Probe-field spectroscopy in atomic media optically oriented in the ground state

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The propagation of a probe light beam in an atomic medium (an atomic beam or a low-density gas) is analyzed for the case in which the atoms are optically oriented in their ground state by a "strong" light wave. The polarization state of the atoms is determined under these conditions by the local value of the polarization vector in a given region. It is independent of the intensity of the strong field. The problem in which the strong field is at resonance with a transition $J_0 \rightarrow J_0 + \Delta J_0$ and generally has a (spatial) polarization gradient, while the probe field is at resonance with another transition, $J_0 \rightarrow J_0 + \Delta J$, is analyzed. The structure of the electric susceptibility tensor is analyzed. The problem of normal waves in a low-density gas optically oriented in the ground state by a field with a uniform polarization is solved. The problem of Bragg scattering of a probe wave by an atomic beam is analyzed in two models, used as examples: the $1 \rightarrow 2$ transition in a $\sigma^+ - \sigma^-$ configuration of the strong field and the $1/2 \rightarrow 1/2$ transition in a *lin* $\perp$ *lin* configuration of the strong field.

1. INTRODUCTION

The propagation of an optical field in an optically oriented atomic medium was studied first in a series of papers.[1,2] The polarization characteristics of the transmitted optical field were analyzed as a function of the polarization of atoms in their ground state. The effect of a polarization of the atoms on light propagation in optically dense media was analyzed in Ref. 3 by a Keldysh-diagram technique.

It is worthwhile to formulate this problem for specific topics in polarization spectroscopy of the ground state of atoms which are degenerate with respect to projections of the angular momentum J_0. In the conventional formulation of problems in nonlinear polarization spectroscopy, one studies those anisotropic properties of atomic ensembles (gases or atomic beams) at the frequency of a weak (probe) field which stems from saturation in terms of a strong field in excited states.[4–6] The discussion is also restricted to the case in which the optical anisotropy of the atomic ensemble in the ground state arises in the course of optical pumping by a steady-state "strong" laser field. The probe (readout) wave can, in general, have a different frequency, a different direction, and a different polarization. Ground-state laser spectroscopy is attractive for the following reasons:

1. It is always possible to choose parameters of the strong field such that ordinary saturation effects can be ignored (in this sense, the "strong" field is weak). The formation of multipole moments of rank κ (in general, $\kappa \leqslant 2J_0$) in the ground state is governed exclusively by the polarization of the strong field in this case.[7–9]

2. For ultradeep cooling of atoms in laser fields,[10,11] there are some necessary conditions: there must be (spatial) gradients in the polarization of the optical field, the field intensities must be low, and the atomic ground state must have a degenerate structure. Polarization ground-state laser spectroscopy would clearly be a reliable method for the diagnostics of atomic ensembles, because of the high measurement accuracy in terms of a large number of parameters. The information obtained as a result would be important for clarifying the mechanisms and dynamics of atomic cooling.

With regard to the first of these points we note that optical orientation of atoms in the ground state tends to build up over time because of the low depolarization rates γ_κ^0. Depolarization of multipole moments of rank $\kappa > 0$ in the ground state stems primarily from collisions between atoms, and it occurs only slowly in low-density gases or atomic beams. The intensity of the strong field, $I = (c/8\pi)|E_0|^2$, must therefore satisfy the rather obvious conditions

$$(\gamma t)^{-1} \ll \frac{I}{I_0} \ll 1, \tag{1}$$

where $I_0 = (c/8\pi)|\hbar\gamma/d|^2$ is the saturation intensity, t is the duration of the interaction of the atoms with the field (in most cases the duration is given by $t = D/\langle v \rangle$, where **D** is the size of the light beam, and $\langle v \rangle$ is the directed velocity of the atoms), γ is the rate of radiative relaxation of the excited state of the atom, and d is the transition dipole moment. For the D_1 line of Na, for example, typical values are $I_0 \simeq 64$ μW/mm^2 and $(\gamma t)^{-1} \simeq 10^{-3}$. The intensity range of the strong fields is thus fairly broad and can easily be reached with existing dye lasers. Conditions (1) can be formulated in a different way by introducing as a characteristic parameter the duration of the optical pumping of the ground state,[7,8] t_{op}.

$$t \geqslant t_{\text{op}}; \quad t_{\text{op}} = (\gamma G_0)^{-1}, \tag{2}$$

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where the parameter G_0 characterizes the saturation of the strong field.

In this case the atomic ensemble is optically anisotropic with an electric susceptibility independent of the intensity of the strong field. This is the primary distinction between the problem discussed below and problems of nonlinear polarization spectroscopy. A problem similar to the one we are discussing here was studied in Refs. 7 and 8 for an angular momentum $J_0=1/2$. There the interaction was labeled "quasilinear" for propagation in an optically ordered medium of this sort. We will use that word in the discussion below. In this paper we leave the angular momentum J_0 arbitrary; J_0 could also be understood to be the total angular momentum of a hyperfine component, F_0.

For the problem of the optical orientation of ground-state atoms in fields with a polarization gradient, satisfying the condition for quasilinear interaction (2) requires that the interaction time t be so short that an atom does not have time to undergo a displacement significant in comparison with the length scales of the variations in the polarization of the field. For the field configurations used in experiments on ultradeep cooling (a helically polarized wave[10] or the *lin* ⊥ *lin* configuration[10,11]), these length scales are of the order of the wavelength of the light, λ. The condition (2) for a quasilinear interaction is then satisfied for atoms whose velocities $v_\parallel$ along the field satisfy[10,11]

$$\frac{kv_\parallel}{\gamma G_0} \leqslant 1. \tag{3}$$

The contribution of this group of atoms to the electric susceptibility is then determined exclusively by the local values of the field polarization. In gases, the relative number of atoms which have velocities $v_\parallel$ satisfying (3) is very small: $v_\parallel/\langle v\rangle \approx \gamma G_0/\Omega_D \ll 1$, where Ω_D is the width of the Doppler absorption line. In gases, effects stemming from optical order in fields with polarization gradients are therefore weak.

On the other hand, (3) is a necessary condition for those atoms which are subjected to the ultradeep cooling (below the Doppler limit $k_B T_D=\hbar\gamma/2$) in an optical field.[10] This condition usually holds in atomic beams which have been collimated beforehand.

In accordance with the discussion above, the analysis below takes the following directions:

1. analysis of the tensor structure of the electric susceptibility of an atomic medium which is optically oriented in its ground state in the course of a "quasilinear" interaction with a "strong" field which in general has a polarization gradient,

2. analysis of birefringence in a gas which is optically oriented in the course of a quasilinear interaction with a strong field which is uniform in terms of polarizations, and

3. analysis of Bragg scattering by spatial gratings of multipole moments of ranks $\kappa=0$, 1, and 2 which arise in atomic beams in optical fields with polarization gradients.

