2.12) and (3.1) to calculate the lifetime τ_1 of the excited state of an atom near a surface in the wave and near-field regions, using the known coordinate dependence of the effective dipole moment.

In the adiabatic approximation we have

$$\tilde{r}_\pm(t',\mathbf{r}_0)=\tilde{r}^{(0)}_\pm(t',\mathbf{r}_0)\exp(\pm i\omega_0 t),\quad \tilde{r}_\pm=\tilde{r}_1\pm i\tilde{r}_2, \tag{3.2}$$

where $\tilde{r}^{(0)}_\pm(t',\mathbf{r}_0)$ denotes unknown operators, whose time dependence is so slow in comparison to $\exp(\pm i\omega_0 t)$, so that they may be moved outside of the integral sign in (2.12). After this transformation the annihilation operator of the $\mathbf{k}_\lambda$th mode is represented in the form

$$\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r}_0,t)=\tilde{c}^{\text{v}}_{\mathbf{k}_\lambda}(\mathbf{r}_0,t)+i\,\frac{\omega_0}{\hbar c}\,(d^{\text{eff}}_{\mathbf{k}_\lambda})^*_{\mathbf{r}_0}[\tilde{r}_+(t,\mathbf{r}_0)\zeta^*(\omega_k+\omega_0)-\tilde{r}_-(t,\mathbf{r}_0)\zeta^*(\omega_k-\omega_0)], \tag{3.3}$$

where ζ is the Heitler function and is given by the expression[19]

$$-i\zeta^*(\omega_k-\omega_0)=-i\,\frac{\mathscr{P}}{\omega_k-\omega_0}+\pi\delta(\omega_k-\omega_0). \tag{3.3a}$$

Here $\mathscr{P}$ signifies that the next integration is carried out in the sense of a principal value.

We find the operator product $\tilde{A}\tilde{r}_1$ in Eq. (3.1c), using operator (3.3). Expressing $\tilde{A}$ in terms of $\tilde{c}_{\mathbf{k}_\lambda}$ and $\tilde{c}^+_{\mathbf{k}_\lambda}$ with consideration of the fact that products of operators must be normally ordered, we arrive at the equation

$$\begin{aligned}d^{\text{eff}}\tilde{A}\tilde{r}_1=&\sum_{\mathbf{k}_\lambda} g_{\mathbf{k}_\lambda}[d^{\text{eff}}_{\mathbf{k}_\lambda}\tilde{c}^{\text{v}+}_{\mathbf{k}_\lambda}\tilde{r}_1+(d^{\text{eff}}_{\mathbf{k}_\lambda})^*\tilde{r}_1\tilde{c}^{\text{v}}_{\mathbf{k}_\lambda}]\\&+\frac{\pi}{2}\,\frac{\omega_0}{\hbar c}\sum_{\mathbf{k}_\lambda}|d^{\text{eff}}_{\mathbf{k}_\lambda}|^2 g^2_{k_\lambda}[\delta(\omega_k-\omega_0)-\delta(\omega_k+\omega_0)]+\pi\,\frac{\omega_0}{\hbar c}\sum_{\mathbf{k}_\lambda}|d^{\text{eff}}_{\mathbf{k}_\lambda}|^2 g^2_{k_\lambda}[\delta(\omega_k-\omega_0)+\delta(\omega_k+\omega_0)]\tilde{r}_3.\end{aligned} \tag{3.4}$$

We now perform vacuum averaging of Eq. (3.1c) by plugging the operator (3.4) into it. Vacuum averaging, which may be denoted by $\langle\ldots\rangle$ is understood to be averaging with respect to the state $|\psi\rangle=|\text{vacuum}\rangle|\varphi\rangle$, where $|\varphi\rangle$ is either state of the two-level atom and $|\text{vacuum}\rangle$ is the state in which there are no photons. As a result we obtain the equation

$$\frac{d}{dt}\langle\tilde{r}_3(t)\rangle=-\frac{1}{\tau_1}\left(\frac{1}{2}+\langle\tilde{r}_3\rangle\right), \tag{3.5}$$

where

$$\frac{1}{\tau_2}=2\pi\left(\frac{\omega_0}{\hbar c}\right)^2\sum_{\mathbf{k}_\lambda}|d^{\text{eff}}_{\mathbf{k}_\lambda}|^2 g^2_{k_\lambda}[\delta(\omega_k-\omega_0)-\delta(\omega_k+\omega_0)]. \tag{3.6}$$

The vacuum averages of terms containing the vacuum parts of photonic operators vanish only because of the normal ordering of the operators according to the properties of free-field operators.

