Screening properties of inhomogeneous superconducting YBaCuO films in magnetic field

S. A. Vitkalov

P. N. Lebedev Physics Institute, Russian Academy of Sciences
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A model of an inhomogeneous superconductor is proposed and used to find the field-induced changes in the conductivity at a frequency ω and the critical current. These changes arise due to field-induced variations of the correction to the conductivity related to superconducting loops with weak links shunting critical links of a percolation superconducting cluster. For strongly inhomogeneous systems the magnetic field dependence is stable to variations of sample microstructure and completely determined by the maximum size R_φ of the shunting loops, which is much larger than the grain size. The model is compared with experimental results on the magnetic field dependence of screening by YBaCuO films, when the sample is divided into pieces of size R, and the size R_φ is found. This size R_φ grows as $T \to T_c$.

1. INTRODUCTION

One of the interesting properties of polycrystalline high-T_c superconductors is the high sensitivity of their electromagnetic response to a weak external magnetic field.[1–4] The majority of the models explaining the magnetic field dependence of conductivity can be provisionally divided into two groups.[5] In the models belonging to the first group the response is determined by the dynamics of vortices moving in a viscous medium[1] and strongly depends on the external electromagnetic field frequency. Thus, according to Ref. 1, the characteristic value of the field for which the quadratic field dependence of the surface impedance becomes linear is proportional to frequency. As the frequency changes from 10^6 to 10^{10} Hz, this value should change by four orders of magnitude, which disagrees with experimental results.[3]

The models of the other group attribute the observed dependence to the presence of "effective" loops in inhomogeneous superconducting media.[2–4,6] The size of these loops gives the scale of constant magnetic fields in which characteristic conductivity changes occur. This characteristic scale has a weak frequency dependence, which agrees with experiment.[2,3] Therefore further studies of the consequences of the effective loop models and their comparison with the experiment are of interest. To verify directly the existence of effective loops and to find their size, we can study the magnetic field dependence of screening in samples varying in size. According to the model suggested, the magnetic field dependence of electromagnetic response is determined by the size of the effective loops, which we denote by R_φ. If the sample dimensions are larger than R_φ, the dependence mentioned above is not affected. If, however, these dimensions become of order or smaller than R_φ, the maximum loop size is given by the sample dimensions and the form of the response should change.

In the present study we examine the changes in screening of electromagnetic field by a YBaCuO film subjected to magnetic field, when the film is cut into smaller and smaller pieces. It is shown that the changes occur when the size of the pieces becomes of the order of the maximum size R_φ of the effective loops found from fitting the theoretical response dependence to the experimental curve. The temperature dependence of R_φ is also studied. It is shown that the size R_φ increases as $T \to T_c$. A model is suggested which allows us to calculate the correction to the conductivity arising from shunting loops with weak links[3] for two-dimensional strongly inhomogeneous superconducting systems. The magnetic field dependence of the critical current, $J_c(H)$, found experimentally for YBaCuO is compared with the dependence following from the proposed model.

2. EXPERIMENTAL PROCEDURE

We studied the screening of electromagnetic field of frequency $\omega/2\pi = 10$–100 kHz by YBaCuO films in a constant or slowly changing magnetic field of intensity up to 50 Oe at temperatures from 77 to 92 K. The samples were placed near a receiving coil at a distance of 0.5 mm from the latter. An alternating magnetic field H_ω was created by Helmholtz coils and was uniform when the sample was absent. We measured the amplitude P_ω of the emf signal in the receiving coil as a function of the magnetic field H and temperature T. When we introduced the sample in the normal state, the amplitude of the emf signal in the receiving coil practically did not change. When the sample experienced the transition into the superconducting state, the amplitude became several times smaller. The constant or slowly changing field H was created by an external solenoid. The alternating magnetic field H_ω was parallel to the external constant field H and was perpendicular to the YBaCuO film.

To interpret the experimental results correctly, we need to know the distribution of the constant magnetic field in the sample. We approximate the film by an oblate spheroid. If the film area R^2 is approximately 1×1 cm^2 and its thickness $d = 10^{-4}$ cm, then the demagnetization coefficient n along the minor spheroid axis, which is equal to $1 - \pi d/2R$ (see Ref. 7), is almost unity. This means that for $H_{c1} = 10^2$ Oe the magnetic field penetrates such a film even in an external field $H > \pi d/2RH_{c1} = 10^{-2}$ Oe, which is much smaller than the fields used in the experiment. Thus, the film is in a mixed state. To check that the distribution of the constant magnetic field in the sample is uniform, we studied the field dependence of the screening with the help of receiving coils of various diameter placed under different parts of the sample surface. In all cases, the variations were less than 10–20%, which is evidence for the uniformity of the constant magnet-

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ic field in the films studied. When we measured the constant magnetic field near the film using the Hall generator, we found that in the magnetic field and temperature range studied the constant field is essentially unscreened by the samples. It follows from these measurements that the magnetic flux pinning is weak, which is typical for YBaCuO at $T > 77$ K. The absence of hysteresis when the magnetic field grows and decreases also indicates that the pinning is weak.

