Sources of charge asymmetry in the cross sections for inelastic transitions excited by fast particles and antiparticles in helium

V. A. Sidorovich

Nuclear Physics Research Institute, Moscow State University
(Submitted 19 November 1992)
Zh. Eksp. Teor. Fiz. **103**, 806–831 (March 1993)

Inelastic transitions excited by helium collisions with fast particles and antiparticles are examined within the framework of the impact-parameter method, with allowance for configuration interaction. The sources of the charge asymmetry in the total cross sections for one- and two-electron excitation and ionization of helium are studied. It is shown that in fast ion–atom collisions, when polarization of the helium-ion electron-shell polarization by the incident ion and the influence of the competing charge-exchange process can be neglected in the scattering of a positively charged ion by helium, the difference between the cross sections of the indicated inelastic transitions for particles and antiparticles can be attributed to interference between different inelastic-transition mechanisms. Specific processes responsible for the charge asymmetry in the integral scattering cross section are determined for each of the considered inelastic transitions, and those making the decisive contribution are identified. The published theoretical and experimental data on the cross sections for one- and two-electron excitation and ionization are analyzed from the standpoint of the described approach.

1. INTRODUCTION

Study of the scattering of electron, positrons, protons, and antiparticles by helium atoms[1–27] points to the presence of charge asymmetry in the elastic-collision cross sections. The charge asymmetry is particularly pronounced in the double ionization process.[2] Investigations[1,3,22] have shown that the dependence of the cross section for double ionization of helium on the sign of the ion charge is contained in the interference term of the cross section, a term proportional to the cube Z^3 of the incident-particle charge. One of the possible causes of the term proportional to Z^3 in the double-ionization cross section is interference between the mechanism for independent separation of the electrons and the double-ionization mechanism, in which the electron ejected by the interaction with the incident ion is scattered by another helium electron.[3,22]

The strong dependence of one-electron excitation of helium on the sign of the charge of the incident ion in the collision-energy interval from 50 to 1000 keV was determined by calculations, based on the multichannel diffraction approximation,[21] of the cross sections for helium excitation by protons and antiprotons. It was ascertained that the excitation cross sections are more asymmetric for optically forbidden transitions than for optically allowed ones, owing to the significant contribution of two-step transitions to excitation of optically forbidden transitions.

Many calculations exist[16,18,20,23,26] of the cross sections for excitation of the lowest autoionization states of helium by protons and antiprotons. The results of tight-binding calculations using a discrete basis of atomic orbitals relative to one center[16] and a Sturm basis of wave functions,[20] calculations in the second Born approximation including only discrete states and transitions outside the energy surface,[18] and also calculations carried out in the second Born approximation and in the tight-binding method using hyperspherical wave functions with allowance for only discrete states,[26] yielded only insignificant ($\sim 30\%$) differences of the proton and antiproton cross sections in the energy interval E from 1.5 to 10 MeV. Exceptions are the calculated cross sections for the excitation of the $2p^2\ ^1D$ states at 1.5 MeV energy,[16] when the cross section for the protons is almost 1.8 times larger than that for the antiprotons. A significantly higher (by up to six times in the case of the $2p^2\ ^1D$ state) cross sections of two-electron excitation of helium by antiprotons compared with those for protons at energies $E = 1.5$–10 MeV was obtained by perturbation-theory calculations[23] of second order in the potential of interaction of an incident ion with helium electrons. Only transitions on the energy surface were taken into account in these calculations. A conflicting situation arises: some calculations show an insignificant discrepancy between the cross sections for particles and antiparticles, while others, in the same collision-energy range, predict a large excess of the antiparticle cross sections over the corresponding ones for particles.

Thus, notwithstanding the large number of original research (both experimental and theoretical) and published reviews devoted to scattering of fast charged particles and antiparticles by inert-gas atoms, there is no adequately verified physical explanation of the charge asymmetry observed in the inelastic-scattering cross sections.

To advance further in the research into the nature of the charge asymmetry in ion–atom collisions, a single approach is presented here, based on the use of the impact-parameter method, with allowance for the configuration interaction, to consideration of various inelastic transitions excited in helium by collisions with fast particles and antiparticles. The causes are investigated of the charge asymmetry in the total cross sections of one- and two-electron excitation

$$Z + \mathrm{He}(1s^2\ ^1S) \to Z + \mathrm{He}^*(1snl), \tag{1a}$$

$$Z + \mathrm{He}(1s^2\ ^1S) \to Z + \mathrm{He}^{**}(n_1l_1n_2l_2\ ^1LM) \tag{1b}$$

and of one- and two-electron ionization

 1063-7761/93/030396-12$10.00

$$Z + \mathrm{He}(1s^2\,{}^1S) = \begin{cases} Z + \mathrm{H}^+(nl) + e^- \\ Z + \mathrm{He}^{**}(n_1 l_1 n_2 l_2\,{}^1LM)\,, \\ \qquad \to \mathrm{He}^+(1s) + e^- \end{cases} \tag{1c}$$

$$Z + \mathrm{He}(1s^2\,{}^1S) \to Z + \mathrm{He}^{++} + 2e^-, \tag{1d}$$

as well as inelastic-transition excitation mechanisms responsible for the charge asymmetry in the scattering cross sections. Polarization of the helium-atom electron shell by the incident ion, which is significant only at low collision velocities (see, e.g., Ref. 3), is neglected here. Since the results of Ref. 28 have revealed a weak influence of charge exchange in helium ionization even at ion energies $E \sim 0.2$ MeV/nucleon, no account is likewise taken here of the competing process of charge exchange in scattering of a positively charged ion by helium. The approach developed is used to analyze the published calculated and experimental data on the cross sections of inelastic transitions in helium.

