Hydrodynamic theory of plasma ion heating by the beats of oppositely directed electromagnetic waves

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The mechanism for plasma ion heating by electromagnetic radiation is considered. Two oppositely directed electromagnetic waves in the plasma can create a beat wave, which gives rise to oscillatory perturbations in the density and velocity profiles. When a plasma element undergoes compression and expansion over a large range, viscous heating of the ions occurs. In the absence of electron–ion collisions the electrons oscillate without acquiring any energy, so that the energy of the electromagnetic field is converted directly to thermal energy of the ions. An explanation of this heating mechanism is presented, based on the laws of momentum and energy conservation.

1. FORMULATION OF THE PROBLEM

Demchenko *et al.*[1] have shown that it is possible to achieve anomalously high ion heating in an expanding laser plasma under conditions such that the expansion takes place in an oscillatory high-frequency potential (an hf potential), which is produced when the laser radiation is reflected from the critical surface. When plasma moves in an oscillatory hf potential a Lagrangian plasma element is subject to repeated compression and expansion, as the result of which the viscosity of the ions causes them to be heated. Plasma flow through the hf potential "lattice" can also occur in a plasma at rest, producing a beat wave in it.

We assume that two electromagnetic waves with frequencies ω_0 and ω_1, respectively, are incident from the left and from the right, respectively, on a uniform plasma slab (Fig. 1) confined in the region $x_0 \leqslant x \leqslant x_1$, and that the frequencies differ by a small amount: $\omega_1 = \omega_0 + \Delta\omega$, $\omega_1 > \omega_0$, $\Delta\omega/\omega_0 \ll 1$. In this work we will not consider Fresnel reflection of waves from the plasma boundaries, assuming that the boundaries are smeared out and that the characteristic width of the spreading is much greater than the wavelength. In this case the waves penetrate into the plasma without reflection. In addition we assume that the width of the spreading in the boundary is much less than the slab thickness and that the waves interact with the plasma primarily in the uniform region. Neglecting the ponderomotive force, we can write the solution for the fields $\mathbf{E} = (0, E_y, 0)$ and $\mathbf{H} = (0, 0, H_z)$ in the slab in the form (in what follows we drop the coordinate subscripts from the fields):

$$E = E_0 \exp[-i(\omega_0 t - k_0 x)] + E_1 \exp[-i(\omega_1 t + k_1 x)], \quad (1)$$

$$H = H_0 \exp[-i(\omega_0 t - k_0 x)] + H_1 \exp[-i(\omega_1 t + k_1 x)], \quad (2)$$

where we have written $H_0 = \beta_0 E_0$, $H_1 = -\beta_1 E_1$, $k_{0,1} = \omega_{0,1}\beta_{0,1}/c$, $\beta_{0,1} = \sqrt{\varepsilon_{0.1}}$, $\varepsilon = 1 - \omega_p^2/\omega^2$, and $\omega_p = (4\pi e^2 n_e/m_e)^{1/2}$ is the plasma frequency. From (1) we find that the square of the field amplitude $|E|^2$ is a beat wave. Let v be its phase velocity, so that the maxima of the function $|E|^2$ move with velocity v relative to the plasma. Then the problem can conveniently be treated in the K' coordinate frame moving with velocity v relative to the laboratory frame, so that both waves have the same frequency in this system. Making a Lorentz transformation $x = \alpha(x' + vt')$, $t = \alpha(t' + vx'/c^2)$, in Eqs. (1)–(2) where $\alpha = (1 - v^2/c^2)^{-1/2}$, we obtain expressions for the frequencies and wave numbers in the moving system of coordinates:

$$\omega_0' = \alpha\omega_0\left(1 - \frac{v}{c}\beta_0\right), \quad k_0' = \alpha\frac{\omega_0}{c}\beta_0\left(1 - \frac{v}{c\beta_0}\right), \quad (3)$$

$$\omega_1' = \alpha\omega_1\left(1 + \frac{v}{c}\beta_1\right), \quad k_1' = \alpha\frac{\omega_1}{c}\beta_1\left(1 + \frac{v}{c\beta_1}\right). \quad (4)$$

Then we neglect the terms that are quadratic in v/c, since $v/c \ll 1$. From the condition $\omega_0' = \omega_1'$ we find for the velocity v

$$v = c\frac{\omega_0 - \omega_1}{\omega_0\beta_0 + \omega_1\beta_1} \approx -c\frac{\Delta\omega}{2\omega_0\beta_0}. \quad (5)$$

