Suppression of photon fluctuations in a light flux nonlinearly amplified in a layer of a resonance medium

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Under conditions of saturated gain in a layer of a resonant medium the excess noise (positive or negative) of the initial light is suppressed and the photons become Poissonian. If, however, the amplifying atoms are regularly excited into the working state, then, on the one hand, the light becomes sub-Poissonian in the entire observation band and, on the other, the initial picture of the photocurrent spectrum remains completely unchanged. The latter feature is also characteristic for saturated absorption under conditions such that the absorbing atoms undergo regular excitation. Two methodological results are obtained: It is shown how the normal (Glauber) diagonal representation for the density matrix can be used together with the kinetic equations when the electromagnetic field exhibits quantum features, and it is shown how the required averages can be constructed from the kinetic equations for the density matrix of the electromagnetic field.

Two phenomena will be discussed in this paper. First, it will be shown that a quantum amplifier operating in the saturation regime, unlike a linear amplifier,[1] is capable of effectively suppressing excess noise in the initial radiation, transforming super-Poissonian light into Poissonian light. Second, by combining saturated gain with regular excitation of amplifying atoms it is also possible to suppress the shot noise, i.e., to regularize the photon flux. Since the predicted suppression can be complete, such a system could be a promising source of quantum light.

The first phenomenon can be understood qualitatively on the basis of quite simple arguments based on semiclassical phenomenology, according to which a Langevin equation of the following form can be written for the intensity of a single-mode field (written here for the number of photons):

$$\dot{n} = A(1 + \beta n)^{-1} n + f(t) .$$

Here the stochastic source $f(t)$ determines the contribution of spontaneous emission to the mode. It is well known that at saturation the contribution of amplifying atoms to this source vanishes, and the contribution of the absorbing atoms saturates and becomes independent of the intensity of the field. We assume that the intensity fluctuates weakly around its average value: $n = \bar{n} + \varepsilon$, $\varepsilon \ll \bar{n}$. Then we write the following equation:

$$\dot{\varepsilon} = A(1 + \beta\bar{n})^{-2}\varepsilon + f(t) \quad \text{with} \quad \dot{\bar{n}} = A(1 + \beta\bar{n})^{-1}\bar{n} .$$

It is now easy to see that if only amplifying atoms are present, the intensity fluctuations do not change in magnitude $\dot{\varepsilon} = 0$, though the average intensity itself increases linearly in time. But this means that as the system evolves the excess noise, whose contribution is determined by the quantity ε, becomes weaker than the Poisson shot noise. At the same time, the presence of absorbing atoms changes the picture, since then $\dot{\varepsilon} = f(t)$.

It is obvious that due to the fluctuation of the number of amplifying atoms, which stems from the stochastic character of the excitation of the medium, amplification should transform the initial, well-regulated (sub-Poissonian) photon flux into a Poissonian flux. From this standpoint, it is also obvious that with saturated gain the noise cannot become weaker than shot noise. The obvious question arises: what happens if there are no fluctuations in the number of amplifying atoms? We can then expect quantum effects to appear, especially since this has already been observed in the case of a quantum generator.

Great difficulties usually arise in attempts to describe the spatial evolution in quantum statistical theory. These difficulties can probably be overcome by different methods. For example, it is widely believed that the problem involving a layer of a medium can be solved as an intracavity problem, replacing the time variable by the spatial variable. In this approach the Lamb–Scully theory[2] for a laser can be used to formulate the problem of amplification in a layer. The drawback is that since time completely vanishes from the theory, this theory does not permit constructing multitime averages, which are important for spectral analysis. The arbitrariness of the approach is also unsatisfactory. Obviously, such an approach is entirely valid in some problems. But, at the same time, we note that "forced" methods can distort subtle effects, such as statistical effects.

Here we employ the theory of the space-time density matrix for the electromagnetic field and the kinetic equation for this matrix.[3] We have already employed this approach, in part, in Ref. 4 in order to describe the evolution of light passing through a layer of nonlinear medium with parametric mixing of the waves. However, the specific nature of this problem allowed the Heisenberg representation to be used, which greatly simplified the calculations. In the present case, however, we are forced to work with the equations for the density matrix. Two basic methodological results will be obtained: First, the limits within which the normal (Glauber) diagonal representation can be employed in the diffusion approximation will be indicated for both classical and quantum (sub-Poissonian, squeezed, etc.) fields. This result is especially important precisely for the space-time theory, since it significantly simplifies the calculations. Second, we will show how to handle the kinetic equations in order to obtain the required averages. Incidentally, this approach will also be correct in the standard, purely temporal theory of the Lamb–Scully type.[2]

First, the fundamentals of the theory of transfer of quantized radiation will be presented. Next, some practical

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recommendations and prescriptions will be considered. Finally, on the basis of all this, the behavior of light interacting with a layer of matter in the saturation regime will be investigated.

1. LOCAL OSCILLATORS OF THE ELECTROMAGNETIC FIELD

The concept of local field oscillators (this term did not appear in Ref. 3) is central for transport theory. It can be introduced by different methods. Here a method based on the formal construction of operator packets will be presented. First, we discuss the free electromagnetic field.