2. THEORY

2.1. Electron wave function of helium atom

Consider scattering of a structureless charged particle by a helium atom. The scattering problem is solved on the basis of the impact-parameter method. It is assumed within the framework of this method that the charged particle moves along a straight line $\mathbf{R}(t) = \boldsymbol{\rho} + \mathbf{v}t$ (where $\boldsymbol{\rho}$ is the impact parameter and $\mathbf{v}$ is the ion velocity relative to the nucleus of the target atom), and the electron wave function Ψ of the helium atom is independent of the relative distance between the colliding particles and is determined by the Schrödinger equation, which takes in the interaction representation the form

$$\begin{aligned} i\frac{\partial}{\partial t}\Psi^{int}(t) &= \tilde{V}(t)\Psi^{int}(t)\,, \\ \Psi^{int}(t)|_{t\to\pm\infty} &= \Psi_{I,F}(\mathbf{r}_1, \mathbf{r}_2)\,, \end{aligned} \tag{2}$$

where

$$\tilde{V}(t) = \exp(iH_0 t)V(\mathbf{R}, \mathbf{r}_1, \mathbf{r}_2)\exp(-iH_0 t) \tag{3}$$

is the operator of the interaction of the ion with the target atom in the interaction representation, and $V(\mathbf{R},\mathbf{r}_1,\mathbf{r}_2) = -\Sigma_{j=1}^{2} Z/|\mathbf{R}(t) - \mathbf{r}_j|$. The wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2)$ is an eigenfunction of the atomic Hamiltonian H_a with an eigenvalue E:

$$H_a\Psi(\mathbf{r}_1, \mathbf{r}_2) = E\Psi(\mathbf{r}_1, \mathbf{r}_2). \tag{4}$$

Here $H_0 = H_a + K$ is the free-motion Hamiltonian; $K = \hat{\mathbf{P}}^2/2M$ and M are the kinetic-energy operator and the reduced mass of the colliding particles, respectively; $\hat{\mathbf{P}}$ is the relative-motion momentum operator; $\mathbf{R}$ and r_j are the radius vectors of the ion and of the jth electron of the helium relative to the nucleus of the target atom; Z is the ion charge.

To determine the wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2)$ it is convenient to express the atomic Hamiltonian H_a as a sum of two terms: the Hamiltonian H_a^0 corresponding to the self-consistent-field approximation, and the proper correlation part W^c

$$H_a = H_a^0 + W^c, \tag{5}$$

$$H_a^0 = \sum_{j=1}^{2}\left(\frac{\hat{\mathbf{p}}_j^2}{2} - \frac{Z_1}{r_j} + w_j\right), \qquad W^c = V^c - \sum_{j=1}^{2} w_j(r_j), \tag{6}$$

$V^c = 1/|\mathbf{r}_1 - \mathbf{r}_2|$ is the interelectron-interaction potential, Z_1 is the charge of the target-atom nucleus; w_j is the screening potential applied to the jth electron of the helium by the other electron; $\hat{\mathbf{p}}_j$ is the momentum operator of the jth electron of the helium atom.

We seek the wave function $\Psi_\mu(\mathbf{r}_1,\mathbf{r}_2)$ of the helium in the form

$$\Psi_\mu(\mathbf{r}_1, \mathbf{r}_2) = \Phi_\mu(\mathbf{r}_1, \mathbf{r}_2) + \left(\sum_\nu{}' + \int d\nu\right)\lambda_\nu\Phi_\nu(\mathbf{r}_1, \mathbf{r}_2), \tag{7}$$

where $\Phi_K(\mathbf{r}_1,\mathbf{r}_2)$ is the wave function of the helium in the self-consistent field approximation

$$H_a^0\Phi_K(\mathbf{r}_1, \mathbf{r}_2) = E_K^0\Phi_K(\mathbf{r}_1, \mathbf{r}_2), \tag{8}$$

E_K^0 is the energy of the Kth state of the helium without allowance for correlation effects. For the discrete (d) and continuous (c) states of the helium, $\Phi_K(\mathbf{r}_1,\mathbf{r}_2)$ is determined respectively by the expressions

$$\Phi_K^{(d)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{m_1,m_2} C^{LM}_{l_1 m_1 l_2 m_2}\frac{1 + \hat{P}_{12}}{\sqrt{2}}\varphi_{k_1}(\mathbf{r}_1)\varphi_{k_2}(\mathbf{r}_2), \tag{9a}$$

and

$$\Phi_K^{(c)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1 + \hat{P}_{12}}{\sqrt{2}}\varphi_{k_1}(\mathbf{r}_1)\varphi_{k_2}(\mathbf{r}_2). \tag{9b}$$

Here k_j denotes the aggregate of quantum numbers describing the state of an individual electron (n, l, m—for a discrete spectrum, $\mathbf{u}$—for continuum states); L and M are the total orbital electron moment and its projection on the direction of the relative-velocity vector $\mathbf{v}$ of the colliding particles; $C^{LM}_{l_1 m_1 l_2 m_2}$ is a Clebsch–Gordan coefficient; φ_k is the wave function of an individual helium electron. The last term in the right-hand side of (7) defines a correction to the wave function of the μth state for configuration mixing. The prime on a summation sign denotes elimination from the sum of states that can lead to self-mixing (i.e., to self-mixing of a configuration); $\hat{P}_{12}$ is the permutation operator.