Since $\omega_1 > \omega_0$ holds, the K' system moves from right to left. Let us show that in the K' system the wave numbers are also equal (to within terms $\sim v^2/c^2$). Defining the dielectric constants in the K' system by means of the relations

$$k_0' = \frac{\omega_0'}{c}\sqrt{\varepsilon_0'}, \quad k_1' = \frac{\omega_1'}{c}\sqrt{\varepsilon_1'}, \quad (6)$$

using Eqs. (3)–(4), and retaining only terms $\sim v/c$ we find

$$\varepsilon_0' = \varepsilon_0 - 2\frac{v}{c}\beta_0(1 - \varepsilon_0), \quad \varepsilon_1' = \varepsilon_1 + 2\frac{v}{c}\beta_1(1 - \varepsilon_1). \quad (7)$$

From expansion $\varepsilon_1 \approx \varepsilon_0 + (d\varepsilon/d\omega)_0(\omega_1 - \omega_0)$ and expression (5) for v we find to within terms of order $\sim v^2/c^2$ that $\varepsilon_0' = \varepsilon_1'$ holds. Consequently, it follows from (6) that when the frequencies are equal so are the wave numbers. Thus in the K' system the fields have only one frequency $\omega_0' = \omega_1' = \omega$, the plasma has a dielectric constant $\varepsilon_0' = \varepsilon_1' = \varepsilon$, and it moves with velocity $u_0 = -v$ from left to right along the x' axis.

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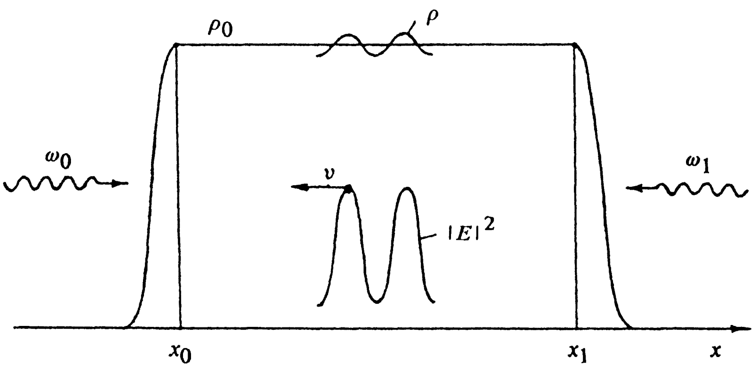

FIG. 1.

Now let us treat the problem including the hf potential. We use the equations of plasma hydrodynamics with a high-frequency field.[2] The solution of the equations given by Gorbunov[2] remains unchanged even when ion viscosity is included, since in the high-frequency motion only electrons take part. According to the hydrodynamic model the process of nonlinear interaction between the radiation and the plasma occurs as follows. The ponderomotive force (the low component of the Lorentz force) acts on the electrons, which are collisionless ($\nu_{ei} \ll \omega$) in the high-frequency motion. When plasma quasineutrality holds the ions are coupled to the electrons by the self-consistent field, so that under the action of the ponderomotive force the plasma density and velocity profiles acquire components that are oscillatory in x'. For an oscillatory density profile the interaction of the waves with the plasma gives rise to a slow x' and t' dependence of the field amplitudes. If the wavelength λ is much less than the characteristic spatial scale on which the field amplitudes vary, then we can assume that the plasma flow through the oscillating hf potential is locally steady, i.e., $\rho u = \rho_0 u_0$, where $\rho = m_i n_i$ is the plasma density, u is its velocity, and ρ and u have oscillatory components with slowly varying amplitudes. We will treat the case in which the energy exchange time $\tau_{ei}^{\varepsilon} \approx m_i/m_e \nu_{ei}$ between electrons and ions is much larger than the time over which the waves act on the plasma.[3] Let the plasma be quasineutral (the Debye radius satisfies $r_D \ll \lambda$) and suppose that the thermal pressure $p_T = n_e T_e + n_i T_i = \rho c_s^2$ (where c_s is the sound speed) is primarily due to electrons, which is valid for $Z T_e \gg T_i$ (where Z is the ion charge). We will also neglect collisional absorption and will assume $T_e = \text{const}$. Then in the K' coordinate frame the process by which waves interact with the plasma is described by the equations

$$\rho u = \rho_0 u_0, \tag{8}$$

$$\frac{\partial}{\partial x'}\left(\rho u^2 + \rho c_s^2 + p_r - \mu \frac{\partial u}{\partial x'}\right) = 0, \tag{9}$$

