Bragg reflection of sound in a periodic structure of piezoelectric-crystal layers with superconducting or metallized interlayers

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The reflection of a transverse elastic wave from a periodic structure of piezomagnetic layers with thin superconducting interlayers and also from a periodic structure of piezoelectric layers with metallized boundaries is analyzed. The conditions for Bragg resonance are analyzed. In this case, the reflection coefficient approaches unity in spectral intervals which are narrow to the extent that the magneto- or electromechanical coupling parameter q^2 is small. The resonant enhancement of reflection is seen particularly vividly in the case of a periodic structure formed by layers of the same material, since in this case the reflection coefficient far from the resonant region of frequencies (or of angles of incidence) is on the order of q^2. In the case of a piezomagnetic layered structure, the reflection essentially vanishes when the interlayer coatings undergo a transition to the *n* state. In other words, the reflection coefficient can jump abruptly from zero to values close to unity when the phase state of the superconducting interlayers changes. The latter change could be caused by a temperature change or an electric current, for example.

INTRODUCTION

As elastic waves propagate through a piezomagnetic (or piezoelectric) medium, quasistatic magnetic (or electric) wave fields, moving at the velocity of sound, are excited.[1] These waves are correspondingly sensitive to the magnetic (or electric) properties of the boundary of the medium, so some distinctive acoustic effects may arise. Even more interesting are coherent effects which occur when magnetoacoustic (or electroacoustic) waves propagate in layered media with special coatings—superconducting or metallic—at the interfaces.

One consequence of the discovery of the high T_c superconductors has been an increase in interest in the diagnostics of superconductors and in applied aspects of phenomena related to superconductivity. For example, various acoustic effects which arise at the interface between a piezomagnet and a superconductor were examined theoretically in Refs. 2 and 3. The basic idea is that when the superconductor goes into the *s* state the boundary conditions for an elastic wave propagating in the piezomagnet change. Specifically, because of the Meissner effect, the normal component of the magnetic induction, B_n, accompanying the elastic wave must vanish at the interface. As a result, the parameters of the elastic wave are modified in a certain way.

This idea was examined in Refs. 3 for the situation in which a horizontally polarized transverse elastic wave is incident on a thin interlayer of a superconducting material in a piezomagnetic crystal. The thickness of the interlayer was much smaller than the wavelength but much larger than the London penetration depth.[2] If the interlayer is in the normal state, then in a first approximation the incident wave "does not notice" the interface; i.e., no reflection occurs. When the interlayer goes into the superconducting state, the boundary condition $B_n = 0$ gives rise to a reflected wave. The reflection from the interlayer is thus a consequence of the phase state of this interlayer; this state can be controlled by varying the temperature, an electric current, etc.

Unfortunately, the effects discussed in Refs. 2 and 3 are generally weak, since they are proportional to the magnetomechanical coupling parameter, which is usually small. This comment applies in particular to reflection from a superconducting interlayer in a piezomagnet. In the present paper we discuss one possibility for intensifying this effect: using the Bragg resonance in the reflection of a transverse elastic wave from a periodic structure of piezomagnetic layers with superconducting coatings. For generality, we consider different situations: (a) The periodic structure is formed by alternating layers of two different hexagonal piezomagnetic materials. (b) The periodic structure is formed by superconducting interlayers in a hexagonal piezomagnet, so that the layers of the two types have identical material constants and differ only in thickness (in a particular case, the layers may also be identical in terms of thickness). In case (b), which is one of particular interest, the reflection coefficient may jump abruptly from zero to a value close to unity under Bragg-resonance conditions when the interlayers go into the *s* state.

In this paper we also solve another problem: that of the reflection of a transverse elastic wave from a periodic structure of piezoelectric layers of hexagonal symmetry with metallized boundaries. A similar problem was taken up in Ref. 4, but the results of Ref. 4 imply that when the piezoelectric layers are made of the same material the Bloch wave number is identically equal to the longitudinal component of the wave vector of the elastic wave. This means that either the forbidden Bloch modes have zero width or there is no reflection at all from metallized boundaries of identical layers. We regard both of these conclusions as dubious. Indeed, the analysis of this problem below leads to results different from those of Ref. 4.

STATEMENT OF THE PROBLEM

We consider a periodic structure formed by repeating cells, each consisting of two different hexagonal piezomagnetic layers of thicknesses d_1 and d_2 with mutually parallel *6* axes, which lie in the plane of the interface, which is the *xz* plane ($z \| 6$; Fig. 1). The layers have thin superconducting coatings in the *s* state. In the quasistatic approximation, the propagation of sound in a piezomagnetic medium is described by the standard equations

$$\rho \ddot{\mathbf{u}} = \nabla \hat{\sigma}, \qquad \nabla \mathbf{B} = 0,$$

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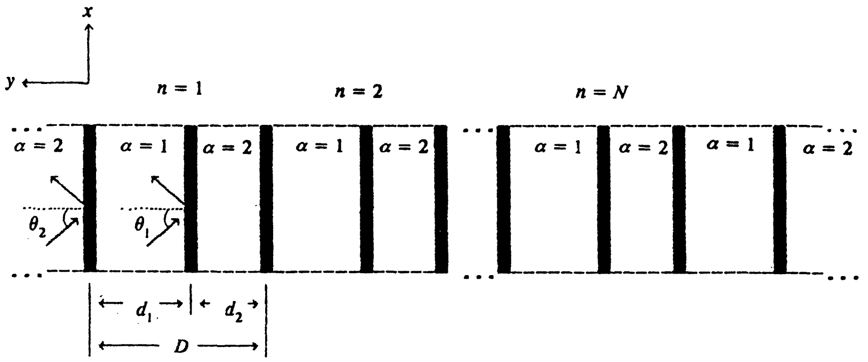


FIG. 1. Geometry of the problem.