2. ELECTRIC SUSCEPTIBILITY TENSOR OF AN ATOMIC ENSEMBLE ORIENTED IN THE GROUND STATE

Let us formulate a problem of the propagation of a probe wave **E** in an optically oriented atomic ensemble. This probe wave does not alter the deviation from equilibrium in the ground state created by the strong field. We assume that the probe wave, of frequency ω, is at resonance with the transition $J_0 \to J_0+\Delta J$ ($\Delta J=0,\pm 1$) with a resonant frequency ω_0, while the strong field, $\mathbf{E}_0$, causes a redistribution of the populations of the Zeeman sublevels of the ground state as it interacts in a resonant fashion with, in general, another transition $J_0 \to J_0+\Delta J_0$ ($\Delta J_0=0,\pm 1$).

The deviation from equilibrium caused in the ground state is determined by the steady-state density matrix of the ground state, $\rho^0_{mm'}(\mathbf{r},\mathbf{v})$, which depends on only the polarization of the strong field in the case of a quasilinear interaction. In this paper we assume that the form of $\rho^0_{mm'}$ is known. A method for calculating $\rho^0_{mm'}$ for gases in a case of a pump field of uniform polarization was presented in Refs. 8, 12, and 9. The density matrix $\rho^0_{mm'}(\mathbf{r},\mathbf{v})$ for atomic beams has been determined for certain field configurations and optical transitions $J_0 \to J_0+\Delta J_0$ in fields with polarization gradients.[10,11]

The electrodynamic part of the problem is to calculate the susceptibility tensor $\hat{\chi}$ at the frequency ω. Since we are taking the medium to be a low-density gas or an atomic beam, in calculating the resonant part of $\hat{\chi}(\omega)$ it is sufficient to use a linear approximation in the density of atoms, N. Using the standard calculation methods, we find

$$\chi^{ij}(\mathbf{k},\mathbf{k}') = -N\int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\frac{X^{ij}(\mathbf{k}-\mathbf{k}',\mathbf{v})}{i\gamma/2+(\Delta\omega-\mathbf{k},\mathbf{v})}, \tag{4}$$

$$X^{ij}(\mathbf{k},\mathbf{v}) = \int d^3\mathbf{r}\exp(-i\mathbf{k}\mathbf{r})\sum_{\mu,m,m'} d^i_{m\mu}d^j_{\mu m'}\rho^0_{m'm}(\mathbf{r},\mathbf{v}). \tag{5}$$

Here

$$d^i_{m\mu} = \langle J_0,m\,|\,(\mathbf{d}\,\mathbf{e}^i)\,|\,J_0+\Delta J,\mu\rangle$$

is the matrix element of the projection of the dipole moment operator $\mathbf{d}$ onto the unit vector $\mathbf{e}^i$ ($i=1,2,3$) of a Cartesian coordinate system, γ is the rate of radiative relaxation of the excited state with angular momentum $J_0+\Delta J$, $\Delta\omega=\omega-\omega_0$ is the offset of the field frequency from resonance, and $\mathbf{k}$ is the wave vector of the field. The density matrix ρ^0 is normalized:

$$\mathrm{Sp}\,\rho^0 = \sum_m \int \frac{d^3\mathbf{r}\,d^3\mathbf{p}}{(2\pi\hbar)^3}\rho^0_{mm}(\mathbf{r},\mathbf{p}) = 1. \tag{6}$$

When the spatial variation of ρ^0 in strong fields with a polarization gradient is taken into account, the polarizability of the atomic ensemble is found from

$$P_i(\mathbf{k},\omega) = \int \frac{d^3\mathbf{k}'}{(2\pi)^3}\chi^{ij}(\mathbf{k},\mathbf{k}')E_j(\mathbf{k}',\omega). \tag{7}$$

This expression is similar to that for the polarizability of a crystal in an external electromagnetic field. Here $E_j(\mathbf{k}',\omega)$ is a Fourier component of this field.

To put (4) in invariant form, we expand the density matrix in terms of the polarization operators $\rho^0_{\kappa q}$ (in the κq representation),[13]

$$\rho^0_{mm'}(\mathbf{r},\mathbf{v})=\sum_{\kappa,q}(-1)^{J_0-\mu}\sqrt{2\kappa+1}\begin{pmatrix} J_0 & \kappa & J_0\\ -m & q & m'\end{pmatrix}\rho^0_{\kappa q}, \tag{8}$$

and we sum (5) over matrix sublevels, using the Wigner–Eckart theorem. We find

$$X^{ij}(\mathbf{k},\mathbf{v})=3d^2(-1)^{2J_0+\Delta J}\sum_{\kappa}\begin{Bmatrix} J_0+\Delta J & J_0 & 1\\ \kappa & 1 & J_0\end{Bmatrix}\int d^3\mathbf{r}$$
$$\times\exp(-i\mathbf{kr})(\{\mathbf{e}^i\otimes\mathbf{e}^j\}\rho^{0*}_{\kappa}). \tag{9}$$

Here we are using the standard notation for the $3jn$ symbols and irreducible tensors: $\{\mathbf{e}^i\otimes\mathbf{e}^j\}_\kappa$ is an irreducible tensor product of rank κ of the first-rank tensors $\mathbf{e}^i$, $(\ldots\cdot\ldots)$ is the scalar product of tensors,[13] the Hermitian tensor $X^{ij*}=X^{ji}$ incorporates all of the information on the anisotropy of the medium, and d is a reduced dipole moment.

For certain models of the interaction of atoms with a pump field, it is possible to choose a local basis of unit vectors which move with an atom (an internal coordinate system). In this basis we can choose a quantization axis $\boldsymbol{\nu}(\mathbf{r})$ such that

$$\rho^0_{\kappa q}=\delta_{q0}\rho_\kappa,\quad \rho^0_{mm'}=\delta_{mm'}\rho^0_m. \tag{10}$$

For example, $\boldsymbol{\nu}$ is parallel to the polarization in the case of a linearly polarized field $\mathbf{E}_0$, and it is parallel to the wave vector $\mathbf{k}_0$ in the case of a circularly polarized field. For certain bleaching transitions ($\Delta J_0=-1$) it is also possible to choose the quantization axis along the axis of a cylinder constructed on the polarization ellipse for an elliptically polarized and otherwise arbitrary field.[14]

The scalar product of tensors in (9) can be simplified in these cases, since the product $\{\mathbf{e}^i\otimes\mathbf{e}^j\}_{\kappa q}$ is projected onto the quantization axis $\boldsymbol{\nu}(\mathbf{r})$. In terms of Cartesian components we have

$$\{\mathbf{e}^i\otimes\mathbf{e}^j\}_{\kappa 0}=-\frac{\delta_{ij}}{3}\delta_{\kappa 0}+\frac{i}{\sqrt{2}}\varepsilon_{ijk}\nu_k\delta_{\kappa 1}+\sqrt{\frac{3}{2}}\left(\nu_i\nu_j-\frac{\delta_{ij}}{3}\right)\delta_{\kappa 2},$$
$$\nu_i(\mathbf{r})=(\boldsymbol{\nu}(\mathbf{r})\mathbf{e}^i). \tag{11}$$

Here and below, a repeated index implies summation.