The summation over the modes in (3.6) can be converted to integration using the following replacement

$$\sum_{\mathbf{k}_\lambda}\to\frac{V_R}{(2\pi c)^3}\int\omega^2 d\omega d\Omega\sum_{\lambda=1}^{2},$$

where $d\Omega$ is an element of the solid angle in the direction in which the spontaneous photon leaves. Taking into account the properties of the integrals (2.10a), we can calculate the total lifetime of the excited state of the atom at various distances from the surface. Since the observation point $\mathbf{r}_0$ at which we determine the lifetime of the excited state of the atom is outside the optical medium, the volume integrals (2.10a) which determine the effective dipole moment of the atom are calculated by integrating over the entire volume of the optical medium. If the observation point is located in the wave region $[(\omega_0/c)R\gg 1]$, we have a single spatial scale λ_0 for the interaction of the atoms and instead of the volume integrals (2.10a) we have integrals over the surface Σ of the optical medium. These surface integrals can be calculated by the method of stationary phase.[14] A different situation is observed in the case of $(\omega_0/c)R\lesssim 1$, in which two spatial scales λ_0 and l_0 must be taken into account. In this case the volume integrals in (2.10a) are replaced by surface integrals over a part of the surface Σ and over the truncated Lorentzian sphere of radius L_0 and by a sum over the discretely distributed dipoles within the sphere. This gives rise to a power dependence $(\sim 1/R_0^3)$ of the lifetime in (3.6) on the position of the emitting atom, which we regard as a manifestation of the near-field effect. The coordinate dependence of the lifetime in (3.6) is completely determined by the effective dipole moment of the atom, which, in turn, depends on the values of the surface integrals. Below we shall present the necessary expressions with consideration of summation over the discrete distribution of atoms near the observation point. It can be shown that the probability of the emission of a spontaneous photon per unit time parallel to the z axis coincides with the value of Cook and Milonni,[3] which follows from Eq. (1.1) of the one-dimensional boundary problem only when the following conditions are satisfied: 1. The electron polarizing field is considerably stronger than the positron polarizing field. 2. The excited atom is found in the wave region relative to the surface Σ (Fig. 1). Only in this case can the near-field effect be neglected. Applying Green's theorem and the method of stationary phase,[14] we can express the volume integral $\mathbf{I}^{(1)}_{\mathbf{k}_\lambda}$ in terms of the Fresnel reflection coefficient and the phase multiplier $\cos[2(\omega_k/c)d]$, which is stipulated by the additional retardation in the interaction of the atom with the medium caused by virtual photon exchange. This property of the interaction of the atom with the medium may also be interpreted as an interaction of the atom with its own image in the medium. In the near-field region, the probability of the emission of a photon per unit time differs significantly from $K_0[1+R\cos(2k_0d)]$ owing to the near-field effect and is determined mainly by the electrostatic part of the polarizing field of the discretely distributed atoms within the truncated sphere of radius L_0. In the single-mode approximation this causes the photon emission probability to contain not only an oscillating term of the form $\cos(2k_0d)$, but also a sum over the power functions $1/R_a^3$, where the subscript a labels the atoms within the truncated sphere of radius L_0, and this sum specifies the coordinate dependence of the lifetime given by (3.6) in the near-field region. Thus, we note an inconsistency in the theory in Ref. 3, in which the near-field effect was totally ignored, although Cook and Milonni[3] proposed using Eq. (1.1) specifically for the near-field region, where $k_0d\sim 1$.

B. Energy shift of an atom near a surface

Let us calculate the influence of the radiative interaction of an atom and a dielectric medium on the energy spectrum of the atom. This interaction is determined by the polarizing fields (2.10a).

We define the operator $\tilde{A}\tilde{r}_3$ in Eq. (3.1a), using the photonic operators (3.3) for this purpose:

$$\begin{aligned}d^{\text{eff}}\tilde{A}\tilde{r}_3=&\sum_{\mathbf{k}_\lambda}[d^{\text{eff}}_{\mathbf{k}_\lambda}g_{k_\lambda}\tilde{r}_3\tilde{c}^{\text{v}}_{\mathbf{k}_\lambda}+(d^{\text{eff}}_{\mathbf{k}_\lambda})^* g_{k_\lambda}\tilde{c}^{\text{v}+}_{\mathbf{k}_\lambda}\tilde{r}_3]\\&+\frac{1}{2}\sum_{\mathbf{k}_\lambda}|d^{\text{eff}}_{\mathbf{k}_\lambda}|^2 g^2_{k_\lambda}\frac{i\omega_0}{\hbar c}\{\zeta^*(\omega_k+\omega_0)-\zeta(\omega_k\\&-\omega_0)\}\tilde{r}_+ +\frac{1}{2}\sum_{\mathbf{k}_\lambda}|d^{\text{eff}}_{\mathbf{k}_\lambda}|^2 g^2_{k_\lambda}\frac{i\omega_0}{\hbar c}\{\zeta^*(\omega_k-\omega_0)\\&-\zeta(\omega_k+\omega_0)\}\tilde{r}_-.\end{aligned} \tag{3.7}$$