where

$$\sigma_{ij} = c_{ijkl}\nabla_k u_l + m_{kij}\nabla_k F, \qquad B_i = m_{ikl}\nabla_k u_l - \mu_{ij}\nabla_j F,$$

ρ is the density, $\mathbf{u}$ is the elastic displacement, $\hat{\sigma}$ is the stress, $\hat{c}$ is the elastic constants, $\hat{m}$ are the piezomagnetic moduli, F is the magnetic potential, and $\hat{\mu}$ is the magnetic permeability. We assume that a transverse wave is excited in the first layer and propagates in the xy plane. It is polarized along the z axis. The total wave field in each layer consists of the incident and reflected transverse elastic waves, which are accompanied by a quasistatic magnetic field, and two nonuniform magnetic-field modes, which cause corresponding elastic forces via the piezomagnetic effect (there are no nonuniform elastic displacements because of the symmetry; see Ref. 2, for example). Omitting a common factor of $\exp[i(k_x x - \omega t)]$, we write the parameters of the wave field for the αth layer ($\alpha = 1,2$) of the nth cell, the displacement $u_z^{(n,\alpha)}(y)$, the elastic force $f_z^{(n,\alpha)} = \sigma_{zy}^{(n,\alpha)}$, and the normal component of the magnetic induction, $B_y^{(n,\alpha)}$:

$$\begin{pmatrix} u_z(y) \\ ik_x^{-1} f_z(y) \\ ik_x^{-1} B_y(y) \end{pmatrix}^{(n,\alpha)} = b_i^{(n,\alpha)} \begin{pmatrix} 1 \\ \varkappa_i^{(\alpha)} \\ m_{14}^{(\alpha)} \end{pmatrix} e^{-ik_{\alpha y}(y+nD)} + b_r^{(n,\alpha)} \begin{pmatrix} 1 \\ \varkappa_r^{(\alpha)} \\ m_{14}^{(\alpha)} \end{pmatrix} e^{ik_{\alpha y}(y+nD)} + b_s^{(n,\alpha)} \begin{pmatrix} 0 \\ \gamma_\alpha \\ i\mu_\alpha \end{pmatrix} e^{-k_x(y+nD)} + b_{s'}^{(n,\alpha)} \begin{pmatrix} 0 \\ \gamma_\alpha^* \\ -i\mu_\alpha \end{pmatrix} e^{k_x(y+nD)}. \qquad (1)$$

A subscript i specifies the incident wave, r the reflected wave, and s and s' the nonuniform modes. The asterisk (*) means complex conjugation;

$$D = d_1 + d_2, \qquad k_x = \omega(\bar{c}_{44}^{(\alpha)}/\rho_\alpha)^{-1/2}\sin\theta_\alpha,$$

$$k_{\alpha y} = k_x \operatorname{ctg}\theta_\alpha, \qquad \gamma_\alpha = -m_{14}^{(\alpha)} - i m_{15}^{(\alpha)},$$

$$\varkappa_i^{(\alpha)} = \bar{c}_{44}^{(\alpha)} \operatorname{ctg}\theta_\alpha - \frac{m_{14}^{(\alpha)} m_{15}^{(\alpha)}}{\mu_\alpha},$$

$$\varkappa_r^{(\alpha)} = -\bar{c}_{44}^{(\alpha)} \operatorname{ctg}\theta_\alpha - \frac{m_{14}^{(\alpha)} m_{15}^{(\alpha)}}{\mu_\alpha}, \qquad (2)$$

$$\bar{c}_{44}^{(\alpha)} = c_{44}^{(\alpha)} + (m_{15}^{(\alpha)})^2/\mu_\alpha, \qquad \alpha = 1, 2.$$

The wave fields in the layers $(n,1)$, $(n,2)$, and $(n+1,1)$ are related by the boundary conditions

$$\begin{pmatrix} u_z(y) \\ f_z(y) \\ B_y(y) \\ 0 \end{pmatrix}^{(n,1)} = \begin{pmatrix} u_z(y) \\ f_z(y) \\ 0 \\ B_y(y) \end{pmatrix}^{(n,2)} \quad \text{at} \quad y = -(n-1)D - d_1, \qquad (3)$$

$$\begin{pmatrix} u_z(y) \\ f_z(y) \\ 0 \\ B_y(y) \end{pmatrix}^{(n,2)} = \begin{pmatrix} u_z(y) \\ f_z(y) \\ B_y(y) \\ 0 \end{pmatrix}^{(n+1,1)} \quad \text{at} \quad y = -nD. \qquad (4)$$

Substituting (1) into (3) and (4), we find

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ \varkappa_i^{(\alpha)} & \varkappa_r^{(\alpha)} & \gamma_\alpha & \gamma_\alpha^* \\ m_{14}^{(\alpha)} & m_{14}^{(\alpha)} & i\mu_\alpha & -i\mu_\alpha \\ 0 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} e^{-ik_{\alpha y}d_2} & 0 & 0 & 0 \\ 0 & e^{ik_{\alpha y}d_2} & 0 & 0 \\ 0 & 0 & e^{-k_x d_2} & 0 \\ 0 & 0 & 0 & e^{k_x d_2} \end{pmatrix} \begin{pmatrix} b_i \\ b_r \\ b_s \\ b_{s'} \end{pmatrix}^{(n,1)}$$

$$= \begin{pmatrix} 1 & 1 & 0 & 0 \\ \varkappa_i^{(\alpha)} & \varkappa_r^{(\alpha)} & \gamma_\alpha & \gamma_\alpha^* \\ 0 & 0 & 0 & 0 \\ m_{14}^{(\alpha)} & m_{14}^{(\alpha)} & i\mu_\alpha & -i\mu_\alpha \end{pmatrix} \begin{pmatrix} e^{-ik_{\alpha y}d_2} & 0 & 0 & 0 \\ 0 & e^{ik_{\alpha y}d_2} & 0 & 0 \\ 0 & 0 & e^{-k_x d_2} & 0 \\ 0 & 0 & 0 & e^{k_x d_2} \end{pmatrix}$$

