Direct "ionization" of the hydrogen atom by photons in the presence of an electric field

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We show that in the presence of an electric field F the hydrogen-atom spectrum must be approximated not only by Stark states but also by a set of "ionization" curves. We compare the proposed theory with the experimental data on the behavior of the levels of Li and Na atoms in a constant field and with the results of microwave ionization of highly excited Rydberg states of the hydrogen atom.

1. INTRODUCTION

With the development of laser techniques the detailed study of highly excited Rydberg states of atoms began. In interpreting the experimental data both experimenters and theoreticians assumed, without even discussing the problem, that all behavior of atoms in a field can be understood by considering only Stark levels.[1] Very general considerations show that this is not the case, The results of the action of photons on an atom at $F=0$ cannot be understood only by considering discrete states, since if the photon energy exceeds the ionization energy, the atom becomes ionized. Stark states, however, are simply a natural continuation of the discrete levels of an atom at $F=0$ into the region with $F\neq 0$. The presence of a field lifts the degeneracy of the states in the parabolic quantum numbers n_1, n_2, and m. A method for such a continuation was suggested by Erwin Schrödinger, who used a perturbation-theory expansion in the small parameter F (Refs. 2 and 3). This continuation is asymptotically exact as $F\rightarrow 0$. The question of what happens with the ionization limit of an atom when the field F is switched on, which at $F=0$ has $E=0$ and is infinitely degenerate in n_1, n_2, and $|m|$, has not even been fully posed. The Stark states alone provide a fairly full picture of the behavior of an atom in a field only up to energies at which direct ionization of the atom becomes possible. But for $F\neq 0$ we cannot even introduce the notion of an ionization limit determined by a natural value of energy E. Only one mechanism of direct ionization of an atom in a field has so far been suggested in Sec. 54 of Bethe and Salpeter's monograph.[3] These authors did not consider it in detail but did show that it was impossible to identify it using the experimental data available at the time. The possibility emerged only when it became possible to gather data on the behavior of highly excited Rydberg states in Li and Na atoms at $m=1,2$ in the presence of a constant electric field.[4,5]

The ionization mechanism suggested in Ref. 3 was analyzed in detail in a previous paper of the present authors.[6] In particular it was shown that as the principal (total) quantum number n grows (for $m\geqslant 1$), all the Stark sublevels belonging to a given value of n begin to reach the ionization limit at values of F for which the probability of their tunneling in the field is negligible. This leads to the apparent disappearance of the Stark sublevels in such critical fields, a phenomenon observed in Li with the magnetic quantum number m equal to unity and the principal quantum numbers $n=18$, 19, and 20.

In this paper we consider other mechanisms for "direct" ionization of the hydrogen atom in the presence of a constant electric field, mechanisms that are "technically" much simpler than the one mentioned above. Here there is also the possibility of comparing theoretical results with the experimental data on the excitation of the Li atom at $m=0$ and $m=1$ (Ref. 4) and of the Na atom at $m=2$ (Ref. 5), and with the results of microwave ionization of highly excited hydrogen atoms ($n\sim 32$–90 and $m=0$; Refs. 7–10).

The Schrödinger equation for the hydrogen atom in the presence of a homogeneous field separates if it is written in parabolic coordinates, and for this reason finding the asymptotic solution for it is elementary. For our purposes we can restrict ourselves to two terms in the perturbation-theory expansions for the separation constants β_1 and β_2 and the coupling condition which makes it possible to find the energy E (Refs. 2 and 3):

$$\beta_1=\sqrt{-2E}\left[n_1+\frac{m+1}{2}+(6n_1^2+6n_1m+6n_1+m^2+3m+2)\times\frac{F(-2E)^{-3/2}}{4}-\cdots\right], \tag{1}$$

$$\beta_2=\sqrt{-2E}\left[n_2+\frac{m+1}{2}-(6n_2^2+6n_2m+6n_2+m^2+3m+2)\times\frac{F(-2E)^{-3/2}}{4}-\cdots\right], \tag{2}$$

$$\beta_1+\beta_2=1. \tag{3}$$

We can find the Stark energies $E_{n_1n_2m}(F)$ by requiring that Eqs. (1)–(3) hold simultaneously:

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$$E_{n_1,n_2,m}(F) = -\frac{1}{2n^2} + \frac{3}{2}n(n_1-n_2)F - \cdots, \tag{4}$$

where $n=n_1+n_2+m+1$ is the principal (total) quantum number.