To determine the coefficient λ_ν we substitute expression (7) for the wave function $\Phi_\mu(\mathbf{r}_1,\mathbf{r}_2)$ in Eq. (4), multiply the right-and left-hand sides of the result by the function $\Phi_\gamma(\mathbf{r}_1,\mathbf{r}_2)$ and integrate over the coordinates of both electrons. The result is the following equation for the coefficients λ_γ:

$$\langle\Phi_\gamma|W^c|\Phi_\mu\rangle + \lambda_\gamma(E_\gamma^0 - E_\mu) + \left(\sum_\nu{}' + \int d\nu\right)\lambda_\nu\langle\Phi_\gamma|W^c|\Phi_\nu\rangle = 0. \tag{10}$$

If the elements of the system (10) are written in matrix form, the terms with λ_γ are located on a diagonal (since the system (10) includes continuum states, the matrix of the system of equations has an infinite dimensionality). Confining ourselves in the solution of (10) to diagonal elements only (arguments favoring this confinement are given in Ref. 29), we obtain for the coefficient λ_γ

$$\lambda_\gamma = \frac{\langle \Phi_\gamma | W^c | \Phi_\mu \rangle}{E_\mu - E_\gamma + i0}, \tag{11}$$

where the imaginary increment $i0$ specifies, in the case when $\Phi_\mu(\mathbf{r}_1,\mathbf{r}_2)$ describes a state in a continuous spectrum, the direction of circuiting the pole in the integration $E_\gamma = E_\gamma^0 + \langle \Phi_\gamma | W^c | \Phi_\gamma \rangle$.

In the case of single-electron ionization, transitions into states with one electron in the continuum $\Phi_\mu(\mathbf{r}_1,\mathbf{r}_2) = (1 + \hat{P}_{12}/\sqrt{2})F(\mathbf{r}_1)\varphi_0(\mathbf{r}_2)$ (where $\varphi_0(r)$ describes the ground state for one of the helium electrons, and $F(\mathbf{r})$ describes the continuum state for the other electron) can proceed through resonance states with energy equal to the energy of a state with one electron in the continuum. In this case one cannot use the expression (11) for the coefficients λ_γ that determine the contribution of the resonance states to the wave function (7), since λ_γ becomes infinite. We shall determine the contribution of the resonant states to the wave function Ψ_μ for the coefficients λ_γ by using an expression obtained in the framework of the diagonalization approximation,[29]

$$\lambda_\gamma = \frac{\langle \Phi_\gamma(\mathbf{r}_1,\mathbf{r}_2) | W^c | [(1 + \hat{P}_{12})/\sqrt{2}] F^+(\mathbf{r}_1)\varphi_0(\mathbf{r}_2) \rangle}{E_\mu - E_\gamma^0 - \Delta_\gamma + i\Gamma/2}, \tag{12}$$

where

$$\Delta_\gamma = \sum_{j=1}^{2} \sum_{l \neq j} \langle \Phi_\gamma(\mathbf{r}_1,\mathbf{r}_2) | W^c \varphi_0(\mathbf{r}_j) \frac{P}{K^2 - h_l} \varphi_0(\mathbf{r}_j) W^c | \Phi_\gamma(\mathbf{r}_1,\mathbf{r}_2) \rangle \tag{13}$$

is the energy shift of the position of the autoionization resonance E_R relative to the level E_γ^0 that results from the coupling of the resonant state with the channel Φ_μ; $h_l = -\Delta_l - (Z_1/r_l) + (1/|\mathbf{r}_j - \mathbf{r}_l|)$; $K^2 = 2(E_\mu - \varepsilon_0)$; ε_0 is the ground-state energy of the singly charge ion He^+;

$$\Gamma = 2|\langle \Phi_\gamma(\mathbf{r}_1,\mathbf{r}_2) | W^c | \frac{1 + \hat{P}_{12}}{\sqrt{2}} F^+(\mathbf{r}_1)\varphi_0(\mathbf{r}_2) \rangle|^2 \tag{14}$$

is the autoionization width of the resonance $\Phi_\gamma(\mathbf{r}_1\mathbf{r}_2)$; $F^+(\mathbf{r}_1)$ is the solution, regular at zero, of the equation

$$\frac{\Delta_1}{2} F^+(\mathbf{r}_1) + (E_\mu - \varepsilon_0)F^+(\mathbf{r}_1) + \left(\frac{Z_1}{r_1} - \langle \varphi_0(\mathbf{r}_2) | W^c | \varphi_0(\mathbf{r}_2) \rangle \right) F^+(\mathbf{r}_1) = 0. \tag{15}$$

We take hereafter F^+ to mean a continuum single-electron wave function defined in the self-consistent-field approximation.