The quantized electromagnetic field in an auxiliary volume $V = L_x L_y L_z$ can be represented in the form of the well-known superposition

$$E(\mathbf{r}, t) = \sum_{\mathbf{k}} i\left(\frac{\omega_{\mathbf{k}}}{2V}\right)^{1/2} a_{\mathbf{k}}(t) e^{i\mathbf{k}\mathbf{r}}, \tag{1}$$

$$\mathbf{k}_i = \frac{2\pi}{L_i} n_i, \qquad n_i = 0, \pm 1, \pm 2, \ldots, \qquad i = x, y, z.$$

The operators $a_{\mathbf{k}}^+$ and $a_{\mathbf{k}}$ are creation and annihilation operators for the $\mathbf{k}$th photon and they satisfy the commutation relation $[a_{\mathbf{k}}, a_{\mathbf{k}'}^+] = \delta_{\mathbf{k}\mathbf{k}'}$. It is well known that for each of these operators the following equation can be written in the case of a free field:

$$i\dot{a}_{\mathbf{k}} = \omega_{\mathbf{k}} a_{\mathbf{k}}. \tag{2}$$

Forming packets of these operators, we construct new operators of the form

$$a_{\mathbf{m}}(\mathbf{r}, t) = \sum_{\mathbf{k}-\mathbf{m}} \left(\frac{v}{V}\right)^{1/2} a_{\mathbf{k}}(t) e^{i(\mathbf{k}-\mathbf{m})\mathbf{r}}, \tag{3}$$

$$m_i = \frac{2\pi}{l_i} n_i, \qquad n_i = 0, \pm 1, \pm 2, \ldots, \qquad i = x, y, z.$$

We divide the entire lattice of wave vectors $\mathbf{k}$ into cells, whose centers we designate by the set of vectors m, which are a subset of the set of wave vectors $\mathbf{k}$. The summation in Eq. (3) is limited to wave vectors in a single, distinguished cell. The distance between neighboring vectors $\Delta m_i = 2\pi/l_i$ is given by a new spatial parameter $v = l_x l_y l_z$. Thus the number of terms in Eq. (3) is V/v. In what follows, we assume that the spatial parameters l_i are limited below by the wavelength of the radiation $\lambda \ll l_i$. In the case of a free field, no upper limits appear (neglecting the natural restriction associated with the size of the auxiliary volume V, which, however, is set equal to infinity in the final results). In the presence of a material medium, however, we must require $l_i \ll L_0$, where L_0 is the distance over which variations of the field can no longer be neglected.

It is easy to prove the following commutation relation:

$$[a_{\mathbf{m}}(\mathbf{r}, t), a_{\mathbf{m}'}(\mathbf{r}', t)] = \delta_{\mathbf{m}\mathbf{m}'} v \delta_v^{(3)}(\mathbf{r} - \mathbf{r}'), \tag{4}$$

where

$$v\delta_v^{(3)}(r) = \prod_{i=x,y,z} l_i \delta_{l_i}(i), \tag{5}$$

$$\delta_l(z) = \sum_{k_z - m_z} \frac{l_z}{L_z} e^{i(k_z - m_z)z} \underset{L_z \to \infty}{\to} \frac{\sin(\pi z/l_z)}{\pi z}.$$

In what follows we interpret the function $l\delta_l(z)$ as a step function: $l\delta_l(z) = 1$ for $z < l$ and $l\delta_l(z) = 0$ for $z > l$. There exist other variants of the theory. It follows from these variants that this interpretation does not impose any restrictions on this theory. But, then for the operators $a_{\mathbf{m}}, a_{\mathbf{m}}^+$ within the spatial cells v commutation relations of the form $[a_{\mathbf{m}}(\mathbf{r},t), a_{\mathbf{m}'}^+(\mathbf{r},t)] = \delta_{\mathbf{m}\mathbf{m}'}$ arise (if we neglect the difference between $\mathbf{r}$ and $\mathbf{r}'$ within the cell), which enables us to treat these operators as annihilation or creation operators. It is this latter fact that enables us to introduce into the analysis a field oscillator, having wave vector m and localized in the spatial cell with radius vector $\mathbf{r}$. Here, it is important to note that since our conclusion that a local oscillator can be introduced depends on the character of $a_{\mathbf{k}}(t)$, it is valid for a free field (monochromatic dependence on t) as well as any other field.

In concluding this section we make a remark regarding the equation for $a_{\mathbf{m}}(\mathbf{r},t)$ which can be easily derived with the help of Eq. (2):

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{m}}{m}\frac{\partial}{\partial \mathbf{r}}\right) a_{\mathbf{m}} = i\omega_{\mathbf{m}} a_{\mathbf{m}}. \tag{6}$$

This equation is valid to the extent that the equality

$$k \approx m + \mathbf{m}(\mathbf{k} - \mathbf{m})/m \tag{7}$$

is valid and the approximation is associated with dropping the higher-order derivatives with respect to the spatial variables. For example, in the one-dimensional approximation, in the case of plane waves traveling along some axis, when all diffraction phenomena are dropped at the outset, Eq. (6) is exact.