Tensor (9) can thus be decomposed into irreducible components, which are scalar, antisymmetric, and symmetric:

$$X^{ij}=C_0\delta_{ij}+i\varepsilon_{ijm}C_{1,m}+C_{2,ij}, \tag{12}$$

where

$$C_0=-\frac{1}{3}\Pi_0\int d^3\mathbf{r}\exp(-i\mathbf{kr})\rho^0_0(\mathbf{r},\mathbf{v}), \tag{13}$$

$$C_{1,m}=\frac{1}{\sqrt{2}}\Pi_1\int d^3\mathbf{r}\exp(-i\mathbf{kr})\rho^0_{1,0}(\mathbf{r},\mathbf{v})\nu_m(); \tag{14}$$

$$C_{2,ij}=\sqrt{\frac{3}{2}}\Pi_2\int d^3\mathbf{r}\exp(-i\mathbf{kr})\rho^0_{2,0}(\mathbf{r},\mathbf{v})\left(\nu_i(\mathbf{r})\nu_j(\mathbf{r})-\frac{\delta_{i,j}}{3}\right),$$

$$\Pi_\kappa=3d^2(-1)^{2J_0+\Delta J}\begin{Bmatrix} J_0+\Delta J & J_0 & 1\\ \kappa & 1 & J_0\end{Bmatrix}. \tag{15}$$

The quantities $C_{1,m}$, which stem from the orientation, and $C_{2,ij}$, which stem from the alignment of the atoms in the ground state, determine, in particular, the degree of natural optical activity and elliptical birefringence for a signal propagating through such a medium (these effects were analyzed in Refs. 7, 8, and 15) with $\mathbf{k}=\mathbf{k}'$ in (14) and (15). In the limiting case of a quasilinear interaction, ρ^0_κ is independent of the intensity of the pump field, being determined exclusively by the polarization, as we mentioned above.

3. BIREFRINGENCE IN AN OPTICALLY ORDERED GAS

For the reasons set forth in the Introduction, the spectroscopy of a low-density gas which is optically oriented in its ground state is of interest in the case of a spatially uniform polarization of the strong field. In this case the quantization axis $\boldsymbol{\nu}$ in (11) is independent of $\mathbf{r}$, and we also have

$$\rho^0_{mm'}(\mathbf{r},\mathbf{v})=\rho^0_{mm'}f_0(\mathbf{v}),\quad \sum_m\rho^0_{mm}=1, \tag{16}$$

where $f_0(\mathbf{v})$ is a Maxwellian velocity distribution in the gas. In other words, the gas as a whole is at equilibrium if we ignore recoil effects in the interaction of the atoms with the strong field, but in terms of internal degrees of freedom the deviation from equilibrium is substantial:

$$\rho^0_{mm}\neq\frac{\delta_{mm'}}{2J_0+1}. \tag{17}$$

The calculation of susceptibility tensor (5) then simplifies considerably:

$$X^{ij}(\mathbf{k},\mathbf{k}')=\delta^3(\mathbf{k}-\mathbf{k}')\chi_0(\omega)\xi^{ij}, \tag{18}$$

where

$$\chi_0(\omega)=-\frac{Nd^2}{\hbar(2J_0+1)}\int\frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\frac{f_0(\mathbf{v})}{\Delta\omega-\mathbf{kv}+i\gamma/2} \tag{19}$$

is the ordinary linear electric susceptibility, while the tensor

$$\xi^{ij}=\delta^{ij}+i\varepsilon_{ijm}\nu_m\tilde{C}_1+(\nu_i\nu_j-\tfrac{1}{3}\delta^{ij})\tilde{C}_2 \tag{20}$$

determines the anisotropic properties of the gas. In an ordinary gas we would have $\xi^{ij}=\delta^{ij}$. The explicit expression for the constants in (20) is

TABLE I. Expressions for the constant $\tilde{C}_1$ of the anisotropy tensor ξ^{ij} for various transitions as a function of the angular momentum of the ground state and the polarization of the orienting field.

ΔJ	$\Delta J_0 = -1$	$\Delta J_0 = 0$	$\Delta J_0 = +1$
-1	$q_0 \frac{3}{4}\frac{(2J_0-1)}{J_0}$	$\frac{3}{2}q_0$	$\frac{3}{2}q_0$
0	$q_0 \frac{3}{4}\frac{(2J_0-1)}{J_0(J_0+1)}$	$\frac{3}{2}\frac{q_0}{(J_0+1)}$	$\frac{3}{2}\frac{q_0}{(J_0+1)}$
$+1$	$-q_0 \frac{3}{4}\frac{(2J_0-1)}{(J_0+1)}$	$-\frac{3}{2}q_0\frac{J_0}{(J_0+1)}$	$-\frac{3}{2}q_0\frac{J_0}{(J_0+1)}$

Note. The cells along the diagonal correspond to the case in which the probe and strong fields are at resonance with transitions of a common type, $\Delta J=\Delta J_0$. The off-diagonal cells correspond to the case in which the probe field is at resonance with a transition of a different type, $\Delta J\neq\Delta J_0$.

$$\tilde{C}_\kappa=\frac{3}{\sqrt{2}}\sqrt{2\kappa+1}(2J_0+1)(-1)^{2J_0+\Delta J}\begin{Bmatrix} J_0+\Delta J & J_0 & 1 \\ \kappa & 1 & J_0\end{Bmatrix}\rho^0_\kappa. \tag{21}$$

Tables I and II show the results of a calculation of the magnetic dipole constant $\tilde{C}_1$ and the quadrupole constant $\tilde{C}_2$ for various values of ΔJ, ΔJ_0, and q_0 (this is the polarization of the strong field; we have $q_0=0$ in the case of linear polarization, and $q_0=\pm 1$ in the case of circular polarization). The method for calculating $\tilde{C}_\kappa$ and ρ^0_κ is given in Refs. 12, 8, and 9.

We turn now to the problem of finding the normal waves, their velocities, and their absorption coefficients for a medium described by susceptibility tensor (18), as is typically done in problems in the optics of crystals.[16]

In the linear approximation in terms of the density, the

TABLE II. Expressions for the constant $\tilde{C}_2$ of the anisotropy tensor ξ^{ij} for various transitions as a function of the angular momentum of the ground state and the polarization of the orienting field.

