Emission from a neutral fermion with anomalous magnetic moment in a nonlinear plane-wave field

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We use the exact solution of the Dirac equation in a nonlinear plane-wave field to calculate the emission from a neutral fermion. Our analysis of the radiative characteristics—in particular, the conservation laws obtained—suggests a fundamentally new way to measure the anomalous magnetic moment of a fermion experimentally.

The influence of a fermion's anomalous magnetic moment (AMM) on its interaction with an external field is currently of interest in high-energy physics. In this paper, we examine the emission from a fermion with an AMM in a nonuniform external field.

The construction of exact solutions of the Dirac equation

$$(i\partial - e\hat{A} - i\mu\sigma^{\mu\nu}F_{\mu\nu}/2 - m)\Psi = 0, \tag{1}$$

in which the AMM is taken into account phenomenologically for a variety of fields, is an important step in the calculation of such processes. Occasionally, a knowledge of the exact solution of the quantum equation can lead to the prediction of nontrivial physical consequences. One explicit example is the theory of synchrotron radiation propounded by Sokolov, Ternov, and their students,[1] which is based upon an exact solution of the relativistic Dirac equation in a constant, uniform magnetic field.

In Ref. 2, we constructed an exact solution of Eq. (1) in the field

$$A^{\mu} = a[a_1^{\mu}\mathrm{cn}(\varphi,r) + a_2^{\mu}\mathrm{sn}(\varphi,r)], \tag{2}$$

where $\varphi = \mathbf{k}x$, $\mathbf{k}^2 = 0$, $(\mathbf{a}_1,\mathbf{a}_2) = (\mathbf{k},\mathbf{a}_1) = (\mathbf{k},\mathbf{a}_2) = 0$, $\mathbf{a}_1^2 = \mathbf{a}_2^2 = -1$, and $\mathrm{cn}(\varphi,r)$ and $\mathrm{sn}(\varphi,r)$ are Jacobi elliptic functions[3] with modulus $r \in [0,1]$.

The selection of this external field configuration was discussed in some detail in Ref. 2. Briefly, the elliptic functions are by definition the solution of the nonlinear pendulum equation, with r being the analog of the initial energy. At $r=0$ the pendulum becomes a simple harmonic oscillator, and (2) then describes a plane monochromatic field, in which the solution of the Dirac equation is well known.[4] In the opposite limit $r=1$, the pendulum sits at the separatrix, $\mathrm{cn}(\varphi) \to 1/\mathrm{ch}(\varphi)$, $\mathrm{sn}(\varphi) \to \mathrm{th}(\varphi)$, and the field (2) describes a soliton configuration in which the E and H fields are solutions of the Korteweg–de Vries (KdV) equation.[5] Thus, the vector potential of the external field is a solution of a nonlinear KdV-type equation, in contrast to the customary plane wave, whose vector potential is a solution of the linear differential equation of the pendulum. This is precisely the context in which the "nonlinearity" of the plane-wave field (2) is to be understood. This question too was discussed in more detail in Ref. 2.

Such fields obviously make sense if we recall that actual plane waves, such as those produced nowadays by lasers, can only tentatively be described by solutions of linear differential equations.[6]

The solution of Eq. (1) in the field (2) takes the form[2]

$$\Psi = U(\varphi)\psi_0,$$

where ψ_0 is the solution of the free-particle Dirac equation,[1] and

$$U(\varphi) = \left(1 + e\frac{\hat{k}\hat{A}}{2kp}\right)\exp\left\{-i\int d\varphi\left[e\frac{pA}{kp} - e^2\frac{A^2}{2kp}\right]\right\}T(\varphi) \tag{3}$$

contains the well-known Wolkow exponential[7] and the operator $T(\varphi)$. The latter describes precession of the particle's spin due to an AMM, and can be expressed in terms of linear and quadratic combinations of unit-vector operators

$$F_{\pm} = \frac{1}{4kp}(\hat{p}\hat{k}\hat{a}_{\pm} + \hat{k}\hat{a}_{\pm}\hat{p}), \quad F_+F_- + F_-F_+ = -1,$$

$$F_{\pm}F_{\pm} = 0, \quad a_{\pm} = a_1 \pm ia_2$$

in the following manner:

$$T(\varphi) = Y_+(\varphi)F_- + Y_-(\varphi)F_+ - Z_+(\varphi)F_-F_+ - Z_+(\varphi)F_+F_-,$$

where $Y_{\pm}(\varphi) = (\delta_-/z)e^{\pm i\delta_+ \mathrm{am}\varphi}$ and $Z_{\pm}(\varphi) = e^{\mp i\delta_- \mathrm{am}\varphi}$, $z = \mu a$, $\delta_{\pm} = \pm 1 + \sqrt{1+4z^2}/2$. Alternatively, adopting the normalization $\bar{T}T = 1$ and $\lim_{a\to 0} T = 1$, we finally obtain

$$\sqrt{\varepsilon}T(\varphi) = \frac{\delta_-}{z}(F_-e^{i\delta_+\mathrm{am}\varphi} + F_+e^{-\delta_+\mathrm{am}\varphi}) - F_-F_+e^{-i\delta_-\mathrm{am}\varphi} - F_+F_-e^{i\delta_-\mathrm{am}\varphi}, \tag{4}$$

where $\varepsilon = 1 + \delta_-^2/z^2$.

It is clear from (3) and (4) that the solution consists of two multiplicative components; the first results from the fact that the particle has an electric charge, and essentially gives rise to the Wolkow exponential $[U(\varphi) = T(\varphi)$ for $e=0]$, while the second is due to the presence of an anomalous magnetic moment and is responsible for the preces-

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sion of the fermion's spin [$T(\varphi)=1$ *for* $\mu=0$]. *We shall examine the radiation generated by a neutral fermion, with* $e=0$ *and* $U(\varphi)=T(\varphi)$, *since it is in fact the operator* $T(\varphi)$ *that embodies all of the novel features of the solution.*

We consider the radiation from a neutral fermion in the field (2), taking the external field into account exactly, and the radiative field to a first approximation. The neutral fermion current in the field (2) is

$$S_{fi}=\sqrt{\frac{4p}{2\omega' 2E' 2E}}\frac{\mu}{\varepsilon}\times\int d^4x e^{i(p'+k'-p)x}\bar{u}_{p'}\bar{U}_{p'}(\varphi)\hat{k}'\hat{e}^{*}U_p(\varphi)u_p, \tag{5}$$

where $\mathbf{p}$ and E ($\mathbf{p}'$ and E') are the initial (final) neutron momentum and energy, $k'=(\omega',\mathbf{k}')$ is the four-momentum of the emitted photon, and $\mathbf{e}$ is its polarization vector.

The main problem in working with the current (5) comes in separating out an overall exponential factor of the form $e^{-if(k)\varphi}$, which in conjunction with the exponential in (5) specifies the conservation laws for the given process. It can easily be seen upon analyzing (4) that the feasibility of making this separation is dictated entirely by the form of the coefficients $Y_\pm$ and $Z_\pm$, and accordingly by the possible expansions of the elliptic amplitude $\mathrm{am}(\varphi,r)$ in terms of $r\in[0,1]$. For $r=0$ (plane-wave limit) there is no particular problem, since $\mathrm{am}(\varphi,0)=\varphi$. A similar expansion is also feasible when the elliptic modulus is small, $r\ll1$, a case we consider below.

When $r\ll1$, the external field is a plane wave with nonlinear corrections in φ, and we can expand the amplitude of the elliptic function as[3]

$$\mathrm{am}\,\varphi=\frac{\pi}{2K}\varphi+2q\sin\left(\frac{\pi}{K}\varphi\right), \tag{6}$$

where $K(r)=\int_0^{\pi/2}d\phi/\sqrt{1-r^2\sin^2\phi}$ is the complete elliptic integral of the first kind ($r=0$, $K=\pi/2$; $r=1$, $K=+\infty$), and q is the coefficient of the Θ function,[3] which is related to the modulus r via a well known inversion problem, and which can be constructed, for example, using the procedure discussed in Ref. 8. With $r\ll1$ in the present case, these are simply related by $r^2=16q$.