$$\partial E'/\partial x' - ikH' = 0, \tag{10}$$

$$\partial H'/\partial x' - ik\varepsilon E' = 0, \tag{11}$$

$$p_r = (|E'|^2 + |H'|^2)/16\pi, \tag{12}$$

where μ is the ion viscosity (we assume $\mu = \text{const}$, $c_s = \text{const}$); $k = \omega/c$; p_r is the electromagnetic stress tensor averaged over an oscillation period,[4] taken with the minus sign. It also includes the ponderomotive stress tensor. The quantity p_r can be called the radiation pressure. From Eqs. (10)–(11) we obtain the relation

$$-\frac{\partial p_r}{\partial x'} = (\varepsilon - 1)\frac{\partial}{\partial x'}\left(\frac{|E'|^2}{16\pi}\right), \tag{13}$$

which expresses the relationship between p_r and the ponderomotive force in the plasma.[5]

For the hydrodynamic approximation to be valid for the ion component it is necessary that the conditions $l_i < L_h$ and $\tau_{ii} < t_h$ hold, where $l_i = v_{Ti}\tau_{ii}$ and $\tau_{ii} = 3\sqrt{m_i}T_i^{3/2}/4\sqrt{\pi}Z^4 e^4 \Lambda_{ii} n_i$ are the ion collision length and collision time,[6] $v_{Ti} = \sqrt{T_i/m_i}$ is the ion thermal velocity, L_h and t_h are the characteristic length and time on which the hydrodynamic quantities vary (the time is treated in the Lagrangian approach). Assuming that the perturbation wavelength in the plasma is on the order of the radiation wavelength, we can write these conditions in the form $kl_i < 1$ and $kl_i < v_{Ti}/u_0$. Thus, we are considering the case in which the ion perturbations undergo collisional dissipation. When $kl_i < 1$ holds we can use the expression $\mu = \rho_0 v_{Ti} l_i$ for the ion viscosity coefficient.[6]

We make a comment here in connection with the use of the Maxwell equations in the form (10)–(11), where the derivatives of the amplitudes with respect to time are omitted and only a high-frequency dependence in the form of $\exp(-i\omega t)$ is assumed. The slow time dependence in the K' system is related to the motion of the plasma boundaries on which the boundary conditions for the wave amplitudes are imposed. Since both boundaries move with velocity u_0, this amplitude dependence on x' and t' must have the form of a propagating wave, i.e., x' and t' must enter in the form of the combination $x = x' - u_0 t'$. Differentiation of this dependence with respect to time yields the correction terms in Eqs. (10)–(11), which are small (of order u_0/c) compared to the main terms.

2. SOLUTION OF THE EQUATIONS

We will look for a solution for the fields in the form[7,8]

$$E' = Pe^{i\varphi} + Re^{-i\varphi}, \tag{14}$$

$$H' = \beta(Pe^{i\varphi} - Re^{-i\varphi}), \tag{15}$$

where $\beta = \sqrt{\varepsilon}$, $P=P(x)$, $R=R(x)$, $x=x'-u_0t'$, and the phase φ is given in the form

$$\varphi = \varphi(x') = k\int_0^{x'} \beta dx \approx k\beta_0 x'. \tag{16}$$

From Eqs. (10)–(11) we find equations for the amplitudes:

$$\frac{dP}{dx} = \frac{1}{4\varepsilon}\frac{d\varepsilon}{dx}(Re^{-2i\varphi} - P), \tag{17}$$

$$\frac{dR}{dx} = \frac{1}{4\varepsilon}\frac{d\varepsilon}{dx}(Pe^{2i\varphi} - R). \tag{18}$$

Writing the amplitudes in the form $P=|P|\exp(i\alpha)$, $R=|R|e^{i\beta}$ and expressing the density in the form of a sum $\rho=\rho_0+\rho_1$, we find for the correction ρ_1 from Eqs. (8)–(9)

$$\rho_1 = a(x)\cos(2\varphi+\alpha-\beta+\delta), \tag{19}$$

where $a(x)$ is a slowly varying amplitude:

$$a(x) = \rho_0 \frac{|P(x)|\,|R(x)|}{8\pi\rho_c c_s^2} \times\left[(M^2-1)^2 + \left(\frac{2\mu k\beta_0 M}{\rho_0 c_s}\right)^2\right]^{-1/2}, \tag{20}$$

and δ is a phase shift introduced by the viscosity:

$$\operatorname{tg}\delta = \frac{2\mu k\beta_0 M}{\rho_0 c_s(M^2-1)}, \tag{21}$$

