Thresholdless Auger recombination mechanism in semiconductors in a quantizing magnetic field

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Auger recombination in narrow-bandgap semiconductors in the presence of a quantizing magnetic field is studied theoretically. It is shown that for certain values of the magnetic field the Auger process is a thresholdless process. The absence of a threshold is related directly to Landau quantization. It is established that in a quantizing magnetic field the Auger transitions of electrons from a lower Landau level into a highly excited level is of a resonance character. The Auger recombination rate is an oscillating function of the magnetic field. The effect of Auger recombination processes on the emission spectra of narrow-bandgap semiconductors at low temperatures in a quantizing magnetic field is analyzed. Recombination rates are calculated for InSb and HgCdTe at different temperatures.

1. INTRODUCTION

Optical phenomena in narrow-bandgap semiconductors in the presence of a magnetic field have been studied for a long time.[1,2] In particular, in Refs. 2 and 3 it was observed that at low temperatures the maximum luminescence intensity oscillates as a function of the magnetic field intensity. This behavior of the radiation intensity could be associated with the activation of nonradiative electron-hole recombination channels in the presence of an external magnetic field.[2,3] It should be noted that even at quite low temperatures an external magnetic field can significantly influence the Auger recombination rate. Auger recombination is a threshold process,[4] and as we have shown, a magnetic field removes the constraints imposed on the interelectronic collision processes by the laws of conservation of energy and momentum, as a result of which the threshold is removed and the Auger recombination rate thereby increases significantly.

The objective of the present work is to investigate theoretically nonradiative Auger recombination in the presence of a quantizing magnetic field in narrow-bandgap semiconductors with a Kane dispersion law. In the presence of a quantizing magnetic field, as shown in the present paper, the Auger recombination rate has a significantly weaker temperature dependence—a power law and not the exponential dependence observed for the ordinary Auger process.[4] As shown below, the thresholdless nature of the Auger process in a quantizing magnetic field is directly related to Landau quantization.

We shall investigate the Auger process in which two electrons from the conduction band and a light hole from the valence band participate. One electron recombines with the light hole and the second electron is transferred by the Coulomb interaction into a highly excited state. In contrast to Ref. 5, where an Auger process with the participation of a heavy hole was considered in the case of a simple band, we solved the problem in the Kane three-band model, taking into account the nonparabolic nature of the spectrum of the highly excited electron. In Ref. 6, the Auger recombination rate in a magnetic field was also calculated taking into account the nonparabolic nature of the electron spectrum. However, the expression obtained there for the rate contained an exponential (threshold) temperature dependence. As we have noted above, a strong magnetic field removes the threshold for the Auger process. The threshold-elimination method qualitatively is as follows. In the presence of a strong magnetic field the electron spectrum in a direction perpendicular to the magnetic field is quantized. As a result of Coulomb interaction, the Auger electron, having acquired an energy of order E_g (the width of the forbidden band) executes a vertical transition to a high Landau level without a change in quasimomentum (Fig. 1). Large quasimomentum transfer is not required because of the Coulomb collision of the two electrons. The only requirement is that the excited Auger electron must occupy a Landau level. Thus in a quantizing magnetic field, when the conditions indicated above are satisfied, the Auger process has a resonant character. Therefore, the rate of the Auger process oscillates as a function of the magnetic field, and these oscillations are related to the breakdown of resonance.

2. MATRIX ELEMENT OF THE AUGER PROCESS IN THE PRESENCE OF A MAGNETIC FIELD

In order to find the Auger recombination rate we employ the Kane model in the presence of a magnetic field:

$$\left(E-E_g-g-\frac{\hat{\Phi}^2}{2m_{hh}}\right)U-\gamma\frac{\hat{\Phi}}{\hbar}\mathbf{V}=0,$$
$$\left(E+g-\frac{\hat{\Phi}^2}{2m_{hh}}\right)\mathbf{V}-ig[\hat{\sigma},\mathbf{V}]-\gamma\frac{\hat{\Phi}}{\hbar}U=0. \tag{1}$$

Here $\hat{\Phi}=\hat{\mathbf{P}}+(e/c)\mathbf{A}$ is the generalized momentum of the particles; $\hat{\mathbf{P}}$ is the quasimomentum of the particles; $\mathbf{A}$ is the vector potential of the magnetic field; E is the carrier energy, measured from the top of the valence band; γ is the Kane matrix element; $g=\Delta_{so}/3$, where Δ_{so} is the spin-orbit interaction constant; m_{hh} is the mass of the heavy hole; U

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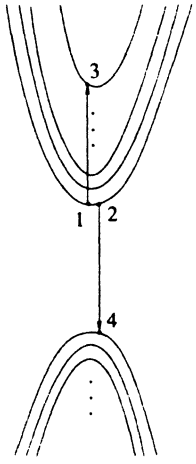

FIG. 1. Schematic view of the band diagram of a semiconductor in a quantizing magnetic field. The arrows mark electron transitions from the lower Landau level.

and **V** are, respectively, the s- and **p**-envelopes, and $\Psi = U|s\rangle + \mathbf{V}|\mathbf{p}\rangle$; and $\hat{\sigma}$ are the Pauli matrices.