$$\times \begin{pmatrix} b_i \\ b_r \\ b_s \\ b_{s'} \end{pmatrix}^{(n,2)}, \qquad (5)$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ \varkappa_i^{(\alpha)} & \varkappa_r^{(\alpha)} & \gamma_\alpha & \gamma_\alpha^* \\ 0 & 0 & 0 & 0 \\ m_{14}^{(\alpha)} & m_{14}^{(\alpha)} & i\mu_\alpha & -i\mu_\alpha \end{pmatrix} \begin{pmatrix} b_i \\ b_r \\ b_s \\ b_{s'} \end{pmatrix}^{(n,2)}$$

$$= \begin{pmatrix} 1 & 1 & 0 & 0 \\ \varkappa_i^{(\alpha)} & \varkappa_r^{(\alpha)} & \gamma_\alpha & \gamma_\alpha^* \\ m_{14}^{(\alpha)} & m_{14}^{(\alpha)} & i\mu_\alpha & -i\mu_\alpha \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \begin{pmatrix} e^{-ik_{\alpha y}D} & 0 & 0 & 0 \\ 0 & e^{ik_{\alpha y}D} & 0 & 0 \\ 0 & 0 & e^{-k_xD} & 0 \\ 0 & 0 & 0 & e^{k_xD} \end{pmatrix} \begin{pmatrix} b_i \\ b_r \\ b_s \\ b_{s'} \end{pmatrix}^{(n+1,1)} \tag{6}$$

We know that one way to calculate the reflection coefficient of a periodic structure is to construct the matrix propagator, i.e., a matrix which can be used to express the wave fields in neighboring cells in terms of each other. When the boundary conditions are incorporated in the requirement that the wave characteristics be continuous, this matrix can be constructed directly by joining the wave fields at the interfaces. In particular, this algorithm can be applied to acoustic waves in piezomagnetic or piezoelectric layers with interfaces which are magnetically or electrically "open," i.e., interfaces which maintain the continuity of the elastic displacements and forces and which also maintain the continuity of the normal component of the magnetic induction (or electric displacement) and the tangential component of the accompanying magnetic (or electric) field. The corresponding calculation of the 4×4 matrix propagator for a transverse wave propagating through a periodic system of piezoelectric layers was carried out in Ref. 5 (this problem was also solved, by other methods, in Refs. 6 and 7).

For "piezolayers" with magnetically "closed" (superconducting) or electrically closed (metallized) interfaces, in contrast, the standard joining method cannot be used. Formally, the reason is that in expressions (5) and (6) (in the case of piezoelectrics, B_y should be replaced by the electric potential φ) the matrices multiplied by the columns of amplitudes of partial wave modes are from the outset degenerate, i.e., do not have inverses. It is thus not possible to construct a 4×4 propagator which couples the four-component columns of partial amplitudes.

On the other hand, it is not difficult to see that a propagator modified in a certain way can be defined even in the case of piezolayers with metallized or superconducting boundaries. For this purpose, in each (n,α)th layer the boundary conditions $B_y^{(n,\alpha)}=0$ or $\varphi^{(n,\alpha)}=0$ at the two boundaries of the layer should be used to express the amplitudes of the nonuniform modes, $b_s^{(n,\alpha)}$, $b_{s'}^{(n,\alpha)}$, in terms of the amplitudes of the incident and reflected waves, $b_i^{(n,\alpha)}$, $b_r^{(n,\alpha)}$. The amplitudes $b_s^{(n,\alpha)}$, $b_{s'}^{(n,\alpha)}$ can thus be eliminated from the boundary conditions stating the continuity of the elastic displacements and forces. It then becomes possible to construct, in the standard way, a 2×2 propagator which acts on a column consisting of the amplitudes $b_i^{(n,\alpha)}$, $b_r^{(n,\alpha)}$. (In this case of magnetically or electrically closed boundary conditions, it is difficult to use Fourier analysis, which is an alternative to the matrix-propagator method in the case of open boundary conditions.)

GENERAL SOLUTION FOR A PERIODIC LAYERED STRUCTURE OF TWO DIFFERENT PIEZOMAGNETIC MATERIALS

Let us apply this calculation procedure to the problem described by Eqs. (1)–(6). From the two equations $B_y^{(n,2)}=0$ at $y=-(n-1)D-d_1$ and $y=-nD$ [see the last rows of the matrix equations (5) and (6)] we find[1)]

$$\begin{pmatrix} b_s \\ b_{s'} \end{pmatrix}^{(n,2)} = \frac{im_{14}^{(2)}}{2\mu_2 \mathrm{sh}\, k_x d_2}$$

$$\times \begin{pmatrix} e^{k_xd_2}-e^{-ik_{2y}d_2}, & e^{k_xd_2}-e^{ik_{2y}d_2} \\ e^{-k_xd_2}-e^{-ik_{2y}d_2}, & e^{-k_xd_2}-e^{ik_{2y}d_2} \end{pmatrix} \begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n,2)}. \tag{7}$$

In a similar way we find the corresponding expression for the $(n,1)$st layer:

$$\begin{pmatrix} b_s \\ b_{s'} \end{pmatrix}^{(n,1)} = \frac{im_{14}^{(1)}}{2\mu_1 \mathrm{sh}\, k_x d_2}$$

$$\times \begin{pmatrix} e^{k_xD-ik_{1y}D}(e^{ik_{1y}d_1}-e^{-k_xd_1}), & e^{k_xD+ik_{1y}D}(e^{-ik_{1y}d_1}-e^{-k_xd_1}) \\ e^{-k_xD-ik_{1y}D}(e^{ik_{1y}d_1}-e^{k_xd_1g}), & e^{-k_xD+ik_{1y}D}(e^{-ik_{1y}d_1}-e^{k_xd_1}) \end{pmatrix}$$

$$\times \begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n,1)} \tag{8}$$

Because of the periodicity, Eqs. (7) and (8) of course hold for any cell index n.

