Resonant tunneling through a fluctuating level

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We examine the effect of fluctuations of the surrounding medium on the resonant tunneling through a nonstationary barrier containing localized states. The models employed are those of a Gauss–Markov process and a discontinuous Markov process. We establish the effect of dynamic narrowing of the barrier penetration probability, which manifests itself in the narrowing of the curve representing the dependence of the penetration probability on the energy of the tunneling electron as the correlation time of the fluctuations increases.

1. INTRODUCTION

Resonant tunneling through local energy levels has been well studied both theoretically and experimentally (see, e.g., Ref. 1). This phenomenon has seen an upsurge of interest in the development of solid-state microelectronics, field-effect transistors based on one-dimensional metal–insulator–semiconductor structures, mesoscopic semiconductor junctions, etc.[2–5] The probability of an electron with an energy ε tunneling from region 1 on the left of the barrier to region 3 on the right of the barrier through a localized state with an energy ε_2 inside the barrier (the barrier's transmission coefficient) is described, as is known,[6] by a formula of the Breit–Wigner type:

$$D(\varepsilon-\varepsilon_2)=\frac{\Gamma_1\Gamma_3}{(\varepsilon-\varepsilon_2)^2+\Gamma^2/4}, \tag{1}$$

where $\frac{1}{2}\Gamma$ is the width of the level formed by the localized state, and $\Gamma=\Gamma_1+\Gamma_3$, with Γ_1 and Γ_3 the partial widths describing the tunneling decay of the electron in the regions on the left and right of the barrier, respectively. This paper examines how fluctuations of the energy ε_2 of the level brought on by the surrounding medium affect the Breit–Wigner resonance (1). This is part of the general problem of tunneling through a nonstationary barrier.[7–9] The reason for the fluctuations may, for instance, be electron–phonon processes, which are known to broaden the lines in the optical spectra of local centers. For phononless lines the contributions of the zero-point vibrations of phonons[10] lead to a Lorentzian shape of the optical absorption lines and correspond to a random-force correlator proportional to $\exp\{-|t-t'|/\tau\}$. The interaction of an electron on a local center inside the barrier with the optical phonons within the phonon dispersion band also leads to random forces that change the position of the energy level in time. Another example of such fluctuations is two possible positions of the energy level ε_2 related to a change in the charge of the localized state in the barrier brought on by electrons hopping onto the closest (nonresonance) states and back. Such a situation may have been realized in experiments.[2–5] A likely source of random forces is an external source of noise, say, an ultrasonic generator acting on the insulator layer of the barrier, which has piezoelectric properties.

This paper considers Markov processes that describe fluctuations in the position of a resonance level, $\varepsilon'(t)=\varepsilon_2(t)-\langle\varepsilon_2\rangle$, with a correlation function

$$\langle\varepsilon'(t)\varepsilon'(t')\rangle=\Delta^2\exp\left\{-\frac{|t-t'|}{\tau}\right\}, \tag{2}$$

where $\langle\cdots\rangle$ denotes an average over the ensemble of fluctuations of the local-level position, Δ is the characteristic width of the fluctuation distribution function, and τ is the correlation time. The choice of the correlation function in the form of (2) makes it possible to explore nonstationary electron–phonon processes, which sets the statement of the problem in this paper apart from that in Refs. 11–15 and other papers, which allow only for steady-state electron–phonon processes. As will shortly be shown, in the presence of fluctuations the barrier transmission coefficient $D(\varepsilon-\varepsilon_2)$ is highly dependent on the ratio of the width δ of the fluctuation distribution function and the reciprocal correlation time τ^{-1}. In the quasistatic limit $\Delta\tau\gg 1$, the curve representing the D vs $\varepsilon-\varepsilon_2$ dependence is a set of infinitely close Lorentzian profiles with a halfwidth $\frac{1}{2}\Gamma$ and with an envelope following the shape of the distribution function of the fluctuations of positions of the resonance level (say, a distribution function of the Gaussian type $D(\varepsilon-\varepsilon_2)\propto\exp\{-(\varepsilon-\langle\varepsilon_2\rangle)^2/2\Delta^2\}$). In the opposite limiting case of rapid fluctuations, $\Delta\tau\ll 1$, the curve representing the dependence of the tunneling transmission coefficient on the energy of the tunneling particle, $D(\varepsilon-\varepsilon_2)$, has a Lorentzian profile with a halfwidth $\frac{1}{2}\Gamma+\Delta^2\tau$. In the most interesting case, $\frac{1}{2}\Gamma\ll\Delta^2\tau\ll\Delta$, the width of this curve is determined by $\Delta^2\tau$ and diminishes sharply as the correlation time τ decreases (the effect of dynamic narrowing of the barrier penetration probability, similar to the well-known effect of dynamic narrowing of electromagnetic-radiation absorption lines in stochastic fields[16,17]). A further decrease in the correlation time τ, such taht $\Delta^2\tau\ll\frac{1}{2}\Gamma$ holds, switches off the effect of fluctuations, since the tunneling particle has no time to follow