$M=u_0/c_s$ is the Mach number, and ρ_c is the critical density $(\varepsilon = 1-\rho/\rho_c)$. From Eqs. (17)–(18) we can find equations for $|P|^2$ and $|R|^2$:

$$\frac{d|P|^2}{dx} = \frac{1}{2\varepsilon}\frac{d\varepsilon}{dx}[|P|\,|R|\cos(2\varphi+\alpha-\beta) - |P|^2], \tag{22}$$

$$\frac{d|R|^2}{dx} = \frac{1}{2\varepsilon}\frac{d\varepsilon}{dx}[|P|\,|R|\cos(2\varphi+\alpha-\beta) - |R|^2]. \tag{23}$$

Substituting the solution (19) in the right-hand sides of (22)–(23) and averaging the latter spatially over a wavelength, and also rewriting in terms of the fluxes

$$q_0 = \frac{c}{8\pi}\beta_0\overline{|P|^2}, \tag{24}$$

$$q_1 = \frac{c}{8\pi}\beta_0\overline{|R|^2}, \tag{25}$$

where the bar indicates an average with respect to x, we find

$$dq_0/dx = Bq_0q_1, \tag{26}$$

$$dq_1/dx = Bq_0q_1, \tag{27}$$

$$B = \frac{\mu u_0 k^2 G}{c\beta_0\rho_c^2 c_s^4}, \quad G = \left[(M^2-1)^2 + \left(\frac{2\mu k\beta_0 M}{\rho_0 c_s}\right)^2\right]^{-1}. \tag{28}$$

The dimensionless factor G has a resonant dependence on the Mach number. Equations (26)–(27) were treated by Gorbunov[9] for the case of steady expansion of a nonuniform plasma. They describe attenuation of a wave moving from the right due to backscattering on the oscillating density profile, accompanied by amplification of the oppositely directed wave.

As an example we consider the special case of the system (26)–(27) with $q_0(x)\equiv q_1(x)$. In this case the field amplitude at the minima vanishes. The solution takes the form

$$q_0(x) \equiv q_1(x) = \frac{q_{11}}{1+q_{11}B(x_1-x)}, \tag{29}$$

where $q_{11}=q_1(x_1)$. In order to obtain this case it is necessary that the fluxes $q_{00}=q_0(x_0)$ and $q_{11}=q_1(x_1)$ incident from the left and the right satisfy a relation which follows from (29): $q_{11}/q_{00} = 1+q_{11}B(x_1-x_0)$.

Consider the energy equation for the ions including viscosity, neglecting thermal conductivity and energy exchange with the electrons. In Lagrangian form the equation can be written as

$$\frac{\partial\varepsilon_i}{\partial t'} = -p_{Ti}\frac{\partial V}{\partial t'} + \frac{\mu}{V}\left(\frac{\partial V}{\partial t'}\right)^2, \tag{30}$$

where $\varepsilon_1 = 3T_i/2m_i$ is the specific internal energy of the ions, m_i and T_i are the ion mass and temperature, $V=1/\rho$ is the specific volume, and $p_{Ti}=T_i/m_iV$ is the thermal pressure. For steady motion with oscillatory density and velocity profiles the specific volume of this Lagrangian particle oscillates in time. Averaging (30) over time we obtain

$$\frac{\partial\bar{\varepsilon}_i}{\partial t'} = \overline{\frac{\mu}{V}\left(\frac{\partial V}{\partial t'}\right)^2} \approx \frac{1}{2}\frac{\mu}{V_0}\left(\frac{\partial V}{\partial t'}\right)^2_{\max}, \tag{31}$$

where the bar indicates a time average and $(\partial V/\partial t')_{\max}$ is the amplitude of the derivative $\partial V/\partial t'$. Using the equation for the volume,

$$\frac{\partial V}{\partial t'} = \frac{1}{\rho}\frac{\partial u}{\partial x'}, \tag{32}$$

and also Eqs. (8) and (19) we find for the viscous heating rate of the ions

$$Q \equiv \frac{\partial\bar{\varepsilon}_i}{\partial t'} = \frac{2\mu k^2 u_0^2 G}{\rho_0 c^2\rho_c^2 c_s^4} q_0 q_1. \tag{33}$$

This heating can be regarded as work done by the effective frictional force arising between antinodes of the electromagnetic field and the plasma when the relative velocity between them is u_0. The frictional force per ion F_i can be expressed by means of the relation

$$F_i u_0 = m_i Q\ . \tag{34}$$

The occurrence of ion heating and an effective frictional force imply that the transmission of momentum and energy of plasma radiation must be treated.

