Atom-field phasing dynamics in coherent interactions

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A system of non-truncated Maxwell–Bloch equations is used to analyze coherent-resonance interactions of light in a two-level system of atoms. The integral of motion of the non-truncated equations is determined and used to demonstrate the evolution of the phase difference between the light wave and the polarization wave in generation and absorption of a π pulse and in propagation of a 2π pulse.

1. INTRODUCTION

In connection with the progress in the method of generating ultrashort laser pulses consisting of only a few optical-oscillation periods, solutions of non-truncated Maxwell–Bloch equations are attracting ever increasing attention.[1–5] The change from equations for slowly varying amplitudes to non-truncated equations is due to refutation of a number of traditional approximations, such as narrowness of the pulse spectrum compared with its carrier frequency, or weakness of the nonlinearity of the medium, as well as to promulgation of theories of new physical phenomena. Noteworthy among the latter is the unipolar soliton, the feasibility of which is demonstrated in Ref. 6, and results[3] of numerical simulation and analysis of the stability of such excitations. Under discussion are the possibility of amplifying pulses by nonresonant media[1,4,5] and a "blue" shift of the frequency of propagating pulses.[1,4] The features of diffractive spreading of ultrashort pulses are under investigation.[7]

The study of the properties of non-truncated Maxwell–Bloch equations is also of purely theoretical interest, since such an analysis can lead both to new laws governing the course of the processes, and provide a better understanding of well-known phenomena. Thus, for example, the existing theories of coherent interaction of radiation with resonant media[8–11] not only lead to a qualitative description of the phenomena but permit also a calculation of the field-strength profile in a medium. Since, however, these theories are based as a rule on equations for slowly-varying amplitudes, they cannot claim to yield a consistent description of the coherence dynamics, i.e., of the establishment of phase relations between the field and the medium. Even though the threshold processes evolve only during the very start of the interaction, they can influence also all the succeeding stages. Study of the dynamics of coherence dynamics is also of general theoretical interest. In the present paper, using a system of non-truncated equations, we investigate the dynamics of the phase difference in the generation of π-pulses and propagation of 2π-pulses in a resonant medium.

2. BASIC EQUATIONS

The system of equations for the interaction of an ensemble of two-level atoms with radiation can be written in the form

$$\Delta A^{\pm} - \frac{1}{c^2}\frac{\partial^2 A^{\pm}}{\partial t^2} = -\frac{4\pi}{c} j^{\pm},$$

$$\frac{\partial j^{\pm}}{\partial t} \mp i\omega_0 j^{\pm} = \pm \frac{i\omega_0^2 |d|^2}{\hbar c} A^{\pm}\rho,$$

$$\frac{\partial \rho}{\partial t} = \frac{2i}{\hbar c}(j^+A^- - j^-A^+). \tag{1}$$

The system (1) pertains to a transition between the ground level $|1\rangle = |l,m\rangle$ and an excited one $|2\rangle = |l+1, m+1\rangle$, where l and m are the quantum numbers of the angular momentum and of its projection. Here $d = \langle 2|\hat{d}_+|1\rangle$ is the matrix element of the transition,

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{e}^{(+)}A^+(\mathbf{r}, t) + \mathbf{e}^{(-)}A^-(\mathbf{r}, t),$$
$$\mathbf{e}^{(\pm)} = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2},$$

$$j^+(\mathbf{r}, t) = i\omega_0 d\sum_{i=1}^{N}\langle\sigma_{i+}\rangle\delta(\mathbf{r}-\mathbf{r}_i), \quad \rho(\mathbf{r}, t) = \sum_{i=1}^{N}\langle\sigma_{i3}\rangle\delta(\mathbf{r}-\mathbf{r}_i),$$

where σ_+ and σ_3 are Pauli spin matrices.

The system (1) has an integral of motion in the form

$$j^+(\mathbf{r}, t)j^-(\mathbf{r}, t) + \frac{|m|^2}{4}\rho^2(\mathbf{r}, t)$$
$$= j^+(\mathbf{r}, 0)j^-(\mathbf{r}, 0) + \frac{|m|^2}{4}\rho^2(\mathbf{r}, 0), \tag{2}$$

where $m = i\omega_0 d$. Therefore, putting

$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r})\cos\theta(\mathbf{r}, t), \tag{3}$$

we obtain for $j^{\pm}(\mathbf{r},t)$

$$j^{\pm}(\mathbf{r}, t) = \frac{|m|}{2}\rho_0(\mathbf{r})\sin\theta(\mathbf{r}, t)\exp(\pm i\varphi(\mathbf{r}, t)). \tag{4}$$