2. KINETIC EQUATION FOR THE DENSITY MATRIX OF A LOCAL OSCILLATOR

It is more convenient to switch from the Heisenberg form of the theory, in which the entire time dependence is contained in the operators and the state of the field is given initially and then remains constant in time, to an unusual interaction representation in order to describe the free electromagnetic field and transfer the slow time dependence to the density matrix of the electromagnetic field. In order for this to be understandable, we formulate the Hamiltonian formalism for the field. We start with the energy operator of the field

$$H = \int_{(V)} d^3r [E^+(\mathbf{r}, t)E(\mathbf{r}, t) + E(\mathbf{r}, t)E^+(\mathbf{r}, t)]. \tag{8}$$

Substituting an expansion of the form (1) into Eq. (8) we can easily show that the standard expression

$$\sum_{\mathbf{k}} \frac{1}{2} \omega_{\mathbf{k}} (a_{\mathbf{k}}^+ a_{\mathbf{k}} + a_{\mathbf{k}} a_{\mathbf{k}}^+)$$

is obtained. Now we employ the expansion (3) in operator packets, which has the form

$$E(\mathbf{r}, t) \approx \sum_{\mathbf{m}} i\left(\frac{\omega_{\mathbf{m}}}{2v}\right)^{1/2} \left[\left(1 - \frac{i\mathbf{m}}{2m^2}\frac{\partial}{\partial \mathbf{r}}\right) a_{\mathbf{m}}\right] e^{i\mathbf{m}\mathbf{r}} \tag{9}$$

and we obtain the following expression for the Hamiltonian, written in terms of the Heisenberg operators of the form (3):

$$H = \frac{1}{v}\int\limits_{(V)} d^3r \sum_{\mathbf{m}} [H_{0\mathbf{m}}(\mathbf{r}) + H_{1\mathbf{m}}(\mathbf{r})] , \qquad (10)$$

where

$$H_{0\mathbf{m}}(\mathbf{r}) = \frac{1}{2}\omega_{\mathbf{m}} [a^+_{\mathbf{m}}(\mathbf{r}, t)a_{\mathbf{m}}(\mathbf{r}, t) + a_{\mathbf{m}}(\mathbf{r}, t)a^+_{\mathbf{m}}(\mathbf{r}, t)] \qquad (11)$$

is the Hamiltonian of a free local oscillator, and the term

$$H_{1\mathbf{m}}(\mathbf{r}) = \frac{im}{2m}\left[\frac{\partial a^+_{\mathbf{m}}}{\partial \mathbf{r}} a_{\mathbf{m}} - a^+_{\mathbf{m}}\frac{\partial a_{\mathbf{m}}}{\partial \mathbf{r}}\right] \qquad (12)$$

determines the interaction of a local oscillator in neighboring spatial cells. In this theory the interaction term determines the radiation transfer in space and for this reason it can never be neglected.

In calculating Eqs. (10)–(12) it is necessary to calculate a volume integral e.g., of the following expression:

$$a^+_{\mathbf{m}_1}(r)a_{\mathbf{m}_2}(r)\exp[i(\mathbf{m}_1 - \mathbf{m}_2)\mathbf{r}] .$$

For this, we divide the auxiliary volume V into smaller cells of volume v, in which the field oscillators are localized, and in integrating over the cells we neglect the spatial dependence of the operators $a_{\mathbf{m}}$ and $a^+_{\mathbf{m}}$:

$$\int\limits_{(V)} d^2r a^+_{\mathbf{m}_1}(\mathbf{r})a_{\mathbf{m}_2}(\mathbf{r})e^{[(\mathbf{m}_1-\mathbf{m}_2)\mathbf{r}} = v\,\delta_{\mathbf{m}_1\mathbf{m}_2}\sum_j a^+_{\mathbf{m}_1}(\mathbf{r}_j)a_{\mathbf{m}_2}(\mathbf{r}_j) .$$

Finally, we switch to a "coarse" spatial scale, replacing summation over the cells by integration:

$$\sum_j \ldots \to \frac{1}{v}\int\limits_{(V)} d^3r \ldots$$

Now, we can write the Heisenberg equation for any operator. In particular, Eq. (6) follows from the equation

$$i\dot{a}_{\mathbf{m}} = [a_{\mathbf{m}}, H_{0\mathbf{m}} + H_{1\mathbf{m}}] \qquad (13)$$

As should be, the Hamiltonian $H_{0\mathbf{m}}$ of a free local oscillator gives the fast time dependence with frequency $\omega_{\mathbf{m}}$, and the interaction Hamiltonian causes the operator to change slowly, transforming the excitation of the local oscillator. At the same time, the state of the local oscillator, determined, for example, by the density matrix, remains constant in time: $\dot{\rho}_{\mathbf{m}} = 0$. Time and space appear symmetrically in this theory (this refers to the slow change), so that we can treat them identically and require that the density matrix in the Heisenberg representation remain constant in both time and space, i.e., we must add the requirement $\operatorname{div}\rho_{\mathbf{m}} = 0$. We thereby assert that the entire space-time dependence in the Heisenberg representation is concentrated in the operators, i.e., in equations of the type (6) and (13). In order to transfer the slow space-time dependence to the density matrix, we must perform the unitary transformation

$$\tilde{\rho}_{\mathbf{m}}(\mathbf{r}, t) = S^+\left(t - \frac{\mathbf{mr}}{m}\right)\rho_{\mathbf{m}}S\left(t - \frac{\mathbf{mr}}{m}\right) ,$$
$$\tilde{a}_{\mathbf{m}}(t) = S\left(t - \frac{\mathbf{mr}}{m}\right)a_{\mathbf{m}}(\mathbf{r}, t)S^+\left(t - \frac{\mathbf{mr}}{m}\right) , \qquad (14)$$
$$S(t) = \exp(iH_{1\mathbf{m}}t) .$$