Substituting (7), (8) into (5) and (6), we find

$$\begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n,1)} = \hat{W}^{(1)} \begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n,2)}, \quad \begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n,2)} = \hat{W}^{(2)} \begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n+1,1)}, \tag{9}$$

where

$$W_{11}^{(1)} = W_{22}^{(1)*} = e^{i(k_{1y}-k_{2y})d_2}(1-p_1),$$

$$W_{12}^{(1)} = W_{21}^{(1)*} = e^{i(k_{1y}+k_{2y})d_2}p_1^*, \tag{10}$$

$$W_{11}^{(2)} = W_{22}^{(2)*} = e^{-ik_{1y}D}(1-p_2),$$

$$W_{12}^{(2)} = W_{21}^{(2)*} = e^{ik_{1y}D}p_2^*. \tag{11}$$

Here we are using the notation

$$p_\alpha = \frac{1}{2}\left[\bar{c}_{44}^{(\alpha)}\mathrm{ctg}\,\theta_\alpha + \frac{(m_{14}^{(\alpha)})^2 \sin k_{\alpha y}d_\alpha}{\mu_\alpha \mathrm{sh}\, k_x d_\alpha}\right]^{-1}$$

$$\times\left[\bar{c}_{44}^{(\alpha)}\mathrm{ctg}\,\theta_\alpha - \bar{c}_{44}^{(\beta)}\mathrm{ctg}\,\theta_\beta\right.$$

$$\left. + \frac{i(m_{14}^{(\alpha)})^2(e^{-ik_{\alpha y}d_\alpha}-\mathrm{ch}\,k_xd_\alpha)}{\mu_\alpha \mathrm{sh}\,k_xd_\alpha} + \frac{i(m_{14}^{(\beta)})^2(e^{ik_{\beta y}d_\beta}-\mathrm{ch}\,k_xd_\beta)}{\mu_\beta \mathrm{sh}\,k_xd_\beta}\right].$$

$$\alpha,\beta=1,2; \quad \alpha\neq\beta. \tag{12}$$

Expressions (10) and (11) can be used to calculate the matrix propagator $\hat{U}$ in which we are interested. This propagator couples the amplitudes of the incident and reflected waves in neighboring cells:

$$\begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n,1)} = \hat{U} \begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(n+1,1)}, \quad (13)$$

where

$$\hat{U} = \hat{W}^{(1)} \hat{W}^{(2)}. \quad (14)$$

We assume that the periodic structure contains $2N+1$ layers, i.e., has N cells, each consisting of a pair of a layers, $\alpha = 1$ and $\alpha = 2$, and a last "unpaired" $\alpha = 1$ layer. We also assume that this periodic structure has, on each side, a superconducting boundary with a semi-infinite medium (substrate) of piezomagnetic material corresponding to $\alpha = 2$ (this configuration is therefore symmetric along the y axis; Fig. 1). The amplitudes $b_i^{(0)}$, $b_r^{(0)}$ of the incident and reflected waves propagating in the "entrance" substrate are then related to the amplitude b_t of the transmitted wave in the "exit" substrate by

$$\begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(0)} = \hat{W}^{(en)} \hat{U}^N \hat{W}^{(ex)} \begin{pmatrix} b_t \\ 0 \end{pmatrix}, \quad (15)$$

where the matrix $\hat{W}^{(en)}$ couples the amplitudes of the bulk waves at the interface between the substrate and the $\alpha = 1$ layer, and $\hat{W}^{(ex)}$ does the same at the interface between the $\alpha = 1$ layer and the substrate. These matrices are found from the matrices $\hat{W}^{(2)}$ and $\hat{W}^{(1)}$, respectively, by taking the limit $d_2 \to \infty$. We thus have

$$W_{11}^{(ex)} = W_{22}^{(ex)*} = e^{i(k_{1y}-k_{2y})d_2}(1 - p^{(ex)}),$$
$$W_{12}^{(ex)} = W_{21}^{(ex)*} = e^{i(k_{1y}+k_{2y})d_2} p^{(ex)*}, \quad (16)$$

$$W_{11}^{(en)} = W_{22}^{(en)*} = e^{-ik_{1y}D}(1 - p^{(en)}),$$
$$W_{12}^{(en)} = W_{21}^{(en)*} = e^{ik_{1y}D} p^{(en)*}, \quad (17)$$

where

$$p^{(en)} = (2\bar{c}_{44}^{(2)} \operatorname{ctg}\theta_2)^{-1} \left[\bar{c}_{44}^{(2)} \operatorname{ctg}\theta_2 - \bar{c}_{44}^{(1)} \operatorname{ctg}\theta_1 - \frac{i(m_{14}^{(2)})^2}{\mu_2} + \frac{i(m_{14}^{(1)})^2 (e^{ik_{1y}d_1} - \operatorname{ch} k_x d_1)}{\mu_1 \operatorname{sh} k_x d_1} \right], \quad (18)$$

$$p^{(ex)} = \frac{1}{2} \left[\bar{c}_{44}^{(1)} \operatorname{ctg}\theta_1 + \frac{(m_{14}^{(1)})^2 \sin k_{1y} d_1}{\mu_1 \operatorname{sh} k_x d_1} \right]^{-1} \times \left[\bar{c}_{44}^{(1)} \operatorname{ctg}\theta_1 - \bar{c}_{44}^{(2)} \operatorname{ctg}\theta_2 + \frac{i(m_{14}^{(1)})^2 (e^{-ik_{1y}d_1} - \operatorname{ch} k_x d_1)}{\mu_1 \operatorname{sh} k_x d_1} - \frac{i(m_{14}^{(2)})^2}{\mu_2} \right]. \quad (19)$$

The components of the matrix $\hat{U}^N$ are expressed in terms of those of the unimodular matrix $\hat{U}$ ($\det \hat{U} = 1$) as follows (Ref. 9, for example):

$$(\hat{U}^N)_{11} = (\hat{U}^N)_{22}^* = U_{11} \frac{\sin NKD}{\sin KD} - \frac{\sin(N-1)KD}{\sin KD},$$
$$(\hat{U}^N)_{12} = (\hat{U}^N)_{21}^* = U_{12} \frac{\sin NKD}{\sin KD}. \quad (20)$$

The quantity K, the "Bloch wave number," is given by

$$\cos KD = (U_{11} + U_{11}^*)/2. \quad (21)$$

Using the calculated values of the components of the matrices $\hat{U}$, $\hat{W}^{(en)}$, and $\hat{W}^{(ex)}$, we can work from (15), (20), and (21) to find the reflection coefficient $R = b_r^{(0)}/b_i^{(0)}$.