From (1) we obtain for the variables $\theta(\mathbf{r},t)$ and $\varphi(\mathbf{r},t)$ the equations

 1063-7761/93/040568-05$10.00

$$\frac{\partial\theta}{\partial t} = i\frac{|m|}{\hbar c}(A^+e^{-i\varphi} - A^-e^{i\varphi}), \tag{5a}$$

$$\left(\frac{\partial\varphi}{\partial t} - \omega_0\right)\sin\theta = \frac{|m|}{\hbar c}(A^+e^{-i\varphi} + A^-e^{i\varphi})\cos\theta. \tag{5b}$$

Putting

$$A^{\pm}(\mathbf{r}, t) = B(\mathbf{r}, t)\exp(\pm i\psi(\mathbf{r}, t)), \tag{6}$$

we can represent Eqs. (5a) and (5b) in the form

$$\frac{\partial\theta}{\partial t} = \frac{2|m|}{\hbar c}B\sin(\varphi - \psi), \tag{7a}$$

$$\left(\frac{\partial\varphi}{\partial t} - \omega_0\right)\sin\theta = \frac{2|m|}{\hbar c}B\cos(\varphi - \psi)\cos\theta. \tag{7b}$$

The equations for the variables $B(\mathbf{r},t)$ and $\psi(\mathbf{r},t)$ follow from the first two equations of the system (1):

$$2\left[(\nabla B\nabla\psi) - \frac{1}{c^2}\frac{\partial B}{\partial t}\frac{\partial\psi}{\partial t}\right] + B\left(\Delta\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}\right) = -\frac{2\pi|m|}{c}\rho_0\sin\theta\sin(\varphi - \psi), \tag{8a}$$

$$B\left[(\nabla\psi)^2 - \frac{1}{c^2}\left(\frac{\partial\psi}{\partial t}\right)^2\right] - \left(\Delta B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}\right) = \frac{2\pi|m|}{c}\rho_0\sin\theta\cos(\varphi - \psi). \tag{8b}$$

Equation (8a) can be transformed into

$$\nabla(B^2\nabla\psi) - \frac{1}{c^2}\frac{\partial}{\partial t}\left(B^2\frac{\partial\psi}{\partial t}\right) = \pi\hbar\frac{\partial\rho}{\partial t}, \tag{9}$$

which is the law of variation of the field current density. In fact, from the first two equations of the system (1) we have

$$\frac{i\hbar}{c}\frac{\partial}{\partial t}\left[A^+\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}A^-\right) - \left(\frac{i\hbar}{c}\frac{\partial}{\partial t}A^+\right)A^-\right] + i\hbar\nabla[A^+(-i\hbar\nabla A^-) - (-i\hbar\nabla A^+)A^-] = -2\pi c\hbar^2\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}\rho\right). \tag{10}$$

It is evident from (10) that the components of the field current-density 4-vector are

$$J_0 = A^+\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}A^-\right) - \left(\frac{i\hbar}{c}\frac{\partial}{\partial t}A^+\right)A^- = \frac{2\hbar}{c}B^2\frac{\partial\psi}{\partial t},$$
$$\mathbf{J} = A^+(-i\hbar\nabla A^-) - (-i\hbar\nabla A^+)A^- = -2\hbar B^2\nabla\psi. \tag{11}$$

The relations (11) cast light on the physical meaning of the new field variables $B(\mathbf{r},t)$ and $\psi(\mathbf{r},t)$ used in the exposition that follows and replacing the vector-potential components $A^{\pm}(\mathbf{r},t)$ contained in (1).

3. PARTICULAR SOLUTIONS

The variable $\psi(\mathbf{r},t)$ determines the values of the quadrature components of the field, and $\varphi(\mathbf{r},t)$ the polarization current density. This is made particularly clear by the use of the following transformations

$$A(\mathbf{r}, t) = \frac{1}{2}(A^+ + A^-) = B(\mathbf{r}, t)\cos\psi(\mathbf{r}, t),$$
$$A_1(\mathbf{r}, t) = \frac{1}{2i}(A^+ - A^-) = B(\mathbf{r}, t)\sin\psi(\mathbf{r}, t),$$
$$j(\mathbf{r}, t) = \frac{1}{2}(j^+ + j^-) = p(\mathbf{r}, t)\cos\varphi(\mathbf{r}, t),$$
$$j_1(\mathbf{r}, t) = \frac{1}{2i}(j^+ - j^-) = p(\mathbf{r}, t)\sin\varphi(\mathbf{r}, t), \tag{12}$$

where

$$p(\mathbf{r}, t) = \frac{|m|}{2}\rho_0(\mathbf{r})\sin\theta(\mathbf{r}, t). \tag{13}$$