Let us now consider the continuous spectrum. Obviously, at $F=0$ we can approach the discrete states starting from the continuous spectrum, i.e., $\beta_1>1$ or $\beta_2>1$, satisfying Eqs. (1)–(3), and attaining the value $\beta_1=1$, i.e., $E=0$, in the limit $n_1\to\infty$, or the value $\beta_2=1$, i.e., $E=0$, in the limit $n_2\to\infty$.

But when the field F is switched on, everything changes. We can satisfy the condition $\beta_1=1$ by selecting the energy in the form

$$E^{+}_{n_1m}(F) = -\frac{1}{2(n_1+(m+1)/2)^2} + \frac{6n_1^2+6n_1m+6n_1+m^2+3m+2}{4}F + \cdots, \tag{5}$$

and condition $\beta_2=1$ by selecting the energy in the form

$$E^{-}_{n_2m}(F) = -\frac{1}{2(n_2+(m+1)/2)^2} - \frac{6n_2^2+6n_2m+6n_2+m^2+3m+2}{4}F + \cdots. \tag{6}$$

Since, strictly speaking, the spectrum of the problem for $F\neq0$ is continuous, the condition (3) can in principle be satisfied at any energy, although we do not need to carry out such a procedure here. Since the conditions $\beta_1=1$ and $\beta_2=1$ are the initial "points" for ionization at $F=0$, their natural continuations into the region where $F\neq0$ yield energy values, with direct ionization "switched on" in the presence of a field. We see that the situation with the ionization curves (5) and (6) is quite similar to the one for the Stark sublevels (4). Excitation of "discrete" and "continuous" states of the spectrum for $F\neq0$ is most effective for values of E obtained by perturbation techniques, but experimentally they manifest themselves differently. At energies E satisfying condition (4) the atom is most stable with respect to decay in the field. The states corresponding to such energies became known as quasistationary. To characterize such states, one is forced to introduce the concept of lifetime in relation to decay in the field, in addition to the concept of energy. The quantum number n_2 does not exist for states $E^{+}_{n_1m}(F)$, and n_1 does not exist for states $E^{-}_{n_2m}(F)$. For this reason we may speak of ionization states. At such energies the third quantum number disappears; hence ionization states decay instantly.

As the field F becomes stronger, the Stark lines $E_{n_1n_2m}(F)$ may reach the ionization curves $E^{+}_{n_1m}(F)$ or $E^{-}_{n_2m}(F)$. The process of direct atomic ionization occurs at the points where these curves intersect in addition to the excitation of Stark states. Hence the strength of the signal indicating the presence of a Stark state drops sharply, so that the state seems to simply disappear.

Our previous paper,[6] which examined a different mechanism for direct ionization of atoms, demonstrated that for small principal quantum numbers the Stark states reach the intersection points with the ionization curves at values of F for which they have already practically decayed due to quantum tunneling. However, as n grows, the picture changes dramatically, and by $n\approx12$ (for n_2 not too large) the Stark curves reach the ionization curves, being already practically stable with respect to decay in the field. As can easily be verified, the situation is similar for the cases considered in this paper. Hence clear experimental observation of direct ionization of atoms in the presence of an electric field is possible only for fairly high Rydberg states. For weakly ionized states such ionization is completely masked by the tunneling effect.

2. BEHAVIOR OF THE STARK LEVELS OF THE HYDROGEN ATOM IN THE PRESENCE OF A CONSTANT FIELD *F*

There is still no experimental data on the behavior of highly excited Stark levels of the hydrogen atom caused by variations in F. We therefore examine the results obtained for Li and Na (Refs. 4 and 5), which for $m\geqslant1$ are close to those for hydrogen.