As a result, we obtain the following equations:

$$i\dot{\tilde{a}}_{\mathbf{m}} = \omega_{\mathbf{m}}a_{\mathbf{m}}, \qquad \operatorname{div} a_{\mathbf{m}} = 0 ,$$
$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{m}}{m}\frac{\partial}{\partial \mathbf{r}}\right)\rho_{\mathbf{m}} = 0 . \qquad (15)$$

The last equation is the desired kinetic equation, describing the state of the local oscillator. It is suitable for describing a free electromagnetic field, but it can be easily extended to the case when a medium with which the field can interact is present. Using the kinetic equation we can say that the state of a local oscillator can change for different reasons, for example, because of interaction with atoms present in the same spatial cell as the field oscillator and, of course, because of interaction with neighboring local oscillators, i.e., radiation transfer:

$$\dot{\rho}_{\mathbf{m}} = (\dot{\rho}_{\mathbf{m}})_{\text{transfer}} + (\dot{\rho}_{\mathbf{m}})_{\text{res. medium}} , \qquad (16)$$

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{m}}{m}\frac{\partial}{\partial r}\right)\rho_{\mathbf{m}} = (\dot{\rho}_{\mathbf{m}})_{\text{res. medium}} . \qquad (17)$$

The right-hand side of this equation must be calculated independently for each specific physical situation. In what follows, we shall explain why in our case of a layer of resonant medium (amplifying or absorbing) the right-hand side can be taken from the Lamb–Scully theory[2] for single-mode lasing.

3. RADIATION TRANSFER THROUGH A LAYER OF A RESONANCE MEDIUM: BASIC KINETIC EQUATION

We have in mind the following physical situation. Light from a known source or with known statistical properties passes through a layer of a resonant medium, in which atoms are somehow excited into active levels, and strikes a photocathode, whose photocurrent spectrum is then investigated. According to Refs. 5 and 6, the photocurrent spectrum is given by the following formula:

$$i^{(2)}_{\omega} = \frac{q}{\omega_0}\int\limits_{(S)} d^2r\langle E^+(\mathbf{r}, t)E(\mathbf{r}, t)\rangle + \left(\frac{q}{\omega_0}\right)^2\int\limits_{(S)}\int d^2r_1 d^2r_2$$
$$\times 2\mathrm{Re}\int\limits_0^{\infty} d\tau\langle E^+(\mathbf{r}_1, t)E^+(\mathbf{r}_2, t+\tau)E(\mathbf{r}_2, t+\tau)E(\mathbf{r}_1, t)\rangle e^{i\omega\tau} . \qquad (18)$$

Here q is the quantum efficiency of a photocathode with surface area S (q is equal to unity, if each photon incident on the photocathode generates one photoelectron).

We now expand the field in the local oscillators (9) and rewrite Eq. (18) in the single-mode (in the sense of the m modes but not the $\mathbf{k}$ modes) and one-dimensional approximation, making the assumption that we have only a single plane wave traveling in the positive direction along the z axis:

$$i^{(2)}_{\omega} = i^{(2)}_{\text{shot}}\left[1 + \frac{2q}{\ln(z)}\mathrm{Re}\int\limits_0^{\infty} dt g(z, t)e^{i\omega t}\right] . \qquad (19)$$

The quantity

$$g(z, t) = \langle a^+(z)a^+(z, t)a(z, t)a(z)\rangle, \qquad a(z) = a(z, 0) \qquad (20)$$

characterizes the excess noise of the photocurrent (the index m is dropped everywhere below). The quantity

$$n(z) = \langle a^+(z)a(z)\rangle \qquad (21)$$

is the average number of photons for the local oscillator at the point z, at the location of the photocathode. The length l is the length of the spatial cell containing the local oscillator of the field. Along the z axis we have $l = l_z$.

The averages (20)–(21) can be calculated on the basis of a transport theory of the form (17). But, before writing down the basic equation, we note that the characteristics of a local oscillator of the form (20) and (21) are usually convenient for theoretical constructions, but they nonetheless are of an auxiliary character and cannot enter into the final results. For example, instead of the average number of photons of the local oscillator, it is more reasonable to introduce the radiation power, which can be represented in the form

$$W(z) = \omega n(z)/l\,, \qquad [W] = \text{ energy/sec.} \qquad (22)$$

The single-mode approximation, which was employed in order to write down Eq. (19), requires discussion. In order for this approximation to be correct it is sufficient to assume that a filter with spectral width $\Delta\omega \sim l^{-1}$ is present in front of the photocathode. In the averages, however, the other modes can also play a very significant role. The single-mode approximation can be preserved, if it is assumed that we are interested in a single strong mode, which is not influenced by any other modes that are weak in our theory.