ΔJ	q_0	$\Delta J_0 = -1$	$\Delta J_0 = 0$		$\Delta J_0 = +1$
			J_0 is an integer	J_0 is a half-integer	
-1	± 1	$-\frac{3}{4}\frac{2J_0-3}{J_0}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$
	0	$-\frac{3}{2}$	$\frac{3}{2}\frac{(J_0+1)}{(2J_0-1)}$	$\frac{3}{2}\frac{(J_0+1)}{(2J_0-1)}\alpha_0$	$\frac{3}{2}\frac{(J_0+1)(2J_0+3)}{J_0(2J_0-1)}\alpha_1$
0	± 1	$\frac{3}{4}\frac{(2J_0-3)(2J_0-1)}{J_0(J_0+1)}$	$\frac{3}{2}\frac{(2J_0-1)}{J_0+1}$		$\frac{3}{2}\frac{(2J_0-1)}{(2J_0+3)}$
	0	$\frac{3}{2}\frac{(2J_0-1)}{J_0+1}$	$-\frac{3}{2}$	$\frac{3}{2}\alpha_0$	$-\frac{3}{2}\frac{(2J_0+3)}{J_0}\alpha_1$
$+1$	± 1	$-\frac{3}{4}\frac{(2J_0-1)(2J_0-3)}{(J_0+1)(2J_0+3)}$	$-\frac{3}{2}\frac{J_0(2J_0-1)}{(J_0+1)(2J_0+3)}$		$-\frac{3}{2}\frac{J_0(2J_0-1)}{(J_0+1)(2J_0+3)}$
	0	$-\frac{3}{2}\frac{J_0(2J_0-1)}{(J_0+1)(2J_0+3)}$	$\frac{3}{2}\frac{J_0}{(J_0+3)}$	$\frac{3}{2}\frac{J_0}{2J_0+3}\alpha_0$	$\frac{3}{2}\alpha_1$

$$\alpha_0=\left\{1-\frac{3(2J_0+1)}{J_0(J_0+1)}\left(\sum_n\frac{1}{m^2}\right)^{-1}\right\}>0,$$

$$\alpha_1=\left\{\frac{6(J_0+1)(2J_0+1)}{(2J_0+3)(4J_0+3)(1-\varepsilon)-1}\right\},\qquad \varepsilon=\frac{2[(2J_0+2)!]^2}{(4J_0+4)!}$$

Note. The cells along the diagonal correspond to the case in which the probe and strong fields are at resonance with transitions of the same type, $\Delta J=\Delta J_0$. The off-diagonal cells correspnd to the case in which the probe field is at resonance with the transition of a different type, $\Delta J\neq\Delta J_0$.

longitudinal component of the probe field is small, as is the spatial birefringence angle (more on this below). In a calculation of the normal waves we can thus ignore the longitudinal component, and we can project the tensor ξ^{ij} onto the plane perpendicular to the wave vector of the probe wave, $\mathbf{k}$:

$$\xi_{\alpha\beta}=(\mathbf{x}_\alpha\hat{\xi}\mathbf{x}_\beta),\quad \sum_\alpha x^i_\alpha x^j_\alpha=\delta^{ij}-n^i_{\mathbf{k}}n^j_{\mathbf{k}},\quad \mathbf{n}_{\mathbf{k}}=\frac{\mathbf{k}}{k}.$$

The Greek indices $\alpha,\beta=1,2$ refer to the coordinate system $\{\mathbf{x}_\alpha\}$, moving with the probe wave, while the Latin indices (i,j) refer to the Cartesian coordinate system $\{\mathbf{e}^i\}$ which we defined earlier. We then have

$$\xi_{\alpha\beta}=\delta_{\alpha\beta}\left[1-\frac{1}{2}\tilde{C}_2\left(\nu_3^2-\frac{1}{3}\right)\right]+i\tilde{C}_1\varepsilon_{\alpha\beta3}\nu_3$$
$$+\tilde{C}_2\left[\nu_\alpha\nu_\beta-\frac{\delta_{\alpha\beta}}{2}(1-\nu_3^2)\right],$$
$$\nu_\alpha=(\boldsymbol{\nu}\mathbf{x}_\alpha),\quad \nu_3=(\boldsymbol{\nu}\mathbf{n}_{\mathbf{k}}). \tag{22}$$

In a gas with tensor (22), the normal waves are elliptically polarized orthogonal waves whose complex unit vectors

$$\mathbf{e}_+=\frac{\mathbf{y}_2+i\,\mathbf{y}_1\tan\alpha}{\sqrt{1+\tan^2\alpha}},\quad \mathbf{e}_-=\frac{\mathbf{y}_1+i\,\mathbf{y}_2\tan\alpha}{\sqrt{1+\tan^2\alpha}}, \tag{23}$$
$$\mathbf{y}_1=\mathbf{x}_1\cos\Psi+\mathbf{x}_2\sin\Psi,\quad \mathbf{y}_2=\mathbf{x}_2\cos\Psi-\mathbf{x}_1\sin\Psi$$

diagonalize tensor (22). Here Ψ is the angle through which the polarization ellipse rotates in the $\{\mathbf{x}_1,\mathbf{x}_2\}$ plane, and $\alpha(-\pi/4\leqslant\alpha\leqslant\pi/4)$ is the ellipticity angle. These parameters of the polarization ellipse are related to the constants $\tilde{C}_1$ and $\tilde{C}_2$, defined earlier, by

$$\Psi=\varphi+\frac{\pi}{2}\theta(\tilde{C}_2),$$
$$\tan\alpha=2\tilde{C}_1\cos\vartheta\left(\sqrt{\tilde{C}_2^2\sin^4\vartheta+4\tilde{C}_1^2\cos^2\vartheta}+|\tilde{C}_2|\sin^2\vartheta\right)^{-1}. \tag{24}$$

Here $\theta(\tilde{C}_2)$ is the unit step function, ϑ is the angle between the quantization axis $\boldsymbol{\nu}$ and the direction of the probe wave, $\mathbf{n}_{\vec{k}}$, and φ is the polar angle specifying the orientation of the quantization $\boldsymbol{\nu}$ in the coordinate system $\{\mathbf{x}_1,\mathbf{x}_2,\mathbf{n}_{\vec{k}}\}$. Figure 1 shows one of the normal ellipses in the case $\tilde{C}_2<0$; Fig. 2 shows the relationship between the normal ellipses and the angles specifying the quantization axis $\boldsymbol{\nu}$.