Comparing Eqs. (5) and (6) with (4), we see that the ionization curves behave differently for odd and even values of m. For instance, for m odd the ionization curves $E^{+}_{n_1'm}(F)$ and $E^{-}_{n_2'm}(F)$ encompass the set of Stark sublevels $E_{n_1n_2m}(F)$ belonging to the same principal quantum number $n=n_1+n_2+m+1$. Hence, at the intersections of Stark levels whose principal quantum numbers differ by unity, the strength of the signal indicating the presence of a level drops sharply. This subtle effect was clearly identified in the experiment reported in Ref. 4 for Stark lines of Li at $m=1$. It was interpreted, however, as a rule according to which the Stark levels of all atoms except hydrogen do not cross. Hydrogen states, however, must behave in exactly the same manner. For m even and a field increasing in strength, the ionization curves begin to intersect the Stark states at a certain field strength, and at this field strength the signal from the Stark state drops off sharply and in some cases can practically disappear.

Kleppner and his collaborators[5] began their studies of the behavior of Stark levels caused by varying F for Na at $m=2$, since this case resembles most closely the situation with hydrogen. They discovered that the level with quantum numbers $n_1=6$, $n_2=3$, and $m=2$, which is almost completely stable with respect to decay in the field, suddenly disappears at an energy $E\sim730\,\mathrm{cm}^{-1}$ and a field strength $F\sim15.2\,\mathrm{kV\,cm}^{-1}$ and then, as the field grows in strength, reappears. This phenomenon was discussed in detail in Ref. 5, where the researchers explained the observed effect by the intersection of the $E_{6,3,2}(F)$ state with the $E_{0,11,2}(F)$ state, which, however, did not manifest itself directly in the experiment owing to its extremely short lifetime with respect to decay in the field. Actually, the temporary disappearance of the $E_{6,3,2}(F)$ state is caused by the intersection of this state with the ionization curve $E^{-}_{13,2}(F)$.

When the authors of Ref. 5 moved from Na atomic states with $m=2$ to Li atomic states with $m=1$, they found no similarity in the behavior of the states. Not expecting the states with $m=1$ and $m=2$ to behave differently, Littman, Zimmerman, and Kleppner apparently arrived at the conclusion that there was an error of some kind in the measurements for Na, with the result that in a detailed review (see Ref. 4) there is no reference to the case with $m=2$. The most complete experimental information about ionization states can be found by examining the Stark curves of the Li atom for $m=0$ and the principal quantum number n equal to 15. We cannot *a priori* expect very good agreement between the theoretical formulas obtained for hydrogen and the experimental data, since the case with $m=0$ for Li is decidedly not hydrogenlike. Nevertheless, here too it is possible to easily verify that the ionization curves $E^{-}_{15,0}(F)$ and $E^{+}_{14,0}(F)$ almost quantitatively describe the most important feature in the behavior of the set of Stark sublevels $E_{n_1n_20}(F)$ when the principal quantum number n is equal to 15; namely, temporary disappearance of the signals from all lines as the field F varies from 1.5 to 3 kV cm^{-1} (see Fig. 3.9 in Ref. 4). As the field strength increases, the signals from the Stark sublevels reappear. There is also an alternative experimental verification of the theory, a verification not directly linked to the formulas (5) and (6). In the experiment reported in Ref. 4 the lithium atoms, being excited to Stark states, did not spontaneously decay, i.e., their lifetimes with respect to quantum tunneling in such fields were too long. The presence of excited atoms can be assessed only by ionizing them by strong field pulses after a certain time lag and fixing the Li^+ ions that appear as result of this ionization. If the experimental method is changed to detecting the Li^+ ions immediately after the atoms have been excited, without ionizing the atoms with field pulses, the ions appear only in such fields F and at such energies E for which the signals corresponding to the presence of Stark states disappear. Another (also correct) explanation of the temporary disappearance of signals from the Stark levels of the lithium atom with $n=15$ and $m=0$ was given in Ref. 4. According to this, in the field and energy ranges specified above the oscillator strengths responsible for the excitation of the specified levels sharply decrease. This in no way contradicts our explanation and, strictly speaking, only the two together provide a full picture of the behavior of the atom in a field. The probability of an atom being directly ionized in a field increases in those fields and at those energies for which the probability of exciting quasidiscrete states drastically decreases.