The experiments were carried out in a regime linear in the amplitude of the alternating magnetic field, i.e., the signal magnitude P_ω in the receiving coil was proportional to H_ω and the shape of the curve $P_\omega(H)$ was independent of H_ω. The amplitude H_ω^0 of the alternating field was ≈ 15 mOe. The quantity H_ω consists of two terms: the magnetic field H_ω^0 created by excitation coils in the sample absence and the field H_ω^1 created by screening currents in the sample. The latter depends on the external constant field H. The field H_ω^1 depends on the form of the constitutive relation between the screening current I_ω and the alternating field, as well as on the sample size and shape. For a thin disc of conductivity $\sigma_\square$ per unit area the field H_ω is given by an integral expression[8] and is not reduced to elementary functions. If, however, we study small variations of the screening currents I_ω in the constant uniform magnetic field H, the variations of H_ω are proportional to the conductivity variations in the field H. In the case of a thin film of conductivity $\sigma_\square$ per unit area the variations of H_ω are proportional to the variations of $\sigma_\square$: $\Delta H_\omega = \alpha \Delta\sigma_\square(H)$, where α is independent of the external constant magnetic field. In our experiments which are conducted at $T = 77$ K, the changes of ΔH_ω in the magnetic field were insignificant. Therefore, when we processed our experimental data, we assumed $\Delta H_\omega \propto \Delta\sigma_\square(H)$. In the case of strong variations of $\Delta H_\omega(H)$ observed directly at the superconducting transition, such an assumption can give rise to systematic errors. In the present study, however, this fact was disregarded and all the curves were processed with the help of the simple relation mentioned above.

The experiments were carried out on textured YBaCuO films with the c axis along the (100) direction obtained both by laser and magnetron evaporation on ZrO_2 and MgO substrates. The resistive superconducting transition temperatures T_c were in the range of 85–91 K. The film thickness amounted to 0.3–1 μm. The samples had linear dimensions of $\sim 10 \times 10$ mm^2. To find the effect of the sample dimensions on the magnitude and form of the magnetic field dependence of screening, the sample was cut into isolated squares of smaller dimensions either by a diamond needle (the square dimensions ranged from 2 mm to 50 μm) or by means of chemical etching (with squares from 1 mm to 5–6 μm).

For temperature control the sample was mounted on a sapphire rod with a heater at the other end. The temperature was controlled by the resistance of a thin copper wire wound around the sapphire rod.

3. EFFECTIVE LOOP MODEL

In a previous study[3] we suggested a model by means of which we found the correction to the conductivity of a granular superconducting two-dimensional system in magnetic field. The resulting expression gives the form of the magnetic field dependence of the conductivity observed in the experiments on YBaCuO.[3] In the approach suggested in Ref. 3 the screening changed due to changes in the interference contribution to the film conductivity of the "effective" superconducting loops containing Josephson links and having a certain maximum length L_φ which, in fact, was postulated and found from experiment. This length turned out to be much larger than the grain size d. In this section we present a further development of the approach mentioned above and suggest a model in which the maximum loop length L_φ arises in a natural way and turns out to be larger than the size of separate grains.

In granular superconductors of the YBaCuO type the critical current is several orders of magnitude smaller than that in monocrystals. This is attributed to weak Josephson links between grains. On the other hand, if all the intergranular links were weak, the temperature of the superconducting transition would be on the order of the Josephson energy E_J and much lower than T_c in monocrystals. However, the temperature of the superconducting transition in granular superconductors of the YBaCuO type is close to that in monocrystals, which means that there are good superconducting links between grains. Thus, we can assume that the granular superconductors of the YBaCuO type are systems with a very nonuniform distribution of the strength of superconducting intergranular links. Consider now such a superconductor.

We assume that the superconducting Josephson link between the grains i and j is given by the conductivity $\sigma(i,j)$, which depends on a random parameter ξ with the distribution function $F(\xi)$. As shown below, a specific form of $F(\xi)$ for a highly nonuniform medium does not affect the magnetic field dependence of the conductivity. For simplicity, we assume that the parameter ξ is uniformly distributed in the interval $-\xi_0 \leqslant \xi \leqslant \xi_0$ ($\xi_0 \gg 1$) and the conductivity is given by the expression[9]

$$\sigma(i, j) = \sigma_0 \exp[-\xi(i, j)] \ . \tag{1}$$

Exponentially large variations of the link conductivity can arise due to changes in the thickness of dielectric or metallic interlayers between different pairs of grains, leading to exponential changes in σ. We need to find how the conductivity of such a nonuniform system is affected by magnetic field. Let ξ be a number in the interval $-\xi_0 \leqslant \xi \leqslant \xi_0$ and all the links with $\xi' > \xi$ be broken. Let the conductivity of the system be $\sigma(\xi)$. The number ξ gives the probability of an arbitrarily chosen link not being broken:

$$x(\xi) = \int_{-\xi_0}^{\xi} F(\xi')d\xi' \ . \tag{2}$$

Here $x(\xi)$ is the concentration of the unbroken links and $F(\xi)$ is the distribution function of ξ, assumed to be uniform:

$$F(\xi) = \begin{cases} 1/2\xi_0 \, , & |\xi| \le \xi_0 \, , \\ 0 \, , & |\xi| > \xi_0 \, . \end{cases} \tag{3}$$