According to (7a) and (7b), the angle $\varphi(\mathbf{r},t)$ can be written in the form

$$\varphi(\mathbf{r}, t) = \omega_0 t + \eta(\mathbf{r}, t), \tag{14}$$

where

$$\tan\eta(\mathbf{r}, t) = \frac{\nu\int_0^t B(t')\cos(\omega_0 t' - \psi)\cos\theta(t')dt' + \sin\theta_0\sin\varphi_0}{\nu\int_0^t B(t')\sin(\omega_0 t' - \psi)\cos\theta(t')dt' + \sin\theta_0\cos\varphi_0}. \tag{15}$$

Here

$$\nu = \frac{2|m|}{\hbar c}, \qquad \theta_0 = \theta(\mathbf{r}, 0), \qquad \varphi_0 = \varphi(\mathbf{r}, 0).$$

From the definition of the angles $\theta(\mathbf{r},t)$ and $\varphi(\mathbf{r},t)$ it follows that $\theta(\mathbf{r},t)$ determines the change of the atomic-subsystem energy, and $\varphi(\mathbf{r},t)$ determines the polarization phase, i.e., the dispersive properties of the medium. It is seen from Eqs. (7a) and (7b) that the field-polarization field difference $\varphi - \psi$ determines the relation between the rates of change of the angles θ and φ. At $\varphi = \psi$ the rate of the radiative transitions is zero, $\partial\theta/\partial t = 0$, and the frequency dispersion is a maximum:

$$\frac{\partial\varphi}{\partial t} = \omega_0 + \frac{2|m|}{\hbar c}B\operatorname{ctg}\theta_0.$$

It follows from Eq. (9) that at $\partial\theta/\partial t = 0$ the field amplitude at the point $\mathbf{r}$ is altered only as a result of radiation transport. At $\varphi - \psi = \pi/2$ the maximum rate of the radiative transitions is

$$\frac{\partial\theta}{\partial t} = \frac{2|m|}{\hbar c}B, \text{ and } \frac{\partial\varphi}{\partial t} = \omega_0.$$

Note that if $\theta_0 = 0$ and $\psi(\mathbf{r},t) = \omega_0 t + \Delta\psi(\mathbf{r})$ it follows from (15) that

$$\operatorname{tg}\eta(\mathbf{r}, t) = \operatorname{ctg}\Delta\psi(\mathbf{r}),$$

i.e.,

$$\eta(\mathbf{r}, t) = \pi/2 + \Delta\psi(\mathbf{r}). \tag{16}$$

Let $\psi(\mathbf{r},t) = \omega_0 t - kx$ and $\varphi - \psi = \pi/2$, then the system of equations (7a), (7b), (8a), and (8b) takes the form

$$\frac{\partial B}{\partial t} + c\frac{k}{\varkappa}\frac{\partial B}{\partial x} = \frac{\pi|m|}{\varkappa}\rho_0 \sin\theta,$$

$$\frac{\partial^2 B}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} + (\varkappa^2 - k^2)B = 0, \qquad \frac{\partial\theta}{\partial t} = \frac{2|m|}{\hbar c}B, \tag{17}$$

where $\varkappa = \omega_0/c$. From the first and last equations of (17) we obtain

$$\frac{\partial^2\theta}{\partial t^2} + cn\frac{\partial^2\theta}{\partial t\partial x} = \frac{2\pi|m|^2}{\hbar\omega}\rho_0\sin\theta, \tag{18}$$

where $n = k/\varkappa$. The solution of this equation has the well-known form

$$\theta = 4\,\mathrm{arctg}\left[\exp\left(\frac{1}{\tau_0}\left(t - \frac{x}{v}\right)\right)\right], \tag{19}$$

where the relation between the pulse duration τ_0 and its velocity v is

$$\frac{v}{cn} = 1 - \frac{\tau_0^2}{\tau_c^2}R_0. \tag{20}$$

Here

$$\frac{1}{\tau_c^2} = \frac{2\pi|m|^2}{\hbar\omega}\frac{N}{V}, \qquad R_0 = \frac{\rho_0}{N/V}, \tag{21}$$