3. MICROWAVE IONIZATION OF HIGHLY EXCITED STATES OF THE HYDROGEN ATOM

Let us apply the above theory to a time-dependent process. We start by briefly discussing the experiment described in Refs. 7–10. A high-energy beam of excited hydrogen atoms interacted with radiation in a cavity. The atoms were "created" in strictly defined "one-dimensional" states with $n_1=n-1$ or $n_2=n-1$ and $m=0$ (here we employ the terminology of Ref. 9). The values of n varied from 32 to 90. Linearly polarized radiation did not change the magnetic quantum number $m=0$. The temporal variation of the field obeyed the law $A(t)F\sin(\omega t+\varphi)$, where F is directed along the z axis, that is, in the direction of the beam, and $A(t)$ is a slowly varying function characterizing the moment when the field was switched on and off, the periods lasting about 60 oscillations.

We can assume that the quantity $A(t)F$ is time-independent, $A(t)F=\text{const}$, for about 300 oscillations in the cavity[8] and is determined to within 5%. We denote this product by F, and in what follows we will assume that the microwave field has its peak value in the cavity unless otherwise specified. In the first experiments, reported in Refs. 7 and 8, the frequency $\nu=\omega/2\pi$ was maintained at 9.9233 GHz, while F varied from 1 V $^{-1}$ to several volts per centimeter, depending on n. In recent years other frequencies have been used.[9,10]

As a result of interacting with radiation the excited hydrogen atoms were found to decay. The percentage of decayed atoms in relation to the initial number of atoms was measured for every value of n as a function of F. Of all the data obtained so far, the literature most often gives the dependence on n and F of 10% and 90% "ionization," i.e., the percentage of decayed atoms.

Let us examine the main feature of the experimental data more closely. The first experiments conducted at $n=66$ showed that an atom with a binding energy of roughly 3.1 meV effectively decays at a frequency of 9.92 GHz, at which the photon energy equals 41 μeV and the reduced field strength is $n^4F\sim0.09$ a.u. (see Ref. 7). For an excited atom to reach the ionization limit, for which the authors of Refs. 7–10 always took $E=0$, it must absorb approximately 80 photons simultaneously as a result of a multiphoton process. The probability of this is, of course, negligible. There would seem to be the possibility of the atom decaying via quantum mechanical tunneling at the peak value of the oscillating field. But at $n^4F\sim0.09$ a.u. and lower the probability of this process is negligible, too, which follows from Eq. (72) of Ref. 11. A thorough study, lasting many years and involving a broad range of principal quantum numbers n, revealed several factors whose presence had no apparent explanation. For instance, as n increased (starting at $n=32$) and at a frequency of 9.92 GHz, the value of field F at which 10% and 90% ionization takes place falls, which appears natural since the energy with the binding energy of the electron in the atom decreases in the process. But examining the overall shape of the ionization curves with n varying from 32 to 90 (Ref. 8), we discover at least six or seven sections where an increase in n by several units does not lead to a decrease in F. Such behavior in ionization curves was named "ladder-like" in Ref. 8.

This effect manifests itself even more vividly on the ionization curves at $\nu=36.02$, 30.36, and 26.43 GHz when