We assume that the amplifying (or absorbing) medium consists of stationary atoms, whose energy structure is identical to that of Lamb and Scully:[2] They have two levels, from which spontaneous decay into auxiliary levels occurs and into which the atoms are excited. In what follows, we allow for both random and regular pumping of the working levels. We recall how the right-hand side of the kinetic equation in the Lamb–Scully theory was calculated.[2] First, the system of equations for the matrix elements of the density matrix of the system consisting of "one atom + field oscillator" was written down. On the basis of this, the increment to the field density matrix due to interaction over a sufficiently long time with one atom was calculated. Next, summation over many atoms was performed. This made it possible to write out the time derivative explicitly (on the "coarse" time scale) of the field density matrix. The situation is essentially identical in our case. The only difference is that the oscillator enters into the theory not as a characteristic oscillation of a high-Q optical cavity but rather as a local oscillator, and when summing over atoms it is not necessary to include all atoms, but only those atoms which are present in the same spatial cell as the local oscillator. Thus, we can employ the results of Refs. 2 and 7 and write the basic kinetic equation in the form

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\rho = (\underrightarrow{a^+a} - \underrightarrow{aa^+})\hat{\mathcal{R}}_a\rho + (\underrightarrow{a\,a^+} - \underrightarrow{a^+a})\mathcal{R}_b\rho + \text{H.c.} \qquad (23)$$

The operator

$$\hat{\mathcal{R}}_a = \frac{1}{2}A_a[1 + \frac{1}{2}\beta_b(\underrightarrow{aa^+} - \underleftarrow{aa^+})][1 + \frac{1}{2}\beta(\underrightarrow{aa^+} + \underleftarrow{aa^+}) + \frac{1}{4}\beta_a\beta_b(\underrightarrow{aa^+} - \underleftarrow{aa^+})^2]^{-1}$$

determines the excitation of a field oscillator of the resonance medium taking into account nonlinear processes. The operator $\hat{\mathcal{R}}_b$ is obtained from $\hat{\mathcal{R}}_a^+$ by interchanging all indices ($a \rightleftarrows b$) and operator combinations ($\underrightarrow{aa^+}$) by ($\underleftarrow{a^+a}$). The arrows under the operators indicate where these operators must be positioned with respect to the operators standing farther to the right. Other notations are:

$$A_a = \frac{r_a}{\gamma_a}\frac{2|g|^2}{\gamma_{ab}}\,, \qquad A_b = \frac{r_b}{\gamma_b}\frac{2|g|^2}{\gamma_{ab}}\,,$$

r_a and r_b are the average rate of incoherent excitation of the top and bottom levels in the actual atomic transition; γ_a and γ_b are the widths, due to decay into secondary levels, of the top and bottom levels in the actual transition; γ_{ab} is the transverse relaxation constant;

$$g = i\left(\frac{\omega_m}{2v}\right)^{1/2} d_{ab}e^{imz}$$

is the dipole interaction constant of the atom and the mth wave,

$$\beta_a = \frac{2|g|^2}{\gamma_a\gamma_{ab}}\,, \qquad \beta_b = \frac{2|g|^2}{\gamma_b\gamma_{ab}}\,, \qquad \beta = \beta_a + \beta_b$$

are the saturation parameters characterizing the efficiency of nonlinear processes; and $A_a = r_a\beta_a$ and $A_b = r_b\beta_b$.

The linear (unsaturated) gain $A = A_a - A_b$ appears below. This gain, as we can see, is given by the difference of the populations of the active levels $N = (r_a/\gamma_a) - (r_b/\gamma_b)$.

The equation (23) (even neglecting the spatial derivative, which determines the radiation transfer) is a generalization of the Lamb–Scully theory,[2] which is contained, for example, in Ref. 7.

We now transform to the diagonal representation of the density matrix $P(\alpha,z,t)$, which following Glauber[5] we write in the form

$$\rho(z,t) = \int d^2\alpha P(\alpha,z,t)\cdot|\alpha\rangle\langle\alpha|\,,$$

$$a|\alpha\rangle = \alpha|\alpha\rangle, \qquad d^2\alpha = d(\mathrm{Re}\,\alpha)d(\mathrm{Im}\,\alpha), \qquad \alpha = |\alpha|e^{i\varphi}$$

and we confine our attention to the approximation of small relative fluctuations in the number of photons:

$$|\alpha|^2 = \bar{n}(z) + \varepsilon, \qquad \varepsilon \ll \bar{n}\,.$$

Then the density matrix can be factored in the form

$$P(\alpha,z,t) = R(\varepsilon)\Phi(\varphi)$$

and the following equation can be written down for $R(\varepsilon,z,t)$:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)R = -\Gamma\frac{\partial}{\partial\varepsilon}(\varepsilon R) + D\frac{\partial^2 R}{\partial\varepsilon^2} + \sum_{s=3}^{\infty} D_s\frac{\partial^s R}{\partial\varepsilon^s}\,. \qquad (24)$$

The coefficient Γ can be expressed as follows in terms of the parameters of the problem:

$$\Gamma(z,t) = \frac{A}{1 + I(z,t)}\,, \qquad I(z,t) = \beta\bar{n}(z,t)\,. \qquad (25)$$

It is well known that in the purely temporal theory of the type given in Ref. 2 Γ determines the spectral width of the photon fluctuations. The coefficient D is given by the relation

$$D(z,t)=\frac{A\bar{n}}{(1+I)^2}\left(\frac{N_a}{N}+\frac{N_b}{N}I\right),\qquad N_a=\frac{r_a}{\gamma_a},\qquad N_b=\frac{r_b}{\gamma_b}. \tag{26}$$

This coefficient determines the intensity of the photon fluctuations. Explicit expressions for other coefficients D_s in the equation are not important in this work, since they will not contribute to the observed photocurrent spectrum (19).