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the fluctuations. Here the halfwidth of the energy dependence of $D(\varepsilon-\varepsilon_2)$ is, as before, equal to $\frac{1}{2}\Gamma$.

When a Gauss–Markov process is used to describe random forces, an expression for $D(\varepsilon-\varepsilon_2)$ can only be found for the limiting cases of rapid and slow fluctuations. Using the example of a discontinuous Markov process without history, we are able to find an analytical expression for $D(\varepsilon-\varepsilon_2)$ when the relation between Δ and τ^{-1} is arbitrary. This method of description also allows the criterion for the correctness of the two limiting cases to be sharpened. The expressions obtained for the tunneling transmission coefficient make it possible to calculate the tunnel current in the barrier system and clarify the role of fluctuations in the structure the current–voltage characteristic of the system.

2. CALCULATING THE TUNNELING TRANSMISSION COEFFICIENT $D(\varepsilon-\varepsilon_2)$ FOR A GAUSS–MARKOV PROCESS

We examine resonant tunneling in a system with a barrier containing a localized energy level whose fluctuations in time, $\varepsilon'(t)$, are described by a Gauss–Markov process with a correlation function (2). The Schrödinger equation for the electron amplitudes in the tunneling Hamiltonian approximation[1] is

$$\left[i\frac{\partial}{\partial t}-\varepsilon_{1p}\right]c_{1p}(t)=T_{12}c_2(t), \tag{3}$$

$$\left[i\frac{\partial}{\partial t}-\varepsilon_2\right]c_2(t)=\varepsilon'(t)c_2(t)+\sum_p T_{21}c_{1p}(t)+\sum_k T_{23}c_{3k}(t), \tag{4}$$

$$\left[i\frac{\partial}{\partial t}-\varepsilon_{3k}\right]c_{3k}(t)=T_{32}c_2(t), \tag{5}$$

where $C_{1p}(t)$ is the amplitude of an electron in state p to the left of the barrier, $c_2(t)$ is that of an electron on a local center with an average energy ε_2, $C_{3k}(t)$ is the amplitude of an electron in state k to the right of the barrier, and T_{12} and T_{23} are the Hamiltonian matrix elements between the respective states (here and in what follows we ignore the dependence of T_{12} and T_{23} on the energies ε_{1p}, ε_2, and ε_{3k}).

The probability per unit time $w_{1p\to 3k}$ of a transition from state p into state k can be found from

$$w_{1p\to 3k}=\frac{d}{dt}\bigg|_{t\to\infty}<|c_{3k}(t)|^2>.$$

The system of equations (3)–(5) was solved by applying methods of the theory of perturbations on the assumption that the matrix elements T_{12} and T_{23} are small compared to the characteristic energies of the electron states (see also Refs. 11 and 18). The result is