BRAGG RESONANCE IN THE REFLECTION FROM A PERIODIC STRUCTURE OF LAYERS MADE OF A COMMON PIEZOMAGNETIC MATERIAL

According to Refs. 3, which we referred to back in the Introduction, no reflection of an elastic wave occurs in the case of identical piezomagnetic layers with thin superconducting coatings if these coatings are in the n state. When they go into the s state, the reflection from each coating is proportional to the small magnetomechanical coupling parameter

$$q_M^2 = m_{14}^2/\mu\bar{c}_{44} \ll 1. \quad (22)$$

We wish to analyze the conditions for Bragg resonance, i.e., to determine the relationship among the period D, the angle of incidence θ, and the magnitude of the wave vector k at which the reflection coefficient of a periodic structure of this sort (with a sufficiently large number of layers) reaches a value on the order of unity.

To avoid some complicated calculations below, we assume that the quantity $\exp(k_x d_\alpha)$ is a large parameter. It is easy to verify by means of the expressions derived below [see in particular (36) and (40); see also Fig. 2a] that this assumption is approximately correct even near the first maximum of the reflection coefficient (corresponding to the longest Bragg-resonance wavelength), provided that the angle of incidence is far from grazing and provided that the layers do not differ too greatly in thickness. Adopting the condition $\exp(k_x d_\alpha) \gg 1$, we find from (16) and (17)

$$\hat{W}^{(en)} \approx \hat{W}^{(2)}, \quad \hat{W}^{(ex)} \approx \hat{W}^{(1)}. \quad (23)$$

We can thus approximate expression (15) by

$$\begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(0)} = \hat{V}^{N+1} \begin{pmatrix} b_t \\ 0 \end{pmatrix}, \quad (24)$$

where

$$\hat{V} = \hat{W}^{(2)} \hat{W}^{(1)} \quad (25)$$

is the matrix propagator which couples the incident and reflected waves in the $(n,2)$nd and $(n+1,2)$nd layers [in contrast with (13) and (14)]. From (24) we find

$$R = b_r^{(0)}/b_i^{(0)} = (\hat{V}^{N+1})_{21}/(\hat{V}^{N+1})_{11}. \quad (26)$$

Another approximation used below is that terms of higher than first order in the small parameter q_M^2 are discarded. In this case we find the following results from (10)–(12), (25) for this structure of layers of the same material:

$$V_{11} = V_{22}^* \approx e^{-ik_y D}(1 - p + p^*), \quad (27)$$

$$V_{12} = V_{21}^* \approx e^{ik_y(d_2-d_1)}p^* - e^{ik_yD}p, \quad (28)$$

where

$$p = \frac{iq_M^2}{2\,\mathrm{ctg}\,\theta}\left(\frac{e^{-ik_yd_1} - \mathrm{ch}\,k_xd_1}{\mathrm{sh}\,k_xd_1} + \frac{e^{ik_yd_2} - \mathrm{ch}\,k_xd_2}{\mathrm{sh}\,k_xd_2}\right). \quad (29)$$

From (21) and (27) we find a relation which determines the Bloch wave number $K = K(\theta,k)$:

$$\cos KD \approx \cos k_yD + q_M^2\mathrm{tg}\,\theta\sin k_yD\left(\frac{\mathrm{ch}\,k_xd_1 - \cos k_yd_1}{\mathrm{sh}\,k_xd_1} + \frac{\mathrm{ch}\,k_xd_2 - \cos k_yd_2}{\mathrm{sh}\,k_xd_2}\right). \quad (30)$$

It can be seen from (30) that there are narrow regions (narrow to the extent that the parameter q_M^2 is small) of the values of θ and k for which the condition $|\cos KD| > 1$ holds, i.e., for which $K(\theta,k)$ is a complex quantity. We call these "forbidden zones." The boundaries of these zones are specified by the condition

$$\cos K(\theta, k)D = \pm 1. \quad (31)$$

Inside a forbidden zone and in a small neighborhood of it, the reflection coefficient is found from (20), (21), and (26) to be[2)] (Ref. 9):

$$|R|^2 \approx \frac{|V_{12}|^2}{|V_{12}|^2 + [\sin KD/\sin(N+1)KD]^2}. \quad (32)$$

Here, according to (28) and (29), we have

$$|V_{12}|^2 \approx \frac{q_M^4}{\mathrm{ctg}^2\theta}\left(\frac{\cos k_yD - \cos k_yd_1\,\mathrm{ch}\,k_xd_2}{\mathrm{sh}\,k_xd_2} + \frac{1 - \cos k_yd_1\mathrm{ch}\,k_xd_1}{\mathrm{sh}\,k_xd_1}\right)^2. \quad (33)$$

Since we have $|V_{12}| \sim q_M^2$, the maximum values of the reflection coefficient $|R(\theta,k)|$, which are in the forbidden zones according to the general theory, have a value on the order of unity at $N \sim 1/q_M^2$ [a more accurate estimate follows from (39) in the discussion below].