In writing down Eqs. (24) and (25) we assumed that the quantity $\bar{n}$ satisfies the same equation as the semiclassical solution

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)\bar{n}=A\bar{n}(1+\beta\bar{n})^{-1}. \tag{27}$$

This restriction rules out, for example, studying the problem in the absence of the external field incident on the layer of medium or for not too high intensities of the incident field.

4. CALCULATION OF AVERAGES USING THE KINETIC EQUATION

We first discuss when the diffusion approximation is adequate in Eq. (24), i.e., up to second derivatives with respect to ε. This is valid for classical fields, i.e., for fields which we describe on the basis of quantum theory, but which could also be described classically. The functions $R(\varepsilon)$ are then smooth functions and the higher-order derivatives are small. Another example, when the diffusion equation is simply exact (higher-order derivatives do not arise), does not depend on whether or not the fields are quantum or classical, and it is associated with two-photon processes or three- and four-wave mixing processes (in the transmission zone or in the case of resonance, but in the "linear" approximation). In any case, it is obvious that a large class of physical situations is described very well by Eq. (24) with $D_S=0$. This is the Fokker–Planck equation for the probability (quasiprobability) $R(\varepsilon,z,t)$, and the solution of the equation can be written down, imposing real boundary or initial conditions, if we are interested in the density matrix itself, or δ-function conditions, if we are interested in the Green's function of the equation. This program is entirely feasible, and this is exactly how we proceeded in the purely time-dependent variant of the theory. However, the presence of the spatial aspect makes this approach very cumbersome, which, of course, is not justified, since the solution obtained in this manner will contain all information about the field, whereas we are interested only in a single average of the form $\langle\varepsilon(1)\varepsilon(2)\rangle$.

Gorbachev and Trubilko[8] propose switching from the Fokker–Planck equation for the probability $R(\varepsilon,z,t)$ to the corresponding Langevin equation for the random quantity $\varepsilon(z,t)$. In so doing, however, the formulation of the properties of the random source becomes indeterminate due to the presence of an additional (spatial) degree of freedom. In order to eliminate this indeterminateness, the authors were forced to appeal to comparisons with solutions obtained by other methods.[4] The results of Ref. 8 are certainly correct, but the methodology remains unsatisfactory, since it is obvious *a priori* that an equation of the form (24) contains all required information about the field. Moreover, strictly speaking, derivatives of all orders with respect to ε must be included in Eq. (24). For quantum fields this approach on the basis of the Glauber diagonal representation, strictly speaking, becomes incorrect.

Here we propose a variant of the calculations of the averages that follows logically from the transport theory and is free of the drawbacks indicated above.

We now discuss how to construct the average $\langle\varepsilon(z_1t_1)\varepsilon(z_2t_2)\rangle$. This average can be represented in the form[5]

$$\langle\varepsilon(z_1t_1)\varepsilon(z_2t_2)\rangle=\int\int d\varepsilon_1 d\varepsilon_2\varepsilon_1\varepsilon_2 R(\varepsilon_1 z_1 t_1)G(\varepsilon_1 z_1 t_1|\varepsilon_2 z_2 t_2). \tag{28}$$

Here $G(\varepsilon_1 z_1 t_1|\varepsilon_2 z_2 t_2)$ is a conditional probability, which, as is well known, satisfies the same equation (24)

$$\left(\frac{\partial}{\partial t_2}+\frac{\partial}{\partial z_2}\right)G=-\Gamma\frac{\partial}{\partial\varepsilon_2}(\varepsilon_2 G)+D\frac{\partial^2 G}{\partial\varepsilon_2^2}+\sum_{s=3}^{\infty}D_s\frac{\partial^2 G}{\partial\varepsilon^s} \tag{29}$$

and is different from zero for $t_2>t_1$ and for $z_1=z_2-t_2+t_1$, due to the properties of the kinetic equation. The difference between z_1 and z_2 is connected only with the retardation and can be neglected. The solution of Eq. (29) must be sought with the boundary condition

$$G(\varepsilon_1 0 t_1|\varepsilon_2 0 t_2)=G_0(\varepsilon_1 t_1|\varepsilon_2 t_2), \tag{30}$$

where $G_0(\varepsilon_1 t_1|\varepsilon_2 t_2)$ is a known function, characterizing the light at the front boundary of the layer, and in addition

$$G_0(\varepsilon_1 t_1|\varepsilon_2 t_1)=\delta(\varepsilon_1-\varepsilon_2). \tag{31}$$

From Eq. (29) and the formula (28) there follows an equation for the desired average in the form

$$\left(\frac{\partial}{\partial t_2}+\frac{\partial}{\partial z_2}\right)\langle\varepsilon(z_1t_1)\varepsilon(z_2t_2)\rangle=2\Gamma\langle\varepsilon(z_1t_1)\varepsilon(z_2t_2)\rangle+2Dl\,\delta_l(t_2-t_1). \tag{32}$$