Making use of Eq. (6) and the familiar expansion[3] $e^{ix\sin\Theta}=\sum_{s=-\infty}^{+\infty}J_s(x)e^{is\Theta}$, it can be shown that the current S_{fi} is an infinite sum of terms that individually satisfy the conservation law

$$p'+k'=p\pm\alpha_i^s k, \tag{7}$$

where $\alpha_i^s=\pi/K[(\alpha_i/2)+s]$ and $\alpha_i=\{1,\sqrt{1+4z^2}-1,\sqrt{1+4z^2},\sqrt{1+4z^2}+1\}$; s varies within limits such that we always have a plus sign on the right-hand side of Eq. (7).

Formally, Eq. (7) comprises eight distinct series (ranging over s) of conservation laws, but depending on the value of $\sqrt{1+4z^2}$, the actual number may vary. What we mean here by a series is a set of coefficients $\{\alpha_i^s\}$ with constant i and variable s preceding the momentum of the absorbed photon; these are essentially absorption weighting factors. The Sth term of such a set describes the emission of a photon with momentum k' resulting from the absorption of α_i^s photons (i.e., the appropriate fraction of their energy and momentum) with four-momentum k. The only series that are physically reasonable under these circumstances are those that ultimately result in a plus sign on the right-hand side of Eq. (7).

The net result of a straightforward mathematical analysis of Eq. (7) is that four fundamentally different cases can be distinguished:

a) $\sqrt{1+4z^2}$ is neither an integer nor half an odd integer. In this least restrictive case, we have five distinct series.

b) $\sqrt{1+4z^2}$ is half an odd integer. Two pairs of series merge, leaving only three distinct series.

c) $\sqrt{1+4z^2}=n=2k+1$, yielding two series.

d) $\sqrt{1+4z^2}=n=2k$, again yielding two series, which, however, differ from those in (c).

Note that the fewer the conservation laws among the distinct series, the greater the contributions to the amplitude of the radiation, since identical series make an additional contribution to the final amplitude when the current is squared. Consequently, the probability amplitude for emission in cases (b)–(d) can just be added to the amplitude for the most general case (a). This simplifies calculations substantially, enabling one to treat (b)–(d) as a supplement to the fundamental case (a).

We previously noted[2] this decomposition of the process into resonant cases based on $\sqrt{1+4z^2}$, proposing that it might provide for an experimental measurement of the AMM of a neutral fermion. The foregoing analysis and the following calculation actually explore this possibility, expressing the total probability of emission as a function of the value $z=\mu a$ of the characteristic.

We therefore first consider case (a), in which $\sqrt{1+4z^2}$ is neither an integer nor half an odd integer. Squaring the current in (5), summing over final states of the neutral fermion and photon, and averaging over initial states of the neutral fermion, we have

$$\sum_{\gamma,e}|S_{fi}|^2=\frac{4\pi\mu^2}{2\omega' 2E2E'\varepsilon^2}\times\int d^4x d^4x' e^{i(p'+k'-p)(x-x')}\mathrm{Sp}[a], \tag{8}$$

where

$$\begin{aligned}\mathrm{Sp}[a]=\tfrac{1}{2}(&Y_+^2Y_+'^{*2}+Y_-^2Y_-'^{*2}+Z_-^2Z_-'^{*2}+Z_+^2Z_+'^{*2}\\&-2Y_+Z_-Y_+'^{*}Z_-'^{*}-2Y_-Z_+Y_-'^{*}Z_+'^{*}\\&+2Y_+Z_+Y_+'^{*}Z_+'^{*}\\&+2Y_-Z_-Y_-'^{*}Z_-'^{*})\mathrm{Sp}(p'F'_-F'_+k'F_-F_+pk')\\&+(Y_+Z_-Y_+'^{*}Z_-'^{*}+Y_-Z_+Y_-'^{*}Z_+'^{*})\Big[4(p'k')\end{aligned}$$

$$\times(pk')-\frac{m^2}{2}\mathrm{Sp}(F'_-k'F_+k'+F'_+k'F_-k')\Big]$$

$$+\frac{m^2}{2}(Y_-^2Z_+'^{*2}+Z_-^2Y_+'^{*2}-2Y_-Z_-Y_+'^{*}Z_-'^{*})$$

$$\times\mathrm{Sp}(F'_+k'F_+k'+F'_-k'F_-k').$$

We obtain the total emission probability by integrating over final particle states:

$$dW=\frac{d^3k'd^3p'}{(2\pi)^6}\sum_{\gamma,e}|S_{fi}|^2. \tag{9}$$

Four of the six integrals in (9) can be evaluated via the δ function in (8), yielding the conservations laws in (7). Furthermore, all of the traces in the expression for Sp[a], consisting of eight γ matrices, can be calculated with the help of these conservation laws. Note that these are all the nontrivial combinations that can be formed from the $F_\pm$ operators and the momenta of the participating particles:

$$\mathrm{Sp}(p'F'_-F'_+k'F_-F_+pk')$$

$$=2(p'k)(pk)\left[2(\alpha_i^s)^2+m^2\left(\frac{p}{pk}-\frac{p'}{p'k}\right)^2\right],$$

$$\mathrm{Sp}(F'_\pm k'F_\mp k')=2(p'k)(pk)\left(\frac{p}{pk}-\frac{p'}{p'k}\right)^2,$$

$$\mathrm{Sp}(F'_\pm k'F_\pm k')=-2(p'k)(pk)\left(\frac{pa_\pm}{pk}-\frac{p'a_\pm}{p'k}\right)^2.$$

As one of the two remaining variables of integration, we choose the invariant $u=kk'/kp'$. Evaluating the δ-function integrals, we then have

$$\frac{d^3p'd^3k'}{E'\omega'}\delta^{(4)}(p'+k'-p-\alpha_i^sk)\to\frac{dud\varphi}{(1+u)^2},$$

where φ varies from 0 to 2π, and u varies from 0 to $u_{\alpha_i^s}=2\alpha_i^skp/m^2$.

With this result in hand, we obtain the angular spectrum of the radiation:

$$d\omega=\frac{dud\varphi}{(1+u)^2}\frac{\mu^2}{8\pi E\varepsilon^2}\sum_{i=1}^{5}W_i(u,\varphi), \tag{10}$$

where

$$W_1=2\frac{\delta_-^2}{z^2}\sum_{s>-1/2}\Bigg[(J_s^2(2q)+J_{s+1}^2(2q))\frac{m^4}{2}\frac{u_\alpha^2}{1+u}$$

$$\times\left[1-2\frac{u}{u_\alpha}+2\frac{u^2}{u_\alpha^2}\right]-(-1)^sJ_s(2q)J_{s+1}(2q)$$

$$\times2m^2(1+u)\left(m-\frac{E_\alpha}{1+u}\right)^2\cos2\varphi\Bigg],$$

$$W_2=\sum_{s<-\delta_+}\Bigg[\left(\frac{\delta_-^4}{z^4}J_s^2(4\delta_+q)+J_{s+1}^2(4\delta_-q)\right)$$

$$\times\frac{m^4}{2}\frac{u_\alpha^2}{1+u}\left[1-2\frac{u}{u_\alpha}+2\frac{u^2}{u_\alpha^2}\right]+\frac{\delta_-^2}{z^2}J_s(4\delta_+q)$$

$$\times J_{s+1}(4\delta_-q)2m^2(1+u)\left(m-\frac{E_\alpha}{1+u}\right)^2\cos2\varphi\Bigg],$$

$$W_3=\sum_{s>-\delta_+}\Bigg[\left(\frac{\delta_-^4}{z^4}J_s^2(4\delta_+q)+J_{s+1}^2(4\delta_-q)\right)$$

$$\times\frac{m^4}{2}\frac{u_\alpha^2}{1+u}\left[1-2\frac{u}{u_\alpha}+2\frac{u^2}{u_\alpha^2}\right]+\frac{\delta_-^2}{z^2}J_s(4\delta_+q)$$

$$\times J_{s+1}(4\delta_-q)2m^2(1+u)\left(m-\frac{E_\alpha}{1+u}\right)^2\cos2\varphi\Bigg],$$

$$W_4=4m^4\frac{\delta_-^2}{z^2}\sum_{s<-\sqrt{1+4z^2}/2}J_s^2(2\sqrt{1+4z^2}q)\frac{u(u-u_\alpha)}{1+u},$$

$$W_5=4m^4\frac{\delta_-^2}{z^2}\sum_{s>-\sqrt{1+4z^2}/2}J_s^2(2\sqrt{1+4z^2}q)\frac{u(u-u_\alpha)}{1+u},$$

$$E_\alpha=E+\alpha_i^s\omega=E'+\omega'.$$

In analyzing the angular dependence (10) of the radiation, it must be noted that in contrast to the plane-wave case,[9] the axial symmetry of the radiation is broken in the expressions for W_1-W_3, which is a natural consequence of the nonlinearity of the external field, whose vector potential is the solution of a nonlinear differential equation.[2]

