

# X-ray free-electron laser in the quantum regime

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The action of free electron lasers (FELs) operating in a fundamentally new regime is examined. In this regime the size of an emitted photon exceeds the radiation line width. It is shown that such an FEL consists of a two-level quantum generator with a completely inverted active medium. Analysis of the nonlinear dynamics of an FEL in this regime shows, in analogy with the process by which a  $\pi$ -pulse is formed in a coherently amplifying medium, that the inversion can be completely removed in a single pass. The nature of the physical processes in a "two-level" FEL is essentially quantum-mechanical and cannot be described using classical electrodynamics. Lasing in this quantum regime can be achieved in an x-ray FEL with an optical undulator.

## 1. INTRODUCTION

One of the most pressing issues in laser physics is the task of extending the range within which lasing occurs into the x-ray region of the spectrum. Free electron lasers (FELs) are regarded as promising sources of coherent XUV radiation.<sup>1,2</sup> In contrast with conventional schemes for x-ray lasers using multicharged ions, the frequency range within which FELs lase is not restricted by the structure of discrete energy levels. Moreover, the wavelengths of radiation from an FEL can be tuned over a broad range. The use of the mechanism of induced free-free transitions also allows the properties of FEL radiation to be superior to undulator and synchrotron XUV sources.<sup>3</sup>

Since the demonstration of the first FEL,<sup>4</sup> lasing in these devices has been achieved over a broad frequency range (from microwave to UV). The possibility of extending FELs with magnetostatic undulators into the XUV spectral range is usually associated with raising the electron energy to the GeV level.<sup>3,5</sup> As the electron energy increases, however, the FEL gain rapidly drops  $\sim \gamma^{-3}$  (here  $\gamma$  is the Lorentz factor). This effect must be compensated for by increasing the length of the undulator and the beam current, which in turn impose severe restrictions on the quality of the electron beam.<sup>5,6</sup> Furthermore, a number of significant difficulties arise, associated with the need to use large accelerator systems. At the present time it is felt that a promising technique for overcoming these problems is the use of microundulators and techniques for generating higher harmonics of short-wavelength radiation. Ongoing programs for using FELs to generate XUV radiation are based on these approaches.<sup>5-8</sup>

An alternative approach for achieving short-wavelength FELs is based on using intense laser radiation propagating in the opposite direction from the electron beam (a so-called optical undulator).<sup>9,10</sup> In order to reach the soft x-ray range in this case it is sufficient to use electrons with low energies (a few MeV), which means that we can use relatively compact and high-current low-voltage accelerators as electron-beam sources.

In the present work we analyze the nonlinear dynamics

of x-ray generation in an FEL with an optical undulator. We will treat the quantum regime, corresponding to electrons whose state changes considerably as a result of recoil associated with the emission of an x-ray photon. We show that in the high-frequency region of the spectrum an FEL with an optical pump acts as a two-level quantum generator with a fully inverted active medium. This means it is possible to have an interaction with maximum efficiency, corresponding to complete removal of the inversion. The lasing of a "two-level" FEL is essentially quantum-mechanical and cannot be described using classical electrodynamics.

## 2. THE RELATION BETWEEN THE CLASSICAL AND QUANTUM APPROACHES IN FEL THEORY

Theoretical studies of free electron lasers begun in the early 1970s originally employed the quantum representation of physical processes in an FEL.<sup>11-15</sup> In this picture the effect of laser amplification results from stimulated electron Compton scattering of a photon beam in an oppositely propagating electromagnetic pump wave. Subsequently the results first obtained using quantum theory were rederived by various authors<sup>16-19</sup> using classical electrodynamics. It was shown that the amplification in an FEL can be treated as a consequence of resonant energy exchange between a slow beat wave and an electron beam periodically modulated in density. The complete agreement between the results of the classical and quantum approaches convinced researchers that an FEL is essentially a classical device, more akin to conventional devices of classical electronics (like the traveling-wave tube, klystron, etc.), rather than an intrinsically quantum-mechanical generator. This viewpoint has become firmly established in recent years. As it happens, the equivalence of the two approaches in FEL theory is far from universal; it is valid only in a certain range of FEL frequencies. To confirm this assertion we direct our attention to the following.