From (2) and (3) we get

$$x(\xi) = (\xi_0 + \xi)/2\xi_0 \ . \tag{4}$$

For ξ close to $-\xi_0$ the concentration $x(\xi)$ is small and unbroken links form isolated clusters. The conductivity $\sigma(\xi)$ equals zero. Let ξ grow. When it reaches a threshold value ξ_c for which the concentration $x(\xi)$ equals the percolation threshold x_c for the percolation bond problem, an infinite cluster is formed. When ξ increases further from ξ_c to $\xi_c + 1$ the conductivity becomes finite and grows according to a power law:

$$\sigma(\xi) \propto [x(\xi) - x(\xi_c)]^t , \qquad (5)$$

where $t > 0$. In the interval $\xi_c < \xi \leqslant \xi_c + 1$ we can neglect changes in the conductivity of the connected links. However, for further growth of ξ, i.e., for $\xi > \xi_c + 1$, the link conductivity becomes exponentially small, and the conductivity of the whole system, in spite of the increase in the density of the link network, does not change. Thus, the conductivity of the system is determined by the critical subnetwork, i.e., by the infinite cluster arising for $\xi - \xi_c \approx 1$.[9,10] The conductivity of the critical subnetwork is determined by its highest resistances, i.e., by the critical links of conductivity on the order of $\sigma_0 \exp(-\xi_c)$. The density of the critical subnetwork is

$$\rho \approx (x - x_c)^\beta \approx (1/\xi_0)^\beta \ll 1 , \qquad (6)$$

where $\beta > 0$.

As is well-known,[11] the conductivity of a homogeneous (formed by equal resistances) percolation network near the percolation threshold grows slower than the density of the infinite cluster, i.e., $t > \beta$. This means that most of the mass of an infinite cluster is in dead ends or in long, and, therefore, high-resistance, chains in parallel with short segments of cluster chains.[9] In the case of a very nonuniform system, when ξ changes in the interval $\xi_c \leqslant \xi \leqslant \xi_c + 1$, the resulting long chains which shunt critical links have almost the same conductivity as the latter, i.e., are, in fact, low-resistance. This is related to the fact that the probability of two links from the interval $\xi_c \leqslant \xi \leqslant \xi_c + 1$ being on the same shunting chain is small in proportion to the parameter $\alpha \approx 1/\xi_0 \ll 1$.[1)] Therefore the chains shunting a critical link almost always have only one high-resistance link determining the conductivity of these chains. In the case of an arbitrary distribution function $F(\xi)$, the parameter α is

$$\alpha = F(\xi_c) / \int^{\xi_c} F(\xi) d\xi .$$

For a function $F(\xi)$ which has no resonance singularities at $\xi \approx \xi_c$ and is finite in a sufficiently wide interval of ξ values (in the strongly inhomogeneous case this interval is much larger than unity) this parameter satisfies $\alpha \ll 1$. The conductivity of long chains in parallel with a critical link is comparable to the conductivity of the latter and is insensitive (to order α) to the specific form of the distribution function $F(\xi)$.

When a highly nonuniform superconducting system is placed in a magnetic field, the shunting chains must substantially change their conductivity in fields of order Φ_0/S, where Φ_0 is the flux quantum, and S is the area of the shunting loop. For long chains the area S is much larger then the area of a single Josephson junction. Consequently, the correction to the conductivity of the critical subnetwork connected with long loops in parallel with a critical link must be much more sensitive to the magnetic field than the conductivity of a single Josephson junction. In the fields $H \sim \Phi_0/S$ we can therefore disregard the changes in $F(\xi)$ and assume that the response of the system does not depend on the form of the distribution function $F(\xi)$ and is determined by the change in the correction related to the shunting loops. This is one of the important differences between the suggested approach and other models of the Josephson medium.[12]

Thus, in the case of a strongly inhomogeneous medium long shunting chains can determine the magnetic field dependence of the conductivity. Since a highly nonuniform system is essentially at the percolation transition [$\rho \ll 1$; see Eq. (6)], its topological properties must be universal. On the other hand, the conductivity of long loops in parallel with a critical link is essentially insensitive to the form of the distribution function $F(\xi)$ and is determined by its value at $\xi = \xi_c$, which does not change in the fields $H \sim \Phi_0/S$ which are important for the correction. We should therefore expect that a large group of highly nonuniform superconducting systems would exhibit universal behavior in the magnetic field. This is also one of the differences between the suggested approach and other models of a Josephson medium[12,13] in which the magnetic field dependence of the response is determined by the distribution function $F(\xi)$ and is therefore sensitive to the size and position of separate grains, the orientation of the Josephson transition with respect to the magnetic field, etc. From the standpoint of experiment universality should decrease the number of fitting parameters used to interpret the experimental results. As shown below, the field dependence is given by a single parameter R_φ which is the largest size of the shunting loops.