The wave vectors $\mathbf{k}_\pm$ corresponding to the normal polarizations in (23) are

$$\mathbf{k}_\pm=k_\pm\mathbf{n}_{\vec{k}},\quad k_\pm=\frac{\omega}{c}(1+2\pi\xi_0\Delta n_\pm);$$
$$\Delta n_\pm=1-\tfrac{1}{2}\tilde{C}_2(\cos^2\vartheta-\tfrac{1}{3})\pm\sqrt{\tilde{C}_2^2\sin^4\vartheta+4\tilde{C}_1^2\cos^2\vartheta}. \tag{25}$$

In general, an arbitrary probe wave can be decomposed into two normal waves:

$$\mathbf{E}(\mathbf{k},\omega,z)=e^{-i\omega t}(E_+(0)\mathbf{e}_+e^{ik_+z}+E_-\mathbf{e}_-e^{ik_-z})+\text{c.c.} \tag{26}$$

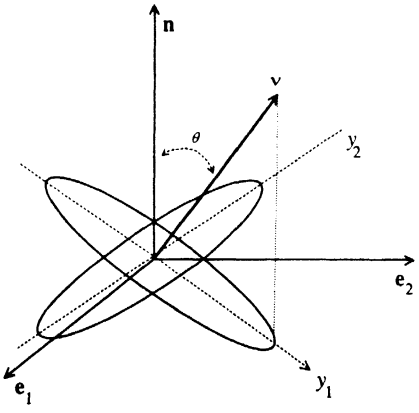

FIG. 1. Orientation of the quantization axis $\boldsymbol{\nu}$ and the polarization ellipses of the normal waves with respect to the laboratory coordinate system.

Solution (26) makes it possible to find the ellipse rotation angle $\Psi(z)$ and the ellipticity $\alpha(z)$ for an arbitrary probe wave which has passed through a medium as a function of the geometric path length in the gas, z, under the condition $N\chi_0 z<1$:

$$\Psi(z)=\Psi(0)+2\pi\frac{\omega}{c}(\Delta n_+-\Delta n_-)z\,\mathrm{Re}\chi_0(\omega), \tag{27}$$
$$\tan\left(\alpha(z)+\frac{\pi}{4}\right)=\tan\left(\alpha(0)+\frac{\pi}{4}\right)\exp\left\{2\pi\frac{\omega}{c}(\Delta n_+-\Delta n_-)z\,\mathrm{Im}\chi_0(\omega)\right\}. \tag{28}$$

We have thus found a complete description of the propagation of an arbitrary polarized probe wave in an optically oriented gas.

Let us look at some particular cases.

If a single wave propagates through a gas and creates an anisotropy, then this wave is naturally one of the normal waves. The anisotropy reduces in this case to dichroism with the absorption coefficients

$$\kappa_\pm=2\pi\frac{\omega}{c}\Delta n^0_\pm\,\mathrm{Im}\,\chi_0.$$

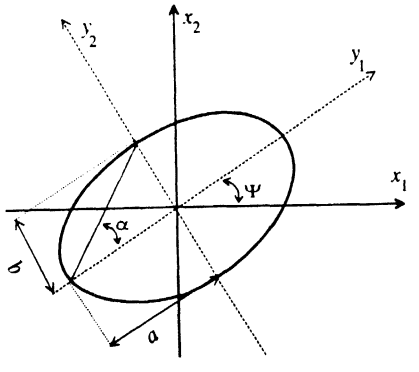

FIG. 2. One of the normal-wave polarization ellipses. α—Ellipticity angle; Ψ—ellipse rotation angle.

1. For a linearly polarized wave ($\widetilde{C}_1=0$, $\vartheta=\pi/2$, $\varphi=0$, $\alpha=0$) we have

$$\Delta n^0_{\pm}=1+\tfrac{1}{6}\widetilde{C}_2\pm\tfrac{1}{2}|\widetilde{C}_2|, \quad (\mathbf{x}_1\mathbf{e}_{\pm})=\Theta(\pm\widetilde{C}_2). \tag{29}$$

In the case $\widetilde{C}_2>0$, this wave is the normal wave with maximum absorption, $\Delta n^0_+=1+(2/3)\widetilde{C}_2$ in the case $\widetilde{C}_2<0$ it is the normal wave with minimum absorption, $\Delta n^0_-=1-(2/3)\widetilde{C}_2$. From Tables I and II we find that the first case holds for transitions with $\Delta J_0=\pm 1$, and the second for transitions with $\Delta J_0=0,-1$.

2. For a circularly polarized wave ($\vartheta=0$, $q_0=\pm 1$) we find

$$\Delta n^0_{\pm}=1-\frac{1}{3}\widetilde{C}_2\pm|\widetilde{C}_1|, \quad |(\mathbf{e}_{q0}\mathbf{e}^*_{\pm})|=\frac{1}{2}\left(1\mp q_0\frac{\widetilde{C}_1}{|\widetilde{C}_1|}\right) \tag{30}$$

$$\mathbf{e}_{q0}=\frac{1}{\sqrt{2}}(-q_0\mathbf{x}_1-i\mathbf{x}_2).$$

For $\widetilde{C}_1q_0>0$, the wave experiences its minimum absorption, while for $\widetilde{C}_1q_0<0$ it experiences its maximum absorption.

3. For an unpolarized field ($\widetilde{C}_1=0,\vartheta=0$) we find

$$\Delta n^0_{\pm}=1-\frac{\widetilde{C}_2}{3};$$

the absorption is the same for the two normal waves.

4. We now look at an interesting case which follows from Tables I and II under the condition $|\widetilde{C}_1|=|\widetilde{C}_2|=3/2$. We now have $\Delta n^0_-=0$, regardless of the direction in which the probe wave is propagating. In other words, over a sufficiently large propagation distance the probe wave becomes completely polarized, with polarization-ellipse parameters $\Psi=\varphi$, and $\tan\alpha=q_0\cos\vartheta$.

Changes occurring in transmission for various transitions and polarizations because of a redistribution of the populations of Zeeman sublevels in the ground state have been studied in astrophysical problems[17] and also in Ref. 15.

The particular cases listed above are simply instructive examples. Using Eqs. (26)–(28) and the values of $\widetilde{C}_1$ and $\widetilde{C}_2$ from Tables I and II, we can study a set of very interesting combinations for various polarizations of the probe field and of the pump field and for various atomic transitions.

Let us look at some approximate quantitative estimates of the effects which arise.

For low-density gases with densities $N\sim 10^{14}$ cm^{-3}, a crude estimate of the linear susceptibility is $\chi\sim 10^{-10}$. Since we have $\Delta n_{\pm}\sim 1$, the arguments of the exponential functions in (26) are on the order of unity for propagation distances

$$z\sim\frac{\lambda}{\chi}\approx 10^{10}\lambda\approx 10^3 \text{ m}$$

of the probe field in the gas.

In other words, the polarization effects described above during the propagation of a probe wave are seen most vividly in astrophysical media and in the upper atmosphere. In particular, we can treat solar radiation as the pump field here: at the very low spectral intensities of solar radiation, the duration of the interaction of the atom with the field is long, so we can determine spectral regions in which the condition (2) for quasilinear interaction clearly holds. A method for calculating the quadrupole constant $\widetilde{C}_2$ in an unpolarized, incoherent, but directed optical pump field is described in Refs. 12 and 15.