N/V is the density of the resonant atoms, $R_0 = 1$ for an initially excited medium, and $R_0 = -1$ for an unexcited one. The pulse amplitude B is given by

$$B(x,t) = \frac{2}{\nu\tau_0}\left[\mathrm{ch}\left(\frac{1}{\tau_0}\left(t - \frac{x}{v}\right)\right)\right]^{-1}. \tag{22}$$

Expression (22), however, must satisfy simultaneously also the second equation of (17). This is possible only if $k = \varkappa$ and $v = c$. It follows from (20) that in this case $\tau_0 = 0$, so that a soliton described by (22) cannot propagate in a nondispersive medium.

4. PHASE-MODULATED PULSE

The system (7a)–(7b) can be transformed into

$$\frac{\partial}{\partial t}(\sin\theta\cos(\varphi)) - \left(\frac{\partial\psi}{\partial t} - \omega_0\right)\sin\theta\sin(\varphi - \psi) = 0,$$

$$\frac{\partial}{\partial t}(\sin\theta\sin(\varphi - \psi)) + \left(\frac{\partial\psi}{\partial t} - \omega_0\right)\sin\theta\cos(\varphi - \psi) = \nu B\cos\theta. \tag{23}$$

Introducing the notation

$$u = R_0\sin\theta\cos(\varphi - \psi), \qquad v = R_0\sin\theta\sin(\varphi - \psi),$$

$$w = R_0\cos\theta, \tag{24}$$

we can transform the equations in (23) into

$$\frac{\partial u}{\partial t} = \left(\frac{\partial\psi}{\partial t} - \omega_0\right)v,$$

$$\frac{\partial v}{\partial t} + \left(\frac{\partial\psi}{\partial t} - \omega_0\right)u = \nu Bw, \qquad \frac{\partial w}{\partial t} = -\nu Bv. \tag{25}$$

The last equation is a consequence of the first two, since the system has an obvious integral of the motion

$$u^2 + v^2 + w^2 = 1. \tag{26}$$

Let $\partial\psi/\partial t$ be defined as

$$\frac{\partial\psi}{\partial t} = \omega_0 + \Delta\,\mathrm{th}\,\Phi\frac{\partial\Phi}{\partial t}, \tag{27}$$

so that the solution of Eqs. (25) can be represented as

$$u(\mathbf{r},t) = -R_0\frac{\mathrm{th}\,\gamma(\mathbf{r},t)}{\mathrm{ch}\,\Phi(\mathbf{r},t)}, \qquad v(\mathbf{r},t) = R_0\frac{1}{\mathrm{ch}\,\gamma(\mathbf{r},t)\mathrm{ch}\,\Phi(\mathbf{r},t)},$$

$$w(\mathbf{r},t) = -R_0\mathrm{th}\,\Phi(\mathbf{r},t). \tag{28}$$

The variables $\Phi(\mathbf{r},t)$ and $\gamma(\mathbf{r},t)$ satisfy the equations

$$\frac{\partial\Phi}{\partial t} = \nu B\frac{\mathrm{ch}\,\Phi}{\mathrm{ch}\,\gamma}, \qquad \frac{\partial\gamma}{\partial t} = \nu B\,\mathrm{ch}\,\Phi(\mathrm{sh}\,\gamma - \Delta). \tag{29}$$

It follows from the last equation that $\partial\gamma/\partial t = 0$ for $\sinh\gamma = \Delta$. In this case

$$u(\mathbf{r},t) = -R_0\frac{\Delta}{(1+\Delta^2)^{1/2}}\frac{1}{\mathrm{ch}\,\Phi},$$

$$v(\mathbf{r},t) = R_0\frac{1}{(1+\Delta^2)^{1/2}}\frac{1}{\mathrm{ch}\,\Phi},$$

$$B = \frac{(1+\Delta^2)^{1/2}}{\nu}\frac{1}{\mathrm{ch}\,\Phi}\frac{\partial\Phi}{\partial t}. \tag{30}$$

By analogy with the last equation of the system (17), we introduce an angle $\vartheta(\mathbf{r},t)$ in the form

$$B = \frac{1}{\nu}\frac{\partial\vartheta}{\partial t}, \tag{31}$$

then

$$\frac{1}{\nu}\frac{\partial\vartheta}{\partial t} = \frac{(1+\Delta^2)^{1/2}}{\nu}\frac{1}{\mathrm{ch}\,\Phi}\frac{\partial\Phi}{\partial t},$$

from which it follows that

$$\vartheta = 2(1+\Delta^2)^{1/2}\mathrm{arctg}(\exp(\Phi)), \tag{32}$$

or

$$\frac{1}{\mathrm{ch}\,\Phi} = \sin\left(\frac{\vartheta}{(1+\Delta^2)^{1/2}}\right). \tag{33}$$