The coefficients λ_γ [expression (11)] differ by an energy denominator from those obtained in first-order perturbation theory with respect to the potential W^c in Ref. 19. Our present coefficients contain correlated values of the electron energy, while the coefficients used in Ref. 19 contain values determined in the self-consistent-field approximation. The use of our present wave functions to determine the characteristics describing the scattering process is preferable in virtue of the fact that the expressions obtained on their basis will contain only correlated values of the electron energies.

2.2. Amplitudes of electronic transitions

The amplitude of an inelastic transition in helium from state I to state F, effected by a fast charged particle, is defined as a function of the impact parameters by the expression

$$A(F \leftarrow I; \rho) = \langle \Psi_F | S(+\infty, -\infty) | \Psi_I \rangle, \tag{16}$$

where the scattering matrix S is of the form

$$S(+\infty, -\infty) = T \exp[-i \int_{-\infty}^{\infty} d\tilde{t} \tilde{V}(t)]. \tag{17}$$

Expanding the exponential above in a perturbation-theory series and using the completeness of the eigenfunctions of the Hamiltonian H_a, we obtain for $A(F \leftarrow I; \rho)$

$$A(F \leftarrow I; \rho) = \langle \Psi_F | - i \int_{-\infty}^{\infty} d\tilde{t} \tilde{V}(t) + (-i)^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \times \tilde{V}(t_1)\tilde{V}(t_2) + \ldots | \Psi_I \rangle = -i \int_{-\infty}^{\infty} dt \exp[i(E_F - E_I)t]$$

$$\times \langle \Psi_F | \sum_{j=1}^{2} \frac{-Z}{|\mathbf{R}(t) - \mathbf{r}_j|} | \Psi_I \rangle - \left(\sum_K + \int dK \right) \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \times \exp[i(E_F - E_K)t_1] \exp[i(E_K - E_I)t_2]$$

$$\times \langle \Psi_F | \sum_{j=1}^{2} \frac{-Z}{|\mathbf{R}(t_1) - \mathbf{r}_j|} | \Psi_K \rangle \langle \Psi_K | \sum_{j=1}^{2} \frac{-Z}{|\mathbf{R}(t_2) - \mathbf{r}_j|} | \Psi_I \rangle + \ldots, \tag{18}$$

where $\{\Psi_K(\mathbf{r}_1,\mathbf{r}_2)\}$ is the complete set of eigenfunction of the Hamiltonian H_a and E_K is the eigenvalue corresponding to the eigenfunction $\Psi_K(\Gamma_1,\Gamma_2)$. Since we are interested in the charge asymmetry of ion–atom collisions, we need take into account in the definition of the inelastic-transition amplitude only the perturbation-theory series up to second order in the potential V inclusive. These very terms are responsible for the charge asymmetry, for only they can contribute to the term proportional to Z^3 in the scattering cross section. We neglect the contribution of the terms proportional to Z^n with $n \geqslant 5$ to the scattering cross section.

Substituting in (18) expressions (7), (9), and (11)–(14), which define the wave function $\Psi_\mu(\mathbf{r}_1,\mathbf{r}_2)$ and the coefficients λ_ν, and using the Bethe integral[30]

$$\frac{-Z}{|\mathbf{R}(t) - \mathbf{r}_j|} = \frac{-Z}{2\pi^2} \int \frac{d^3q}{q^2} \exp\{-iq[\mathbf{R}(t) - \mathbf{r}_j]\}, \tag{19}$$

we write for the amplitudes (1) of the inelastic translations:

a) the single-electron excitation amplitude

$$A(1snlm \leftarrow 1s^2\,{}^1S; \rho) = a^{I1}(1snlm; \rho) + a^{c1}(1snlm; \rho) + a^{I2}(1snlm; \rho) + d(1snlm; \rho) + a^{c2}(1snlm; \rho), \tag{20}$$

b) the two-electron excitation amplitude

$$A(n_1l_1n_2l_2\,{}^1LM \leftarrow 1s^2\,{}^1S; \rho) = \sum_{m_1,m_2} C^{LM}_{l_1m_1l_2m_2} \times \{a^{c1}(n_1l_1m_1n_2l_2m_2; \rho) + a^{I2}(n_1l_1m_1n_2l_2m_2; \rho) + a^{c2}(n_1l_1m_1n_2l_2m_2; \rho)\}, \tag{21}$$

c) the single-electron ionization amplitude

$$A(nlmu \leftarrow 1s^2\,{}^1S;\rho) = a^{I1}(nlmu;\rho) + a^{c1}(nlmu;\rho) + a^{I2}(nlmu;\rho) + d(nlmu;\rho) + a^{c2}(nlmu;\rho) + a^{R}(nlmu;\rho), \tag{22}$$

d) the two-electron ionization amplitude

$$A(u_1u_2 \leftarrow 1s^2\,{}^1S;\rho) = a^{c1}(u_1u_2;\rho) + a^{I2}(u_1u_2;\rho) + a^{c2}(u_1u_2;\rho). \tag{23}$$

The amplitudes a^{I1}, a^{c1}, a^{I2}, d, a^{c2} and a^R are defined, respectively, as

$$a^{I1}(f_1f_2;\rho) = \frac{Z\sqrt{2}i}{\pi}\sum_{j=1}^{2}\sum_{t\neq j}\int\frac{d^3q'}{q'^2}\exp(iq_\perp'\rho) \times\delta(E_F - E_I - q_\parallel'v)\langle f_j|\exp(iq'r)|1s\rangle\langle f_t|1s\rangle, \tag{24}$$