For simplicity we consider below the case of strong spin-orbit interaction, $\Delta_{so} \gg E_g$, and infinite heavy-hole mass $m_{hh} = \infty$. Taking into account the finiteness of the mass of the heavy hole and letting the ratio of Δ_{so} and E_g be arbitrary does not change the result qualitatively. In order to calculate the Auger-recombination rate it is convenient to switch to the basis $u_i(\mathbf{r})$ $(i=1,2,...,8)$, in which the wave functions of the conduction band are s-type spherical functions and the wave functions of the valence band are eigenfunctions of the operators J^2 and J_z^2 (J is the total angular momentum operator).[7,8] Then, in the Landau gauge ($\mathbf{A}_x = -yH$, $\mathbf{A}_y = \mathbf{A}_z = 0$) the complete coordinate wave functions have the form

$$\Psi(\mathbf{r}) = \exp[i(k_x x + k_z z)] \sum_i C_i \chi_{n_i}(y) u_i(\mathbf{r}). \tag{2}$$

Here $\chi_{n_i}(y)$ are oscillator functions with the number n_i and the coefficients C_i are functions of k_z and n_i.

The Auger-recombination rate is calculated in the first order of perturbation theory in the electron–electron interaction:

$$G = \frac{2\pi}{\hbar} \sum_{n,\sigma} \int |V|^2 \delta(E_1 + E_2 - E_3 - E_4) \times f(E_1) f(E_2) f(E_4) [1 - f(E_3)] d\tau, \tag{3}$$

where $f(E_i)$ is the distribution function of the ith particle; E_1 and E_2 are the initial and E_3 and E_4 are the final energy states of the electrons (the hole state is the final state for one of the electrons participating in the process); V is the matrix element of the electron-electron interaction, calculated using antisymmetrized electron wave functions of the initial and final states;

$$d\tau = \prod_{\nu=1}^{4} \left(\frac{dk_x^{(\nu)}}{2\pi}\right)\left(\frac{dk_z^{(\nu)}}{2\pi}\right)$$

is an element of the phase space. The summation in Eq. (3) extends over all numbers of the Landau levels and spin states of the particles participating in the process.

The matrix element can be represented in the form

$$V = V_{\mathrm{I}} - V_{\mathrm{II}}, \tag{4}$$

$$V_{\mathrm{I}} = \sum_{\sigma,\sigma'} \int\int \Psi_1^*(\mathbf{r}_1,\sigma) \Psi_3(\mathbf{r}_1,\sigma) \times \hat{V} \Psi_2^*(\mathbf{r}_2,\sigma') \Psi_4(\mathbf{r}_2,\sigma') d\mathbf{r}_1 d\mathbf{r}_2, \tag{5}$$

$$V_{\mathrm{II}} = \sum_{\sigma,\sigma'} \int\int \Psi_1^*(\mathbf{r}_1,\sigma) \Psi_4(\mathbf{r}_1,\sigma) \times \hat{V} \Psi_2^*(\mathbf{r}_2,\sigma') \Psi_3(\mathbf{r}_2,\sigma') d\mathbf{r}_1 d\mathbf{r}_2. \tag{6}$$

The square of the absolute value of the matrix element separates into a sum of the direct and exchange parts:

$$|V|^2 = (|V_{\mathrm{I}}|^2 + |V_{\mathrm{II}}|^2) - (V_{\mathrm{I}} V_{\mathrm{II}}^* + V_{\mathrm{I}}^* V_{\mathrm{II}}). \tag{7}$$

Here

$$|V_{\mathrm{I}}|^2 = \frac{4e^4}{\varkappa^2} \int\int \frac{dq dq' \delta_{\sigma_1\sigma_3} \delta_{\sigma_2\sigma_4}}{(q^2 + k_{31}^2 + \lambda^2)(q'^2 + k_{31}^2 + \lambda^2)} \times \delta(K_x)\delta(K_z) \sum_{\substack{i,i' \\ j,j'}} [C_i^{*(1)} C_i^{(3)} I_i^{13}(q)] \times [C_{i'}^{(1)} C_{i'}^{*(3)} I_{i'}^{13}(-q')] [C_j^{*(2)} C_j^{(4)} I_j^{24}(-q)] \times [C_{j'}^{(2)} C_{j'}^{*(4)} I_{j'}^{24}(q')], \tag{8}$$

where

$$k_{31}^2 = (k_x^{(3)} - k_x^{(1)})^2 + (k_z^{(3)} - k_z^{(1)})^2,$$

$$K_x = k_x^{(1)} + k_x^{(2)} - k_x^{(3)} - k_x^{(4)};$$

$$K_z = k_z^{(1)} + k_z^{(2)} - k_z^{(3)} - k_z^{(4)},$$

$$I_i^{\alpha\beta}(q) = \int e^{iqy} \chi_{n_i}^{(\alpha)}(y) \chi_{m_i}^{(\beta)}(y) dy,$$

is the static permittivity of the medium; λ is the Debye screening radius; q is the transferred momentum; α and β designate the numbers of the particle states ($\alpha, \beta = 1, 2, 3, 4$); $\delta_{\sigma_\alpha\sigma_\beta} = 1$ if $\sigma_\alpha = \sigma_\beta$ (if the spins of the particles α and β are equal) and zero if $\sigma_\alpha \neq \sigma_\beta$.