$$w_{1p\to 3k}=2|T_{12}|^2|T_{23}|^2\lim_{t\to\infty}\left\{\exp[-i(\varepsilon_2-\varepsilon_{3k})t-\tfrac{1}{2}\Gamma t]\times\int_0^t dt_1\exp[-i(\varepsilon_{1p}-\varepsilon_2)t_1+\tfrac{1}{2}\Gamma t_1]\times\int_0^{t_1}dt_2\exp[i(\varepsilon_2-\varepsilon_{3k})t_2-\tfrac{1}{2}\Gamma t_2]\times\int_0^{t_2}dt_3\exp[i(\varepsilon_{1p}-\varepsilon_2)t_3+\tfrac{1}{2}\Gamma t_3]\times\langle S(t,t_1,t_2,t_3)\rangle\right\}, \tag{6}$$

where

$$\Gamma=\Gamma_1+\Gamma_3,$$

$$\Gamma_1=2\pi\sum_p|T_{12}|^2\delta(\varepsilon_2-\varepsilon_{1p}), \tag{7}$$

$$\Gamma_3=2\pi\sum_k|T_{23}|^2\delta(\varepsilon_2-\varepsilon_{3k}),$$

and the function $S(t,t_1,t_2,t_3)$ has the form

$$S(t,t_1,t_2,t_3)=\exp\left\{-i\left[\int_0^t dt'\,\varepsilon'(t)-\int_0^{t_1}dt_1'\,\varepsilon'(t_1')-\int_0^{t_2}dt_2'\,\varepsilon'(t_2)+\int_0^{t_3}dt_3'\,\varepsilon'(t_3')\right]\right\}. \tag{8}$$

We now use a well-know property of Gaussian processes:

$$\langle S(t,t_1,t_2,t_3)\rangle=\exp\left\{-\frac{1}{2}\left\langle\left[\int_0^t dt'\,\varepsilon'(t)-\int_0^{t_1}dt_1'\,\varepsilon'(t_1')-\int_0^{t_2}dt_2'\,\varepsilon'(t_2)+\int_0^{t_3}dt_3'\,\varepsilon'(t_3')\right]^2\right\rangle\right\}. \tag{9}$$

The final result is

$$\langle S(t,t_1,t_2,t_3)\rangle=\exp\left\{2\Delta^2\tau^2--\Delta^2\tau[t-t_1-t_2+t_3+2\min(t_1,t_2)-2\min(t_1,t_3)]-\Delta^2\tau^2\left[\exp\left\{-\frac{t-t_1}{\tau}\right\}+\exp\left\{-\frac{t-t_2}{\tau}\right\}-\exp\left\{-\frac{t-t_3}{\tau}\right\}-\exp\left\{-\frac{|t_1-t_2|}{\tau}\right\}+\exp\left\{-\frac{|t_1-t_3|}{\tau}\right\}+\exp\left\{-\frac{t_2-t_3}{\tau}\right\}\right]\right\}, \tag{10}$$

where we have used a correlation function of the Gauss–Markov process of the form (2). The quasistatic limit $\Delta\tau\gg 1$ means ignoring $\varepsilon'(t)$ in the course of the quantum–transition time. We find that

$$w_{1p\to 3k}=\frac{1}{\Delta\sqrt{2\pi}}\int d\varepsilon' \exp\left\{-\frac{\varepsilon'^2}{2\Delta^2}\right\}2\pi|T_{12}|^2|T_{23}|^2 \times\frac{\delta(\varepsilon_{1p}-\varepsilon_{3k})}{(\varepsilon_{1p}-\varepsilon_2-\varepsilon')^2+\Gamma^2/4}. \tag{11}$$

The probability of an electron with an energy ε resonantly tunneling, per unit time, through a barrier (penetration probability, or transmission coefficient), which we define as

$$D(\varepsilon-\varepsilon_2)=2\pi\sum_{p,k} w_{1p\to 3k}\delta(\varepsilon-\varepsilon_{1p}),$$

in the quasistatic limit has the form

$$D(\varepsilon-\varepsilon_2)=\frac{1}{\Delta\sqrt{2\pi}}\int d\varepsilon' \exp\left\{-\frac{\varepsilon'^2}{2\Delta^2}\right\} \times\frac{\Gamma_1\Gamma_3}{(\varepsilon-\varepsilon_2-\varepsilon')^2+\Gamma^2/4}. \tag{12}$$

This formula implies that for $\Delta\tau\gg 1$ the penetration probability $D(\varepsilon-\varepsilon_2)$ considered as a function of ε is a set of infinitely close Lorentzian profiles of type (1) (each of width $\frac{1}{2}\Gamma$) with a Gaussian envelope of width $\sim\Delta$.