δ being the amplitude of the transition of one of the helium electrons from the state $|1s\rangle$ into the state $|f_{1,2}\rangle$ ($|f_j\rangle = |nlm\rangle$ or $|u\rangle$ for transitions into the states of the discrete or continuous spectra, respectively) without change of the state of the second electron (inasmuch as the one-electron wave functions are orthogonal, we have $\langle f_t|1s\rangle = 0$ for $|f_t\rangle \neq |1s\rangle$, or $\langle f_t|1s\rangle = 1$ for $|f_t\rangle = |1s\rangle$), defined in first-order perturbation theory;

$$a^{c1}(f_1f_2;\rho) = \frac{Z2\sqrt{2}i}{\pi}\sum_{j=1}^{2}\sum_{t\neq j}\left(\sum_\gamma{}' + \int d\gamma\right)\int\frac{d^3q'}{q'^2}\exp(-iq_\perp'\rho) \times\delta(E_F - E_I - q_\parallel'v)\left\{\left[i\pi\delta(E_F - E_\Gamma) + \frac{P}{E_F - E_\Gamma}\right] \times\langle f_jf_t|W^c|\gamma 1s\rangle\langle\gamma|\exp(iq'r)|1s\rangle + \frac{1}{E_I - E_\Gamma} \times\langle f_j|\exp(iq'r)|\gamma\rangle\langle\gamma f_t|W^c|1s^2\rangle\right\} \tag{25}$$

is the amplitude of the inelastic transitions of helium electrons as a result of single scattering of an ion by helium and of the correlation interaction of the atomic electrons;

$$a^{I2}(f_1f_2;\rho) = \frac{-Z^2\sqrt{2}}{\pi^2}\sum_{j=1}^{2}\sum_{t\neq j}\int\frac{d^3q}{q^2}\exp(-iq_\perp\rho) \times\langle f_j|\exp(iqr)|1s\rangle\delta(\varepsilon_{f_j} - \varepsilon_{1s} - q_\parallel v) \times\int\frac{d^3p}{p^2}\exp(-ip_\perp\rho)\langle f_t|\exp(ipr)|1s\rangle\delta(\varepsilon_{f_t} - \varepsilon_{1s} - p_\parallel v) \tag{26}$$

is the amplitude of the transition of two helium electrons as a result of interaction of each of them with the incident ion. The amplitude a^{I2} coincides with the amplitude determine in the independent-electron approximation in second order of perturbation theory, and corresponds to the amplitude TS-2 introduced in Ref. 3 (it is shown in the Appendix that in second-order perturbation theory the two-electron transition amplitude determined in the independent-electron approximation contains no off-energy part);

$$d(f_1f_2;\rho) = \frac{Z^2\sqrt{2}i}{\pi^3}\sum_{j=1}^{2}\sum_{t\neq j}\left(\sum_k + \int dk\right)\int\frac{d^3q}{q^2}\exp(-iq_\perp\rho) \times\langle f_j|\exp(iqr)|k\rangle\int\frac{d^3p}{p^2}\exp(-ip_\perp\rho)\left\{\left[i\pi\delta(\varepsilon_k - \varepsilon_{1s} - p_\parallel v) + \frac{P}{\varepsilon_k - \varepsilon_{1s} - p_\parallel v}\right] \times\langle k|\exp(ipr)|1s\rangle\delta(\varepsilon_{f_j} - \varepsilon_{1s} - q_\parallel v - p_\parallel v)\langle f_t|1s\rangle\right\} \tag{27}$$

is the amplitude of the steplike transitions of one helium electron as a result of its interaction with the incident electron;

$$a^{c2}(f_1f_2;\rho) = \frac{Z^22\sqrt{2}i}{\pi^3}\sum_{j=1}^{2}\sum_{t\neq j}\left(\sum_k + \int dk\right) \times\left(\sum_\gamma{}' + \int d\gamma\right)\int\frac{d^3q}{q^2}\exp(-iq_\perp\rho) \times\int\frac{d^3p}{p^2}\exp(-ip_\perp\rho)\delta(E_F - E_I - q_\parallel v - p_\parallel v) \times\left[i\pi\delta(E_K - E_I - p_\parallel v) + \frac{P}{E_K - E_I - p_\parallel v}\right]\left\{\frac{1}{E_F - E_\Gamma - i0}\left[\langle f_jf_t|W^c|\gamma 1s\rangle \times\langle\gamma|\exp(iqr)|k\rangle\langle k|\exp(ipr)|1s\rangle + \langle f_jf_t|W^c|k\gamma\rangle\langle\gamma|\exp(iqr)|1s\rangle\langle k|\exp(ipr)|1s\rangle\right] + \left[\langle f_j|\exp(iqr)|k\rangle\frac{\langle kf_t + f_tk|W^c|\gamma 1s\rangle}{E_K - E_\Gamma - i0}\langle\gamma|\exp(ipr)|1s\rangle + \langle f_j|\exp(iqr)|\gamma\rangle\frac{\langle\gamma f_t + f_t\gamma|W^c|k1s\rangle}{E_K - E_\Gamma + i0}\langle k|\exp(ipr)|1s\rangle\right] + \frac{1}{E_I - E_\Gamma + i0}\left[\langle f_j|\exp(iqr)|k\rangle\langle f_t|\exp(ipr)|\gamma\rangle\langle k\gamma|W^c|1s^2\rangle + \langle f_j|\exp(iqr)|k\rangle\langle k|\exp(ipr)|\gamma\rangle\langle\gamma f_t|W^c|1s^2\rangle\right]\right\} \tag{28}$$