The expression for $|V_{\mathrm{II}}|^2$ is obtained from Eq. (7) by interchanging the indices 3 and 4. Similarly, the exchange term is

$$V_I V_{II}^* = \frac{4e^4}{\varkappa^2} \int\int \frac{dq dq' \delta_{\sigma_1\sigma_2}\delta_{\sigma_1\sigma_3}\delta_{\sigma_1\sigma_4}}{(q^2+k_{31}^2+\lambda^2)(q'^2+k_{41}^2+\lambda^2)}$$
$$\times \delta(K_x)\delta(K_z) \sum_{\substack{i,i' \\ j,j'}} [C_i^{*(1)} C_i^{(3)} I_i^{13}(q)]$$
$$\times [C_{i'}^{(1)} C_{i'}^{*(4)} I_{i'}^{14}(-q')][C_j^{*(2)} C_j^{(4)} I_j^{24}(-q)]$$
$$\times [C_{j'}^{(2)} C_{j'}^{*(3)} I_{j'}^{23}(q')]. \qquad (9)$$

The integrals $I_i^{\alpha\beta}(q)$ reduce to a well-known tabulated integral for the Hermite polynomials.[9] The presence of the delta function in Eqs. (8) and (9) expresses the law of conservation of momentum along the x and z axes. There is no conservation law in the y direction, since Landau quantization applies:

$$E = \frac{1}{2}E_g \pm \sqrt{\frac{1}{4}E_g^2 + \frac{2}{3}\gamma^2\left[k_z^2 + \frac{2}{a_H^2}\left(n+\frac{1}{2}\pm\frac{1}{4}\right)\right]}. \qquad (10)$$

Here $a_H = \sqrt{(\hbar c/eH)}$ is the magnetic length, the + and − signs in front of the square root correspond to electrons and holes, and the ± in the radicand correspond to the two spin orientations. If the magnetic field is made to approach zero, then the oscillator functions become plane waves and the integrals $I_i^{\alpha\beta}(q)$ become delta functions, giving the law of conservation of momentum along the y axis. Therefore, $|V_I|^2$ becomes the well-known expression given in Refs. 4 and 10.

We now integrate $|V_I|^2$ over $d\tau_x$:

$$\int |V_I|^2 d\tau_x = \frac{e^4}{\pi a_H \varkappa^2} \frac{\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4}}{(2\pi)^3} \prod_{\nu=1}^{4} \frac{E^{(\nu)}}{2E^{(\nu)}-E_g} \delta(K_z)$$
$$\times \int d\eta_1 d\eta_2 d\eta_3 d\xi d\xi'$$
$$\times \exp\left\{-\frac{\xi^2+\xi'^2}{2} - \eta_1^2 + i\eta_2(\xi-\xi')\right\}$$
$$\times (\eta_1 - i\xi)^{n_4-n_2}(\eta_1+i\xi')^{n_4-n_2}$$
$$\times (-\eta_1 - i\xi')^{n_3-n_1}(-\eta_1+i\xi)^{n_3-n_1}$$
$$\times \frac{P_{13}(r)P_{13}(r')P_{24}(r)P_{24}(r')}{(r+R)(r'+R)}$$
$$\times \frac{2^{n_1+n_2}n_1!n_2!}{2^{n_3+n_4}n_3!n_4!}. \qquad (11)$$

Here

$$\xi = a_H q, \quad \xi' = a_H q', \quad \eta_1 = a_H(k_x^{(4)} - k_x^{(2)}),$$
$$\eta_2 = a_H(k_x^{(4)} - k_x^{(1)}), \quad \eta_3 = a_H(k_x^{(4)} + k_x^{(3)}),$$
$$r = \xi^2 + \eta_1^2, \quad r' = \xi'^2 + \eta_1^2, \quad R = a_H^2(k_z^{(3)} - k_z^{(1)})^2,$$

the polynomials $P_{\alpha\beta}(r)$ are expressed in terms of the Laguerre polynomials and the coefficients $C_i^{(\alpha)}C_i^{(\beta)}$. In the expression (11) it is assumed that $n_1 \leqslant n_3$ and $n_2 \leqslant n_4$. The product $C_i^{(\alpha)}C_i^{(\beta)}$ is proportional to the overlap integrals of the particles participating in the recombination process.

We calculated the overlap integrals using the Kane model and found that they depend on the external magnetic field H. In Ref. 5, where the Auger-recombination rate is also calculated, the overlap integrals were calculated using the two-band model and did not depend on the magnetic field. It is shown below that the exact calculation of the overlapping integrals fundamentally influences the Auger recombination rate, since they depend on the magnetic field. The integration of $|V_I|^2$ over $d\tau_x$ in Eq. (11) is easily performed. The integral over $d\eta_2$ gives a delta function $\delta(\xi-\xi')$. Next, switching to polar coordinates in the (η_1,ξ) plane, the remaining integral reduces to a one-dimensional integral. The integral of $V_I V_{II}^*$ over $d\tau_x$ is calculated in the Appendix. We give the final result of the integration of $|V|^2$ over $d\tau_x$:

$$\int |V|^2 d\tau_x = \frac{e^4}{\pi a_H^2 \varkappa^2} \frac{\hbar\omega\theta_4}{E_g} \frac{1}{2^{n_3}} f(n_3,\sigma_\nu)\delta(K_z). \qquad (12)$$