Further calculations in analytical form become more complicated, and we restrict our discussion to the limiting case of rapid fluctuations, when the correlation function (2) becomes

$$\langle\varepsilon'(t)\varepsilon'(t')\rangle=2\Delta^2\tau\delta(t-t'). \tag{13}$$

The integrals in Eqs. (6) and (9) can be evaluated directly. As a result we have

$$w_{1p\to 3k}=\frac{2|T_{12}|^2|T_{23}|^2}{(\varepsilon_{1p}-\varepsilon_2)^2+(\Delta^2\tau+\Gamma/2)^2}\left\{\pi\delta(\varepsilon_{1p}-\varepsilon_{3k}) +\frac{2\Delta^2\tau(\Delta^2\tau+\Gamma/2)}{\Gamma[(\varepsilon_2-\varepsilon_{3k})^2+(\Delta^2\tau+\Gamma/2)^2]}\right\}. \tag{14}$$

This differs from the Breit–Wigner formula by the presence of a second term, which describes an additional inelastic-tunneling channel. This channel results from loss of coherence between the $1p\to 2$ and $2\to 3k$ processes caused by a breakdown in the phase of the electron brought on by fluctuations.

Calculation of the penetration probability yields

$$D(\varepsilon-\varepsilon_2)=\frac{2\Gamma_1\Gamma_3(\Delta^2\tau+\Gamma/2)}{\Gamma[(\varepsilon-\varepsilon_2)^2+(\Delta^2\tau+\Gamma/2)^2]}. \tag{15}$$

As this formula implies, for $\Delta\tau\ll 1$ and $\Delta^2\tau\gg\frac{1}{2}\Gamma$ the Breit–Wigner resonance is extremely broad, with the penetration-probability width determined by the correlation time τ and diminishing as τ decreases. This phenomenon, similar to the effect of dynamic narrowing of electromagnetic-radiation absorption lines and first noted by Dicke[16] and Anderson,[17] could be called the dynamic effect of penetration probability narrowing. But if $\Delta^2\tau\ll\frac{1}{2}\Gamma$ holds (the limiting case of rapid fluctuations), the tunneling particle is unable to follow the fluctuations and the Breit–Wigner resonance retains its shape with a halfwidth $\frac{1}{2}\Gamma$.

We also note that, as Eqs. (12) and (15) imply, the fluctuations $\varepsilon'(t)$, while narrowing the Breit–Wigner resonance (1), do not change the total transmission coefficient D_{tot}:

$$D_{\text{tot}}\equiv\int d\varepsilon\, D(\varepsilon-\varepsilon_2)=\frac{2\pi\Gamma_1\Gamma_3}{\Gamma}.$$

This consequence, similar to the sum rule formulated in Refs. 11 and 14, was first noted in Ref. 8.

Within the formal structure developed here it is difficult to define a criterion for the applicability of the approximation (13) for the correlation function and describe by analytical means the case of intermediate values of the parameter $\Delta\tau$. The problems are resolved if to describe the source of fluctuations we use a discontinuous Markov process without history (a generalized telegraph process).[19] As studies of the spectroscopy of absorption lines in stochastic fields show,[20] the use of discontinuous Markov processes gives a fairly exact picture of the main features of the phenomenon. Therefore, in the section below we examine resonant tunneling in a barrier system through a localized state with an energy whose fluctuations in time are described by a purely discontinuous Markov process without history. We also examine the case of a simple telegraph process, corresponding to only two possible positions of the resonance level. As noted in the Introduction, such a situation can be realized owing to a change in the charge of the localized state in the barrier brought on by electrons hopping onto the closest (nonresonance) states and back.