We consider the equation for $\tilde{r}_+$. After vacuum averaging, this equation takes the form

$$\begin{aligned}\frac{d}{dt}\langle\tilde{r}_+(t)\rangle=&i\left(\omega_0+\delta+\frac{i}{2\tau_1}\right)\langle\tilde{r}_+(t)\rangle-i\left(\delta-i\frac{1}{2\tau_1}\right)\\&\times\langle\tilde{r}_-(t)\rangle,\end{aligned} \tag{3.8}$$

where the shift of the transition line center is

$$\delta=\left(\frac{\omega_0}{\hbar c}\right)^2\sum_{\mathbf{k}_\lambda}\left(\frac{\mathscr{P}}{\omega_k+\omega_0}-\frac{\mathscr{P}}{\omega_k-\omega_0}\right)g^2_{k_\lambda}|d^{\text{eff}}_{\mathbf{k}_\lambda}|^2. \tag{3.9}$$

In the continuum limit a hypothetical upper limit must be introduced into (3.9) to eliminate ultraviolet divergence.

4. NEAR-FIELD EFFECT

We have already offered some preliminary remarks regarding the near-field effect, which is associated with the existence of two spatial scales in the interaction of atoms in a radiation field. We calculate $d^{\text{eff}}_{\mathbf{k}_\lambda}$ (2.10) at various observation points. The properties of the integrals (2.10a) are such that in the wave region, where $kR_0\gg 1$, their dependence on the coordinates of the observation point $\mathbf{r}_0$ is completely de-

termined by the exponential multiplier $\exp(i\mathbf{k}\mathbf{r}_0)$. This was shown in Ref. 14, where an analytical expression for these integrals was obtained. A different situation is observed in the near-field region, where $kR_0 \lesssim 1$. To demonstrate this, we transform the integral $\mathbf{I}^{(1)}_{\mathbf{k}_\lambda}$ using Green's theorem in the following manner:

$$\mathbf{I}^{(1)}_{\mathbf{k}_\lambda} = \frac{1}{(n_0^2-1)(\omega_k/c)^2} \operatorname{curl}\operatorname{curl} \int_\Sigma \left\{ \mathbf{Q}_{\mathbf{k}_\lambda} \frac{\partial G_k}{\partial \nu'} - G_k \frac{\partial \mathbf{Q}_{\mathbf{k}_\lambda}}{\partial \nu'} \right\} dS', \tag{4.1}$$

where the differentiation $\partial/\partial\nu'$ is carried out along an external normal to the surface Σ (Fig. 1) and $G_k(R_0) = \exp(i(\omega/c)R_0)/R_0$.

The derivatives appearing in the integral with respect to Σ have the form

$$\frac{\partial \mathbf{Q}_{\mathbf{k}_\lambda}}{\partial \nu'} = N\alpha \mathbf{e}_{\mathbf{k}_\lambda} \left(\frac{\partial \mathbf{r}'}{\partial \nu'} \mathbf{K} \right) \exp(i\mathbf{K}\mathbf{r}'),$$

$$\frac{\partial G_k}{\partial \nu'} = i \frac{\omega_k}{c} \frac{\partial R_0}{\partial \nu'} G_k \left(1 + \frac{i}{(\omega_k/c)R_0} \right). \tag{4.2}$$

Following Ref. 14, when the condition $kR_0 \gg 1$ is satisfied, we can use the method of stationary phase and obtain an analytical expression for integral (2.10a), which expresses the Fresnel field of the reflected wave at $\mathbf{r}_0$ with a constant amplitude and the phase factor $\exp(i\mathbf{k}\mathbf{r}_0)$.

We calculate the surface integral I_Σ appearing in (4.1) for distances R_0 in both the near-field and wave regions. We assume that the observation point $\mathbf{r}$ is located outside the medium; therefore, we have the following vectors:

$$\mathbf{r} = (x,y,z), \quad \mathbf{r}' = (x',y',0), \quad R = |\mathbf{r}-\mathbf{r}'|. \tag{4.3}$$

We specify the xz plane as the plane of incidence of the photons and assign a certain angle of refraction θ_T to the photons, so that the wave vector of each photon in the medium lies in the plane of incidence and has the following components

$$K_x = -\frac{\omega}{c} n \sin\theta, \quad K_y = 0, \quad K_z = -\frac{\omega}{c} n \cos\theta_T. \tag{4.4}$$

Then in view of (4.2) the surface integral takes on the form

$$I_\Sigma = ik_0 \int_\Sigma \exp\{ik_0(R - nx' \sin\theta_T)\} \frac{1}{R} \left[n\cos\theta_T - \frac{z}{R}\left(1 + \frac{i}{k_0 R}\right)\right] dS', \tag{4.5}$$

where $k_0 = \omega/c$. We introduce a polar coordinate system in the following manner:

$$x' = x - \rho\sin\varphi, \quad y' = y - \rho\cos\varphi, \quad dS' = \rho d\rho d\varphi. \tag{4.6}$$