Here the function $f(n_3,\sigma_\nu)$, $\nu = 1, 2, 3$, and 4, has the form

$$f(n_3,\sigma_\nu) = [\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4} + \delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}]\left[a^2(n_3+1)(n_3+2) + 2ab(n_3+1) + b^2 + 2ac + \frac{2bc}{n_3} + \frac{c^2}{n_3(n_3-1)}\right]$$
$$- \delta_{\sigma_1\sigma_2}\delta_{\sigma_1\sigma_3}\delta_{\sigma_1\sigma_4}\left[a^2 n_3(n_3+1) + 2ab(n_3+1) + b^2 + 2ac\frac{n_3+2}{n_3} + \frac{2bc}{n_3} + \frac{c^2}{n_3^2}[1-s(n_3)]\right]. \qquad (13)$$

The following notation was introduced in Eq. (13):

$$a = \frac{\hbar\omega}{8E_g}\theta_3, \quad b = -\tfrac{3}{2}[1+a(1+n_3)], \quad c = -4(b+1), \qquad (14)$$
$$s(n_3) = \sum_{k=1}^{\infty} \frac{k}{(n+k)2^{k+1}},$$

$\theta_{3,4} = 1$ if the spin of particle 3 or 4 is oriented along the magnetic field and $\theta_{3,4} = 1/3$ if the spin is oriented opposite to the field. It is evident from Eqs. (13) and (14) that $f(n_3,\sigma_\nu)$ is a quadratic function of the magnetic field for fixed values of n_3 and σ_ν.

The nature of the factor $\hbar\omega/E_g$ in Eq. (12) is physically understandable. In the absence of a magnetic field, as is well known,[4,10] the overlap integral of the electrons and holes is proportional to the product of their wave vectors $\mathbf{k}_e$ and $\mathbf{k}_h$; therefore the factor T/E_g, where T is given in energy units, appears in the Auger rate. In the presence of a quantizing magnetic field for which $\hbar\omega \gg T$ holds the spectrum of electrons and holes is quantized, and hence their kinetic energy is of order $\hbar\omega$. Therefore, the overlap integral for the lower Landau levels of the electrons and light holes is greater than the overlap integral in the absence of a magnetic field by the amount $\hbar\omega/T \gg 1$. Thus, neglecting the dependence of the overlap integrals on the magnetic field H, as done in Ref. 5, results in an incorrect

magnetic-field dependence of the matrix element and, therefore, of the Auger recombination rate also.

Let the particles occupy in the initial state the lower Landau level; therefore, $n_1=n_2=n_4=0$. Moreover, it follows from the law of conservation of energy that $n_3 \geqslant 2E_g/\hbar\omega \gg 1$. The factor $1/2^{n_3}$ in Eq. (12) corresponds to the fact that as a result of the Coulomb collision one electron, which has acquired energy of the order of E_g, is transferred into the highly excited Landau level n_3. This state corresponds to a rapidly oscillating wave function with number $n_3 \gg 1$, and for this reason the overlap integral between the ground and excited states is small and of order $1/2^{n_3}$.

3. CALCULATION OF THE AUGER-RECOMBINATION RATE

We write the law of conservation of energy for the Auger process in the following form, taking into account the law of conservation of quasimomentum of the particles along the z axis:

$$\frac{\varepsilon_H}{\mu}\zeta^2+t=\sqrt{l+\varepsilon_H\zeta^2\cos^2\vartheta}. \tag{15}$$

Here $\varepsilon_H=\hbar\omega/2E_g$; $\mu=2m_e+m_{hl}/m_e=3$; the electron mass m_e equals the light-hole mass m_{hl}; ζ, ϑ, and φ are spherical coordinates in momentum space $\zeta_i=\sqrt{\mu}k_z^{(i)}a_H$ $(i=1,2,4)$; and

$$\begin{aligned} t&=\frac{3}{2}+2\varepsilon_H\left(n_1+n_2+n_4+\frac{3}{2}+\frac{\sigma_1+\sigma_2+\sigma_4}{2}\right),\\ l&=\frac{1}{4}+2\varepsilon_H\left(n_3+\frac{1}{2}+\frac{\sigma_3}{2}\right). \end{aligned} \tag{16}$$

Here $\sigma_\alpha=\pm 1/2$.