Consider the interaction of relativistic electrons with the fields of electromagnetic pump waves (frequency  $\omega_i$ ) and a signal (frequency  $\omega_s \gg \omega_i$ ), where we restrict our-

selves to the case of collinear geometry in order to keep from complicating the discussion. The pump and signal waves give rise to the slow difference wave of the ponderomotive potential with frequency  $\omega = \omega_s - \omega_i$  an wave number  $k = (\omega_s + \omega_i)/c$ . The dependence of the emission and absorption probabilities for high-frequency photons on the electron energy  $\varepsilon$  is of a resonant nature. Using the energy and momentum conservation laws in an elementary scattering event, it is easy to show that the locations of the centers of the absorption line (a) and the emission line (e) are determined by  $(\varepsilon_0 \gg \hbar\omega)$

$$\varepsilon_e = \varepsilon_0 + \frac{1}{2}\hbar\omega, \quad \varepsilon_a = \varepsilon_0 - \frac{1}{2}\hbar\omega. \quad (1)$$

Here the energy  $\varepsilon_0 = mc^2(1 - \omega/kc)^{-2}$  corresponds to the condition that an electron be synchronized with the difference wave. The total width of the lines

$$\Delta\varepsilon/\varepsilon_0 \approx \max\{(4N)^{-1}, \Delta\gamma/\gamma, \langle\Delta\theta^2\rangle/2\}$$

is determined by the finite lengths  $N\lambda_i$  of the interaction region [homogeneous broadening,  $(4N)^{-1}$ ] and inhomogeneous broadening associated with both the spread in electron energies  $\Delta\gamma/\gamma$  in the beam and with noncollinearity  $\langle\Delta\theta^2\rangle/2$  of the interacting beams, resulting from the finite emittance of the electron beam and the divergence of the laser radiation.

The quantum properties of an FEL are determined by the ratio of the separation  $\hbar\omega$  between the absorption and emission lines and their effective width

$$\eta = \hbar\omega/\Delta\varepsilon. \quad (2)$$

In the usual FEL design with a magnetostatic undulator, which operates in the visible and microwave bands, the energy of an emitted photon is less than a few eV and almost always satisfies  $\eta \ll 1$ . The emission and absorption profiles practically coincide; the competing processes of direct and inverse induced Compton scattering for each electron occur with almost the same probabilities. It is this last circumstance which gives rise to the classical behavior of lasing in this limit: the results of the analysis in the first nonvanishing order of perturbation theory do not contain the Planck constant and agree with those derived from classical theory.<sup>16-19</sup>

In the opposite limit  $\eta \gg 1$ , however, the emission and absorption lines are completely separated. Then when the energy changes by  $\hbar\omega$  the nature of the interaction between an electron and the laser field changes considerably, and an adequate description thus requires the introduction of quantum theory. As will be shown below, the dynamics of the emission from an FEL in this strong quantum limit reduces to transitions between two energy levels whose positions are given by Eqs. (1).

The "quantum threshold"  $\hbar\omega_s > \Delta\varepsilon$  can be exceeded in an x-ray FEL with an optical pump.<sup>20</sup> For a beam electron energy  $\varepsilon \approx 3$  MeV and line width  $\Delta\varepsilon \approx 100$  eV ( $N \approx 10^4$ ,  $\Delta\gamma/\gamma \approx 3 \times 10^{-5}$ ,  $\sqrt{\langle\Delta\theta^2\rangle} \approx 5 \times 10^{-3}$  rad) this condition corresponds to emitted photons  $\hbar\omega_s \approx 100$  eV (the soft x-ray range). Then the pump frequency  $\omega_i = \omega_s/4\gamma^2$  is in the near-IR range and the length of the optical undulator is  $L = N\lambda_i \approx 1$  cm.

### 3. THE TWO-LEVEL APPROXIMATION

As a basis for the two-level model of the x-ray FEL with optical pumping we consider the interaction between a relativistic electron beam and the field of two oppositely directed plane electromagnetic waves, described by the vector potentials

$$\mathbf{A}_i(z,t) = \frac{1}{2}\mathbf{e}_i A_i(z,t) \exp\{-i(\omega_i t + k_z z)\} + \text{c.c.}, \quad (3)$$

$$\mathbf{A}_s(z,t) = \frac{1}{2}\mathbf{e}_s A_s(z,t) \exp\{-i(\omega_s t - k_z z)\} + \text{c.c.}$$

Here  $A_{i,s}(z,t)$  are the "slow" amplitudes of the pump wave (*i*) and the signal wave (*s*),  $\mathbf{e}_{i,s}$ ,  $k_{i,s}$ ,  $\omega_{i,s}$  are their respective polarization vectors, wave numbers, and frequencies, and  $(\mathbf{e}_i, \mathbf{e}_s) = 1$ . We restrict ourselves to the case of collinear geometry, and we also rule out of consideration effects associated with the transverse structure of the REB and the laser waves. With these approximations the problem becomes one-dimensional.