This allows us to rewrite expression (4.5) as

$$I_\Sigma = 2\pi i k_0 \exp(-ik_0 \sin\theta_I x) \int_0^\infty \frac{\rho}{(\rho^2+z^2)^{1/2}} \times \left[n\cos\theta_T - z\left(\frac{1}{(\rho^2+z^2)^{1/2}} + \frac{i}{k_0(\rho^2+z^2)} \right)\right] \times \exp\{ik_0(\rho^2+z^2)^{1/2}\} J_0(C\rho) d\rho, \tag{4.7}$$

where θ_I is the angle of incidence of the photon, $n\sin\theta_T = \sin\theta_I$,

$$C = k_0 n \sin\theta_T, \quad 0 < C < k_0,$$

and $J(C\rho)$ is a zeroth-order Bessel function.

From Ref. 20, we can find the following integrals:

$$\int_0^\infty \frac{x}{\sqrt{x^2+z^2}} \sin(b\sqrt{x^2+z^2}) J_0(Cx) dx = \frac{1}{\sqrt{b^2-C^2}} \cos(z\sqrt{b^2-C^2}),$$

$$\int_0^\infty \frac{x}{\sqrt{x^2+z^2}} \cos(b\sqrt{x^2+z^2}) J_0(Cx) dx = -\frac{1}{\sqrt{b^2-C^2}} \sin(z\sqrt{b^2-C^2}). \tag{4.8}$$

Therefore, we have

$$I(z) = \int_0^\infty \frac{x}{\sqrt{x^2+z^2}} \exp(ik_0\sqrt{x^2+z^2}) J_0(Cx) dx = \frac{i}{\sqrt{k_0^2-C^2}} \exp(iz\sqrt{k_0^2-C^2}). \tag{4.9}$$

The differentiation of (4.9) with respect to z gives the expression

$$I'(z) = ik_0 \int_0^\infty \frac{x}{\sqrt{x^2+z^2}} \exp(ik_0\sqrt{x^2+z^2}) z \left[\frac{1}{\sqrt{x^2+z^2}} + \frac{i}{k_0(x^2+z^2)} \right] J_0(Cx) dx = \exp(-iz\sqrt{k_0^2-C^2}). \tag{4.10}$$

Thus, taking into account expression (4.10), we find

$$I_\Sigma = -2\pi \exp(-ik_0 \sin\theta_I x) I'(z) + 2\pi i k_0 \times \exp(-ik_0 \sin\theta_I x) n \cos\theta_T I(z),$$

or, after some simple transformations, we ultimately obtain

$$I_\Sigma = -2\pi \exp(ik_0 \mathbf{s}_R \mathbf{r}) \frac{\sin(\theta_I - \theta_T)}{\sin\theta_T \cos\theta_I}, \tag{4.11}$$

where the unit vector $\mathbf{s}_R$ has the components

$$\mathbf{s}_R = (-\sin\theta_I, 0, \cos\theta_I).$$

Expression (4.11) coincides with the corresponding surface integral calculated by the method of stationary phase[14] in the wave region. As we know, this value of the integral makes it possible to obtain the Fresnel laws for the refraction and reflection of light waves. Our calculation of surface in-

tegral (4.5) is also valid for $k_0R\sim1$, and this indicates that strict amplitude-phase matching of the fields appearing in the integrand in (4.5), which may be different functions of R, occurs at the surface Σ. Thus, when there is a continuous distribution of dipoles in the medium, the surface Σ marks the mathematical boundary $z=0$, at which the reflected wave with the exponential phase multiplier $\exp(ik_0\mathbf{s}_R\mathbf{r})$ begins.

We consider separately the case of $k_0R\ll1$, in which the model of a continuous dielectric medium cannot be employed. In this case the model of a physical boundary of a dielectric, whose thickness is specified by the structural parameter L_0, should be used. Strict matching of the fields appearing in the integrand in (4.5) does not occur on the physical surface Σ, and a dependence of integral (4.5) differing from $\exp(ik_0\mathbf{s}_R\mathbf{r})$ appears. The concept of a physical boundary, which we have introduced in our treatment, is associated with the near-field effect.