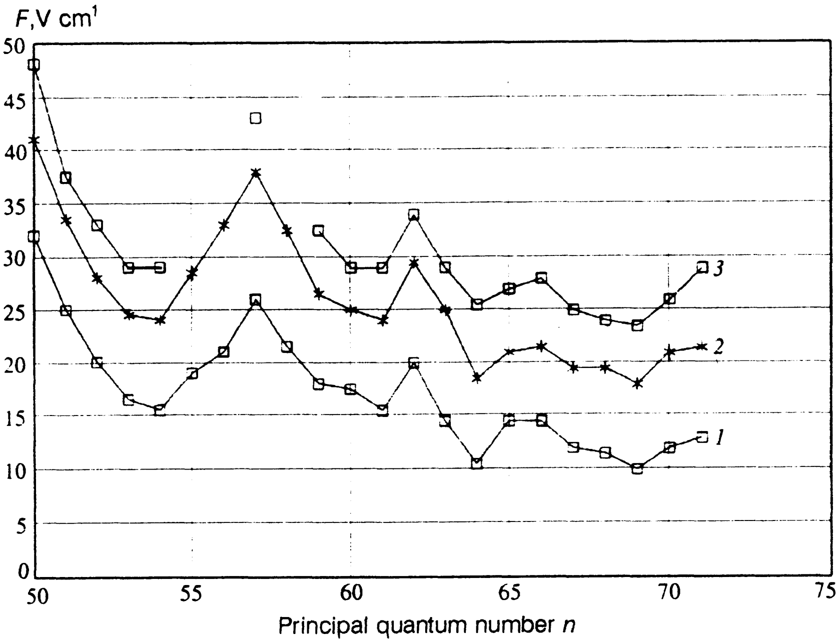

FIG. 1. The threshold field F for 10% (curve *1*), 50% (curve *2*), and 70% (curve *3*) ionization of the hydrogen atom in a microwave field at a frequency $\nu=36.02$ GHz as a function of the principal quantum number n (Ref. 10).

n varies from 50 to 80. An example of this is illustrated by Fig. 1 taken from Ref. 10.

We see that states with $n=57$, 62, and 65–66 are essentially more locally stable under decay in the field than neighboring states.

Alternation of locally stable and unstable states can be seen even more clearly at $\nu=9.92$ GHz if the ionization curves are plotted in the "natural" variables n^4F and $n^3\omega$. In this case the "steps" observed in the ionization curves plotted in the F and n variables transform into alternating maxima and minima.^{8}

In view of their unconventional nature, the experimental data on microwave ionization produced a great number of theoretical papers. Reference to them can be found in Refs. 7–10. We will not discuss the theoretical results arrived at in these papers. Suffice it to note that no simple and physically straightforward mechanism of the phenomena occurring in the microwave ionization of atoms was suggested. An attempt was made to guess and then theoretically substantiate a formula that yielded the sequence of local maxima on the ionization curves.$^{8-10}$ The formula considered was $n^3\omega=p/q$, with p and q integers ($p/q=1$, 2/3, 2/5, 1/3, 1/4, 1/5, 1/6, etc.). This approach, in which the formula was initially "guessed," has been used by theoreticians in many problems and sometimes successfully. However, in the case at hand the suggested formula was on the whole unsatisfactory because only some of the values given by it succeed in explaining the local maxima.

Theoreticians in any case did not assume that microwave ionization can be understood solely by examining the behavior of an atom in a constant field. Hence model time-dependent (classical and quantum) equations were solved and attempts to find a correlation between the obtained numerical and analytical results and the features of microwave ionization were made.

Let us now turn to our study. As shown above, the spectrum of the hydrogen atom for $F\neq 0$ can be approximated by sets of quasidiscrete [Eq. (4)] and quasicontinuous [Eqs. (5) and (6)] states alternating in energy. Mathematically speaking, such an approximation of a basically continuous atomic spectrum is asymptotically meaningful as $F\to 0$ and hence can be used in practical applications of those fields and quantum states where the second terms in Eqs. (4)–(6) prove to be much smaller than the first terms. In discussing the process of microwave ionization we assume that when the field F carries a level in state $n_2=n-1$ changes its energy according to Eq. (4). The change in the sign of F automatically transforms the level into one in the state $n_1=n-1$. By absorbing a number of photons, the quasidiscrete state described by Eq. (4) with $n=n_1+1$ or $n=n_2+1$ and $m=0$ may transform into a state of the continuous spectrum, (5) or (6), and hence become ionized. There is an infinite number of such possibilities, but only some are efficient. For instance, though most preferable energy-wise and requiring small values of F, the transition from state $E_{0n_20}(F)$ to state $E^+_{n_10}(F)$ is entirely ineffective because there is no overlap between the wave functions of the initial and final states (since n_1 and n_2 are extremely large). Hence we must find only the transition energy

$$\Delta E(F)=E^-_{n+k,0}(F)-E_{0,n-1,0}(F)$$
$$=E^+_{n+k,0}(F)-E_{n-1,0,0}(F), \qquad (7)$$

where $n+k$ is the quantum number for the "ionization" curve, and k determines the channel "number."