3. MOMENTUM AND ENERGY CONSERVATION LAWS

Let us consider the momentum conservation law. Using the definition (34) for the frictional force we can find the force density

$$f=n_iF_i=\rho_0Q/u_0 \tag{35}$$

and the total force p_f acting on the plasma slab (per unit area)

$$p_f=\int_{x_0}^{x_1} f dx=\frac{2\mu k^2u_0G}{c^2\rho_c^2c_s^4}\int_{x_0}^{x_1} q_0q_1dx. \tag{36}$$

For the integral which appears in Eq. (36) we find using Eqs. (26)–(27)

$$\int_{x_0}^{x_1} q_0q_1dx=\frac{1}{2B}(q_0+q_1)\Big|_{x_0}^{x_1}. \tag{37}$$

Consider the Maxwell stress tensor averaged over an oscillation period (we omit the minus sign):

$$\overline{T'_M}=\frac{1}{16\pi}(\varepsilon|E'|^2+|H'|^2). \tag{38}$$

This is just the part of the total tensor which corresponds to the momentum transmitted by radiation to the plasma slab. The ponderomotive part gives the redistribution of momentum over the slab.[4] Using (14)–(15) and (24)–(25) we find

$$\overline{T'_M}=\frac{\beta_0}{c}(q_0+q_1). \tag{39}$$

From (36) we find using (37) and (39)

$$p_f=\overline{T'_M}(x_1)-\overline{T'_M}(x_0). \tag{40}$$

The relation (40) implies that the momentum acquired by the plasma slab is equal to the electromagnetic momentum lost due to the interaction between the radiation and the slab. Thus the approximations used in solving Eqs. (8)–(11) and Eq. (30) enable us to find solutions which satisfy the momentum conservation law.

Let us go on to the energy conservation law. The power expended in heating the ions (per unit area) is

$$q_T=\int_{x_0}^{x_1}\rho_0Qdx=\frac{2\mu k^2u_0^2G}{c^2\rho_c^2c_s^4}\int_{x_0}^{x_1} q_0q_1dx. \tag{41}$$

Taking into account (37), (39), and (40) we find

$$q_T=u_0p_f. \tag{42}$$

In the K' coordinate frame the plasma slab loses kinetic energy at a rate u_0p_f due to the force (40), and from (42) the loss of kinetic energy is equal to the increase in the thermal energy.

It is interesting to trace the passage of energy to the slab in the laboratory coordinate frame, since in this system the plasma is at rest and its kinetic energy vanishes. Using the Lorentz transformation for the fields we can find a transformation equation for the energy flux density S when we pass from the moving coordinate frame to the laboratory frame:[4,10]

$$S=S'-u_0U'_M-u_0T'_M, \tag{43}$$

where $U'_M=[\varepsilon(E')^2+(H')^2]/8\pi$ is the electromagnetic energy density and T'_M is the Maxwell stress tensor, taken with a minus sign (in this case we have $T'_M=U'_M$). To within terms $\sim(u_0/c)^2$ we can assume that $U_M=U'_M$ and $T_M=T'_M$ hold. Using Eq. (43) we find the power dissipated in the slab:

$$\bar S_0-\bar S_1=\overline{S'_0}-\overline{S'_1}-u_0(\overline{U'_{M0}}-\overline{U'_{M1}})-u_0(\overline{T'_{M0}}-\overline{T'_{M1}}), \tag{44}$$

where the bar indicates a time average and we have written $S_0=S(x_0)$, $S_1=S(x_1)$ and similarly with the other indices. In order to determine the difference $\bar S'_0-\bar S'_1$ we use the Poynting theorem in the K' system:

$$-\frac{\partial S'}{\partial x'}=\frac{\partial U'_M}{\partial t'}. \tag{45}$$

Integrating (45) with respect to x' from $x'_0(t)$ to $x'_1(t)$ and assuming $dx'_0/dt'=dx'_1/dt'=u_0$ we find

$$S'\Big|_{x'_1}^{x'_0}=\frac{\partial}{\partial t'}\int_{x'_0}^{x'_1}U'_Mdx'+u_0(U'_{M0}-U'_{M1}). \tag{46}$$

Averaging (46) over time and substituting the result in (44) we find

$$\bar S_0-\bar S_1=u_0(\overline{T'_{M1}}-\overline{T'_{M0}})=q_T. \tag{47}$$

Thus, in the laboratory coordinate system ion heating results from the action of the ponderomotive force.