Equation (15) reflects the nonparabolic nature of the spectrum of the highly excited electron. A parabolic spectrum was adopted for the initial states. It is important to take into account the nonparabolic nature of the spectrum for the highly excited electron when calculating the Auger recombination rate. In this case the minimum value of n_3, following from the law of conservation of energy, is $n_3^{\min}\approx 2E_g/\hbar\omega$. If the nonparabolic nature of the spectrum for the excited electron is neglected, then the minimum value of n_3 is two times smaller. Just as in the absence of a magnetic field, it is of fundamental importance to take into account the nonparabolic nature of the spectrum when calculating the Auger-recombination rate.[4,10]

The equation (15), i.e., the law of conservation of energy, holds only if $l \geqslant t^2$. Then the solutions for ζ fall between the values $\zeta_{\min}$ and $\zeta_{\max}$, given by

$$\begin{aligned} \zeta_{\min}^2&=\frac{\mu}{\varepsilon_H}(\sqrt{l}-t),\\ \zeta_{\max}^2&=\frac{\mu}{2\varepsilon_H}[\mu-2t+\sqrt{(\mu-2t)^2+4(l-t^2)}]. \end{aligned} \tag{17}$$

The condition $l \geqslant t^2$ implies that n_3 has a lower limit: $n_3 \geqslant n_3^{\min}\approx 2E_g/\hbar\omega$. As one can see from Eq. (16), l and t are discrete for a given magnetic field. In addition, l and t have lower limits, since n_1, n_2, n_3, and n_4 are nonnegative integers. For values of n_1, n_2, n_3, and n_4 for which $l=t^2$ the longitudinal kinetic energy of the particles is identically zero. This means that for a magnetic field H_0 such that $l(H_0)\approx t^2(H_0)$ the transition of an electron from the lower (zeroth) Landau level into a high Landau level $(n_3 \gg 1)$ occurs without a change of the quasimomentum. Such a transition is vertical; it corresponds to the maximum Auger rate. Therefore, the Auger process in a quantizing magnetic field is of a resonance character and is thresholdless.

A magnetic field removes the constraints imposed on the Auger process by the laws of conservation of energy and momentum. As in the case when a heteroboundary is present, in a quantizing magnetic field the Auger process is thresholdless,[10] i.e., the rate of such a process is exponentially high, in contrast to the case of no magnetic field.

It remains to calculate the integral over $d\tau_z$ in the expression (3). Neglecting the dependence on $k_z^{(\nu)}$ and λ in the matrix element (since $a_H\lambda$, $a_Hk_z^{(\nu)} \leqslant 1$), we have

$$G=\frac{2\pi}{\hbar}\sum_{n,\sigma}\int |V|^2 d\tau_x \int f(E)\delta(E_1+E_2-E_3-E_4)d\tau_z. \tag{18}$$

In Eq. (18) the matrix element V depends on $k_z^{(\nu)}$ via the delta function: $\delta(K_z)=\delta(k_z^{(1)}+k_z^{(2)}-k_z^{(3)}-k_z^{(4)})$.

The integration over $d\tau_z$ must be performed in order to find G. This integration is performed on a surface in momentum space determined by the law of conservation of energy (15). It follows from Eq. (15) that this surface is axisymmetric. Therefore, when we integrate in spherical coordinates (ζ, θ, φ) the integrand does not depend on φ. We find that the integral over $d\tau_z$ reduces to a one-dimensional integral and equals

$$\begin{aligned} &\int f(E)\delta(E_1+E_2-E_3-E_4)d\tau_z\\ &=(4\pi)^4\frac{N_e^2N_{hl}a_H^3}{E_g\varepsilon_H^{1/2}}\operatorname{sh}^3\left(\frac{\hbar\omega}{4T}\right)\\ &\times\left(\frac{\hbar^2}{2\pi\mu m_eT}\right)^{3/2}\int_{\zeta_{\min}}^{\zeta_{\max}}\frac{\zeta d\zeta(t+\varepsilon_H\zeta^2/\mu)}{\sqrt{(t+\varepsilon_H\zeta^2/\mu)^2-l}}\\ &\times\exp\left[-\frac{E_g}{T}\left(t+\frac{\varepsilon_H}{\mu}\zeta^2-\frac{3}{2}\right)\right], \end{aligned} \tag{19}$$

where N_e and N_{hl} are, respectively, the electron and light-hole concentrations.

We recall that the values of $\zeta_{\min}$ and $\zeta_{\max}$ are determined by the expression (17). For $T/E_g \ll 1$ the integral in Eq. (17) reduces to the error function. Then we obtain for the Auger recombination rate the final expression

$$\begin{aligned} G=64\pi^3\omega\frac{\hbar\omega}{T}\frac{\hbar\omega}{E_g}\frac{a_H^2}{a_B^2}N_e^2N_ha_H^6\sum_{n_3,\sigma_\nu}\frac{\theta_4}{2^{n_3}}f(n_3,\sigma_\nu)\operatorname{sh}^3\left(\frac{\hbar\omega}{4T}\right)\\ \times\exp\left[-\frac{E_g}{T}\left(\sqrt{l}-\frac{3}{2}\right)\right]\operatorname{erf}\left(\sqrt{\frac{E_g\Delta}{T}}\right). \end{aligned} \tag{20}$$