On the other hand, it is also possible to observe the effects described above in the laboratory, at higher gas densities. We should point out, however, that the depolarization of the multipole moments of rank $\kappa>0$ in the ground state due to interatomic collisions becomes more substantial as the density of atoms increases. As a result, the values of the magnetic dipole constant $\widetilde{C}_1$ and the quadrupole constant $\widetilde{C}_2$ may be quite different from the values given in Tables I and II.

To conclude this section of the paper, we take up a calculation of the spatial birefringence angle for the normal waves found above, in (23). Here we need to consider the longitudinal component of the electric field, $E_{\parallel}=(\mathbf{E}\mathbf{n}_{\mathbf{k}})$. This component is evidently proportional to the density of atoms; to first order in N it is given by

$$E_{\parallel}=-4\pi\chi_0(\mathbf{n}_{\mathbf{k}}\hat{\xi}\mathbf{E}_{\perp}),$$

where $\hat{\xi}$ and χ_0 are given by (19) and (20).

Our calculations show that the birefringence angle $\Delta\vartheta$ is comparable to the angle of the diffractive divergence of the probe light beam, $\Delta\vartheta_0$:

$$\frac{\Delta\vartheta}{\Delta\vartheta_0}<1, \quad \Delta\vartheta_0\sim\frac{\lambda}{D}, \tag{31}$$

where D is the transverse dimension of the beam. Observing a spatial separation of rays is thus problematic in this case.

On the other hand, this spatial birefringence is of fundamental importance for a classification of gases which are optically oriented in the ground state in accordance with crystal-optics ideas.[16] In particular, it can be shown that if optical anisotropy is produced in a gas by an elliptically polarized strong wave, then the equivalent "crystal" will be biaxial, with gyrotropy and dichroism.

4. DIFFRACTION OF A PROBE WAVE BY ATOMIC BEAMS WHICH ARE OPTICALLY ORIENTED IN THE GROUND STATE

In atomic beams, the transverse velocities of the atoms $v_{\perp}$ are small in comparison with their longitudinal velocity v_0. For a well-collimated beam (with a divergence angle $\beta=v_{\perp}/v_0\approx 10^{-3}$ rad) these velocities are $v_{\perp}\sim 1$ m/s. Experiments on ultradeep cooling have achieved a record atomic-beam collimation $v_{\perp}\sim\hbar k/m\sim 1$ cm/s, where $\hbar k$ is the momentum of a photon of the light wave. For an interaction of an atomic beam with a pump field in the perpendicular geometry, the optical orientation of the atoms in the ground state thus becomes substantial in fields with polarization gradients in which the length scales of the

variations in the polarization are of the order of the wavelength of the light. The condition (3) for a quasilinear optical orientation, with the electric susceptibility of the medium $\chi(\mathbf{r})$ being determined by the local value of the polarization of the strong field (and independent of the intensity of this field), thus holds for pump fields with saturation parameters $G_0 \sim 10^{-1}$–10^{-4} for atomic beams with a collimation $\beta \sim 10^{-3}$–10^{-6}.

In a medium which has an electric susceptibility with spatial variations over distances of the order of the wavelength of the light, Bragg diffraction becomes possible for a probe optical wave. Let us consider the problem of the Bragg reflection of a probe wave from an optically oriented atomic beam with a 1D variation, $\chi(z)$. Situations of this sort arise in pump fields formed by two counterpropagating coherent waves which differ in polarization. Here are some familiar examples of such field configurations.[10]

1. The σ^+–σ^- configuration (i.e., a linearly polarized helix with a pitch equal to the wavelength of the field):

$$\mathbf{E}_0 = E_0[\boldsymbol{\varepsilon}_+ \exp(ik_0 z) + \boldsymbol{\varepsilon}_- \exp(-ik_0 z)]. \tag{32}$$

This configuration is formed by counterpropagating fields with orthogonal circular polarizations $\boldsymbol{\varepsilon}_+$ and $\boldsymbol{\varepsilon}_-$; this z axis is parallel to the wave vector $\mathbf{k}_0$.

2. The *lin* ⊥ *lin* configuration:

$$\mathbf{E}_0 = E_0[\mathbf{e}_x \exp(ik_0 z) + \mathbf{e}_y \exp(-ik_0 z)]. \tag{33}$$

This configuration is formed by two counterpropagating waves with orthogonal linear polarizations $\mathbf{e}_x$ and $\mathbf{e}_y$.

Configurations (32) and (33) are described in detail in Ref. 10. We would like to point out the following distinguishing features of the polarization gradients of these fields. If we characterize the polarization ellipse of a field in the general case by the ellipticity $\alpha(z)$ and the ellipse rotation angle $\Psi(z)$ (Fig. 2), then the field in (32) describes a case in which there is a gradient in the rotation angle but no change in the ellipticity ($\alpha = 0$). The field in (33) is a clear example of a case in which there is a gradient in the ellipticity with no change in rotation angle.

The distribution with respect to Zeeman sublevels of the ground state is then

$$\rho^0_{mm'}(\mathbf{r},\mathbf{v}) = \rho^0_{mm'}(z) f(\mathbf{r},\mathbf{v}), \quad \sum_m \rho^0_{mm}(z) = 1. \tag{34}$$

The tensor $\rho^0_{mm'}(z)$ here characterizes the deviation from equilibrium in terms of internal degrees of freedom. It is determined exclusively by the local value of the pump polarization. The complete distribution function $f(\mathbf{r},\mathbf{v})$ determines the density of the atomic beam, $N(\mathbf{r}) = N \int f(\mathbf{r},\mathbf{v}) d\mathbf{v}$, and the atomic velocity distribution. In general, $f(\mathbf{r},\mathbf{v})$ is quite different from the initial distribution function of the beam, $f_0(\mathbf{r},\mathbf{v})$. In the interaction of atomic beams with optical fields which have polarization gradients, cooling and channeling of the atomic beam can play an important role along with an optical orientation of the atoms in the ground state.[10,11] The channeling consists of a spatial modulation of the density $N(z)$ with a modulation period on the order of the wavelength of the light. This effect must obviously be taken into account in an analysis of the Bragg scattering of a probe wave.

To analyze Bragg scattering, we use as examples the optical transitions

1. $J_0 = 1 \to J_0 + \Delta J_0 = 2$ in field (32) and
2. $J_0 = 1/2 \to J_0 + \Delta J_0 = 1/2$ in field (33).