Let us find the correction to the conductivity of the critical subnetwork connected with the chains (loops) shunting the critical links. We determine how the loops are distributed in length. Let the critical link between the grains i and j be at the origin, $\mathbf{r} = 0$, and an additional, shunting, link be at an arbitrary point $\mathbf{r}$. Let the system consist of grains of size d and assume that an arbitrary chain of the infinite cluster has the form of the Brownian particle trajectory. Let a random trajectory leaving the grain j intersect a trajectory leaving the grain i with some probability p at each step of length d. This probability is proportional to the probability for the chain to intersect the critical subnetwork. Since, according to (6), the critical subnetwork density is small in comparison with unity, we can assume that the probability for an arbitrary chain from the grain j to intersect trajectories from the grain i is small, $p \ll 1$. The probability of a given Brownian trajectory from the grain j not intersecting other chains from the grain i and coming to a point $\mathbf{r}$ after N_k steps is

$$P(j, N_k, \mathbf{r}) = \left(\frac{1}{2\pi d^2 N_k}\right)^{D/2} \exp\left(-\frac{|\mathbf{r}|^2}{2d^2 N_k}\right)(1-p)^{N_k}$$
$$\approx \left(\frac{1}{2\pi d^2 N_k}\right)^{D/2} \exp\left(-\frac{|\mathbf{r}|^2}{2d^2 N_k} - pN_k\right) , \qquad (7)$$

where D is the space dimensionality. The same is true for the trajectories from the grain i. The total probability of two trajectories from the grains i and j (of length N_k and N_m respectively) intersecting at some point r is equal to the sum over $\mathbf{r}$ of the products of the probabilities (7):

$$P(i \to j, N = N_k + N_m)$$
$$= \int dr \left\{\frac{1}{(2\pi d^2)^D (N_k N_m)^{D/2}} \exp\left[-\frac{|r^2|}{2d^2}\left(\frac{1}{N_k} + \frac{1}{N_m}\right) - p(N_k + N_m)\right]\right\} \approx \left(\frac{1}{2\pi d^2 N}\right)^{D/2} \exp(-pN) , \qquad (8)$$

where N is the total length of the trajectory connecting the grains i and j. Thus, Eq. (8) gives the length distribution of shunting loops. It is seen that shunting loops of length $L = Nd > d/p = L_\varphi$ are exponentially unlikely. This means, in fact, that due to the finite density ρ of the critical subnetwork ρ, the intergranular links are shunted by chains that are short in comparison with $L_\varphi \approx d/\rho \gg d$. As calculations show, the contribution of shunting trajectories to the conductivity of a two-dimensional system for a $\rho \ll 1$ low-density critical subnetwork is the most important one; see Eq. (15) below. Hence we call these shunting loops effective from the point of view of conductivity.

To find the correction to the critical link conductivity, we must sum the contributions of all loops shunting this link. As mentioned above, in the limit of a low-density critical subnetwork the contribution of shunting loops containing more than one critical link can be neglected in the parameter $1/\xi_0 \ll 1$. We therefore assume that a shunting loop contains only one critical link. To find the conductance of the loop X_k shunting the superconducting junction between the grains i and j we determine the current I_ω flowing in this loop. Let an alternating voltage $U_\omega = U_0 \exp(i\omega t)$ from an external generator be applied to the points i and j. The phase difference φ_{ij} of the order parameter near the generator outlets i and j oscillates at a frequency ω:

$$\hbar \frac{d\varphi_{ij}}{dt} = 2eU_\omega, \qquad \varphi_{ij} = \varphi_{ij}^0 + \frac{2eU_\omega}{i\omega\hbar}.$$

Let the loop X_k enclose the magnetic flux Φ_k. For the phase difference at the Josephson junction in the loop X_k we get

$$\Delta\varphi_k \approx -\varphi_{ij} - 2\pi\Phi_k/\Phi_0. \tag{9}$$

Here we have assumed that the current in the loop X_k is small and the distance between i and j is much shorter than the length of the loop X_k. The current I_k flowing in the loop X_k is

$$I_k = I_J \sin(\Delta\varphi_k), \tag{10}$$

where I_J is the critical Josephson current. The current I_k at the frequency ω in the linear in U_ω regime is

$$I_\omega = \frac{2ieI_J U_\omega}{\hbar\omega} \cos\left(2\pi \frac{\Phi_k}{\Phi_0}\right). \tag{11}$$

Thus, the correction to the critical link conductivity due to the shunting loops changes in the magnetic field as follows:

$$\langle\delta\sigma\rangle = \langle \sum_{X_k} [\sigma(X_k, H) - \sigma(X_k, 0)]\rangle$$
$$\approx 2ie\langle I_J\rangle \langle \sum_{X_k} \frac{1}{\hbar\omega} \left[\cos\left(\frac{2\pi\Phi_k}{\Phi_0}\right) - 1\right]\rangle. \tag{12}$$

The brackets mean averages over realizations. Since the Josephson junction area is much smaller than the area enclosed by the loop X_k, we neglect the changes in I_J in the magnetic field.