3. CALCULATING THE TUNNELING TRANSMISSION COEFFICIENT $D(\varepsilon-\varepsilon_2)$ FOR A DISCONTINUOUS MARKOV PROCESS WITHOUT HISTORY

A discontinuous Markov process without history[19] presupposes the random quantity ε' changing by jumps, abruptly. These jumps occur independently of each other at arbitrary moments in time $t_1,t_2,\ldots,t_n,\ldots$, with the resulting known distribution over the time intervals between the jumps:

$$dW(t_{n+1}-t_n)=\frac{1}{\tau}\exp\left\{-\frac{t_{n+1}-t_n}{\tau}\right\}dt_{n+1},$$

where τ is the average duration of an interval. Between the jumps the value of the random quantity ε' remains constant and is specified by some distribution $\varphi(\varepsilon')$. The absence of history means that the probability of the random quantity assuming a certain value after the nth jump is in no way affected by the previous value of the quantity. The correlation function for such a process has the form (2), where τ should be interpreted as the average duration of an interval between jumps.

In what follows we will find it convenient to use the density matrix $\rho_{ij}(t)$ instead of electron amplitudes. To simplify notation, we denote the state of an electron with energy ε_{1p} to the left of the junction by 1, that of an electron on level ε_2 by 2, and that of an electron with energy ε_{3k} to the right of the junction by 3. Thus, the subscripts i and j may take on the values 1, 2, and 3. According to the theory of discontinuous Markov processes,[19,21] we introduce the idea of a partial density matrix $\rho_{ij}(t,\alpha)$, which describes the subensemble of systems whose random function $\varepsilon'(t)$ assumes a specified value α after the jump closest to moment t, with $\rho_{ij}(t,\alpha)$ already averaged over all the realizations of the discontinuous Markov process $\varepsilon'(t)$ in all time intervals up to the last jump. Averaging over all such subensembles with the weight function

$$\bar{\rho}_{ij}(t)\equiv\int d\alpha\,\varphi(\alpha)\rho_{ij}(t,\alpha)=\langle\rho_{ij}(t)\rangle$$

yields the density matrix $\langle\rho_{ij}(t)\rangle$ completely averaged over all realizations of the random process $\varepsilon'(t)$. The partial density matrix of our problem satisfies the system of equations[21]

$$i\frac{\partial\hat{\rho}(t,\alpha)}{\partial t}=[\hat{H}(\alpha),\hat{\rho}(t,\alpha)]-i\hat{\gamma}\hat{\rho}(t,\alpha)-i\,\frac{\hat{\rho}(t,\alpha)-\bar{\rho}(t)}{\tau}\tag{16}$$

with the initial conditions $\rho_{ij}|_{t=0}=\delta_{i1}\delta_{j1}$. The Hamiltonian matrix in (16) has the form

$$\hat{H}(\alpha)=\begin{pmatrix}\varepsilon_{1p} & T_{12} & 0\\ T_{21} & \varepsilon_2+\alpha & T_{23}\\ 0 & T_{32} & \varepsilon_{3k}\end{pmatrix}$$

and depends only on α. The $\hat{\gamma}$ matrix describes the tunnel width of the localized state:

$$\gamma_{ij}=\tfrac{1}{2}\Gamma\,\delta_{ij}[\delta_{i2}+\delta_{j2}],$$

where Γ is specified, as before, in Eqs. (7). We wish to calculate the probability $w_{1p\to3k}$ of the tunneling transition $1p\to3k$ per unit time:

$$w_{1p\to3k}=\frac{d}{dt}\bigg|_{t\to\infty}\bar{\rho}_{33}(t).\tag{17}$$

To solve the system of equations (16) we apply the Laplace transformation. This results in the following system of equations for the Laplace transforms $Q_{ij}(s)$ of the components of the density matrix elements $\rho_{ij}(t)$:

$$i[sQ_{11}(s,\alpha)-1]=0,\tag{18}$$

$$isQ_{12}(s,\alpha)=\left[\varepsilon_{12}-\alpha-i\,\frac{\Gamma}{2}\right]Q_{12}(s,\alpha)-\frac{T_{12}}{s}-i\,\frac{Q_{12}(s,\alpha)-\bar{Q}_{12}(s)}{\tau},\tag{19}$$

$$isQ_{22}(s,\alpha)=-i\Gamma Q_{22}(s,\alpha)+T_{21}Q_{12}(s,\alpha)-T_{12}Q_{12}(s,\alpha)-i\,\frac{Q_{22}(s,\alpha)-\bar{Q}_{22}(s)}{\tau},\tag{20}$$

$$isQ_{13}(s,\alpha)=\varepsilon_{13}Q_{13}(s,\alpha)-T_{32}Q_{12}(s,\alpha)-i\,\frac{Q_{13}(s,\alpha)-\bar{Q}_{13}(s)}{\tau},\tag{21}$$

$$isQ_{23}(s,\alpha)=\left[\varepsilon_{23}+\alpha-i\,\frac{\Gamma}{2}\right]Q_{23}(s,\alpha)+T_{21}Q_{13}(s,\alpha)-T_{32}Q_{22}(s,\alpha)-i\,\frac{Q_{23}(s,\alpha)-\bar{Q}_{23}(s)}{\tau},\tag{22}$$

$$isQ_{33}(s,\alpha)=T_{32}Q_{23}(s,\alpha)-T_{23}Q_{32}(s,\alpha).\tag{23}$$

In the system of equations (18)–(23) we have retained only the lowest-order terms in T_{12} and T_{23} and introduced the notation $\varepsilon_{12}=\varepsilon_{1p}-\varepsilon_2$, $\varepsilon_{23}=\varepsilon_2-\varepsilon_{3k}$, and $\varepsilon_{13}=\varepsilon_{1p}-\varepsilon_{3k}$. We have found that

$$\frac{d}{dt}\,\bar{\rho}_{33}(t)=-\frac{i}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}ds\,e^{st}[T_{32}\bar{Q}_{23}-T_{23}\bar{Q}_{32}],\tag{24}$$

$$\bar{Q}_{23}=\frac{i}{1-\bar{R}_{23}/\tau}\int d\alpha\,\varphi(\alpha)[T_{23}Q_{22}-T_{21}Q_{13}]R_{23},\tag{25}$$

$$Q_{13}=-\frac{T_{12}T_{23}}{s(s+i\varepsilon_{13}+1/\tau)(1-\bar{R}_{12}/\tau)}\left[\frac{\bar{R}_{12}/\tau}{s+i\varepsilon_{13}}+R_{12}\right],\tag{26}$$

$$Q_{22}=\frac{|T_{12}|^2}{s(s+\Gamma+1/\tau)}\left\{\frac{1}{1-\bar{R}_{12}/\tau}\left[\frac{\bar{R}_{12}/\tau}{s+\Gamma}+R_{12}\right]+\frac{1}{1-\bar{R}_{21}/\tau}\left[\frac{\bar{R}_{21}/\tau}{s+\Gamma}+R_{21}\right]\right\},\tag{27}$$

where

$$R_{12}(s,\alpha)=\frac{1}{s+i(\varepsilon_{12}-\alpha)+(\Gamma/2+1/\tau)},\tag{28}$$

$$R_{23}(s,\alpha)=\frac{1}{s+i(\varepsilon_{23}+\alpha)+(\Gamma/2+1/\tau)},\tag{29}$$

$$Q_{ij}(s,\alpha)=Q^*_{ji}(s^*,\alpha),R_{ij}(s,\alpha)=R^*_{ji}(s^*,\alpha).$$

Formulas (24)–(29) provide a simple algorithm for calculating the tunneling transition rate $w_{1p\to3k}$ for a distribution function $\varphi(\alpha)$ of arbitrary form and for all values of parameter $\Delta\tau$. They also yield a criterion for the applicability of the rapid-fluctuation limit. For instance, for a Gaussian distribution function $\varphi(\alpha)$ we use the representation of $\bar{R}_{ij}(s)$ in the form of a continued fraction:[22]