We comment regarding the boundary conditions. We have assumed that in the plasma–vacuum transition region the zeroth-order approximation of geometric optics holds, where the width of the region is small and the effects in the next approximation are negligible. In this case we can introduce a local moving coordinate frame whose velocity according to (5) is $v=v(x)\propto 1/\beta_0(x)$. In analogy with (43) we have

$$\bar S=\bar S'+v(x)\overline{U'_M(x)}+v(x)\overline{T'_M(x)}, \tag{48}$$

where the bar indicates a time average. In (48) $\bar S'$ is independence of x, since $\bar S'=c(\overline{E'H'^*+E'^*H'})/16\pi$, $E'\propto\beta_0^{-1/2}$, $H'\propto\beta_0^{1/2}$. Hence we have $\overline{U'_M}$ and $\overline{T'_M}\propto\beta_0(x)$, while $v\propto 1/\beta_0(x)$, Eq. (48) implies that the energy flux is constant in the transition region. If we replace the transition region by a surface we can use a boundary condition for Eqs. (26)–(27) on this surface in the form of the continuity of the fluxes q_0 and q_1.

To conclude this section we consider the limits of applicability of the solutions. The linearization of Eqs. (8)–(11) imposes a restriction on the magnitude of the perturbations $\rho_1=\rho-\rho_0$: $\rho_1/\rho_0\ll 1$, or taking into account (19),

$$\frac{|P||R|G^{1/2}}{8\pi\rho_cc_s^2}\ll 1. \tag{49}$$

It is particularly necessary to consider the resonant case $u_0=c_s$, in which the solution can deviate from linearity. Eliminating the velocity from Eqs. (8)–(9) and retaining the term quadratic in ρ_1 in the expansion we find an equation which yields the relation between p_1 and the radiation pressure:

$$(c_s^2-u_0^2)\rho_1+\frac{u_0^2}{\rho_0}\rho_1^2+\mu\frac{u_0}{\rho_0}\frac{d\rho_1}{dx'}+p_r=\text{const.} \qquad (50)$$

The first term in (50) vanishes at resonance. The relation between ρ_1 and ρ_r remains linear if we can ignore the quadratic term. This yields the condition for the applicability of the solution in the resonant case:

$$\frac{|P||R|}{32\pi\rho_0 c_s^2}\ll\frac{\rho_c}{\rho_0}\left(\frac{\mu k\beta_0}{\rho_0 c_s}\right)^2. \qquad (51)$$

For the case of classical viscosity $\mu=\rho_0 v_{Ti}l_i$, Eq. (51) can be written in the form

$$\frac{|P||R|}{32\pi\rho_0 c_s^2}\ll\frac{\rho_c}{\rho_0}(k\beta_0 l_i)^2\frac{T_i}{ZT_e+T_i}. \qquad (52)$$

The left-hand side of (52) characterizes the ratio of the change in the quantity $|E|^2$ in the beat wave to the thermal pressure. From (52) it follows that the limitation on this ratio may not be very severe. The right-hand side of (52) contains three dimensional factors, of which the ratio ρ_c/ρ_0 is greater than unity, the product $k\beta_0 l_i$ may be less than unity from the condition for the applicability of the classical viscosity model, and the expression containing the temperatures is also less than unity.

4. CONCLUSION

The simplest method for heating a plasma by electromagnetic radiation is based on absorption of the field energy by electrons and conversion into electron thermal energy, with the ions being heated by energy transfer from the electrons in Coulomb collisions. Because of the large difference between the electron and ion masses energy transfer from electrons to ions is inefficient. The heating mechanism considered above is an example of a direct transfer of electromagnetic energy to ion thermal energy. Here the electrons oscillate and do not acquire energy. In actuality, there always exists some collisional heating of electrons, but this heating rate falls off with increasing electron temperature and decreasing plasma density and may be negligibly small. On the other hand, as the ion temperature rises the rate of direct ion heating increases. For example, when the classical collision model holds the ion viscosity is given by $\mu\propto T_i^{5/2}$, and according to (33) the heating rate is proportional to μ. Thus, in the heating process it may be possible to reach the regime of direct ion heating with collisionless ($\nu_{ei}=0$) electrons.

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