is the amplitude of the inelastic transitions of helium electrons as a result of the twofold scattering of the ion by the atom and of the correlation interaction of the atomic electrons;

$$a^R(1su;\rho) = \frac{Z^22\sqrt{2}i}{\pi^3}\int\frac{d^3q}{q^2}\exp(-iq_\perp\rho)\int\frac{d^3p}{p^2}\exp(-ip_\perp\rho) \times\delta(E_F - E_K - q_\parallel v - p_\parallel v)\left[i\pi\delta(E_K - E_I - p_\parallel v) + \frac{P}{E_K - E_I - p_\parallel v}\right] \times\frac{\langle 1su|W^c|\gamma k + k\gamma\rangle}{E_F - E_R + i\Gamma/2}\langle\gamma|\exp(iqr)|1s\rangle\langle k|\exp(ipr)|1s\rangle \tag{29}$$

is the amplitude of the resonant transition of one helium

electron to the continuum via an autoionization state. We have confined ourselves here only to the case of one resonant state.

Here $\mathbf{q}(\mathbf{p})$ and $\mathbf{q}'$ are the momenta transferred respectively to one electron and the helium atom in the collision process, $q_{\parallel}(p_{\parallel})$, $q'_{\parallel}$ and $\mathbf{q}_{\perp}(\mathbf{p}_{\perp})$, $\mathbf{q}'_{\perp}$ are their parallel and orthogonal components relative to the velocity vector $\mathbf{v}$; ε_{ν} is the energy of one helium electron in the state ν; the symbol P in expressions (25) and (27)–(29) denotes that the integrals over ε and $p_{\parallel}$ are calculated in the sense of the principal value.

We continue the transformation of the amplitudes (24)–(29) using the scheme described in Ref. 22. For the wave functions $|g\rangle$ of an individual electron we use Coulomb wave functions defined in the field of the effective charge Z^*. We use an expansion of $\exp(i\mathbf{tr})$ in a series of spherical harmonics[31]

$$\exp(i\mathbf{tr}) = 4\pi\sum_{l,m} i^l j_l(tr) Y^*_{lm}(\Omega_t) Y_{lm}(\Omega_r), \tag{30}$$

and an expansion of the one-electron wave function $|\mathbf{k}\rangle$ in Coulomb functions of the stationary states with definite energy ε, momentum l, and its component m along the quantization axis[31]

$$|\mathbf{k}\rangle = \frac{2\pi}{k}\sum_{l,m} i^l \exp(i\delta_l) R_{kl}(r) Y^*_{lm}(\Omega_k) Y_{lm}(\Omega_r). \tag{31}$$

Here $j_l(tr)$ is a spherical Bessel function of order l; $Y_{lm}(\Omega)$ a spherical harmonic; Ω_t is the spherical angle of the vector $\mathbf{t}$; δ_l is the phase shift; $\mathbf{k}$ the electron momentum in the continuum state; $R_{kl}(r)$ a Coulomb wave function defined in the effective-charge field Z^*. We denote by $Q(f_1 f_2;\rho)$ any of the amplitudes (24)–(29) and introduce the definition

$$Q(f_1 f_2;\rho) = \sum_{l'_1 m'_1 l'_2 m'_2} U_{l'_1 m'_1}(\tilde{n}_1) U_{l'_2 m'_2}(\tilde{n}_2) \tilde{Q}(\tilde{n}_1 l'_1 m'_1 \tilde{n}_2 l'_2 m'_2;\rho), \tag{32}$$

where

$$U_{l'm'}(\tilde{n}) = \begin{cases} \delta_{ll'}\delta_{mm'}, & \tilde{n} = n \\ \dfrac{2\pi}{u}(+i)^{l'}\exp(-i\delta_{l'}) Y_{l'm'}(\Omega_u), & \tilde{n} = |\mathbf{u}| \end{cases}. \tag{33}$$

The variable $\tilde{n}$ denotes here any principal quantum number n of the electron state in the discrete spectrum or else the electron momentum $\varkappa$ in the continuum state, respectively. The functions $\tilde{Q}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho)$ coincide in the case of discrete transitions with the corresponding functions, while in the case of transitions to the continuum they are the amplitudes of transitions to states characterized by a definite energy $\varkappa^2/2$, a momentum l, and its component m along the ion velocity $\mathbf{v}$. The introduction of the functions $\tilde{Q}$ is convenient because they can be resolved into sums of not more than two terms, each of which proportional to the imaginary unity raised to a definite degree (more below), in contrast to the functions Q which do not have this property in the case of transitions into the continuum.