The area enclosed by the loop X_k is proportional to the square of the loop dimension, $S_k \propto R_k^2$. If the loops have the Brownian shape, then the size of the loop X_k consisting of N_k grains is proportional to $N_k^{1/2}$. Thus, we have $S_k = \beta N_k$. To estimate the constant β we set $N = 4$ and consider grains in the form of discs of diameter d in the vertices of a square of side length d. Then $\beta = (d/4)^2$. For other configurations we find similar expressions. In what follows we will use this value of β. For the magnetic field flux through the loop X_k we have the following expression:

$$\Phi_k = S_k H = (d/4)^2 N_k H. \tag{13}$$

As comparison with experimental results shows, the thickness of the films we studied is much less than the greatest length L_φ of the shunting loops. Therefore we find the correction to the conductivity in the two-dimensional case. Substituting (13) into (12) and using (8), we have for the magnetic field variation of the average, in realizations, correction to the critical link conductivity ($D = 2$):

$$\langle\delta\sigma(H)\rangle = A\langle \sum_{X_k} \left[\cos\left(\frac{2\pi\Phi_k}{\Phi_0}\right) - 1\right]\rangle$$
$$\approx A \sum_{N_k} P(i \to j, N_k) \left[\cos\left(\frac{2\pi\Phi_k}{\Phi_0}\right) - 1\right]$$
$$\approx A \int_0^\infty [\cos(\alpha H x) - 1] \exp\left(-\frac{x}{L_\varphi}\right) \frac{dx}{x}, \tag{14}$$

where $A = ei\langle I_J\rangle/\hbar\omega$ and $\alpha = \pi d/8\Phi_0$ are constants. In deriving Eq. (14) we multiplied the probability density $P(i \to j, N_k)$ by a normalization factor πd^2. We thus allow for the fact that the trajectory should come back not to the point $r = 0$, but to the region whose area is of order d^2.

The resistivity of a square network of equal resistances is equal to the resistance of a single element. In our case, small variations of the conductivity per square of a two-dimensional cluster are proportional to small variations of the average conductivity of cluster critical links in the magnetic field. Hence the right-hand side of Eq. (14) is proportional to small variations of the film conductivity $\delta\sigma_\square(H)$ in the magnetic field. For $\alpha H L_\varphi \ll 1$ the cosine in the integrand (14) can be expanded in powers of H. It is seen that $\delta\sigma(H) \propto H^2$ as $H \to 0$. For large values of H the upper limit of the integral (14) can be replaced by $1/\alpha H$. As $H \to \infty$ we have $\delta\sigma(H) \propto \ln H$. For $H = 0$ the correction to the critical link conductivity connected with the shunting loops is

$$\frac{\Delta\sigma}{\sigma_0 \exp(-\xi_c)} = \int_d^\infty \frac{dx}{x} \exp\left(-\frac{x}{L_\varphi}\right) \approx \ln\frac{L_\varphi}{d} \gg 1. \tag{15}$$

It is seen that the contribution of the loops in parallel with critical links determines the conductivity of critical regions belonging to the critical subnetwork.

Since the shape of the shunting loops is Brownian, their greatest size is $R_\varphi = (L_\varphi d)^{1/2}$.

4. RESULTS AND DISCUSSION

Screening in magnetic field versus sample dimensions

As has already been noted, the variation of the screening signal ΔP_ω in the magnetic field H is proportional to the variation of the magnitude of an alternating current I_ω induced by the field H_ω. In terms the model considered in Sec. 3 this means that $\Delta P_\omega = P_\omega(H) - P_\omega(0)$ is proportional to the field variation of the correction to the film conductivity

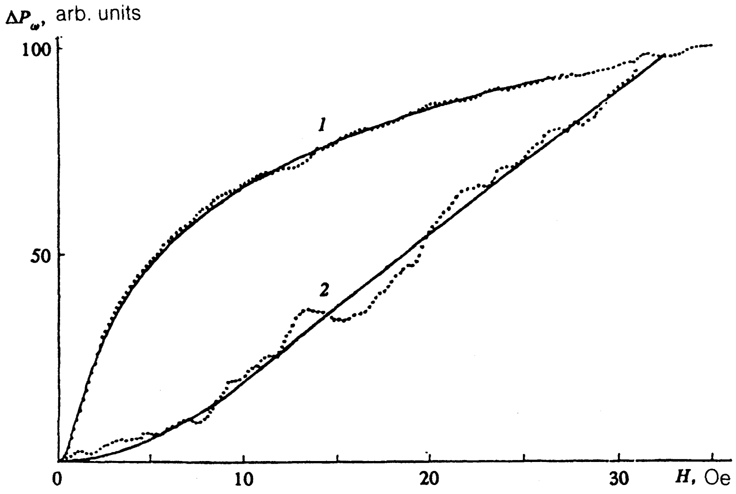

FIG. 1. Magnetic field dependence of the screening for a YBaCuO film ($T_c = 85$ K, magnetron evaporation, $H_\omega = 20$ mOe, $\omega/2\pi = 10$ kHz, and $T = 77$ K). Curve *1*: the whole sample; curve *2*: sample divided into $6 \times 6\,\mu m^2$ squares. Solid curves: approximation by Eq. (14).