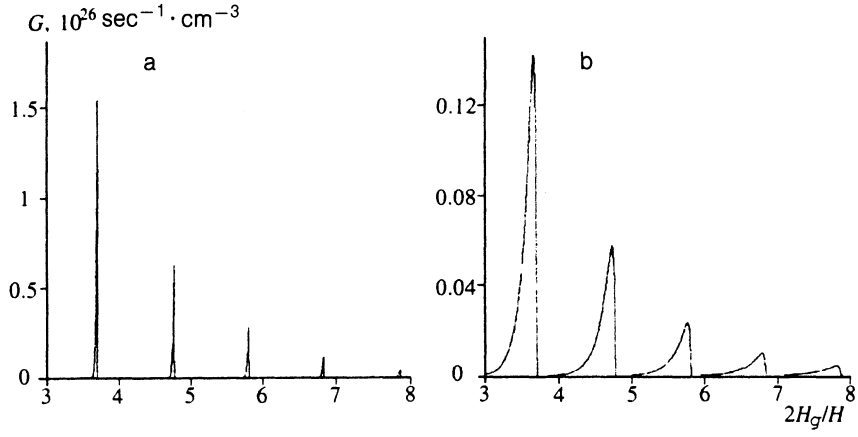

FIG. 2. Auger-recombination rate as a function of the inverse magnetic field for InSb at $T=4.2$ K (a) and $T=50$ K (b); $H_g=(m_c c/\hbar e)E_g\approx 28.7$ T.

Here $\Delta=(\varepsilon_H/\mu)(\xi^2_{\max}-\xi^2_{\min})$, $a_B=\hbar^2\varkappa/m_e e^2$ is the Bohr radius of an electron, and $\operatorname{erf}(x)$ is the error function. The summation over n_3 and σ_v in Eq. (20) extends over values for which $l\geqslant t^2$.

In calculating G we set $n_1=n_2=n_4=0$. This approximation is applicable for the following reasons. As is evident from Eq. (3), the Auger recombination rate equals the sum over n_i, each term of which is proportional to

$$\operatorname{sh}^3\left(\frac{\hbar\omega}{4T}\right)\exp\left[-\frac{E_g}{T}\left(\sqrt{l}-\frac{3}{2}\right)\right].$$

In this sum we retain terms with the minimum value of the exponent of the exponential, to which correspond $n_1=n_2=n_4=0$. The remaining terms are exponentially small—of the order of $\exp(-\hbar\omega/2T)\ll 1$—in the given magnetic field (we recall that we are considering the case of a quantizing field $\hbar\omega>T$).

In the case of strong quantization, $\hbar\omega\gg T$, in the sum over n_3 only one term is significant—the term corresponding to $n_3=n_3^{\min}$, which is exponentially large [or of the order of $\exp(\hbar\omega/2T)\gg 1$] compared to the other terms. Then the rate of the Auger process (its maximum value) in the case of a resonance transition is

$$G^{\max}=\frac{16}{3}-\pi^3\omega\,\frac{\hbar\omega}{T}\,\frac{\hbar\omega}{E_g}\,\frac{a_H^2}{a_B^2}\,N_e^2N_h a_H^6\,\frac{1}{2^{n_3^{\min}}}\,f_1(n_3^{\min}), \tag{21}$$

$$f_1=2a^2(n_3^{\min}+1)+\frac{c}{n_3^{\min}}\left\{-4a+\frac{c}{n_3^{\min}}\left[s(n_3^{\min})+\frac{1}{n_3^{\min}-1}\right]\right\}.$$

4. DISCUSSION

Interesting results follow from the expression which we have derived for the Auger-recombination rate.

1. The Auger-recombination rate does not contain an exponential (threshold) temperature dependence, and therefore the ratio $G(H,T)/G(0,T)\propto\exp(E_{th}/T)$ is exponentially large [$G(0,T)$ is the recombination rate with no magnetic field and E_{th} is the threshold energy of the process[4,10]].

2. The Auger-recombination rate at the maximum increases exponentially with the magnetic field as exp $[-(2Eg/\hbar\omega)\ln 2]$.

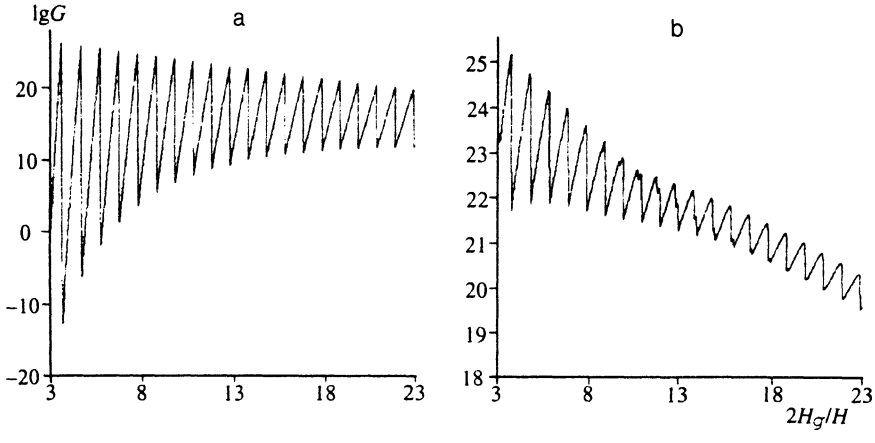

FIG. 3. Logarithm of the Auger-recombination rate as a function of the inverse magnetic field for InSb at $T=4.2$ K (a) and $T=50$ K (b).

3. The rate G is a strongly oscillating function of the magnetic field. Figure 2a displays the Auger-recombination rate as a function of the inverse magnetic field for the compound InSb at $T=4.2$ K.