Assuming that $n\gg k$, we obtain

$$\Delta E=\frac{2k+1}{2n^3}\left[1-\frac{6(k+1)}{2k+1}\,n^4F\right]. \qquad (8)$$

The transition between "discrete" and "ionization" states is most effective when

$$\Delta E=N\omega, \qquad (9)$$

where N is an integer equal to the number of absorbed photons. Naturally, transitions also occur when condition (9) is not met exactly, but their intensity proves to be much lower.

Equating (8) and (9), we get

$$n^4F=\frac{2k+1-2Nn^3\omega}{6(k+1)}. \tag{10}$$

This clearly shows that the process of N-photon ionization through channel k is terminated at the following value of the scaled frequency:

$$n^3\omega=\frac{2k+1}{2N}. \tag{11}$$

Of course, Eq. (11) [as well as Eqs. (8) and (10)] is not exact, if only because for $F=0$ the ionization process is entirely impossible. It does provide, however, a qualitative description of the main features of the experimental data on microwave ionization of the hydrogen atom. Also, the experimental procedure, though ingenious and neat, cannot in any way claim to provide a strictly qualitative description of the process.

Obviously, the dominant processes leading to ionization are those for which N is small. But there is also a second, competing, condition: the field F must be as low as possible. These two factors determine the experimentally obtained results.

Let us first consider the results qualitatively for relatively low values of n and $\nu=9.92$ GHz. The dominating process here is the one with $N=1$ and $k=0$. Using Eq. (10), we find that $n^4F=0.15$ at $n=32$ and $n^4F=0.145$ at $n=35$. This agrees with the experimental data on 90% ionization.[8] When the value of n corresponding to 90% ionization grows, n^4F decreases, but as Eq. (11) shows, the process with $N=1$ and $k=0$ terminates at $n\approx 69.7$ [here we list fractional values of n obtained from Eq. (11), which avoids confusion]. But, in addition to one-photon ionization, two-photon ionization ($N=2$), three-photon ionization ($N=3$), etc. at $k=0$ can also be fairly effective if n is not too large. [As Eq. (10) shows, the two-photon process terminates at $n\sim 55$, the three-photon process at $n\sim 48$, the four-photon process at $n\sim 44$, etc.] The probability of such processes occurring is much lower, but, nevertheless, they would seem to be responsible for 10% ionization.[8] For instance, Eq. (10) yields $n^4F=0.133$ at $n=32$ and $n^4F=0.124$ at $n=35$ for $N=2$ and $k=0$. These values of n^4F are in good agreement with the experimental data on 10% ionization. Qualitatively it is easy to understand why a number of states in the vicinity of $n=67$–70 (and to a great extent in a broader range of 66–72) are ionized at roughly the same field strength F (see Ref. 8). The value of n^4F for $k=0$ and $N=1$ (a one-photon process), for $k=1$ and $N=3$ (a three-photon process), and for $k=2$ and $N=5$ (a five-photon process) reaches zero at $n\sim 69.7$. At $F\equiv 0$ the ionization channel is "closed." Nevertheless, ionization may take place intensively at fairly low values of F, which is actually the case. This is why at the given values of n the ionization curve plotted in the F vs n variables exhibits a step, called a "stability island" in Ref. 8.

For all the values of n (32–90) studied in Ref. 8, one-photon ionization through channels with $k\geqslant 1$ at the frequency $\nu=9.92$ GHz requires too big a value of F and, therefore, is ineffective. A number of other "stability islands" discovered at 9.92 GHz correlate well with the termination of processes at $N=2$–5 and $k=0$–4. The corresponding calculation and comparison with the experimental data can be carried out in elementary fashion, and of all the cases we note only one more, $N=2$, $k=0$, and $n=54.9$, for the "stability island" $n=53$–57.