connected with superconducting loops with weak links: $\Delta P_\omega \propto \delta\sigma_\square(H)$. The function $\delta\sigma_\square(H)$ is determined, according to (14), by the greatest dimensions R_φ of the effective loops. Comparing the experimental curves $P_\omega(H)$ with Eq. (14), we can find R_φ. If R_φ changes under external forces, so does the function $P_\omega(H)$. One of the direct techniques for changing R_φ is dividing the sample into smaller and smaller parts. When the dimensions of the separate parts are much larger than R_φ, the latter does not change. When the dimensions become comparable to R_φ, its value starts changing, which leads to changes in the characteristic scale length of the function $P_\omega(H)$.

Figure 1 shows plots of magnetic field screening versus magnetic field for an intact sample (curve *1*) and a sample divided into separate squares with sides of length about 6 μm. The solid curves are the result of approximation by means of Eq. (14). It is seen that when the sample is divided into small separate squares, the screening differs from that in the undivided sample. In Fig. 2 the size R_φ is plotted against the size of separate parts ($R_0 = R_\varphi$ for the undivided sample). It is seen that when the square size is larger than R_0, R_φ changes only slightly. The change (decrease) becomes significant, when the dimensions of separate parts are on the order of R_φ. To estimate how the size R_φ should change, when the sample is divided into separate parts, we multiply the integrand in (14) by an additional cutoff factor, $\exp(-x/L_R)$, which allows for loops X_k whose length is at most L_R. The length L_R is chosen in such a way that the size of a Brownian trajectory of length L_R is equal to the size R of sample's separate parts, $L_R = R^2/d$. Then the greatest size of effective loops follows from (14):

$$R_0/R_\varphi = (1 + R_0^2/R^2)^{1/2}. \qquad (16)$$

The dependence (16) corresponds to the solid curve plotted without fitting parameters in Fig. 2. The size R_φ found from experimental data processing is, as a rule, several times larger than the mean microcrystal size found by means of an electron microscope. A similar pattern is observed in other inhomogeneous superconducting systems.[14,15]

Let us now compare the response ΔP_ω found in our model and in the model of independent loops[2] with experimental results. Consider the behavior of $\Delta P_\omega = P_\omega(H) - P_\omega(0)$ when the sample is divided into separate parts. According to the independent loop model,[2] the signal $\Delta P_\omega(H)$ is proportional to the variation of the sum of the magnetic moments of independent loops in the magnetic field. When the sample is divided into parts of size R, the number of such loops does not change if $R > R_\varphi$. Therefore the field-induced variation of the total magnetic moment of a set of independent loops should not change either. This conclusion disagrees with the experiment.

According to our model, the resulting signal $\Delta P_\omega(H)$ is determined by conductivity variations and is therefore sensitive to the distribution and magnitude of the superconducting current in the sample. The signal $\Delta P_\omega(H)$ is proportional to the magnitude of the screening current I_ω. According to the Maxwell equations, the current I_ω is proportional to the sample dimensions R. Therefore we have $\Delta P_\omega(H) \propto R$. In the experiment the signal $\Delta P_\omega(H)$ was approximately proportional to the dimensions of separate parts and decreased by several orders of magnitude, when the sample was divided (Fig. 3). This clearly indicates that, in spite of a certain similarity between the approaches of Refs. 2 and 3, the latter gives a more adequate relation between the magnetic field and conductivity in YBaCuO.

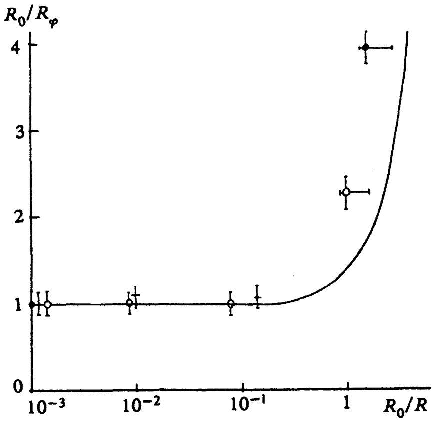

FIG. 2. R dependence of the maximum loop size R_φ, when the sample is divided into squares of side length $2R$. Solid curve: Eq. (16) without fitting parameters ($R_0 = R_\varphi$ for the undivided sample) different points correspond to different YBaCuO samples; $T = 77$ K.