It follows from Eq. (20) that the maxima of the rate G are very sharp on the low-field side and more gently sloping on the high-field side, and in addition the width ΔH_n of the peak is proportional to the temperature T: $\Delta H_n/H \propto T/2E_g$. This behavior of G as a function of $1/H$ corresponds to the factors $\sinh^3(\hbar\omega/4T)\exp[...]\mathrm{erf}[...]$ [see Eq. (20)]. At the points where G is maximum this product is of order unity; at the points where G is minimum the product is of order $\exp(-\hbar\omega/3T)$. We note that for the highly excited electrons arising as a result of the Auger process the distance between the Landau levels equals $\hbar\omega/3$.

The product $\sinh^3(\hbar\omega/4T)\exp[...]$ equals 1 at the point $l(H,n_3)=t^2(H)$, corresponding to the magnetic field $H=H_0(n_3)$. But at this point $\mathrm{erf}(...)=0$. In the case of a very small deviation from $H_0(n_3)$, i.e., $H=H_0(n_3)+\delta H$, where $\delta H/H_0(n_3)\approx T/n_3E_g \ll 1$, we have $\mathrm{erf}(...)\approx 1$. The product $\sinh^3(\hbar\omega/4T)\exp[...]$ is close to 1, as before. It is evident from this analysis that G does not contain a threshold temperature dependence. As the magnetic field increases further up to values $H \leqslant H_0(n_3+1)$ we have $l > t^2$ and $\sinh^3(\hbar\omega/4T)\exp[...]\approx\exp\{-\hbar\omega/3T\}\ll 1$. This corresponds to a minimum value of G. For $H=H_0(n_3+1)$ the pattern repeats. But it is important that at the point of the minimum the value of $G(H,T)$ is exponentially large compared to $G(0,T)$. We can see that in a magnetic field the Auger process is thresholdless.

The oscillatory character of the behavior of G as a function of the magnetic field is physically understandable. In a Coulomb collision between two electrons in the lower Landau level one electron is transferred into the valence band and the other, absorbing the energy transferred to it, occupies a highly excited state. The probability of such a process is different from zero if the electron occupies a Landau level with the number corresponding to the law of conservation of energy (resonant transition) and is small (or even zero at $T=0$) if the excited electron falls between two Landau levels (nonresonant transition).

In order to make a better analysis of our result we present a number of magnetic field dependences of the Auger-recombination rate:

a) Fig. 2b displays G as a function of $1/H$ at $T=50$ K;

b) Figs. 3a and b display $\log(G)$ as a function of $1/H$ at temperatures $T=4.2$ K and 50 K, respectively, for a wider range of magnetic fields.

Figures 2 and 3 convey the main features of the behavior of the Auger recombination rate as a function of the magnetic field at different temperatures. The carrier nonradiative lifetime $\tau_A=N_d/G$ has the same characteristic features as a function of the magnetic field as does G. We note that in narrow-bandgap semiconductors at low temperatures in the presence of a quantizing magnetic field the carrier lifetime is determined mainly by two recombination processes: nonradiative (Auger) and radiative. It follows from what we have said above that at low temperatures the nonradiative recombination lifetime for certain values of the magnetic field corresponding to maximum G is shorter than the radiative-recombination lifetime, i.e., for certain values of the magnetic field, corresponding to resonance Auger transitions, the carrier lifetime is completely controlled by nonradiative Auger recombination. In order to confirm this assertion we shall calculate the radiative recombination rate in a quantizing magnetic field, using the van Roosbroeck–Shockley relation[11] and the expression for the absorption coefficient $\alpha(\omega)$ from Ref. 10:

$$R=12\pi\sqrt{2\pi}\,\frac{\varepsilon_\infty}{n}\,\frac{E_g}{m_ec^2}\,\frac{e^2}{\hbar c}\,\frac{m_e\tilde{E}_g}{\hbar^2}\,\frac{\hbar\omega}{\sqrt{(m_{hh}T)}}\,N_eN_{hh}a_H^4. \tag{22}$$

Here ε_∞ is the high-frequency permittivity; n is the index of refraction of the semiconductor; $\tilde{E}_g=E_g+(eH/2c)(m_e^{-1}+m_{hh}^{-1})$ is the effective width of the forbidden band; and N_{hh} is the concentration of heavy holes. The expression (22) was derived for the case of strong quantization.