Further calculations lead to the following expression for the effective dipole moment

$$d_{\mathbf{k}_\lambda}^{\text{eff}}=d_0\mathbf{u}_d\left\{\frac{N\alpha}{(n_0^2-1)k_0^2}\,\text{curl curl}\,\mathbf{e}_{\mathbf{k}_\lambda}(I_\Sigma-I_{\Sigma_0}+I_{\Sigma_d})+\mathbf{S}\right.$$
$$\left.+\mathbf{I}_{\mathbf{k}_\lambda}^{(2)}+\mathbf{e}_{\mathbf{k}_\lambda}\exp(i\mathbf{kr}_0)\right\},\qquad(4.12)$$

where I_Σ is specified by Eq. (4.11), I_{Σ_0} and I_{Σ_d} are the surface integrals over the circle formed by the intersection of the Lorentzian sphere and Σ and over the part of the Lorentzian sphere inside the medium, respectively, and

$$\mathbf{S}=\alpha\sum_a\exp\{i(\mathbf{Kr}_a+k_0R_a)\}\left[\mathbf{e}_{\mathbf{k}_\lambda}\left(\frac{ik_0}{R_a^2}+\frac{k_0^2}{R_a}\right)\right.$$
$$\left.-k_0^2\mathbf{n}_a\frac{(\mathbf{e}_{\mathbf{k}_\lambda}\mathbf{n}_a)}{R_a}-3ik_0\mathbf{n}_a\frac{(\mathbf{e}_{\mathbf{k}_\lambda}\mathbf{n}_a)}{R_a^2}+\frac{3\mathbf{n}_a(\mathbf{e}_{\mathbf{k}_\lambda}\mathbf{n}_a)-\mathbf{e}_{\mathbf{k}_\lambda}}{R_a^3}\right].\qquad(4.13)$$

Here $\mathbf{R}_a=\mathbf{r}_0-\mathbf{r}_a$, and $\mathbf{n}_a=\mathbf{R}_a/R_a$. As numerical analysis shows, the largest contribution to $d_{\mathbf{k}_\lambda}^{\text{eff}}$ is made by $\mathbf{S}$, which corresponds to the region of the discrete distribution, i.e., by the last term in (4.13), which depends on the coordinate as $1/R_a^3$ and thus corresponds to the electrostatic interaction between the resonant atoms. Upon transition to the wave region, the effective dipole moment takes the form (without consideration of the positron polarizing field)

$$d_{\mathbf{k}_\lambda}^{\text{eff}}=d_0\mathbf{u}_d\left\{\frac{3}{2(n_0^2+2)}\frac{\sin(\theta_T-\theta_I)}{\sin\theta_T\cos\theta_I}\right.$$
$$\left.\times[\mathbf{e}_{\mathbf{k}_\lambda}-\mathbf{s}_R(\mathbf{s}_R\mathbf{e}_{\mathbf{k}_\lambda})]\exp(ik_0\mathbf{s}_R\mathbf{r}_0)+\mathbf{e}_{\mathbf{k}_\lambda}\exp(i\mathbf{kr}_0)\right\}.\qquad(4.14)$$

Using this relation, we can show that in the case of normal incidence ($\theta_I=0$) the expression for the lifetime of an atom in the excited state coincides with the corresponding result in Ref. 3. These relations will be analyzed in greater detail in our next reports.

5. SPATIAL DISTRIBUTION OF SPONTANEOUS PHOTONS NEAR THE SURFACE OF A DIELECTRIC MEDIUM

We use the extinction theorem (2.24) and Eq. (2.25) to describe the photon field near the surface Σ at observation points inside and outside of the medium. We assume here that each observation point is located at a certain distance from Σ, which is greater than the characteristic scale of the medium, so that the refractive index would not depend on the location of the observation point. In the case of a stationary medium, the refractive index also does not depend on the time, i.e., the refractive index in our treatment is a constant real quantity. We also assume that the location of the resonant atom relative to Σ is fixed. We are interested in the field at an arbitrary observation point $\mathbf{r}\neq\mathbf{r}_0$. The distance from the observation point to the atom R_0 must satisfy the condition $R_0\gg a$, where a is the characteristic dimension of the resonant atom. This condition allows us to use the electric dipole approximation and to write the second term on the right-hand side of Eq. (2.14) in the form of a dipole field operator.

The surface integrals $\mathbf{I}_{\mathbf{k}_\lambda}^{(3)}$ and $\mathbf{I}_{\mathbf{k}_\lambda}^{(4)}$ in (2.24) and (2.25) contain the creation (annihilation) operators of the photons $\tilde{c}_{\mathbf{k}_\lambda}^+(\tilde{c}_{\mathbf{k}_\lambda})$ under the integral sign. The operators $\tilde{c}_{\mathbf{k}_\lambda}$ and $\tilde{c}_{\mathbf{k}_\lambda}^+$ depend on the coordinates and on the time, i.e., on the variables $\mathbf{r}'$ and $t-R/c$. It is not possible to solve the quantum-optical boundary problem under these conditions; therefore, as before, we use the approximation of slowly varying amplitudes for the photonic operators (the adiabatic approximation), under which

$$\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r}',t-R/c)\approx\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r},t),\quad\tilde{c}_{\mathbf{k}_\lambda}^+(\mathbf{r}',t-R/c)\approx\tilde{c}_{\mathbf{k}_\lambda}^+(\mathbf{r},t).\qquad(5.1)$$