Let us find the intervals of θ and k which correspond to the forbidden zones. Substituting (30) into (31), we find an equation which has two solutions for each sign in (31). These two solutions specify the upper and lower boundaries, respectively, of the forbidden zones:

$$\cos k_yD = \pm 1, \quad (34)$$

$$\frac{\pm 1 - \cos k_yD}{\sin k_yD} \approx \frac{q_M^2}{\mathrm{ctg}\,\theta}\left[\frac{\mathrm{ch}\,k_xd_1 - \cos k_yd_1}{\mathrm{sh}\,k_xd_1} + \frac{\mathrm{ch}\,k_xd_2 - \cos k_yd_2}{\mathrm{sh}\,k_xd_2}\right]. \quad (35)$$

Making use of the small parameter q_M^2, and assuming that the angle of incidence θ is far from grazing, we find the positions of the boundaries of the forbidden zone of index l as a function of the quantity k_yD [$l = 1, 2, 3,...$; the upper sign in (31), (34), and (35) corresponds to even values of l, and the lower one to odd values]:

$$(k_yD)_l^{(1)} = \pi l, \quad (k_yD)_l^{(2)} = \pi l + \Delta_l. \quad (36)$$

The width of the forbidden zone, Δ_l, is

$$\Delta_l \approx \frac{2q_M^2}{\mathrm{ctg}\,\theta}\left[\frac{\mathrm{ch}(\frac{\pi l d_1}{D}\mathrm{tg}\,\theta) - \cos(\frac{\pi l d_1}{D})}{\mathrm{sh}(\frac{\pi l d_1}{D}\mathrm{tg}\,\theta)} + \frac{\mathrm{ch}(\frac{\pi l d_2}{D}\mathrm{tg}\,\theta) - \cos(\frac{\pi l d_2}{D})}{\mathrm{sh}(\frac{\pi l d_2}{D}\mathrm{tg}\,\theta)}\right]. \quad (37)$$

The maximum value $|R(\theta,k)|_{(\max)_l}$ is realized at the centers of the forbidden zones, where we have

$$(k_yD)_l^{(max)} = \pi l + \frac{\Delta_l}{2}, \quad (38)$$

i.e., $(\mathrm{Im}\,KD)_{\max} = \Delta_l/2$. From (32) we have

$$|R|^2_{(max)_l} \approx \frac{|V_{12}|^2}{|V_{12}|^2 + (\Delta_l/2)^2/\{\mathrm{sh}[(N+1)\Delta_l/2]\}^2}, \quad (39)$$

where the quantity $|V_{12}|$ ($|V_{12}| \sim \Delta_l \sim q_M^2$) is given by (33), in which we have $k_y \approx \pi l/D$, $k_x \approx (\pi l/D)\tan\theta$. The maximum value of the reflection coefficient is evidently close to unity under the condition $\exp(\Delta_l N) \gg 1$.

It is interesting to look at the case in which layers made of the same piezomagnetic material are between two substrates of the same piezomagnet, which also have identical thicknesses d. In principle, the original problem can be solved first; i.e., we can construct the matrix propagator through one layer by replacing the period D by d in the phase factors in wave field (1) [see expressions (45) and (46) below]. On the other hand, we know that the result which we are seeking for a system of N equidistant layers can be derived by taking the corresponding limit in the equations derived above for the case of layers of different thicknesses d_1 and d_2. Substituting the values $d_1 \equiv d$, $d_2 \to 0$, and $D \to d$ into (36) and (37), we find

$$(k_yd)_l^{(1)} = \pi l, \quad (k_yd)_l^{(2)} = \pi l + \Delta_l, \quad (40)$$

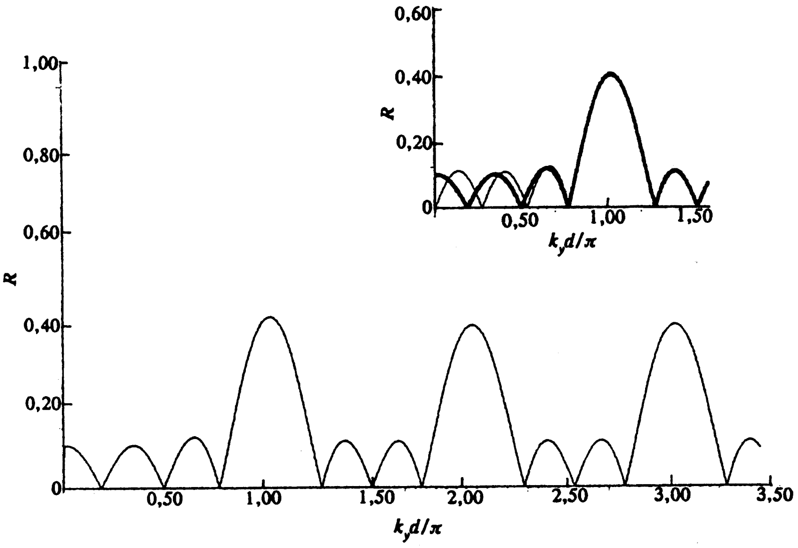
where

$$\Delta_l \approx 2q_M^2\mathrm{tg}\,\theta \times \begin{cases} \mathrm{th}\left(\frac{\pi l\,\mathrm{tg}\,\theta}{2}\right), & \text{for even } l, \\ \mathrm{cth}\left(\frac{\pi l\,\mathrm{tg}\,\theta}{2}\right), & \text{for odd } l. \end{cases} \quad (41)$$