We introduce the angle $\theta_1(\mathbf{r},t)$:

$$\theta_1(\mathbf{r}, t) = \frac{\vartheta}{(1 + \Delta^2)^{1/2}}, \qquad (34)$$

and then Eqs. (28) take the form

$$u(\mathbf{r}, t) = -R_0 \frac{\Delta}{(1 + \Delta^2)^{1/2}} \sin\theta_1,$$

$$v(\mathbf{r}, t) = R_0 \frac{1}{(1 + \Delta^2)^{1/2}} \sin\theta_1, \qquad w(\mathbf{r}, t) = R_0 \cos\theta_1. \qquad (35)$$

Comparing (24) with (35) we get

$$\cos(\varphi - \psi) = \frac{\Delta}{(1 + \Delta^2)^{1/2}}, \qquad \sin(\varphi - \psi) = \frac{1}{(1 + \Delta^2)^{1/2}},$$
$$\theta_1(\mathbf{r}, t) = \theta(\mathbf{r}, t). \qquad (36)$$

Thus, in the stationary case, the difference between the phases of the field $\psi(\mathbf{r},t)$ and the polarization $\varphi(\mathbf{r},t)$ is determined by the value of the dispersion parameter Δ.

5. DYNAMICS OF THE PHASE DIFFERENCE IN A DISPERSIVE MEDIUM

Consider now a general case, when $\partial\gamma/\partial t \neq 0$. The system (30) has the integral of motion

$$\frac{1}{\mathrm{ch}\,\Phi}\left(\frac{|\mathrm{sh}\,\gamma - \Delta|}{\mathrm{ch}\,\gamma}\exp[-\Delta\,\mathrm{arctg}(\mathrm{sh}\,\gamma)]\right)^{1/(1+\Delta^2)}$$
$$= \frac{1}{\mathrm{ch}\,\Phi_0}\left(\frac{|\mathrm{sh}\,\gamma_0 - \Delta|}{\mathrm{ch}\,\gamma_0}\exp[-\Delta\,\mathrm{arctg}(\mathrm{sh}\,\gamma_0)]\right)^{1/(1+\Delta^2)}, \qquad (37)$$

where

$$\Phi_0 = \Phi(\mathbf{r}, 0), \qquad \gamma_0 = \gamma(\mathbf{r}, 0).$$

Comparing Eqs. (24) and (28) and introducing the angle

$$\alpha = \varphi - \psi - \pi/2, \qquad (38)$$

we obtain

$$\sin\theta[|\sin(\alpha - \delta)|\exp(-\Delta\alpha)]^{1/(1+\Delta^2)}$$
$$= \sin\theta_0[|\sin(\alpha_0 - \delta)|\exp(-\Delta\alpha_0)]^{1/(1+\Delta^2)}. \qquad (39)$$

where

$$\sin\delta = \frac{\Delta}{(1 + \Delta^2)^{1/2}}. \qquad (40)$$

It follows from (39) that the phase difference tends with increase of θ to the value

$$\varphi - \psi = \pi/2 + \delta, \qquad (41)$$

and the phase spread $\Delta\alpha = \Delta(\varphi - \psi)$ is proportional to

$$\Delta\alpha = \alpha - \delta \sim \left(\frac{\sin\theta_0}{\sin\theta}\right)^{1+\Delta^2}$$

For example, if $\theta_0 = 10^{-3}$ and $\Delta = 1$, we have at $\theta = 10^{-2}$

$$\Delta\alpha = 10^{-2}.$$

6. 2π-PULSE

Representing the Bloch angle $\theta(\mathbf{r},t)$ in the form

$$\theta = 4\,\mathrm{arctg}(\exp(\Phi)), \qquad (42)$$

we obtain for the variables u, v, and w the expressions

$$u(\mathbf{r}, t) = R_0 \frac{2\,\mathrm{sh}\,\Phi(\mathbf{r}, t)\mathrm{th}\,\gamma(\mathbf{r}, t)}{\mathrm{ch}^2\Phi(\mathbf{r}, t)},$$

$$v(\mathbf{r}, t) = -R_0 \frac{2\,\mathrm{sh}\,\Phi(\mathbf{r}, t)}{\mathrm{ch}\,\gamma(\mathbf{r}, t)\mathrm{ch}^2\Phi(\mathbf{r}, t)},$$

$$w(\mathbf{r}, t) = R_0 \frac{\mathrm{sh}^2\Phi(\mathbf{r}, t) - 1}{\mathrm{ch}^2\Phi(\mathbf{r}, t)}. \qquad (43)$$

