Calculation of the relative dimensions of a region heated by a laser beam

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It is shown that including the thermoelectric effect permits an explanation for the presence of convection in a melted region subject to heating from above. The ratio of the longitudinal and transverse dimensions of this region ($\simeq 50$) is found to be close to the aspect ratio observed experimentally.

1. POSSIBILITY OF THERMOELECTRIC CONVECTION

From experimental investigations[1–3] (see also the review articles in Refs. 4 and 5) it is well known that in the melting of semimetals (e.g., steels in alloys) or semiconductors by long pulses of laser radiation (longer than 5 ms), the melted material is subject to convective mixing. This is especially typical of the case in which the radiation energy is only slightly greater than that necessary for melting ($\simeq 5$ J/mm^3) and vaporization of the material can be disregarded.

On the other hand, it is well known that neither Rayleigh convection[6] excited by buoyant forces due to thermal expansion nor cellular motion excited by surface-tension forces[7,8] due to the temperature dependence of the capillary forces develops when the heat is applied from above. The instability, however, becomes theoretically possible if we take into account thermoelectric forces.[9,10]

We introduce the dimensionless numbers

$$R=\frac{\beta g A h^4}{\kappa\nu},\quad M=\frac{\sigma A h^2}{\rho\kappa\nu},\quad \mathcal{E}=I^2=\frac{\varepsilon\gamma^2A^2h^2}{\rho\kappa\nu},$$

where $A=|\nabla T_0|$ is the effective gradient of the temperature T or the heating; h is the thickness of the liquid layer, i.e., the vertical extent of a cell of the heated surface; κ and ν are the thermal conductivity and viscosity and ρ is the fluid density; $\beta=-(1/\rho)\partial\rho/\partial T$ is the thermal expansion coefficient, $\sigma=-\partial\alpha/\partial T$ is the temperature coefficient of the surface tension, characterized by the coefficient α; and γ is the thermoelectric coefficient. The numbers R, M, and $\mathcal{E}$ characterize the volume force (Rayleigh number), thermocapillary force (Marangoni number), and the thermoelectric force ($E=I^2$).

The first thing to note is that in the case of heating from above the thermoelectric force is responsible for causing motion, while the volume and thermocapillary forces stabilize the motion. Secondly, comparison of the dimensionless numbers reveals that the thermoelectric effect dominates in thin layers of fluid originally formed by melting under the action of laser radiation. In the laser radiation, in addition to the vertical component A_z (the z axis is perpendicular to the surface of the material), there is also a longitudinal component A_x. Under the action of the A_x motion develops[11,12] no matter how small the heating, with velocities $v=v_x\gg v_z$ directed axially (along the layer) and proportional to R and M. In thin layers these numbers are small, so the velocities are also.

But the thermocapillary instability, which is essentially an electroconvection mode,[10] does not depend on the direction of heating and develops for $I\geqslant I_*$. For vertical heating the instability has a threshold,[10] while for lateral heating the instability develops "resonantly" near I_*. This can be seen in the solution[13] which describes purely longitudinal motion. The specific value I_* of the critical number was calculated by Eidel'man;[10,13] it determines the boundary conditions.

In order to make a comparison with experiment it is therefore natural to consider excitation in an infinitely thin (but "macroscopic") initially liquid layer subject to both A_z and A_x, i.e., with heating both from above (in the direction opposite g) and laterally. The effect of the other forces can be disregarded, and we assume that the fluid is initially at rest.

2. CALCULATION OF THE ASPECT RATIO OF THE CONVECTION CELL

The linearized electrostatic and thermal conduction equations (two components) in the "film" approximation $\lambda\gg h$ (here λ is the size of the region parallel to the heated surface) relate the perturbed velocity $v=v_x$, temperature $T_1=T-T_0$, pressure $p_1=p-p_0$, and carrier density $n_1=n-n_0$ (here e is the charge of a carrier particle) to their equilibrium values. We have

$$\frac{\partial p_1}{\partial z}=e\gamma n_1A_z,\quad \frac{\partial p_1}{\partial x}=\rho\nu\frac{\partial^2 v}{\partial z^2},\tag{1}$$

$$en_1=\varepsilon\gamma\frac{\partial^2T_1}{\partial z^2},\quad \kappa\frac{\partial^2T_1}{\partial z^2}=vA_x.\tag{2}$$

The continuity equation enables us to determine the vertical component of velocity, which does not enter into Eqs. (1) and (2) in this approximation. The electric field is assumed to be "purely" thermoelectric, $E=\gamma\nabla T$.

Eliminating all variables except v we have

$$\frac{\partial^3 v}{\partial z^3}=\frac{\varepsilon\gamma^2A_zA_x}{\rho\kappa\nu}\frac{\partial v}{\partial x}.\tag{3}$$

If we introduce the natural unit of length, the layer thickness h, then it is obvious that the onset of instability is characterized by a dimensionless "film number"

$$\mathscr{E}^f=\varepsilon\gamma^2A_xA_zh^2/\rho\nu\kappa,$$

completely analogous to the number $\mathscr{E}$ which characterizes the thermoelectric effect, where A_z and A_x act separately.