In this situation the electron dynamics can be described by the Klein-Gordon equation<sup>21,22</sup>

$$\left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} + \frac{2e^2}{\hbar^2} \mathbf{A}_i \mathbf{A}_s + \frac{m^2 c^4}{\hbar^2} \right) \Psi(z,t) = 0. \quad (4)$$

Here  $m$  is the effective mass of an electron in the external field, given by

$$m^2 = m_0^2 + (e^2/c^4)(A_i^2 + A_s^2)$$

(in contrast to conventional FELs with magnetostatic undulators in real laser fields, this mass shift is negligible).

In consequence of the conservation laws the problem of transitions in the continuum reduces to the equivalent problem of resonant excitation of the system of discrete levels of an anharmonic oscillator,<sup>21,22</sup> characterized by the electron energies  $\varepsilon(p_n) = \sqrt{p_n^2 c^2 + m^2 c^4}$  and momenta  $p_n = p_0 + n\hbar(\omega_i + \omega_s)/c$ ,  $n = 0, \pm 1, \pm 2, \dots$ . Following Ref. 22 we will look for a solution of the Klein-Gordon equation in the form of a superposition of free-electron wave functions corresponding to the specified energy state (here  $\Omega$  is the volume of the quantization region):

$$\Psi(z,t) = \sum_n \sqrt{\frac{mc^2}{2\Omega\varepsilon(p_n)}} a_n \exp\left\{\frac{i}{\hbar} [p_n z - \varepsilon(p_n)t]\right\}. \quad (5)$$

Substituting Eq. (5) into Eq. (4) yields a system of coupled equations describing the excitation of an anharmonic oscillator with an infinite number of levels:

$$i a_n = g_n a_{n-1} \exp\{-i\Delta_n t\} + g_{n+1} a_{n+1} \exp\{i\Delta_{n+1} t\}, \quad (6)$$

$$a_n(0) = \delta_{n0},$$

where  $g_n = (e^2/\hbar) A_i A_s \sqrt{\varepsilon_{n-1} \varepsilon_n}$ . The departure from resonance associated with the transition between neighboring states is determined by

$$\Delta_n = \omega + \frac{1}{\hbar} (\varepsilon_{n-1} - \varepsilon_n) \approx -8\pi N \frac{\varepsilon(p_0) - \varepsilon_e + n\hbar\omega}{\varepsilon_0} \frac{1}{\tau_{in}}. \quad (7)$$

Here  $\tau_{in} \approx N\lambda_i/c$  is the time of flight of an individual electron through the region where it interacts with the laser fields.

In order to be specific we further assume that the electrons are injected into the interaction region with energy close to  $\varepsilon_e$ . Then the probability for emission of a high-frequency photon  $\hbar\omega_s$  is maximized, and hence we should expect optimum conditions for FEL radiation to occur. Then it is easy to see that in the quantum limit  $\eta \gg 1$  of interest the inequality  $\Delta_n \tau_{in} \gg 2\pi$ ,  $n \neq 0$  holds for each electron in the beam. The corresponding exponential terms in (6) are rapidly oscillating and hence drop out under integration. As a result only two states with  $n=0$  and  $n=-1$  survive from the infinite hierarchy of levels.

The possibility of identifying a two-level system is essentially related to the anharmonicity of the equivalent oscillator due to the nonlinear dependence  $\varepsilon(p)$ . The magnitude

$$\Delta \approx 8\hbar\omega_s^{1/2}\omega_i^{3/2}/mc^2$$

of the anharmonicity is a rapidly increasing function of the pump frequency  $\omega_i$ , and although it is small for an FEL with magnetostatic undulators ( $\hbar\omega_i \approx 10^{-5}$  eV,  $\Delta\tau_{in} \ll 1$ ), it becomes fairly large for an FEL with optical pumping: for  $\hbar\omega_i \approx 1$  eV,  $\hbar\omega_s \approx 100$  eV we have  $\Delta\tau_{in} \gg 1$  when  $\tau_{in} > 0.1$  ns holds. Note also that in strong fields we have in addition to the condition  $\eta \gg 1$  a restriction on their intensity, such that the field broadening of the two-level transition is less than the anharmonicity:

$$g_0 \ll \Delta. \quad (8)$$

#### 4. DYNAMICS OF EMISSION FROM A TWO-LEVEL FEL

Since no mirror currently exists which has a high reflection coefficient in the x-ray band, such as is available in the visible band, we anticipate that considerable losses will occur in the cavity during a single pass. Hence in order to reach the threshold for lasing it is necessary to achieve a high gain  $\alpha > 1$ . Assuming mirrors with 50% reflection, amplification in power by a factor  $\approx 4$  in one pass is required to excite an x-ray FEL. This implies that the amplitude of the high-frequency signal and the phase relations vary rapidly through the interaction region. Hence a description of the amplification dynamics in this physical situation requires a self-consistent approach, taking into account the effects of the spatial and temporal evolution of the system.

In the quantum limit we are discussing, the electron wave function in the pulse consists of a superposition of two wave packets with  $n=0, -1$ . In accordance with the Klein-Gordon equation the evolution of the slow amplitudes  $a_0(z,t)$  and  $a_{-1}(z,t)$  satisfies the equations

$$\begin{aligned} \frac{\partial a_0}{\partial t} + V_0 \frac{\partial a_0}{\partial z} &= -\frac{i}{4\hbar} \frac{e^2 A_i A_s^*}{\sqrt{\varepsilon_0 \varepsilon_{-1}}} \exp(-i\Delta_0 t) a_{-1}, \\ \frac{\partial a_{-1}}{\partial t} + V_{-1} \frac{\partial a_{-1}}{\partial z} &= -\frac{i}{4\hbar} \frac{e^2 A_i^* A_s}{(\varepsilon_0 \varepsilon_{-1})^{1/2}} \exp(i\Delta_0 t) a_0, \end{aligned} \quad (9)$$

where  $\Delta_0 = (\omega_s/mc^2\gamma^3)(\varepsilon_0 - \varepsilon_a)$  is the phase shift due to the anharmonicity and  $V_i = c^2 p_i/\varepsilon_i$ ,  $i=0, -1$  are the group velocities of the respective wave packets. Along with Eqs. (9) of the medium we need the wave equation which prescribes the dynamics of the propagation of the laser pulse:

$$\left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{A}_\perp = -\frac{4\pi}{c} \mathbf{j}_\perp, \quad \mathbf{A}_\perp = \mathbf{A}_s + \mathbf{A}_i, \quad (10)$$

where the current  $\mathbf{j}_\perp = (2e^2/mc)\mathbf{A}_\perp \langle \Psi^* \Psi \rangle$  is a quantity averaged over all the electrons in the bunch. In the approximation in which the pump field is specified<sup>1)</sup> it is easy to derive an equation describing the evolution of the slowly varying amplitude of the high-frequency signal:

$$\frac{\partial A_s}{\partial z} + \frac{1}{c} \frac{\partial A_s}{\partial t} = i \frac{2\pi e^2 A_i}{\omega_s \sqrt{\varepsilon_0 \varepsilon_{-1}}} \int_0^\infty d\varepsilon_0 f(\varepsilon_0) \tilde{a}_0 \tilde{a}_{-1}. \quad (11)$$

For convenience on the right-hand side of (11) we have introduced new variables  $\tilde{a}_0$  and  $\tilde{a}_{-1}$  for the two-level medium:

$$\tilde{a}_0 = a_0, \quad \tilde{a}_{-1} = a_{-1} \exp(-i\Delta_0 t) \quad (12)$$

and  $f(\varepsilon_0)$  is the initial electron energy distribution in the bunch, satisfying

$$\int_0^\infty d\varepsilon f(\varepsilon) = 1.$$

Thus, the dynamics of the emission from a two-level FEL is completely described by the closed self-consistent nonlinear system of equations (9), (11). Note that Eqs. (9) and (11) are similar to the Maxwell-Bloch equations describing the interaction between a medium consisting of two-level atoms and a resonant field.<sup>23</sup> This is a consequence of the fact that the processes by which radiation is coherently emitted in free electron lasers and in conventional photoemission in atomic and molecular transitions are physically the same.

The main features in the operation of a two-level FEL can be exhibited clearly in the quasisteady amplification regime, when the pulse length  $\tau_{i,s}$  of the high-frequency signal and pump pulses and the length  $\tau_b$  of the electron bunch are much larger than the characteristic size of the interaction region,

$$\tau_{i,s}, \tau_b \gg L/c. \quad (13)$$

In this geometry the length  $L$  of the interaction region is primarily determined by the focal conditions of the pump laser beam and can be estimated from the Rayleigh length  $L \approx 2k d_i^2$ , where  $d_i$  is the typical dimension of the focal spot.