It is obvious that here higher-order derivatives do not contribute. We can now make the following quite general assertion: The normal (Glauber) diagonal representation of the density matrix can be employed irrespective of whether the fields are classical or quantum, but it must be kept in mind that in order to calculate the averages $\langle\varepsilon^k\rangle$ (multitime or single-time) in Eq. (24) derivatives with respect to ε through order k must be retained. For example, in our case, in order to calculate the photocurrent spectrum (19) it is necessary to know $g(z,\tau)$, which in the diagonal representation can be rewritten in the form

$$g(z,\tau)=\langle|\alpha(z_1t_1)|^2|\alpha(z_2t_2)|^2\rangle=\bar{n}(z_1)\bar{n}(z_2)+\langle\varepsilon(z_1t_1)\varepsilon(z_2t_2)\rangle. \tag{33}$$

Thus the photocurrent spectrum can be expressed in terms of the pair average $\langle\varepsilon^2\rangle$, i.e., in our theory we can confine our attention to the diffusion approximation, though we shall discuss a field with pronounced quantum properties.

5. AMPLIFYING (ABSORBING) LAYER IN THE NONLINEAR-SATURATION REGIME

The physical solution of Eq. (32) can be writen in the form

$$\langle\varepsilon(z,t_1)\varepsilon(z,t_2)\rangle=\langle\varepsilon(0,t_1)\varepsilon(0,t_2)\rangle\exp\{2\int_0^z\Gamma(z')dz'\}+2l\,\delta_l(t_2-t_1)\int_0^z D(z')\exp\{\int_{z'}^z 2\Gamma(z'')dz''\}dz'. \tag{34}$$

Substituting Eqs. (24) and (25) here and using Eq. (27) we can perform all required integrations and obtain the following:

$$\langle \varepsilon(z, t_1)\varepsilon(z, t_2)\rangle = \langle \varepsilon(0, t_1)\varepsilon(0, t_2)\rangle \left(\frac{I}{I_0}\frac{1+I_0}{1+I}\right)^2 + 2l\delta_l(t_2 - t_1)\frac{\bar{n}I}{(1+I)^2}\left[\frac{N_a}{N}\left(\frac{1}{I_0} - \frac{1}{I} + \ln\frac{I}{I_0}\right) + \frac{N_b}{N}\left(I - I_0 + \ln\frac{I}{I_0}\right)\right]. \quad (35)$$

The dimensionless intensity $I(z)$ at the running point z in the layer is related to the intensity at the front boundary by the relation

$$I - I_0 + \ln\frac{I}{I_0} = Az.$$

Thus, the excess noise at the exit from the medium consists of the excess starting noise (multiplied by some factor) and a spectral wide term (proportional to a δ-function of τ) determined by the spontaneous emission of the medium into a preferred mode under conditions when a strong initial field acts on the medium. We can employ Eq. (35) for the case when this action is weak, I, $I_0 \ll 1$, but we cannot set I, I_0 equal to zero, since the condition that the photon fluctuations are small, $\varepsilon \ll \bar{n}$, was important.

Thus far we have always assumed that the excitation of the atoms in the layer of medium into the working levels occurs entirely randomly, and thus the populations of the levels fluctuate in accordance with the Poisson law. The case when regular pumping maintains a nonfluctuating population of the levels should also be of interest. In principle, this can be done on the basis of the method proposed in Ref. 9 or as done in the experiment of Yamamoto *et al.*,[10] who made use of the peculiarities of the electric current flow through a n–p junction. The realization of noise-free pumping made it possible to suppress shot noise in photodetection at low frequencies.[9,10] As we shall see below, quantum features also appear in the case of light passing through a layer of amplifying medium.

Regular excitation can be taken into account as done in Ref. 9: If for completely random pumping the equation for the density matrix of the field has the form $\dot{\rho} = r_a\hat{L}\rho$, then in the case of regular pumping the equation becomes $\dot{\rho} = r_a(L_a - \frac{1}{2}L_a^2)\rho$. As a result, the coefficient D in Eq. (24) changes, and the expression (35) will acquire an additional negative term on the right-hand side:

$$-l\delta_l(\tau)\frac{\bar{n}I}{(1+I)^2}\left(I - I_0 + \ln\frac{I}{I_0}\right) \times \left(\frac{N_a}{N}\frac{\gamma_b}{\gamma_a+\gamma_b} + \frac{N_b}{N}\frac{\gamma_a}{\gamma_a+\gamma_b}\right). \quad (36)$$

6. PHOTOCURRENT SPECTRUM IN THE SATURATED REGIME

We now substitute Eqs. (33), (35), and (36) into the starting formula (19) for the photocurrent and obtain an explicit expression for the photocurrent in terms of the initial physical parameters: for I, $I_0 \gg 1$ ($\omega > 0$):

$$i_\omega^{(2)} \sim W\left\{1 + 2q\xi_L\frac{W_0}{W}\frac{C_L^2}{C_L^2+\omega^2} + 2q\left(1 - \frac{W_0}{W}\right)\frac{N_b}{N} - q\left(1 - \frac{W_0}{W}\right)\left(\frac{N_a}{N}\frac{\gamma_b}{\gamma_a+\gamma_b} + \frac{N_b}{N}\frac{\gamma_a}{\gamma_a+\gamma_b}\right)\right\}. \quad (37)$$