Substituting (30) and (31) in expressions (24)–(29) and introducing the structure functions T, we represent the amplitudes $\tilde{Q}$ in the form:

$$\tilde{a}^{I1}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{Z\sqrt{2}i}{\pi v}\sum_{j=1}^{2}\sum_{t\neq j} T^{(1)}(|\tilde{n}_f l_f m_j 1s(F)\rangle \leftarrow |1s^2(I)\rangle;\rho)\langle \tilde{n}_t l_t m_t|1s\rangle, \tag{34}$$

$$\tilde{d}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \tilde{d}_{on}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) + \tilde{d}_{off}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho), \tag{35}$$

where

$$\tilde{d}_{on}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{-Z^2\sqrt{2}}{\pi^2 v^2} \times \sum_{j=1}^{2}\sum_{t\neq j}\sum_{l,m} \mathop{S}_{\tilde{n}} T^{(2)}(|\tilde{n}_f l_f m_j 1s(F)\rangle \leftarrow |\tilde{n} l m 1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho)\langle \tilde{n}_t l_t m_t|1s\rangle, \tag{35a}$$

$$\tilde{d}_{off}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{-Z^2\sqrt{2}i}{\pi^3 v} \times \sum_{j=1}^{2}\sum_{t\neq j}\sum_{l,m} \mathop{S}_{\tilde{n}} \int dq_{\parallel} \frac{P}{E_F - E_K - q_{\parallel} v} \tilde{T}^{(2)}_{q_{\parallel}}(|\tilde{n}_f l_f m_j 1s(F)\rangle \leftarrow |\tilde{n} l m 1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho)\langle \tilde{n}_t l_t m_t|1s\rangle \tag{35b}$$

are the amplitudes of the direct ($\tilde{a}^{I1}$) and stepwise ($\tilde{d}$) transitions of one helium electron, respectively. The amplitudes $\tilde{d}_{on}$ and $\tilde{d}_{off}$ describe transitions on an off the energy shell, respectively;

$$\tilde{a}^{I2}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{-Z^2\sqrt{2}}{\pi^2 v^2} \times \sum_{j=1}^{2}\sum_{t\neq j} T^{(2)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow |1s\tilde{n}_t l_t m_t(K)\rangle \leftarrow |1s^2(I)\rangle;\rho) \tag{36}$$

is the amplitude of a two-electron transition resulting from interaction of an ion with each of the helium electrons;

$$\tilde{a}^{c1}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \tilde{a}^{c1}_{on}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) + \tilde{a}^{c1}_{off}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho), \tag{37}$$

where

$$\tilde{a}^{c1}_{on}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{-Z8\sqrt{2}\pi^2}{v}\sum_{j=1}^{2}\sum_{t\neq j}\int dk\delta(E_\Gamma - E_F) \times \sum_{l,m}(-1)^l T^{(2)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow |klm1s(\Gamma)\rangle \leftarrow |1s^2(I)\rangle;\rho), \tag{37a}$$

$$\tilde{a}^{c1}_{off}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{Z2\sqrt{2}i}{\pi v} \times \sum_{j=1}^{2}\sum_{t\neq j}\sum_{l,m} \mathop{S'}_{\tilde{n}} \left\{ \frac{P}{E_F - E_\Gamma} T^{(2)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow |\tilde{n} l m 1s(\Gamma)\rangle |1s^2(I)\rangle;\rho) + \frac{1}{E_I - E_\Gamma} T^{(2)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow |\tilde{n} l m \tilde{n}_t l_t m_t(\Gamma)\rangle \leftarrow |1s^2(I)\rangle;\rho) \right\}; \tag{37b}$$

$$\tilde{a}^{c2}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \tilde{a}_1^{c2}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) + \tilde{a}_2^{c2}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho), \quad (38)$$

where

$$\tilde{a}_1^{c2}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{-Z^2 2\sqrt{2}i}{\pi^3 v}$$

$$\times \sum_{j=1}^{2} \sum_{t\neq j} \sum_{l',m'} \sum_{l'',m''} \underset{\tilde{n}'}{S} \underset{\tilde{n}''}{S'} \left\{ \frac{\pi^2}{v} [T^{(3)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |n'l'm'n''l''m''(\Gamma)\rangle \end{matrix} \right\} \leftarrow |\tilde{n}'l'm'1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho)$$

$$\times \delta(E_\Gamma - E_F) \pm T^{(3)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}'l'm'\tilde{n}_t l_t m_t(K)\rangle \\ |\tilde{n}''l''m''\tilde{n}_t l_t m_t(\Gamma)\rangle \end{matrix} \right\} \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |\tilde{n}'l'm'1s(K)\rangle \end{matrix} \right\} \leftarrow |1s^2(I)\rangle;\rho)\delta(E_\Gamma - E_K)]$$

$$+ \int dq_{\parallel} \frac{P}{E_F - E_K - q_{\parallel} v} \left[\frac{1}{E_I - E_\Gamma} \tilde{T}^{(3)}_{q_{\parallel}}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow |\tilde{n}'l'm'\tilde{n}_t l_t m_t(K)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}'l'm'\tilde{n}''l''m''(\Gamma)\rangle \\ |\tilde{n}''l''m''\tilde{n}_t l_t m_t(\Gamma)\rangle \end{matrix} \right\} \leftarrow |1s^2(I)\rangle;\rho) \right.$$