We now study the angular distribution (10) in more detail, writing the expression for W_1 in the form

$$W_1=2\frac{\delta_-^2}{z^2}\sum_{s>-1/2}(J_s^2(2q)+J_{s+1}^2(2q))\frac{m^4}{2}\frac{u_\alpha^2}{1+u}$$

$$\times\left(1-2\frac{u}{u_\alpha}+2\frac{u^2}{u_\alpha^2}\right)\{1-(-1)^sA_s(u,q)\cos2\varphi\},$$

where

$$A_s(u,q)=\frac{J_s(2q)J_{s+1}(2q)}{J_s^2(2q)+J_{s+1}^2(2q)}\frac{4}{m^2}$$

$$\times\frac{(1+u)^2}{u_\alpha^2}\frac{[m-E_\alpha/(1+u)]^2}{(1-2u/u_\alpha+2u^2/u_\alpha^2)}.$$

This makes it clear that for any s,

$$A_s(u,q)>0\ \text{and}\ \lim_{q\to0}A_s(u,q)=0.$$

The axially symmetric radiation pattern normally displayed by a plane wave is thus destroyed, and depending on the parity of s, we have squares that instead are either inscribed within a circle or circumscribed about it.

The remaining φ and u integrals in (10) are elementary, and present no particular difficulty. The net result is then

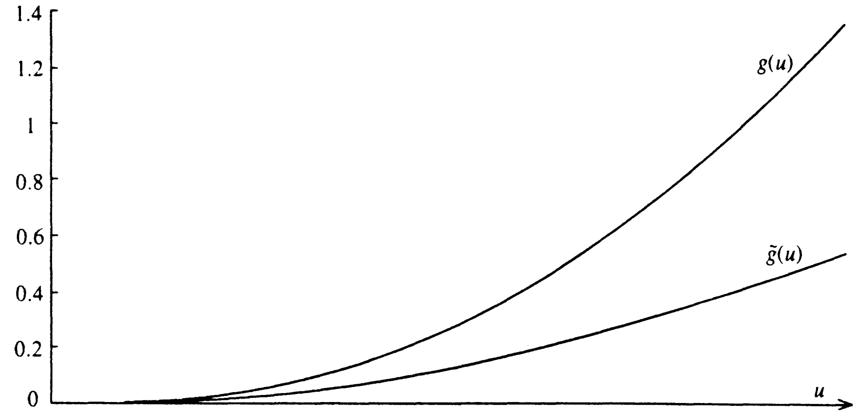

FIG. 1. Emission in a plane-wave field from a neutral fermion with anomalous magnetic moment.

$$\omega=\frac{\mu^2 m^4}{4E\varepsilon^2}\sum_{i=1}^{5}\omega_i, \qquad (11)$$

$$\omega_1=\frac{1}{2}\left(\frac{\delta_-}{z}\right)^2\sum_{s>-1/2}[J_s^2(2q)+J_{s+1}^2(2q)]g(u_{\alpha_1}),$$

$$\omega_2=\frac{1}{4}\sum_{s<-(1+\sqrt{1+4z^2}/2)}\left(\frac{\delta_-^4}{z^4}J_s^2[2(1+\sqrt{1+4z^2})q]\right.$$
$$\left.+J_{s+1}^2[2(-1+\sqrt{1+4z^2})q]\right)g(u_{\alpha_2}),$$

$$\omega_3=\frac{1}{4}\sum_{s>-(1+\sqrt{1+4z^2}/2)}\left(\frac{\delta_-^4}{z^4}J_s^2[2(1+\sqrt{1+4z^2})q]\right.$$
$$\left.+J_{s+1}^2[2(-1+\sqrt{1+4z^2})q]\right)g(u_{\alpha_3}),$$