Let us recall one more experimental result obtained at 9.92 GHz and explain it. Koch[9] gave the dependence of the ionization probability on n^4F for $n=37$ and 38, and for $n=47$ and 48. It was found in the first case ($n=37$ and 38) the ionization curves exhibit a distinct peak at $n^4F\sim 0.12$, after which the ionization probability drops drastically as the field strength grows, but then again increases approaching a value of 100%. In the second case ($n=47$ and 48) the ionization probability monotonically increases with field strength. Such a distinct difference in the behavior of the two pairs of curves can be understood from Eq. (10). At $n=48$ a three-photon process through the channel with $k=0$ is impossible, while a two-photon process at $n=47$ and 48 proceeds most intensively in fields $n^4F\sim 0.05$–0.06 [for such values of n^4F Eq. (10) is satisfied exactly], a range where measurements have yet to be done. At $n=37$ and 38, two-photon ionization proceeds most intensively in a field n^4F of roughly 0.11 and three-photon ionization in fields $n^4F\sim 0.08$–0.09, which qualitatively explains the difference in the experimental data for $n=37$ and 38 and for $n=47$ and 48 (Ref. 10).

The value of n^4F obtained via Eq. (10) with well-chosen values of k and n usually proves to be somewhat lower than the one obtained through experiments. But it would be foolish to expect full agreement because different channels lead to "ionization." Besides, Eq. (10) cannot answer the question of the percentage of atoms actually being ionized.

Let us now turn to the experimental results, which are shown in Fig. 1 for $\nu=36.02$ GHz (Ref. 10). The main details of the spectrum can be understood by considering the values of n at which the most effective one- and two-photon absorption processes terminate; these processes, provided they are possible, on the whole determine the decay of an atom in a microwave field. At these points the ionization cross section must have local minima. Equation (11) yields 64.9 ($k=1$ and $N=1$), 51.1 ($k=1$ and $N=2$), 61.1 ($k=2$ and $N=2$), and 68.4 ($k=3$ and $N=2$). The positions of the maxima, and especially of the most pronounced peak at $n=62$, in this event reveal themselves most clearly.

An additional study of three- and four-photon processes, which terminate at $n\sim 53.4$ ($k=2$ and $N=8$), $n\sim 59.7$ ($k=3$ and $N=3$), $n\sim 54.3$ ($k=3$ and $N=4$), $n\sim 59.0$ ($k=4$ and $N=4$), $n\sim 69.4$ ($k=5$ and $N=3$), and $n\sim 63.1$ ($k=5$ and $N=4$) determine more accurately the behavior of the ionization curves. Of course, here we must bear in mind that as N increases the probability of the processes rapidly drops, which means one cannot expect exact agreement between the theoretical and experimental positions of the minima. Also, experimental measurements do not yield completely reproducible results for all values of n. In particular, at $\nu=9.92$ GHz this fact was noted for $n=82$ and 83 (Ref. 8). It is just at these values that Eq. (11) predicts maxima and minima in n^4F for two close values of the principal quantum number: $n\sim82.3$ ($k=2$ and $N=3$), and 83.4 ($k=3$ and $N=4$). Hence uncontrollable insignificant variations in experimental conditions lead to different results. On the other hand, the data depicted in Fig. 1 were obtained through averaging,[10] without specifying the various spreads in individual measurements. Certain differences in the behavior of ionization curves leading to the disappearance of minute singularities and to shifts in their value of n by unity also emerge in some cases when two alternative methods of calculating the number of decayed atoms were employed in experimental measurements.[8,9] Bearing all this in mind, the agreement between the theoretical interpretation of the spectrum and the experimental data may be assumed to be quite satisfactory.

We note one more small discrepancy that reveals itself at $\nu=36.02$ GHz. From the calculated positions of the minima one can expect a small maximum to appear at $n=52$ on the experimental ionization curves. But such a maximum is most likely to reveal itself on the 5% ionization curve (or even curves representing a lower percentage). A conclusion of this kind can be drawn if we assume that the main process proceeds through the channel with $k=2$ and $N=3$. Then the value of F calculated via Eq. (10) proves to be very close to the experimental value for 10% ionization.

Following a similar line of reasoning, we can analyze the ionization curves for $\nu=30.36$ and 26.43 GHz, given in Ref. 10. The maxima and minima on the ionization curves in all three cases correspond to essentially different values of n. But the agreement between theory and experiment at all these values proves to be equally persuasive.

Thus, we have shown that the experimental data on microwave ionization of highly excited states of the hydrogen atom can be understood qualitatively by employing a simple model of a static electric field if one takes into account that for $E<0$ the atomic spectrum contains states exhibiting the properties of a continuous spectrum in addition to essentially stationary states.

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Translated by Eugene Yankovsky

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