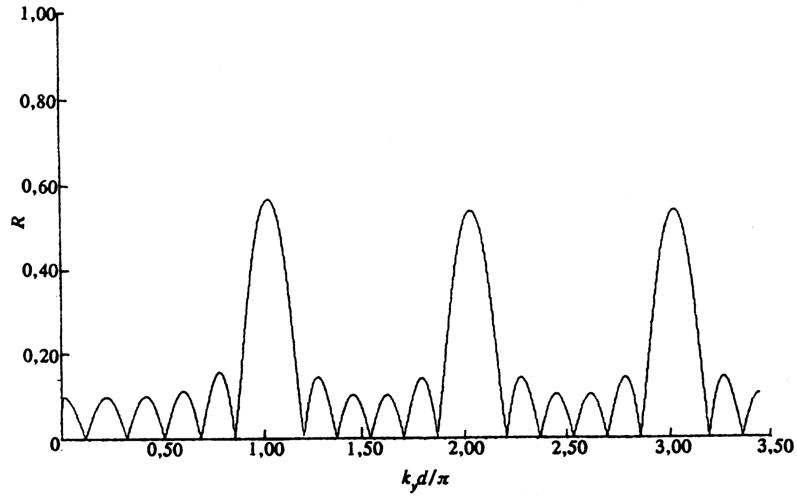
According to (33), in a forbidden zone the quantity $|V_{12}|$ is given by

$$|V_{12}| \approx \Delta_l/2. \quad (42)$$

Substituting (42) into (32) with $D = d$, we find that for values of k_y in (40) which correspond to the edges of the lth forbidden zone ($\sin Kd \to 0$) the reflection coefficient is

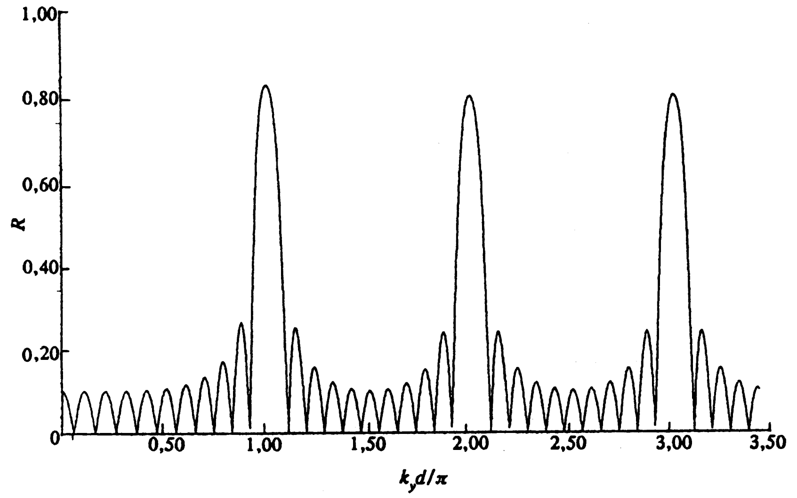


a



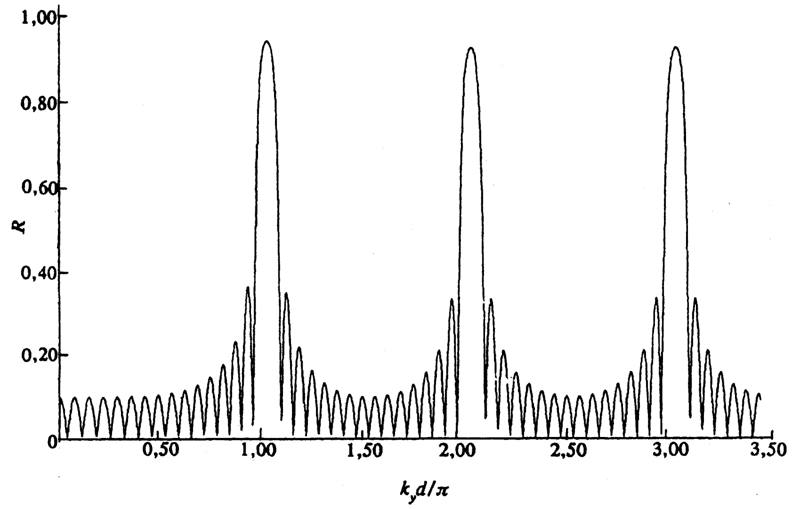
b

FIG. 2. Absolute value of the reflection coefficient, $|R|$, versus $k_y d/\pi$ for N identical piezomagnetic layers with superconducting coatings between two substrates, for $q_M^2 = 0.1$ and $\theta = 45°$. a—$N = 3$; b—5; c—10; d—15. The inset in part a compares results calculated from Eqs. (15′) (the heavy curve) and (24′).



c

(*continued*)



d

FIG. 2.—*continued*.

$$|R|^2_{(\text{edge})_l} \approx 1 - \frac{1}{1 + [\Delta_l(N+1)/2]^2}. \tag{43}$$

The maximum value $|R|^2_{(\max)_l}$ at the centers of the forbidden zones is, according to (39) and (42),

$$|R|^2_{(max)_l} \approx 1 - \frac{1}{1 + [\text{sh}\,\Delta_l(N+1)/2]^2}. \tag{44}$$

The limit $d_1 \equiv d$, $d_2 \to d_1$, $D \to 2d$ obviously has a slightly different physical meaning. The maxima of the reflection coefficient corresponding in the case $d_1 \neq d_2$ to the lth forbidden zones of even index ($l = 2l'$) disappear, while the two maxima corresponding to the l the forbidden zones of odd index ($l = 2l' + 1$) are found from the same expressions, (40)–(44), in which l is to be replaced by l'.