Distributions (34) are known for these cases.[10,11]

For the first model, the quantization axis $\boldsymbol{\nu}$ which diagonalizes the tensor $\rho^0_{mm'}(z)$ in accordance with (10) is directed along the polarization of field (32):

$$\boldsymbol{\nu}(z) = \frac{1}{\sqrt{2}} [\boldsymbol{\varepsilon}_+ \exp(ik_0 z) + \boldsymbol{\varepsilon}_- \exp(-ik_0 z)]. \tag{35}$$

Since polarization (32) is linear at each point z, the atoms have no magnetic moment in their ground state: $\rho^0_{kq} = 0$ for $\kappa = 1$. The quadrupole moment is

$$\rho^0_{20}(\mathbf{r},\mathbf{v}) = -\frac{5}{17}\sqrt{\frac{2}{3}} f(\mathbf{r},\mathbf{v}). \tag{36}$$

There is no channeling in this model, and the distribution function f can be assumed uniform along z over distances on the order of the wavelength of the light.

It follows from (12) and (15) that in this case there is only one contribution to the susceptibility tensor $\chi(\mathbf{k},\mathbf{k}')$ in (4) with $\mathbf{k} \neq \mathbf{k}'$. It describes Bragg reflection and is proportional to tensor (15):

$$C_{2,ij} = -\frac{20\pi^2}{17} \Pi_2 f(\mathbf{v}) [(\boldsymbol{\varepsilon}_+)_i (\boldsymbol{\varepsilon}_+)_j \delta(\mathbf{k} - \mathbf{k}' - 2\mathbf{k}_0) + (\boldsymbol{\varepsilon}_-)_i (\boldsymbol{\varepsilon}_-)_j \delta(\mathbf{k} - \mathbf{k}' + 2\mathbf{k}_0)], \tag{37}$$

where $f(\mathbf{v}) = \int f(\mathbf{r},\mathbf{v}) d\mathbf{r}$ is the velocity distribution of the atoms in the beam.

In the second model, a spatial grating of the magnetic moment is formed in the ground state of the atoms:

$$\rho^0_{1,0}(\mathbf{r},\mathbf{v}) = \cos(2k_0 z) f(\mathbf{r},\mathbf{v}), \tag{38}$$

where the quantization axis $\boldsymbol{\nu}$ is directed along $\mathbf{k}_0$. An important feature of this model is that an atomic beam can be channeled if the atoms interact with the pump field for a sufficiently long time. When the channeling reaches a steady state, the following expression is an accurate approximation of the overall distribution function:[11]

$$\rho^0_0(\mathbf{r},\mathbf{v}) = \mathcal{N}\left\{\frac{4}{3}\frac{\tilde{\gamma}^2}{\tilde{\delta}^2} T_0^2 + \left[\frac{m v_\parallel^2}{2} + T_0 \sin^2(\mathbf{k}_0 \mathbf{r})\right]^2\right\}^{-1/2}, \tag{39}$$

$\tilde{\delta} \gg \tilde{\gamma}$; $T_0^2 = \frac{1}{4}\hbar G_0 \tilde{\delta}$.

Here $\mathcal{N}$ incorporates a normalization constant, a cutoff function in terms of the width of the atomic beam (D), and a functional dependence on the atomic velocities in directions perpendicular to $\mathbf{k}_0$, i.e., $\mathbf{v}_\perp$. In (39), the quantity $\tilde{\delta}$ is the offset of the frequency of the strong field from resonance.

Figure 3 shows the results of numerical calculations of the atomic density $N(z)$ for certain parameters of the pump field.[11]

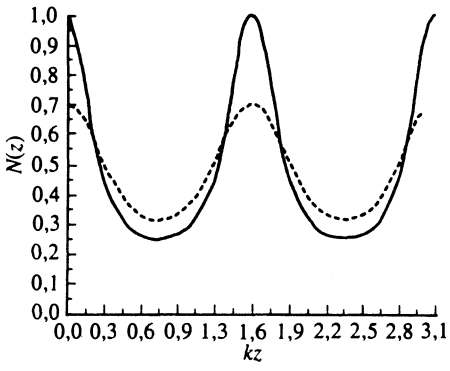

FIG. 3. Density profile $N(z)$ over distances $z=\lambda_0/2$. Solid curve—The offset of the strong field is $\tilde{\delta}=10\tilde{\gamma}$; dashed curve—$\tilde{\delta}=\sqrt{10}\tilde{\gamma}$. The vertical scale is in arbitrary units.

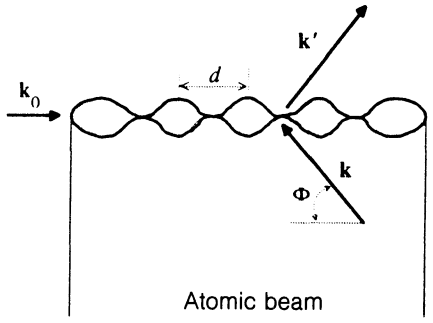

FIG. 4. Bragg scattering of a probe wave by an atomic beam. **k**—Wave vector of the incident probe wave; **k'**—wave vector of the reflected wave; $\mathbf{k}_0'$—wave vector of the strong field; d—period of the spatial grating of multipole moment ρ_κ^0 ($\kappa=0,1,2$) in the ground state of the beam atoms.

The spatial distribution of (39) along z can be written as a Fourier series in spatial harmonics with a maximum period $d=\lambda_0/4$:

$$f(\mathbf{r},\mathbf{v})=\sum_{n=0}^{\infty}\cos(4nk_0z)A_n(\mathbf{v}). \tag{40}$$

For an offset $\tilde{\delta}\approx 10\tilde{\gamma}$ of the pump field, for example, the coefficients in (40) are in the proportions A_0–A_1–A_2–A_3...=1–0.4–0.17–0.08... .

The existence of spatial harmonics as in (40) has the consequence that the diffraction of the probe field (with $\mathbf{k}\neq\mathbf{k}'$) in susceptibility tensor (12) is determined by two series, according to (13) and (14):

$$\chi_{ij}^{(1)}\sim\varepsilon_{ijm}(\mathbf{k}_0)_m\Big\{(2A_0+A_1)[\delta(2\mathbf{k}_0-\mathbf{k}+\mathbf{k}')+\delta(2\mathbf{k}_0+\mathbf{k}-\mathbf{k}')]+\sum_{n=1}^{\infty}(A_n+A_{n+1})[\delta((2+4n)\mathbf{k}_0-\mathbf{k}+\mathbf{k}')+\delta((2+4n)\mathbf{k}_0+\mathbf{k}-\mathbf{k}')]\Big\} \tag{41}$$

a) by virtue of the spatial modulation of the magnetic moment, (38), and

$$\chi_{ij}^{0}\sim\delta_{ij}\Big\{\sum_{n=1}^{\infty}A_n[\delta(4n\mathbf{k}_0-\mathbf{k}+\mathbf{k}')+\delta(4n\mathbf{k}_0+\mathbf{k}-\mathbf{k}')]\Big\} \tag{42}$$

b) by virtue of the spatial modulation of the density, (39).