Equation (3) is soluble by separation of variables. We put

$$v=v(z)\exp(-k_xx/h),$$

where the "separation" constant satisfies $k_x>0$, since the heating in the center (at $x=0$) is very large. In fact, k_x determines the dimension λ of the region in which longitudinal motion occurs from the relation $k_x=2\pi h/\lambda$ for a cell of rectangular shape.

Equations (1) and (2) are supplemented by homogeneous boundary conditions. The lower boundary $z=0$ can always be taken to be "solid,"[6] $v=0$. This is the interface with the solid material. The upper boundary is the air–liquid interface; it must be assumed to be free, so we set $\partial v/\partial z=0$ there. Furthermore, the motion in the film must be closed. The condition for exciting velocity is independent of the thermal conditions on the boundaries.

The calculation shows that motion develops when the film number reaches the value $\mathscr{E}^f_*\simeq 3.4/k_x$.

On the other hand, the results obtained in Refs. 10 and 13 imply that the instability develops only for values of $\mathscr{E}$ close to 20. Hence it is easy to establish that $\lambda=bh\simeq 37h$ must hold.

3. COMPARISON WITH EXPERIMENT

Examples of experiments in which the thermoelectric effect can influence convection include, e.g., heating of melted steel containing a suspension of tungsten carbide[2,3] and liquid films obtained from alloys of silicon and tellurium.[14] For purposes of estimates we can assume that the difference between the temperatures of the hot and cold surfaces, T_h and T_c, is given by $T_h-T_c\simeq Ah\simeq 10^3$–$10^4$ K and that the parameters of the liquid are $\rho=1$–10 g/cm^3; $\beta=(9$–$6)\cdot 10^{-4}$ K^{-1}; N $\nu\simeq\kappa\simeq 5\cdot 10^{-2}$–$1$ mm^2/s; $\sigma=(1$–$3)\cdot 10^{-5}$ N/(m·K).

We also need to know the value of the thermoelectric coefficient γ. In the literature such data are not available at melting temperatures and slightly above. It is well known, however, that the electrical conductivity and the thermoelectric coefficients are essentially constant at the melting point. In solid semiconductors at temperatures above the Debye temperature T_D the thermal emf is determined by entrainment and can be written as $\gamma=a\,k_B/e\;T_D/T$, where k_B is Boltzmann's constant and e is the charge of a carrier. The constant a is such that γ is of order 100 μW/K.

The condition for the thermoelectric effect to induce the liquid to move, such that neither the volume force ($\beta>0$) nor the temperature dependence of the surface tension ($\sigma>0$) can suppress the instability, can be written as

$$A>A_I\simeq\left(\frac{\rho\kappa\nu}{\varepsilon}\right)^{1/2}\frac{I_*}{\gamma h}>\frac{\rho\beta g h^2}{\varepsilon\gamma^2}\frac{\mathscr{E}_*}{R_*}>\frac{\sigma}{\varepsilon\gamma^2}\frac{\mathscr{E}_*}{M_*},$$

where we have $R_*\gtrsim 800$ (cf. Ref. 6), $M_*\gtrsim 80$ (cf. Ref. 8), and $\mathscr{E}_*\gtrsim 40$ (cf. Ref. 9). Hence the thermoelectric effect gives rise to instability if the heating exceeds the value A_I.

Surface waves can be excited in the case of heating from above (cf. Ref. 15). But, firstly, such waves cannot induce motion in a melted layer to the observed depth (up to 150 μm according to Refs. 2 and 3), and therefore cannot be responsible for the penetration of the alloying material (tungsten) there. Secondly, we must compare the excitation conditions. In Ref. 15 it was shown that in order to excite waves the heating must satisfy $A>A_w=\rho gh/\sigma$. Comparing the conditions for the excitation of thermoelectric convection and surface waves, we find[16] that thermoelectric convection is excited first if the thickness of the thin liquid layer is less than

$$h<h_*\simeq\left(\frac{\kappa\nu}{\rho\varepsilon}\right)^{1/2}\frac{I_*\sigma}{\gamma g}.$$

These estimates reveal that $h_*\leqslant 1$ mm holds. Under the experimental conditions the thickness of the alloyed layer is found to be 0.1–0.03 mm, i.e., the thermoelectric instability is the first one to be excited.

It is assumed[6,8] that the properties of the instability which develop first are manifested even after the mode saturates.

The theoretical values $b\simeq 54$ and $b\simeq 37$ for rectangular and hexagonal cells, calculated assuming that two three-dimensional cells are excited right away,[12] agree well with the result $\lambda/h\simeq 47$ obtained by using data from tables of the dimensions of alloying regions.[3]

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