Let us elaborate on the physical meaning of this approximation. The slowly varying amplitudes $\tilde{c}_{\mathbf{k}_\lambda}$ (as well as $\tilde{c}_{\mathbf{k}_\lambda}^+$) under the surface-integral sign satisfy the trivial inequalities

$$e^{-i\omega_kt}\frac{\partial}{\partial t}\tilde{c}_{\mathbf{k}_\lambda}\ll\omega_kc_{\mathbf{k}_\lambda}e^{i\omega_kt},$$
$$e^{i\mathbf{Kr}'}\frac{\partial}{\partial\mathbf{r}'}\tilde{c}_{\mathbf{k}_\lambda}\ll Kc_{\mathbf{k}_\lambda}e^{i\mathbf{kr}'}.\qquad(5.1a)$$

When these inequalities are satisfied in the integrodifferential equation (2.24), two types of fields, viz., the fields at the current points $\mathbf{r}'$ on Σ and the fields at the observation points, can be distinguished. In the near-field region, where $K|\mathbf{r}'-\mathbf{r}|\ll1$ holds, we may assume that the amplitudes of the fields at different points $\mathbf{r}'$ on the surface are identical and that at the observation points $\mathbf{r}$ we have a slowly varying amplitude, which is specified by the superposition of all the polarizing fields and the initial field $\mathbf{E}_{\text{in}}$. When we change the location of the observation point, we automatically alter the identical amplitudes of the fields at the points $\mathbf{r}'$, and as a result of the integration of the surface integrals with rapidly varying exponential functions and multipliers of the forms $1/R$ and $1/R^2$ in the extinction theorem (2.24), we obtain another value for $\tilde{c}_{\mathbf{k}_\lambda}(\mathbf{r},t)$. Note also that as a result of the use of conditions (5.1) and (5.1a), the boundary problem continues to remain self-consistent, since the fields at $\mathbf{r}$ and $\mathbf{r}'$ differ from the initial field. In the wave region, where

$K|\mathbf{r}'-\mathbf{r}|\gg 1$, the amplitudes of the fields are identical at all $\mathbf{r}'$ and $\mathbf{r}$ according to the structure of the surface integrals, and approximations (5.1) and (5.1a) may be omitted. Thus, in our treatment only the field at an observation point has physical meaning. Conditions (5.1) and (5.1a) suppress the weak effects associated with successive consideration of the variations in the amplitudes of the fields at the points $\mathbf{r}'$. Employing conditions (5.1), from Eq. (2.24) we obtain the approximation equation

$$\tilde{c}^{(0)}_{\mathbf{k}_\lambda}(\mathbf{r}_T,t)=-\frac{E^{\text{in}}_{\mathbf{k}_\lambda}(\mathbf{r}_T,t)}{(\text{curl curl }\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}+\mathbf{I}^{(4)}_{\mathbf{k}_\lambda})\mathbf{e}_{\mathbf{k}_\lambda}}\frac{n^2-1}{(c/\omega_k)^2}, \tag{5.2}$$

where $K\neq k$ in accordance with the choice of trial solution (2.17), $\mathbf{e}_{\mathbf{k}_\lambda}$ is the unit vector of the polarization of the initial field $E^{\text{in}}_{\mathbf{k}_\lambda}$, and the values of the surface integrals $\mathbf{I}^{(3)}_{k_\lambda}$ and $\mathbf{I}^{(4)}_{k_\lambda}$ without the photon creation (annihilation) operators were determined at an observation point $(\mathbf{r}_T,T)$ within the medium.

We can now use (5.2) to find the field $E^{\text{R}}_{\mathbf{k}_\lambda}$ at the observation points $(\mathbf{r}_R,t)$ above the surface Σ. Plugging (5.2) into Eq. (2.25), we obtain the following equation for the field amplitude:

$$\mathbf{E}^{\text{R}(0)}_{\mathbf{k}_\lambda}(\mathbf{r}_R,t)=-\frac{\mathbf{E}^{\text{in}}_{\mathbf{k}_\lambda}(\mathbf{r}_T,t)}{(\text{curl curl }\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}+\mathbf{I}^{(4)}_{\mathbf{k}_\lambda})\mathbf{e}_{\mathbf{k}_\lambda}}\times[\text{curl curl }\mathbf{I}^{(3)}_{\mathbf{k}_\lambda}+\mathbf{I}^{(4)}_{\mathbf{k}_\lambda}]_{\mathbf{r}_R,t}\,. \tag{5.3}$$

In this equation there is a definite correspondence between the fields at observation points $\mathbf{r}_T$ within the medium and at an observation point $\mathbf{r}_R$ outside of the medium, which is embodied in the surface integrals.