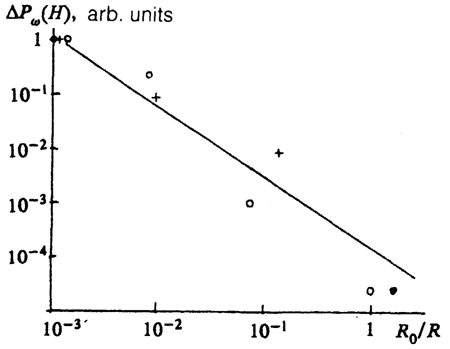

FIG. 3. Field-induced changes in the screening, $\Delta P_\omega(H) = P_\omega(H) - P_\omega(0)$, versus the size $2R$ of separate sample pieces; different points correspond to different YBaCuO samples; $T = 77$ K.

Temperature dependence of screening in magnetic field

In Fig. 4 we have plotted the signal P_ω in the receiving coil at the frequency of 100 kHz versus the magnetic field H for different temperatures. Approximating each curve by Eq. (14), we find the temperature dependence of the greatest size R_φ of effective loops. This dependence for a YBaCuO film produced by laser evaporation is shown in Fig. 5. It is seen that, as $T \to T_c$, the maximum size R_φ of the effective loops grows. A similar behavior is observed in other YBaCuO films. We believe that this can be attributed to the decrease in the density ρ of the superconducting cluster (critical subnetwork) as $T \to T_c$, which reduces the probability of trajectory intersection ($p \propto \rho$) and, as a consequence, increases the maximum size of the shunting loops: $L_\varphi = d/p \propto 1/\rho$ as $T \to T_c$.

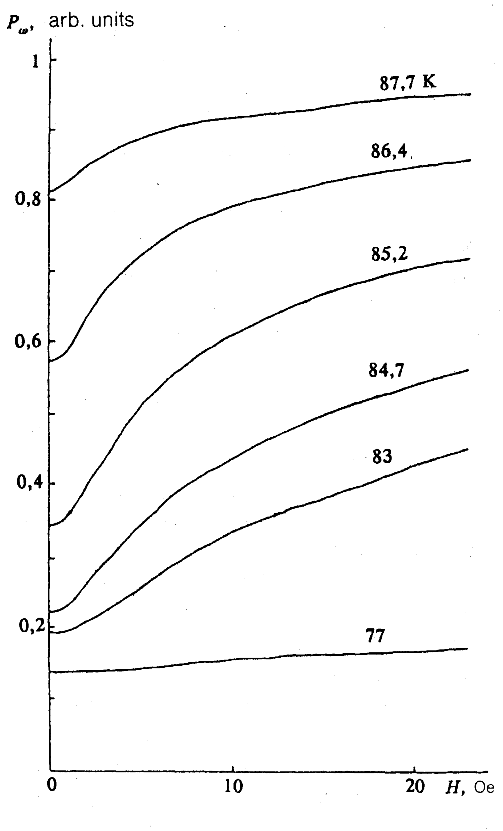

FIG. 4. H magnetic field dependence of P_ω screening in a YBaCuO film for different temperatures; $H_\omega = 20$ mOe, $\omega/2\pi = 100$ kHz, $T_c = 91$ K; laser evaporation.

This dependence $R_\varphi(T)$ disagrees with the results of our brief report,[16] where we argued that R_φ is temperature-independent. In Ref. 16 the experimental curves were processed under the assumption that the constant magnetic field component normal to the film is screened in the same way as the alternating field H_ω. As further studies showed, this assumption is not valid.

Correction to the critical current in the effective loop model

According to the model presented above, the superconducting loops X_k shunt critical links of the superconducting cluster, giving rise to the increase in conductivity at a frequency ω as well as to the changes in the current-carrying ability at zero frequency. The critical superconducting current I_c increases when the shunting superconducting loops are connected, since extra channels for current transport arise. Evidently, when the critical current in all the elements of the superconducting cluster increases, so does the critical current density J_c of the sample.

Let the loop X_k shunt the critical link of the superconducting cluster between the grains i and j and let a constant phase difference φ_{ij} corresponding to the critical link current I_c^0 be given between these grains. The phase difference at the Josephson junction of the loop X_k is defined by (9). Substituting (9) into (10), we have for the current through the loop X_k:

$$I_k = I_J[\sin\varphi_{ij}\cos(2\pi\Phi_k/\Phi_0) + \cos\varphi_{ij}\sin(2\pi\Phi_k/\Phi_0)]\ . \quad (17)$$

To find the total correction to the current, we need to sum the currents in all loops X_k in parallel with the critical link. When we average over realizations, we have to bear in mind that the loop X_k can, with equal probability, be on either side of the chain containing the critical link. This causes the loop to be traversed in the opposite direction and, consequently, reverses the sign of the magnetic flux Φ_k enclosed by this loop. Therefore averaging causes the second term in the right-hand side of Eq. (17) to vanish. Thus, according to (17), the average critical current changes by

$$\langle \Delta I_c \rangle \approx \langle I_J \sin\varphi_{ij} \rangle \langle \sum_{X_k} \cos(2\pi\Phi_k/\Phi_0) \rangle\ . \quad (18)$$