We also restrict ourselves to the case of a homogeneously broadened emission line, assuming that the electron beam is fairly monoenergetic and has a small emittance, so that the inequalities  $\Delta\gamma/\gamma$ ,  $\langle \Delta\theta^2 \rangle/2 \ll 1/4N$  are satisfied. This allows us to neglect the phase shift  $\Delta_0$  in the equations (9) of the medium. Consequently, the self-consistent system (9), (11) is easily integrated. Taking into account the boundary condition at the entrance to the interaction region ( $z=0$ ), we look for a solution for the variables of the medium in the form

$$\tilde{a}_0(z) = \tilde{a}_0(0) \cos[\chi(z)/2], \quad (14)$$

$$\tilde{a}_{-1} = \tilde{a}_{-1}(0) \sin[\chi(z)/2],$$

where

$$\chi(z) = \frac{\mu}{c\hbar} \int_0^z E_s(z) dz$$

is the varying area of the high-frequency pulse. The parameter  $\mu = e^2 E_i / m \omega_i^{1/2} \omega_s^{3/2}$ , in analogy with the classical model of a two-level medium, plays the role of the dipole moment of the resonant transition. Here we have used the amplitudes  $E_{i,s}$  of the laser fields in place of the corresponding vector potentials.

Substituting (14) in (11) yields the equation for the evolution of the area of a pulse as it passes through the interaction region,

$$\frac{\partial^2 \chi}{\partial z^2} = \alpha^2 \sin \chi \quad (15)$$

where

$$\alpha = \sqrt{\frac{2\pi n_e e^4 E_i^2}{\hbar \omega_s^2 \omega_i m^2 c^2}}$$

is the growth rate of the amplitude in the linear ( $\chi \ll 1$ ) regime. Note that Eq. (15) is the familiar pendulum equation which describes the phase of electron synchrotron oscillations in FEL theory.<sup>15</sup>

We can easily write down the solution of (15) in terms of elliptic functions. For the intensity of the high-frequency field we have

$$I_s(L) = dn^{-2} \left( \frac{\alpha L}{\kappa}, \kappa \right) I_s(0). \quad (16)$$

Here  $dn(x, y)$  is the Jacobi elliptic tangent and we have written  $\kappa = \sqrt{1 + I_s(0)/I_b}$ ,  $I_s(0)$  where  $I_s(0)$  is the intensity of the signal wave on entry into the amplifier. As will be seen shortly, the parameter  $I_b = \hbar \omega_s n_e c$ , physically represents the maximum intensity that can be emitted by a beam of "two-level" electrons.

In accordance with (16) the output intensity  $I_s(L)$  of the high-frequency signal as a function of the length of the optical undulator is a periodic function. This is a consequence of the large energy exchange between the electron beam and the radiation field. The maximum gain is achieved for a sequence of optimal values  $L$  given by

$$L_m = \alpha^{-1} \kappa K(\kappa) (2m+1), \quad m=0,1,\dots \quad (17)$$

Here Eq. (16) yields

$$I_s(L_m) = I_s(0) + I_b. \quad (18)$$

The quantity  $K(\kappa)$  is a complete elliptic integral. Physically Eq. (18) means that all of the energy stored in the upper working level of the medium is emitted over interaction regions of length  $L = L_m$ . Thus, lasing occurs in which the inversion is completely removed in one pass; this is analogous to the formation of a  $\pi$ -pulse in a coherently amplifying medium.<sup>23</sup>

Now we discuss the possibility of developing this type of x-ray FEL with optical pumping in an actual experiment. The optimum conditions for excitation obviously correspond to taking the length of the interaction region equal to  $L = L_0$ . If we assume that the mirror reflection coefficient is  $R$ , it is easy to see that the threshold for lasing is determined by

$$\alpha^{-1} \xi K(\xi) = L_0, \quad \xi = \sqrt{1-R^2}. \quad (19)$$

Let us make some estimates. For a neodymium laser ( $\lambda_i = 1.06 \mu\text{m}$ ) focused into a spot of dimensions approximately equal to the size of an electron beam  $r_b \approx 0.01 \text{ cm}$  we have a constriction length (the length of the optical undulator)  $L \approx 6 \text{ cm}$ . For an electron beam with energy  $\varepsilon \approx 2.5 \text{ MeV}$  and density  $n_e \approx 3 \cdot 10^{12} \text{ cm}^{-3}$ , taking the mirror reflection coefficient  $R \approx 0.5$ , we find from (19) an estimate for the power of the pump radiation needed to reach the lasing threshold,  $I_i \approx 2 \cdot 10^{14} \text{ W/cm}^2$ . Hence the frequency  $\omega_s$  is in the soft x-ray range  $\hbar \omega_s = 4\gamma^2 \hbar \omega_i \approx 100 \text{ eV}$  and the density emitted in one pass amounts to  $I_b \approx 1.6 \text{ MW/cm}^2$ .