A. Initial photon field

In Eq. (2.14), as well as in the extinction theorem (2.24), the initial photon field has the form

$$\mathbf{E}_{\text{in}}(\mathbf{r},t)=\tilde{\mathbf{E}}_v(\mathbf{r},t)+\text{curl curl}\frac{\tilde{\mathbf{d}}(\mathbf{r}_0,t-R_0/c)}{R_0}, \tag{5.4}$$

where $R_0=|\mathbf{r}-\mathbf{r}_0|$ is the distance from the fixed location of the resonant atom to the observation point. Here the operator of the local dipole moment $\tilde{\mathbf{d}}$ is determined by the field at $\mathbf{r}_0$, and this field is the solution of the system of equations (2.14) and (3.1). In other words, we have a self-consistent problem of finding the field at different observation points when the resonant atom and the nonresonant medium have some polarizing influence on one another, which is specified by integrodifferential equation (2.14).

To solve the boundary problem posed in this paper, it is important to properly select the corresponding initial conditions. As such an initial condition we choose the field (5.4), in which the local dipole moment of the resonant atom is determined by the self-field of the photons, according to Eq. (2.12), in the assigned polarizing field of the nonresonant dielectric medium. Thus, when $\tilde{\mathbf{d}}$ is calculated, we can disregard the reciprocal influence of the atom on the medium and employ the concept of an effective dipole moment $\mathbf{d}^{\text{eff}}_{\mathbf{k}_\lambda}$. Employing the standard procedure in Ref. 16, which we have already mentioned above in connection with Eqs. (3.6) and (3.9), we obtain the following expression for the initial field (5.4):

$$\langle\mathbf{E}_{\text{in}}(\mathbf{r},t)\rangle=-\text{curl curl}\frac{2d_0}{R_0}\langle\tilde{r}_2\rangle_{\mathbf{r}_0,t-R_0/c}\mathbf{u}_d, \tag{5.5}$$

where the vacuum mean $\langle\tilde{r}_2\rangle$, as a solution of Eqs. (3.1), has the form

$$\langle\tilde{r}_2\rangle=\frac{1}{2i}\exp\left(-\frac{t-R_0/c}{2\tau_1}\right)\{\langle r_+(0)\rangle\exp[i(\omega_0+\delta)\times(t-R_0/c)]-\langle r_-(0)\rangle\exp[-i(\omega_0+\delta)\times(t-R_0/c)]\}. \tag{5.6}$$

Here the lifetime τ_1 and the frequency shift δ were determined at $\mathbf{r}_0$ from Eqs. (3.6) and (3.9) using $\mathbf{I}^{(1)}_{\mathbf{k}_\lambda}$ and $\mathbf{I}^{(2)}_{\mathbf{k}_\lambda}$.

B. Angular distribution of spontaneous photons near a surface

We use Eqs. (5.2) and (5.4) to investigate the principal amplitude-phase characteristics of the field of spontaneous photons at different observation points both within a dielectric medium and outside it in the wave and near-field regions. We ascertain the principal properties of the spontaneous photon field, using the values of the surface integrals specifying the quantum refractive index (5.2) and reflection coefficient (5.3) of spontaneous photons.

1. We assume that the surface of the dielectric medium is uniform. This means that the lifetime τ_1 and the frequency shift δ are not dependent on the coordinates x and y on the surface Σ (Fig. 1). We isolate the spectral component corresponding to the wave vector $\mathbf{k}$ in the field E_{in}. Then the negative-frequency part of field (5.6) has the exponential multiplier $\exp(i\mathbf{k}_\perp r_\perp)$, and the denominators in Eqs. (5.2) and (5.3) contain the exponential function $\exp(i\mathbf{K}_\perp r_\perp)$, where $\mathbf{k}_\perp$ and $\mathbf{K}_\perp$ are projections of the wave vectors $\mathbf{k}$ and $\mathbf{K}$ onto Σ. Owing to the uniformity of Σ, we have

$$k_x=K_x,\quad k_y=K_y, \tag{5.7}$$

or

$$k_x=-\frac{\omega_k}{c}\sin\theta_I\cos\varphi_I,\quad k_y=\frac{\omega_k}{c}\sin\theta_T\sin\varphi_I,$$

$$K_x=-n\frac{\omega_k}{c}\sin\theta_T\cos\varphi_T,\quad K_y=n\frac{\omega_k}{c}\sin\theta_T\sin\varphi_T, \tag{5.8}$$

where θ_I and φ_I are the assigned angles of incidence of the spontaneous photons onto the surface Σ, and θ_T and φ_T are the corresponding angles of refraction. The angles of incidence and reflection in the three-dimensional problem under consideration can be related to one another using Eqs. (5.7). Similarly, the angles of incidence and the angles of reflection θ_R and φ_R can be related to one another using relations (5.7) and Eq. (5.3).