Here the fourth negative term appears because the pumping is regularity and follows from the expression (21). We have assumed that the excess noise of the light at the front boundary of the medium can be expressed by the following formula:

$$\langle \varepsilon(0, t_1)\varepsilon(0, t_2)\rangle = \bar{n}(0)\xi_0\exp\{-\Gamma_0(t_2 - t_1)\}, \qquad t_2 > t_1.$$

As we have already mentioned, $\bar{n}(0)$ and $\bar{n}(z)$ as well as ξ_0 and Γ_0 are auxiliary quantities, which must be replaced by the real characteristics. These are the power of the light W_0 and W, to which we switch according to Eq. (22). In order to move away from the abstract quantities ξ_0 and Γ_0, we assume that the light source is a laser. Then ξ_0 must be replaced by $\xi_L(C_L l)$, and when the lasing saturates Γ_0 must be replaced by C_L, where C_L is the spectral width of the optical cavity of the laser and ξ_L is the Mandel parameter for the intracavity lasing field.[4]

Figure 1 illustrates schematically the case of saturated gain in the absence of absorbing atoms. As we can see from this figure, if the light is initially sub-Poissonian (Fig. 1a), i.e., $\xi_L = -1/2$, then due to saturated gain by randomly excited atoms the entire picture is shifted upward together with the shot noise (Fig. 1b). As a result, in the case of efficient amplification, $W_0 \ll W$, the dip now plays a negligible role, and the light is virtually Poissonian. This fact does not depend on the type of the statistics of the initial field—sub-Poissonian, as in our figure, or some other statistics. The

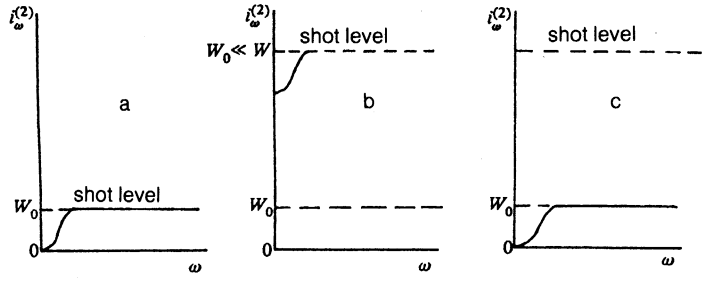

FIG. 1. Photocurrent spectrum with saturated gain in the absence of absorbing atoms $N_b = 0$ and with $q = 1$ and $\gamma_a = 0$: a) for the initial light at the front boundary of the amplifier; b) at the amplifier exit with random excitation of amplifying atoms; and c) at the amplifier exit with regular excitation of amplifying atoms.

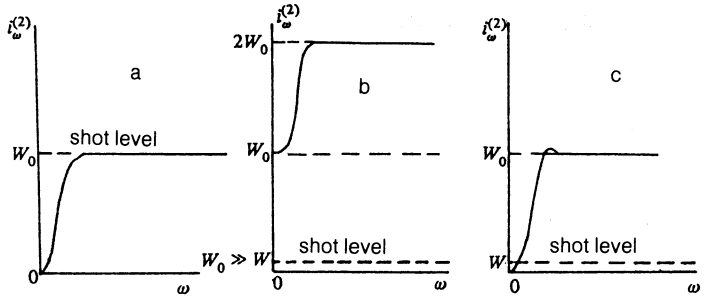

FIG. 2. Photocurrent spectrum with saturated absorption in the absence of amplifying atoms $N_a = 0$ and with $q = 1$ and $\gamma_b = 0$: a) for the initial light at the front boundary of the absorber; b) at the absorber exit with random excitation of absorbing atoms; and, c) at the absorber exit with regular excitation of absorbing atoms.

role of super-Poissonian statistics also decreases in precisely the same manner. Thus such an amplifier could be employed, for example, for preliminary "purification" of the light, i..e, for transforming super-Poissonian light into Poissonian light in order to employ it in existing devices for transforming it into quantum light.

Figure 1c illustrates the situation with saturated gain by regularly excited atoms. The initial picture of the noise appears to remain completely the same. Hence we can conclude that such an amplifier can be employed for efficient amplification without destroying the quantum properties of the light. Moreover, in this sense the situation is improved, since the entire noise curve lies significantly below the shot noise level. Thus an amplifier with regular pumping can be employed, on the one hand, for amplifying quantum light without running the risk of "spoiling" the light and on the other as a source of quantum light, since it transforms super-Poissonian light into sub-Poissonian light while suppressing noise in a wide frequency band. We underscore once again that thus far all projects on converting classical light into quantum light concerned only light which was initially Poissonian. In this paper we have discussed for the first time super-Poissonian light, which with the help of saturated gain by regularly excited atoms is transformed into sub-Poissonian light with complete suppression of noise. Figure 2 illustrates saturated absorption. The most interesting point here is that in the case of regular excitation of absorbing atoms the noise picture remains the same as at the system entrance.

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