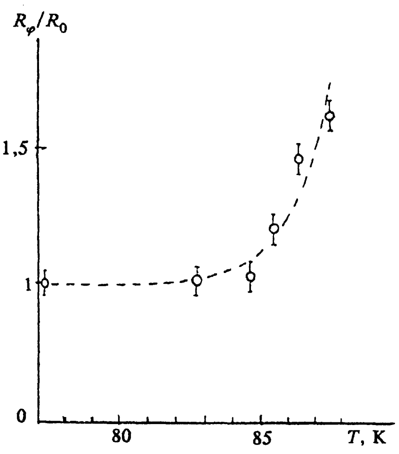

FIG. 5. Temperature dependence of the maximum effective loop size R_φ for a YBaCuO film obtained by laser evaporation; $\omega/2\pi = 100$ kHz.

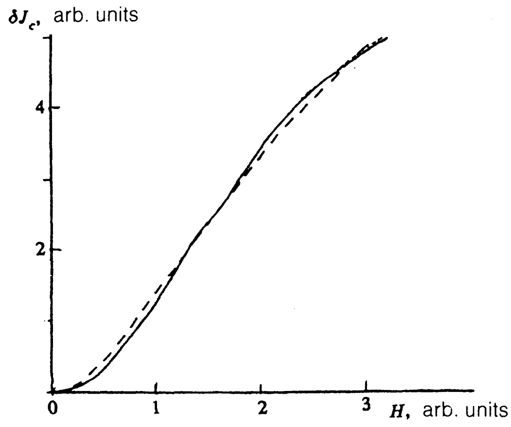

FIG. 6. Critical current density variation $\delta J_c = J_c(0) - J_c(H)$ versus magnetic field H. Solid curve: results reported in Ref. 13; dashed curve: the effective loop model [Eq. (19)].

Using the same reasoning as above, when we derived Eq. (14), we find for the field-induced changes in the correction to the critical current density connected with the superconducting loops X_k:

$$\langle \delta J_c \rangle = \langle J_c(0) - J_c(H) \rangle$$

$$= C \int_0^\infty [1 - \cos(\alpha H x)] \exp\left(-\frac{x}{L_\varphi}\right) \frac{dx}{x}, \qquad (19)$$

where C is a field-independent constant. The relation $\delta J_c(H)$ is the same as for the correction to the conductivity (14). We have compared (19) with the experimental results of Ref. 13 (see Fig. 6).

Note the difference between the effective-loop model and other weak-link models used to account for the magnetic field dependence of the critical current (see, e.g., Refs. 12 and 13). For one thing, they differ in the scales of magnetic fields affecting the critical current. In the model considered in Ref. 13 this scale is $H = \Phi_0/d^2$, while in the effective loop model it is $H = \Phi_0 \rho/d^2$ and for $\rho \ll 1$ is much smaller. The important consequence of our approach is that Eqs. (14) and (19) do not depend on the details of the granular system (i.e., on the distribution of grains in size and shape, their location, the strength of intergranular links, etc.).

5. CONCLUSION

Using a model of a highly nonuniform superconducting medium we have found field-induced changes in the conductivity at frequency ω and the critical current of a granular superconductor. These changes are related to the changes in the correction to the conductivity of the granular superconductor connected with superconducting loops in parallel with critical links. The greatest size R_φ of the shunting loops is inversely proportional to the critical subnetwork density ρ, and for a strongly inhomogeneous system is much larger than the grain size. The magnetic field dependence of the conductivity and critical current does not change with sample microstructure and is completely determined by the size R_φ.

By dividing the sample into smaller and smaller pieces we have obtained direct evidence for the existence of effective loops and measured their maximum size R_φ. In accordance with the model suggested, the changes in the screening in films divided into separate pieces in the magnetic field are proportional to the dimensions of the separate pieces.

When the sample temperature approaches the superconducting transition temperature the size R_φ grows, which is related to the decreasing density of the superconducting critical subnetwork as $T \to T_c$.

The magnetic field dependence of the critical current found in the effective loop model agrees with experimental results on YBaCuO reported in Ref. 13.

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[1)]The probability that there is one weak link on a trajectory consisting of N granules is equal to $P_1 \approx N/\xi_0$; for two weak links we have $P_2 \approx N^2/\xi_0^2$. For $N \lesssim 1/P \approx 1/\rho$ [see Eq. (8)] from (6) we find $P_2/P_1 = N/\xi_0 \lesssim 1/\rho\xi_0 = 1/\xi^{1-\beta}$. Since $\beta = 0.14$ (Ref. 14), for $\xi_0 \gg 1$ we have $P_2/P_1 \ll 1$, and the probability of two weak links in succession can be ignored.

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