$$\int d\alpha\frac{\varphi(\alpha)}{z-\alpha}=\frac{1}{\Delta\sqrt{2\pi}}\int_{-\infty}^{\infty}d\alpha\exp\left\{-\frac{\alpha^2}{2\Delta^2}\right\}\frac{1}{z-\alpha}=\cfrac{1}{z-\cfrac{\Delta^2}{z-\cfrac{2\Delta^2}{z-\cfrac{3\Delta^2}{z-\cdots}}}}.\tag{30}$$

We break off the continued fraction when the small parameter satisfies $\Delta\tau\ll1$ and get

$$\bar{R}_{ij}(s)\simeq\frac{1}{s+i\varepsilon_{ij}+\Gamma/2+1/\tau+\Delta^2\tau}.\tag{31}$$

Employing (31), we see that formulas (24)–(29) give an expression for the transition rate $w_{1p \to 3k}$ that coincides with Eq (14). For $\Delta\tau \gg 1$ Eqs. (24)–(29) yield Eq. (11) for the quasistatic case.

Integration with respect to ε_{3k} yields the following expression for the tunneling transmission coefficient $D(\varepsilon-\varepsilon_2)$:

$$D(\varepsilon-\varepsilon_2)=\frac{2\Gamma_1\Gamma_2}{\Gamma}\mathrm{Re}\left\{\frac{1}{\bar{R}_{12}(0)}-\frac{1}{\tau}\right\}^{-1}. \tag{32}$$

In conclusion we write an analytical expression for the transmission coefficient valid for arbitrary values of parameter $\Delta\tau$ for the simple case of the distribution function

$$\varphi(\alpha)=x\delta\left(\alpha+\Delta\sqrt{\frac{1-x}{x}}\right)+[1-x]\delta\left(\alpha-\Delta\sqrt{\frac{x}{1-x}}\right), \tag{33}$$

when the random quantity α can take on only two discrete values, $-\delta\sqrt{(1-x)/x}$ and $\delta\sqrt{x/(1-x)}$, with probabilities x and $1-x$, respectively, with $\bar{\alpha}=0$ and $\overline{\alpha^2}=\Delta^2$. The formula (32) for $D(\varepsilon-\varepsilon_2)$ then yields

$$D(\varepsilon-\varepsilon_2)=\frac{2\Gamma_1\Gamma_3}{\Gamma}\mathrm{Re}\left\{\frac{[i(\varepsilon-\varepsilon_2-\Delta^-)+(\Gamma/2+1/\tau)][i(\varepsilon-\varepsilon_2-\Delta^+)+(\Gamma/2+1/\tau)]}{i(\varepsilon-\varepsilon_2)+(\Gamma/2+1/\tau)}-\frac{1}{\tau}\right\}^{-1}, \tag{34}$$

where

$$\Delta^-=-\Delta\sqrt{\frac{1-x}{x}},\Delta^+=\Delta\sqrt{\frac{x}{1-x}},$$

Fig. 1 depicts the energy dependence of $D(\varepsilon-\varepsilon_2)$ for different values of parameter $\Delta\tau$ at $x=\frac{1}{2}$. In the range of large values of $\Delta\tau$ the dependence is represented by two Lorentzian profiles of width $\frac{1}{2}\Gamma$ with peaks at $\varepsilon=\varepsilon_2\pm\Delta$ (the quasistatic case). As $\Delta\tau$ decreases to $\frac{1}{2}$, the peaks move closer together and finally merge. In the region $\Delta\tau<\frac{1}{2}$ the curve representing the dependence of $D(\varepsilon-\varepsilon_2)$ is bell-shaped, approaching in form a Lorentzian profile with width of order $\frac{1}{2}\Gamma+\Delta^2\tau$. As the correlation time τ becomes shorter, the peak becomes sharper and grows in height (the effect of dynamic narrowing of $D(\varepsilon-\varepsilon_2)$).