2. When the position of the excited atom $\mathbf{r}_0$ and the photon wave vector are fixed, the initial field (5.5) is a plane

wave with a constant amplitude. The dependence of the surface integrals $\mathbf{I}_{\mathbf{k}_\lambda}^{(3)}$ and $\mathbf{I}_{\mathbf{k}_\lambda}^{(4)}$ in Eq. (5.2) on the coordinate of the observation point $\mathbf{r}$ is such that in the wave region the refracted photons are also represented in the form of a plane wave with a constant amplitude. The amplitude of the reflected photons has the same property. A different situation is observed in the near-field region. When spontaneous photons impinge on a surface in the form of plane waves with a constant amplitude, the reflected and refracted photons are represented in the form of plane waves whose amplitudes vary as the observation point varies. This property of reflected and refracted photons is a manifestation of the near-field effect. We can attribute this property to the linear superposition of electrostatic and retarded polarizing fields having different dependences on the relative distance R in the extinction-inducing field (2.24). It is noteworthy that a similar situation is also observed in classical optics, where the near-field effect results in the appearance of non-Fresnel reflection and refraction laws.

3. When the atom is stationary relative to the surface of the dielectric, the process of the emission of spontaneous photons exhibits anisotropy of the angular distribution, under which the field of spontaneous photons in the upper hemisphere relative to the plane z_0=const differs from the field of spontaneous photons in the lower hemisphere relative to the same plane. This anisotropy is caused by the character of the dynamic interaction of the atoms through the virtual photon field, under which the probability of spontaneous emission from one of a pair of interacting atoms in the direction of the wave vector $\mathbf{k}$ is not equal to the corresponding probability of the emission of a photon in the opposite direction $-\mathbf{k}$ (Ref. 1).

The results of our detailed numerical analysis of Eqs. (5.2) and (5.7) for the complex amplitudes of the field of spontaneous photons at various observation points will be presented separately in special journals.

In conclusion, we wish to thank the participants in the Jubilee Lectures on Photon Echoes (October 1993), as well as V. G. Minogin and Yu. E. Lozovik for a useful discussion of the near-field effect and its role in various boundary problems of classical, nonlinear, and quantum optics.

[1] O. N. Gadomskiĭ, R. A. Vlasov, *Optical Echo Spectroscopy of Surfaces* [in Russian], Nauka i Tekhnika, Minsk, 1990, Chaps. 2 and 3.
[2] L. T. Canham, Appl. Phys. Lett. **57**, 1046 (1990).
[3] R. J. Cook and P. W. Milonni, Phys. Rev. A **35**, 5081 (1987).
[4] A. S. Parkins and C. W. Gardiner, Phys. Rev. A **40**, 3796 (1989).
[5] P. Nordlander and J. C. Tully, Phys. Rev. B **42**, 5564 (1990).
[6] E. A. Hinds and V. Sandoghdar, Phys. Rev. A **43**, 398 (1991).
[7] H. Chew, J. Chem. Phys. **87**, 1355 (1987).
[8] Il-Tong Cheon and Sung Dahm Oh, J. Phys. Soc. Jpn. **58**, 1581 (1989).
[9] G. Barton, Proc. R. Soc. London A **410**, 141 (1987).
[10] G. Björk, Phys. Rev. A **44**, 669 (1991).
[11] A. O. Barut and J. P. Dowling, Phys. Rev. A **36**, 2550 (1987).
[12] S. D. Brorson, H. Yokoyama and E. P. Ippen, IEEE J. Quantum Electron. **26**, 1492 (1990).
[13] A. A. Belov, Yu. E. Lozovik, V. L. Pokrovsky, J. Phys. **22**, L101 (1989).
[14] M. Born and E. Wolf, *Principles of Optics*, 4th edn., Pergamon Press, Oxford-New York, 1970 (Russ. transl. Nauka, Moscow, 1973, p. 109).
[15] O. N. Gadomskiĭ, V. R. Nagibarov and N. K. Solovarov, Zh. Eksp. Teor. Fiz. **70**, 435 (1976) [Sov. Phys. JETP **43**, 225 (1976)].
[16] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms*, Wiley-Interscience, New York, 1975, Chap. 7 (Russ. transl. Mir, Moscow, 1978, Chap. 7).
[17] G. W. F. Drake, Phys. Rev. **5**, 1979 (1972).
[18] L. N. Gadomskiĭ, V. R. Nagibarov, and N. K. Solovarov, Zh. Eksp. Teor. Fiz. **63**, 813 (1972) [Sov. Phys. JETP **36**, 426 (1973)].
[19] W. Heitler, *The Quantum Theory of Radiation*, 3rd edn., Clarendon Press, Oxford, 1954 (Russ. transl. IL, Moscow, 1956, Chap. 8).
[20] A. P. Prudnikov, Yu. A. Brychkov and O. I. Marichev, *Integrals and Series*, Gordon and Breach, New York, 1986.

Translated by P. Shelnitz