In the reflected signal, however, we find only waves which satisfy the dispersion relation for a free wave in vacuum: $k'=k=\omega/c$, where ω is the frequency of the probe field. Taking the 1D nature of these spatial variations into account, we see that this result means that for a given **k** of the probe field, only a single reflected wave can arise, with **k'** directed in accordance with the condition for specular reflection (Fig. 4).

The angle of incidence Φ at which Bragg reflection of the probe wave occurs is determined by the arguments of the δ-functions in (37), (41), and (42).

For the first model, there is one possible value for this angle

$$\cos\Phi_2=\frac{k_0}{k}=\frac{\lambda}{\lambda_0},\quad \lambda\leqslant\lambda_0. \tag{43}$$

For the second model, for a given λ, there may be several of these angles:

$$\cos\Phi_{1,n}=\frac{(1+2n)k_0}{k}+\frac{(1+2n)\lambda}{\lambda_0},\quad n=0,1,2...,\quad (1+2n)\lambda\leqslant\lambda_0, \tag{44}$$

by virtue of reflection from the spatial grating of the magnetic moment, (38), and

$$\cos\Phi_{0,n}=\frac{2nk_0}{k}=\frac{2n\lambda}{\lambda_0},\quad n=1,2,3...,\quad 2n\lambda\leqslant\lambda_0, \tag{45}$$

by virtue of reflection from the spatial grating of the density, (39).

In the optical range, Bragg reflection thus occurs at frequencies $\omega\geqslant\omega_0$ only at the grating of the quadrupole moment (in the first model) and at the grating of the magnetic moment (in the second), when the wave is incident at an angle satisfying

$$\cos\Phi=\frac{\lambda}{\lambda_0}. \tag{46}$$

If the wave reflected from the density grating is to be observed, there must be transitions, $J_0+\Delta J$ with resonant frequencies $\omega\geqslant 2\omega_0$. In this case the angle of incidence, found from

$$\cos\Phi_0=\frac{2\lambda}{\lambda_0}, \tag{47}$$

is not the same as that found from (46).

This result applies directly to our second model. It stems from the small period ($d=\lambda_0/4$) of the spatial grating of the density, (40). In general, however, channeling is also possible in other field configurations, in which the period of the spatial grating of the density is $d=\lambda_0/2$. For

example, it is possible in the field of a standing optical wave.[18] In this case one can also observe Bragg reflection in the optical range.

To find the polarization of the reflected wave, we work from the condition that electromagnetic waves are transverse:

$$(\mathbf{e}(\mathbf{k}')\mathbf{k}')=0, \tag{48}$$

where $\mathbf{e}(\mathbf{k}')$ is the polarization of the probe wave.

The densities of atoms in beams are low ($N\sim 10^{10}$ cm^{-3}), and the region in which the probe field interacts with the atomic beam has dimensions $L\sim D$, where $D\sim 0.1$ cm is the diameter of the atomic beam. Since the components of the electric susceptibility tensor are small, $\chi\sim 10^{-14}$, making the parameter $kL\chi\ll 1$ small, we can content ourselves with the first order of an expansion in this parameter in solving the corresponding Maxwell's equation for the probe wave.[19] The reflected wave $\mathbf{E}(\mathbf{k}')$ is then related to the incident wave $\mathbf{E}(\mathbf{k})$ by

$$E_i(\mathbf{k}')=kLP_{ij}(\mathbf{k}')\chi_{jm}(\mathbf{k}',\mathbf{k})E_m(\mathbf{k}),$$

$$P_{ij}(\mathbf{k}')=\delta_{ij}-\frac{(\mathbf{k}')_i(\mathbf{k}')_j}{k^2}. \tag{49}$$

The operator $\hat{P}$ projects onto the plane perpendicular to the vector $\mathbf{k}'$.

According to (49), the polarization of the reflected wave is

$$\mathbf{e}(\mathbf{k}')\sim\Theta[(\mathbf{k}\mathbf{k}_0)]\{\boldsymbol{\varepsilon}_+-\mathbf{n}_{\mathbf{k}'}(\boldsymbol{\varepsilon}_+\mathbf{n}'_{\mathbf{k}})\}[\boldsymbol{\varepsilon}_+\mathbf{e}(\mathbf{k})]$$
$$+\theta[-(\mathbf{k}\mathbf{k}_0)]\{\boldsymbol{\varepsilon}_--\mathbf{n}_{\mathbf{k}'}(\boldsymbol{\varepsilon}_-\mathbf{n}_{\mathbf{k}'})\}[\boldsymbol{\varepsilon}_-\mathbf{e}(\mathbf{k})], \tag{50}$$

a) where $\theta(x)$ is the unit step function, for the first model and

$$\mathbf{e}(\mathbf{k}')\approx[\mathbf{e}(\mathbf{k})\mathbf{k}_0]-\mathbf{n}_{\mathbf{k}'}(\mathbf{n}'_{\mathbf{k}}[\mathbf{e}(\mathbf{k})\mathbf{k}_0]) \tag{51}$$

b) for the second model in the case of reflection from a grating of the magnetic moment, (38), or

$$\mathbf{e}(\mathbf{k}')=\mathbf{e}(\mathbf{k})-\mathbf{n}_{\mathbf{k}'}[\mathbf{n}_{\mathbf{k}'}\mathbf{e}(\mathbf{k})] \tag{52}$$

for reflection from a density grating, (39).

The quantity $\mathbf{e}(\mathbf{k})$ in (50)–(52) is the polarization of the incident probe wave.

The reflected wave, $E(\mathbf{k}')\sim 10^{-10}E(\mathbf{k})$, is very weak for the parameters of the atomic beam cited above. However, since there is the possibility in principle of arranging large diffractive-reflection angles in (46) and (47) for even wider and more intense atomic beams, it becomes feasible to select a reflected signal in terms of frequency, direction, and polarization against the background of thermal radiation and other noise. We would add that the waves reflected from gratings of the quadrupole moment, the density, and the magnetic moment have different polarizations [see (50), (52), and (51)], and the wave reflected from a density grating also has a reflection angle [see (47)] which is not the same as that in (46).

There is thus the additional possibility of using spectroscopic methods for diagnostics of atomic beams. For example, by comparing the intensities of waves reflected from different regions of an atomic beam, one can extract information on the dynamics of the cooling and channeling of beams. One can furthermore estimate the temperature to which the atoms are cooled in strong optical fields. In this case the temperature determines the velocity at which the atoms diffuse along the z axis ($\delta v_{\|}$) and thus characterizes the extent to which these spatial gratings of the multipole moments become smeared as atoms escape from the region in which they interact with the pump field. As a result, one can work from the extent to which the amplitude of the reflected signal falls off in a scan along the atomic beam to draw conclusions about the magnitude of $\delta v_{\|}$ and thus about the temperature $k_BT=m(\delta v_{\|})^2/2$.

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