$$\omega_4=\sum_{s<-(\sqrt{1+4z^2}/2)}J_s^2(2\sqrt{1+4z^2})\tilde{g}(u_{\alpha_4}),$$

$$\omega_5=\sum_{s>-(\sqrt{1+4z^2}/2)}J_s^2(2\sqrt{1+4z^2})\tilde{g}(u_{\alpha_5}),$$

where $g(u)=u(2+u)/(1+u)^2(u^2-2u-2)+4\ln(1+u)$ and $\tilde{g}(u)=2u(2+u)/(1+u)-4\ln(1+u)$ are monotonically increasing functions of u (see Fig. 1).

The sums in (11) can be evaluated by taking advantage of the properties of Bessel functions of small arguments.[3] Since $J_0(0)=1$ and $J_n(0)=0$ for $n\neq 0$, we can neglect terms in (11) with $s\neq 0$, whereupon only series 1, 3, and 5 survive; in the limit as $q\to 0$, these yield the plane-wave radiation obtained in Ref. 9. Here series 3 comprises two contributions, and we wind up with four channels corresponding to the two spin symmetries and two symmetries of the magnetic moment:

$$\omega_1=\frac{1}{2}\left(\frac{\delta_-}{z}\right)^2 J_0^2(2q)g(u_{\alpha_1^0}),$$

$$\omega_3=\frac{1}{4}J_0^2[2(-1+\sqrt{1+4z^1})q]g(u_{\alpha_3^{-1}})$$
$$+\frac{1}{4}\left(\frac{\delta_-}{z}\right)^4 J_0^2[2(1+\sqrt{1+4z^2})q]g(u_{\alpha_3^0}),$$

$$\omega_5=\left(\frac{\delta_-}{z}\right)^2 J_0^2(2\sqrt{1+4z^2}q)\tilde{g}(u_{\alpha_5^0}).$$

Note that this calculation is in fact consistent with the plane-wave case with the nonlinear correction $2q\sin\varphi$ of (6). The important point here, though, is that we used the exact solution of the Dirac equation in the field (2), and the nonlinear correction corresponds solely to one special case of the resulting solution. In other words, the present calculation was based entirely upon an exact solution. An analysis confirms the conclusions drawn in Ref. 2 about the resonant influence of the factor $\sqrt{1+4z^2}$ on the nature of the radiation, and the corresponding feasibility of measuring the AMM of a fermion experimentally, much as proposed in Ref. 10.

In closing, we offer a number of remarks about the applicability of these results.

First and foremost, we note that Eq. (1) can be derived from the Dirac equation with radiative corrections taken into account. To do so, it is necessary to calculate the self-energy diagram and mass operator of the fermion in an external field, and then to separate out terms linear in the field. Transforming from operators to classical quantities (see the procedure described in Ref. 11), we obtain the desired Eq. (1). This then makes it clear that admissible external fields in (1) should be such that radiative effects are not comparable with the quantities containing the anomalous magnetic moment μ. For the magnetic field, this means that $eH/m^2=H/H_0\ll 1$ (and accordingly $eE/m^2=E/E_0\ll 1$ for the electric field), where H_0 and E_0 are the characteristic Schwinger fields.[12] Under these circumstances, vacuum decay (pair production) is unlikely, and we can utilize Eq. (1) and its consequences.

The present discussion yields a bound on the range of the parameter $z=\mu a$, which must be varied when one measures the AMM. Furthermore, using the actual values $H_0=4.41\times 10^{13}$ G and $E_0=1.3\times 10^{16}$ W/cm of the characteristic fields, we obtain a wide range of possible amplitude variation for an external field pulse.

We have thus demonstrated the feasibility of using "nonlinear" field configurations, in principle, to observe variations in the radiation as a function of the AMM of a fermion. Those variations will be discrete by virtue of a family of conservation laws, and the discreteness may make it possible to improve upon the value of the fermion's AMM using a technique unrelated to a Penning trap. To solve the problem as a whole, of course, it will be necessary to conduct a closer analysis of fermion radiation, paying attention to the manifestations of its charge and spin properties.

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