The quantum x-ray FEL regime is the most efficient. Thus, let us consider an FEL with an optical undulator in the classical and quantum regimes (the transition to the classical regime can be made, in particular, by increasing the energy spread  $\Delta\gamma/\gamma$  of the electron beam). As is well known,<sup>21</sup> the maximum efficiency in the classical regime is  $\approx 1/2N$  per electron; thus, under these conditions the average energy emitted by one electron is  $\langle \delta\varepsilon \rangle \approx \varepsilon_0/2N \ll \hbar \omega_s$ . In a two-level FEL, as we have just seen, at  $L = L_m$  each electron emits an x-ray photon  $\hbar \omega_s$  on the average. Specifically, an electron beam injected into the interaction region with energy close to the energy  $\varepsilon_e$  of the upper working level given by Eq. (1) constitutes a completely inverted medium in the quantum limit  $\eta \gg 1$ , whereas in the classical limit  $\Delta\varepsilon \gg \hbar \omega$  only partial inversion occurs, since all the working levels of the equivalent anharmonic oscillator are "pumped up" simultaneously.

Note that the efficiency of this two-level FEL scheme can be increased by using the technique of phase modulating the pump radiation pulse. The mechanism for this increase is completely analogous to the case of the magneto-static undulator with variable parameters: as the frequency increases adiabatically in the "tail" of the pump pulse, an electron remains in the center of the emission line over the entire interaction region.

As we move into the XUV spectral range the processes of spontaneous relaxation of the "emitting state"  $\varepsilon_e$  of the electron beam can play an important role. Spontaneous emission causes the electron to leave the range of energies where resonant interaction with the laser field occurs, thus effectively reducing the number of particles that take part in generating the induced x-ray signal. For the "two-level" FEL design the characteristic time for spontaneous relaxation is given by<sup>24</sup>

$$\tau^* = \left[ \frac{2}{3} \frac{e^4 E_i^2}{m^2 c^4 \hbar \omega_i} \right]^{-1}. \quad (20)$$

In order to get a larger fraction of the electrons involved in the process of generating a useful signal we must require

$$\alpha\tau^*c > 1. \quad (21)$$

Condition (21) represents a restriction on the allowable power of the optical pump, and consequently on the gain of the FEL. Under the conditions  $\varepsilon \approx 2.5$  MeV,  $\lambda_i = 1.06$   $\mu\text{m}$ ,  $I_i \approx 2 \times 10^{14}$  MW/cm<sup>2</sup> used to obtain the above estimate the characteristic relaxation length of the "emitting state" is equal to  $L^* = c\tau^* \approx 20$  cm  $\gg L_0$  and inequality (21) holds.

## 5. CONCLUSION

In the present work we have considered the physical processes in an x-ray FEL with an optical undulator. We have shown that a device designed according to this scheme allows the electrons to interact with the laser fields in the fully quantum-mechanical regime. Hence the FEL is a two-level quantum generator with a completely inverted active medium. Analysis of the nonlinear dynamics of an FEL in this regime shows that the inversion can be completely removed in one pass, similar to the process by which a  $\pi$ -pulse is formed in coherently amplifying media. The physical processes in a "two-level" FEL are distinctly quantum mechanical and cannot be described using classical electrodynamics.

This analysis shows that a two-level FEL with optical pumping can be a relatively compact and very efficient device for producing tunable coherent short-wavelength radiation. This FEL design, which has a high output power, does not require the use of GeV electron beams to extend lasing to the x-ray region of the spectrum. It should be noted, however, that the requirements imposed on beam quality in FEL x-ray designs<sup>5-8</sup> using magnetic undulators are less severe, and can probably be achieved more readily at the present time.

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<sup>1</sup>Note that energy transferred to the high-frequency component of the laser field is due primarily to the electron energy, and in all cases of practical interest the attenuation in the power of the pump field is extremely negligible.

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