4. DISCUSSION OF RESULTS

We have examined resonant tunneling through a localized state in a system with a potential barrier. The surrounding medium produces fluctuations in the position of the energy level, and these are described by a Gauss–Markov process and a discontinuous Markov process with history. We show that in the presence of fluctuations the barrier transmission coefficient $D(\varepsilon-\varepsilon_2)$ depends significantly on the relation between the dispersion of the random process, Δ, and the reciprocal correlation time τ^{-1}. In the quasistatic limit $\Delta\tau\gg 1$ the energy dependence of $D(\varepsilon-\varepsilon_2)$ follows the shape of the distribution function of the resonance-level position fluctuations. In the opposite limiting case of rapid fluctuations, $\Delta\tau\ll 1$, the dependence of $D(\varepsilon-\varepsilon_2)$ on the energy of the tunneling particle has the shape of a Lorentzian profile with a halfwidth $\frac{1}{2}\Gamma+\Delta^2\tau$. For the most interesting case $\frac{1}{2}\Gamma\ll\frac{1}{2}\Gamma+\Delta^2\tau\ll\Delta$, the width of the curve representing the energy dependence of $D(\varepsilon-\varepsilon_2)$ is determined by $\Delta^2\tau$ and sharply decreases as the correlation time τ shortens (the effect of dynamic narrowing of $D(\varepsilon-\varepsilon_2)$).

These specific features of the tunneling transmission coefficient should manifest themselves in the current–voltage characteristics of the tunneling system under investigation. As is well known, the resonant tunnel current is defined as

$$J=\frac{e}{\pi}\int d\varepsilon\, D(\varepsilon-\varepsilon_2)\times\left[f\left(\varepsilon-\mu-\frac{eV}{2}\right)-f\left(\varepsilon-\mu+\frac{eV}{2}\right)\right]. \tag{35}$$

Here the localized state is assumed to be in the middle of the barrier, $f(\varepsilon-\mu-\frac{1}{2}eV)$ and $f(\varepsilon-\mu+\frac{1}{2}eV)$ are the (Fermi) distribution functions of an electron on the left and right electrodes, respectively, V the voltage applied to the tunneling junction, and μ the Fermi level in the system before the voltage is applied. The possible manifestation of fluctuations of the resonance level is most conveniently studied in the incremental conductance $g(V)=\partial J/\partial V$. In the low-temperature region, where the distribution function can be replaced by a step function,

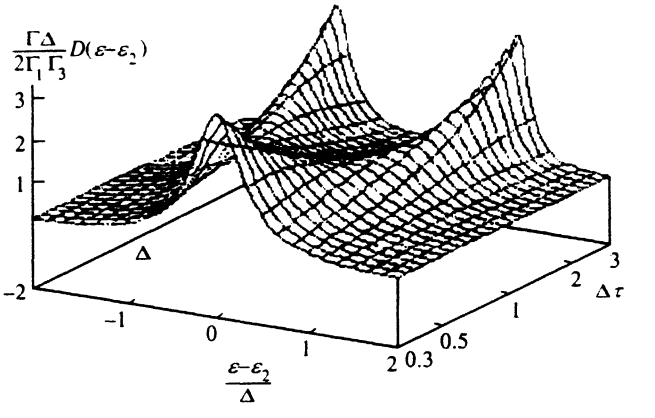

FIG. 1. The tunneling transmission coefficient $D(\varepsilon-\varepsilon_2)$ as a function of the energy ε of a tunneling electron for different values of parameter $\Delta\tau$ in the case of a simple telegraph process corresponding to the distribution function (33) ($x=1/2$ and $\Gamma=0.05\Delta$).

$$g(V)\simeq\frac{e}{2\pi}\left[D\left(\mu+\frac{eV}{2}-\varepsilon_2\right)-D\left(\mu-\frac{eV}{2}-\varepsilon_2\right)\right].$$

Thus, when the Fermi level on one electrode is close to the localized-state energy, the incremental conductance as a function of V reproduces the shape of the energy dependence of the tunneling transmission coefficient.

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