For $\partial\gamma/\partial t = 0$ the frequency modulation is given by

$$\frac{\partial\psi}{\partial t} - \omega_0 = \Delta \frac{\mathrm{sh}^2\Phi(\mathbf{r}, t) - 1}{\mathrm{sh}\,\Phi(\mathbf{r}, t)\mathrm{ch}\,\Phi(\mathbf{r}, t)}\frac{\partial\Phi}{\partial t}. \qquad (44)$$

Substituting (43) and (44) in (25) it is easy to obtain the following equations for the variables $\Phi(\mathbf{r},t)$ and $\gamma(\mathbf{r},t)$:

$$\frac{\partial\Phi}{\partial t} = \frac{1}{2}\nu B \frac{\mathrm{ch}\,\Phi}{\mathrm{ch}\,\gamma}, \qquad \frac{\partial\gamma}{\partial t} = \frac{1}{2}\nu B(\mathrm{sh}\,\gamma - \Delta)\frac{\mathrm{sh}^2\Phi - 1}{\mathrm{sh}\,\Phi}. \qquad (45)$$

The system (45) has the following integral of motion:

$$\frac{\mathrm{sh}\,\Phi}{\mathrm{ch}^2\Phi}\left(\frac{|\mathrm{sh}\,\gamma - \Delta|}{\mathrm{ch}\,\gamma}\exp[-\Delta\,\mathrm{arctg}(\mathrm{sh}\,\gamma)]\right)^{1/(1+\Delta^2)}$$
$$= \frac{\mathrm{sh}\,\Phi_0}{\mathrm{ch}^2\Phi_0}\left(\frac{|\mathrm{sh}\,\gamma_0 - \Delta|}{\mathrm{ch}\,\gamma_0}\exp[-\Delta\,\mathrm{arctg}(\mathrm{sh}\,\gamma_0)]\right)^{1/(1+\Delta^2)} \qquad (46)$$

Recognizing that now

$$\sin\theta = -\frac{2\,\mathrm{sh}\,\Phi}{\mathrm{ch}^2\Phi}, \qquad (47)$$

and, as before, introducing the angle $\alpha = \varphi - \psi - \pi/2$, we get

$$\sin\theta[|\sin(\alpha - \delta)|\exp(-\Delta\alpha)]^{1/(1+\Delta^2)}$$
$$= \sin\theta_0|\sin(\alpha_0 - \delta)|\exp(-\Delta\alpha_0)]^{1/(1+\Delta^2)}. \qquad (48)$$

The substantial difference from the preceding case, however, is that now the angle θ ranges from 0 to 2π, so that α becomes indeterminate when $\theta = \pi$. The cause of this indeterminacy is that at $\theta = \pi$ the quadrature components of the polarization vanish: $u = v = 0$, so that the phase $\varphi(\mathbf{r},t)$ becomes indeterminate. This indeterminacy obtains only for exact equality of the field carrier frequency to the resonance-transition frequency. At any finite detuning these indeterminacies of the polarization phase vanish. Naturally, there is no indeterminacy also in the case of inhomogeneous broadening of the atomic-transition line.

6. CONCLUSION

The reported investigations show how the rate $\partial\theta/\partial t$ of energy emission by the atomic subsystem depends on the phase difference $\varphi - \psi$ between the light wave and the polarization wave. The decay rate $\partial\theta/\partial t$ is a maximum at $\varphi - \psi = \pi/2$ and vanishes at $\varphi - \psi = 0$. This phase difference $\varphi - \psi$ tends quite rapidly to a stationary value with increase of the angle $\theta(\mathbf{r},t)$. The stationary value is determined by the rate of frequency dispersion

$$\varphi - \psi = \pi/2 + \arcsin[\dot{\Delta}/(1 + \Delta^2)^{1/2}].$$

The relations derived are undoubtedly of interest for the development of methods of controlling generation-pulse parameters in superradiance processes, and for the developments of spectroscopy methods in which coherent processes are used.

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