Next, it is convenient to take the ratio of the two recombination rates—Auger and radiative. We designate this ratio by Γ: $\Gamma=G/R$. For most narrow-bandgap semiconductors (InSb, HgCdTe, InAs) at $T=4.2$ K at maximum Auger-recombination rate $\Gamma_{\max}\gg 1$ and at minimum Auger-recombination rate G is exponentially small: $\Gamma_{\min}\approx\Gamma_{\max}\exp(-\hbar\omega/3T)\ll 1$. Therefore the total lifetime at low temperatures is an oscillating function of the magnetic field, the oscillations indicating alternation of the dominant carrier recombination process: nonradiative over radiative and vice versa. For example, for InSb at $T=4.2$ K and $H=11$ T we have $\Gamma_{\min}\approx 5\cdot 10^{-38}$ and $\Gamma_{\max}\approx 44$. For HgCdTe at $T=4.2$ K and $H=5.2$ T we have $\Gamma_{\min}\approx 9.3\cdot 10^{-18}$ and $\Gamma_{\max}\approx 42.4$. In calculating G and R the following parameters were used for HgCdTe: $E_g=0.112$ eV, $m_e=0.010m_0$, $m_{hh}=0.4m_0$, $\varkappa=18.6$, $\varepsilon_\infty=13.4$, $N_e=N_{hh}=3\cdot 10^{15}$ cm^{-3}; for InSb: $E_g=0.237$ eV, $m_e=0.014m_0$, $m_{hh}=0.4m_0$, $\varkappa=16.8$, $\varepsilon_\infty=15.68$, $N_e=N_{hh}=3\cdot 10^{16}$ cm^{-3} (m_0 is the free-electron mass). This behavior of the carrier lifetime in a quantizing magnetic field at low temperatures significantly influences the optical characteristics of narrow-bandgap semiconductors. In particular, the maximum radiative-recombination intensity oscillates as a function of the magnetic field, and these oscillations are directly related to activation of recombination channels for definite values of the magnetic field.[2,3] As noted above, these values of the magnetic field correspond to resonant Auger transitions.

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APPENDIX

We give the expression for the integral of $V_{\mathrm{I}}V_{\mathrm{II}}^*$ over $d\tau_x$ in the case $n_3 \geqslant n_1$, n_2 and $n_4 \geqslant n_1$, n_2:

$$\int V_{\mathrm{I}}V_{\mathrm{II}}^{*}d\tau_x=\frac{e^4}{\pi a_H\varkappa^2}\frac{\delta_{\sigma_1\sigma_2}\delta_{\sigma_1\sigma_3}\delta_{\sigma_1\sigma_4}}{(2\pi)^3}\prod_{\nu=1}^{4}\frac{E^{(\nu)}}{2E^{(\nu)}-E_g}$$
$$\times\delta(K_z)\int d\eta_1 d\eta_2 d\eta_3 d\xi d\xi'$$
$$\times\exp\left[-\frac{\xi^2+\xi'^2}{2}-\frac{\eta_1^2+\eta_2^2}{2}\right.$$
$$\left.+i(\eta_2\xi-\eta_1\xi')\right](-\eta_1+i\xi)^{n_3-n_1}$$
$$\times(\eta_1-i\xi)^{n_4-n_2}(\eta_2-i\xi')^{n_4-n_1}(-\eta_2$$
$$+i\xi')^{n_3-n_2}$$
$$\times\frac{P_{13}(r_1)P_{24}(r_1)P_{14}(r_2)P_{23}(r_2)}{(r_1^2+R)(r_2^2+R')}$$
$$\times\frac{2^{n_1+n_2}n_1!n_2!}{2^{n_3+n_4}n_3!n_4!}. \tag{A1}$$

In Eq. (A1) the same notation was used as in Eq. (11), and

$$r_1^2=\zeta'^2+\eta_1^2,\quad r_2^2=\zeta'^2+\eta_2^2,\quad R'=a_H^2(k_z^{(4)}-k_z^{(1)})^2.$$

We now calculate the integral in Eq. (A1) for $n_1=n_2=n_3=0$ and $k_z^{(\nu)}=0$, $\nu=1,...,4$. It is convenient to switch to polar coordinates: $\eta_1=r_1\cos\varphi_1$, $\zeta=r_1\sin\varphi_1$, $\eta_2=r_2\cos\varphi_2$, $\zeta'=r_2\sin\varphi_2$. Then the integration over $d\varphi_1 d\varphi_2$ gives a Bessel function, and as a result we obtain

$$\int V_{\mathrm{I}}V_{\mathrm{II}}^{*}d\tau_x=\frac{4e^4}{a_H^2\varkappa^2}\frac{\delta_{\sigma_1\sigma_2}\delta_{\sigma_1\sigma_3}\delta_{\sigma_1\sigma_4}}{(2\pi)^3}\prod_{\nu=1}^{4}\frac{E^{(\nu)}}{2E^{(\nu)}-E_g}\frac{1}{2^{n_3}}$$
$$\times\delta(K_z)\int_0^\infty\int_0^\infty dr_1dr_2r_1^{n_3-1}r_2^{n_3-1}P(r_1)$$
$$\times P(r_2)J_{n_3}(r_1r_2)\exp\left(-\frac{r_1^2+r_2^2}{2}\right). \tag{A2}$$

Here $P(r_{1,2})=P_{13}(r_{1,2}^2)P_{24}(r_{1,2}^2)$ is a polynomial of degree four. The integral in Eq. (A2) can be expressed in terms of integrals of the form

$$I_{m_1m_2}=\int_0^\infty\int_0^\infty dr_1dr_2r_1^{n_3+2m_1-1}r_2^{n_3+2m_2-1}J_{n_3}(r_1r_2)$$
$$\times\exp\left(-\frac{r_1^2+r_2^2}{2}\right). \tag{A3}$$

The integral I_{1m} is calculated with the help of the well-known formula for the integral of a Bessel function,[12] and the integrals I_{0m} and I_{2m} can be calculated by integrating and differentiating this formula with respect to a parameter. Then the final expression—Eq. (12)—for the integral of $V_{\mathrm{I}}V_{\mathrm{II}}^{*}$ over $d\tau_x$ can be easily written down.

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