To illustrate the procedure, we will go through an exact numerical calculation of the reflection coefficient for a system of N identical piezomagnetic layers of thickness d between two substrates. We will not resort to the approximations used above, involving the condition $\exp(k_x d) \gg 1$ and the assumption that the parameter q_M^2 is small. In other words, we use an expression analogous to (15):

$$\begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(0)} = \hat{W}^{(en)} \hat{W}^{N-1} \hat{W}^{(ex)} \begin{pmatrix} b_t \\ 0 \end{pmatrix}, \tag{15'}$$

in which we set $D = d_1 \equiv d$ and $d_2 = 0$ for the matrices $\hat{W}^{(en)}$, $\hat{W}^{(ex)}$ in (16)–(19). We also assume that the material constants of the layers are equal, and we specify the matrix propagator $\hat{W}$, which couples the amplitudes in neighboring layers, by

$$W_{11} = W_{22}^* = e^{-ik_yd}(1 - \tilde{p}), \qquad W_{12} = W_{21}^* = e^{ik_yd}\tilde{p}^*, \tag{45}$$

where

$$\tilde{p} = \frac{iq_M^2(\cos k_yd - \text{ch}\,k_xd)}{\text{ctg}\,\theta\,\text{sh}\,k_xd + q_M^2\sin k_yd}. \tag{46}$$

Hence

$$\cos Kd = (W_{11} + W_{11}^*)/2$$
$$= \frac{\text{ctg}\,\theta\cos k_yd\,\text{sh}\,k_xd + q_M^2\sin k_yd\,\text{ch}\,k_xd}{\text{ctg}\,\theta\,\text{sh}\,k_xd + q_M^2\sin k_yd}. \tag{47}$$

Figure 2 shows the results of a numerical calculation of the modulus of the reflection coefficient, $|R|$, as a function of k_yd/π for $q_M^2 = 0.1$ and $\theta = 45°$, for structures of $N = 3$, 5, 10, and 15 identical layers between two substrates. The inset in Fig. 2a shows that even at the first reflection maximum there is essentially complete agreement between the calculations based on exact equation (15′) and on the approximate equation

$$\begin{pmatrix} b_i \\ b_r \end{pmatrix}^{(0)} = \hat{W}^{N+1} \begin{pmatrix} b_t \\ 0 \end{pmatrix} \tag{24'}$$

(the terms quadratic in q_M^2 are not considered in the expression for $\hat{W}^{N+1}$). The latter equation is the basis of the analytic study of the Bragg resonance carried out above. In particular, for $N = 15$ we find from (44) that the value of the first-order maximum is $|R|_{(\max)_1} \approx 0.94$. This result corresponds to the height of the given peak in Fig. 2d. The width of the lth forbidden zone, $\Delta_l \sim q_M^2$ evidently characterizes the width of the corresponding maximum in the reflection coefficient only at large values of N, at which the values of $|R|$ at the edges of the forbidden zone and at the center of the zone are sufficiently different [see (43), (44)]. Since the parameter Δ_l given by (41) is nearly independent of l for $l = 2, 3, \ldots$, the maxima of $|R|$ of order higher than the first ($l > 1$) are essentially identical at a fixed value of N.

REFLECTION FROM A PERIODIC SYSTEM OF PIEZOELECTRIC LAYERS WITH METALLIZED BOUNDARIES

Let us examine the reflection of a transverse elastic wave from a periodic system of piezoelectric layers of hexagonal symmetry with metallized boundaries and mutually parallel **G** axes. These axes lie in the plane of the interfaces and are oriented perpendicular to the plane of incidence (Fig. 1). As we pointed out above, this problem can be solved by the same method as was used in the case of piezomagnetic layers with superconducting coatings. On the other hand, as was pointed out in Ref. 3, these two problems are generally not completely similar, because the tangential component of the electric field and the normal component of the magnetic induction, which vanish at the metallized boundary of the piezoelectric and at the superconducting boundary of the piezomagnetic, respectively, are not symmetric under the interchange of electric and magnetic properties in Maxwell's equations. Nevertheless, after going through calculations similar to those in the preceding sections of this paper, we find that all the final expressions for piezoelectric layers are the same as those for piezomagnetic layers, except for the substitution

$$q_M^2 \to -q_E^2, \tag{48}$$

where $q_E^2 = e_{15}^2/\bar{c}_{44}\varepsilon$ is the electromechanical coupling parameter, $\bar{c}_{44} = c_{44} + e_{15}^2/\varepsilon$, e_{15} is the coefficient of the piezoelectric effect, and ε is the dielectric constant.

CONCLUSION

When a transverse elastic wave is incident obliquely on a periodic structure of N piezomagnetic layers with thin superconducting interlayers, a Bragg reflection resonance arises. The maxima of the reflection coefficient corresponding to the resonant conditions have a spectral width proportional to the small magnetomechanical coupling parameter q_M^2. Their height is close to unity under the condition $\exp(Nq_M^2) \gg 1$ ($|R|^2_{\max} \approx 0.94$ with $N = 15$ and $q_M^2 = 0.1$). If the layers are made of the same piezomagnetic material, the reflection disappears essentially completely when the coatings undergo a transition to the end state. Under these conditions the reflection coefficient can thus jump abruptly from zero to values close to unity as the phase state of the superconducting interlayers is changed, e.g., by means of a change in temperature or electric current.

A similar Bragg-resonance effect arises in the reflection of an elastic wave from a periodic structure of piezoelectric layers with metallized boundaries. Resonant reflection is possible, in particular, for a periodic structure formed by layers of the same piezoelectric material. Since the reflection coefficient under conditions far from resonance is small in this case ($|R| \sim q_E^2$, where $q_E^2 \ll 1$ is the electromechanical coupling parameter), a periodic structure of this sort is more selective than the Bragg reflectors which are ordinarily used in acoustoelectronics, which are made of different piezoelectric layers with unmetallized boundaries.

[1)] Recall that the quasistatic approximation used in describing the magnetic or electric field accompanying the elastic wave in the piezomagnet or piezoelectric breaks down at "ultrasmall" angles of incident $\theta \lesssim v/c$, where v is the sound velocity, and c the velocity of light.[8] It should thus not be surprising to find that the equations derived below are inconsistent with the operation of taking the limit of the case of normal incidence, in which there is no reflection, because the boundary condition $B_y = 0$ (or $\varphi = 0$) holds identically.

[2)] The matrix propagator $\hat{V}$, like $\hat{U}$, is evidently unimodular and satisfies (20) and (21). Incidentally, in verifying the identity $\det \hat{U} = \det \hat{V} = 1$ in the expressions for $U_{11} = V_{11}$ one must incorporate terms $\sim q_M^4$.

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