$$+ \frac{P}{E_K - E_\Gamma} \tilde{T}^{(3)}_{q_{\parallel}}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}'l'm'\tilde{n}_t l_t m_t(K)\rangle \\ |\tilde{n}''l''m''\tilde{n}_t l_t m_t(\Gamma)\rangle \end{matrix} \right\} \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |\tilde{n}'l'm'1s(K)\rangle \end{matrix} \right\} \leftarrow |1s^2(I)\rangle;\rho)$$

$$\left. + \frac{P}{E_F - E_\Gamma} \tilde{T}^{(3)}_{q_{\parallel}}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |\tilde{n}'l'm'\tilde{n}''l''m''(\Gamma)\rangle \end{matrix} \right\} \leftarrow |\tilde{n}'l'm'1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho) \right] \Bigg\}, \quad (38a)$$

$$\tilde{a}_2^{c2}(\tilde{n}_1 l_1 m_1 \tilde{n}_2 l_2 m_2;\rho) = \frac{-Z^2 2\sqrt{2}}{\pi^2 v}$$

$$\times \sum_{j=1}^{2} \sum_{t\neq j} \sum_{l',m'} \sum_{l'',m''} \underset{\tilde{n}'}{S} \underset{\tilde{n}''}{S'} \left\{ \frac{1}{v} \times \left[\frac{1}{E_I - E_\Gamma} T^{(3)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow |\tilde{n}'l'm'\tilde{n}_t l_t m_t(K)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}'l'm'\tilde{n}''l''m''(\Gamma)\rangle \\ |\tilde{n}''l''m''\tilde{n}_t l_t m_t(\Gamma)\rangle \end{matrix} \right\} \leftarrow |1s^2(I)\rangle;\rho) + \frac{P}{E_K - E_\Gamma} T^{(3)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}'l'm'\tilde{n}_t l_t m_t(K)\rangle \\ |\tilde{n}''l''m''n_t l_t m_t(\Gamma)\rangle \end{matrix} \right\} \leftarrow \left\{ \begin{matrix} |1s\tilde{n}''l''m''(\Gamma)\rangle \\ |1s\tilde{n}'l'm'(K)\rangle \end{matrix} \right\} \leftarrow |1s^2(I)\rangle;\rho) \right.$$

$$\left. + \frac{P}{E_F - E_\Gamma} T^{(3)}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |\tilde{n}'l'm'\tilde{n}''l''m''(\Gamma)\rangle \end{matrix} \right\} \leftarrow |\tilde{n}'l'm'1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho) \right] - \int dq_{\parallel} \frac{P}{E_F - E_K - q_{\parallel} v} [\tilde{T}^{(3)}_{q_{\parallel}}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |\tilde{n}'l'm'n''l''m''(\Gamma)\rangle \end{matrix} \right\} \leftarrow |\tilde{n}'l'm'1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho)\delta(E_\Gamma - E_F)$$

$$\pm \tilde{T}^{(3)}_{q_{\parallel}}(|\tilde{n}_f l_f m_f \tilde{n}_t l_t m_t(F)\rangle \leftarrow \left\{ \begin{matrix} |\tilde{n}'l'm'\tilde{n}_t l_t m_t(K)\rangle \\ |\tilde{n}''l''m''\tilde{n}_t l_t m_t(\Gamma)\rangle \end{matrix} \right\} \leftarrow \left\{ \begin{matrix} |\tilde{n}''l''m''1s(\Gamma)\rangle \\ |\tilde{n}'l'm'1s(K)\rangle \end{matrix} \right\} \leftarrow |1s^2(I)\rangle;\rho)\delta(E_\Gamma - E_K)] \Bigg\} \quad (38b)$$

are the amplitudes of inelastic electron transitions resulting from single ($\tilde{a}^{c1}$) and twofold ($\tilde{a}^{c2}$) scattering of anion by an atom, respectively, and of the correlation interaction of atomic electrons. The amplitudes $\tilde{a}^{c1}_{on}$ and $\tilde{a}^{c1}_{off}$ describe transitions respectively on and off the energy shell, whereas each of the amplitudes $\tilde{a}^{c2}_1$ and $\tilde{a}^{c2}_2$ includes both transitions. The amplitude $\tilde{a}^{c1}_{on}$ corresponds to ion scattering by helium such that the electron ejected as a result of the interaction with the incident ion is scattered by another helium electron, and describes transitions via states of the one-electron continuum. Since this transition takes place on the energy shell ($E_F = E_\Gamma$), the amplitude does no contain this term in the case of single-electron excitation of helium. In the case of twofold ionization of helium, the amplitude a^{c1}_{on} responsible for $\tilde{a}^{c1}_{on}$ corresponds to the amplitude $TS-1$ (Ref. 3);

$$\tilde{a}^R(1sulm;\rho) = \tilde{a}_1^R(1sulm;\rho) + \tilde{a}_2^R(1sulm;\rho), \quad (39)$$

where

$$\tilde{a}_1^R(1sulm;\rho) = \frac{-Z^2 2\sqrt{2}}{\pi^3 v[(E_F - E_R)^2 + \Gamma^2/4]}$$

$$\times \sum_{\substack{m_1,m_2,\\ m_1',m_2'}} C^{LM}_{l_1 m_1 l_2 m_2} C^{LM}_{l_1 m_1' l_2 m_2'} \sum_{j=1}^{2} \sum_{t\neq j} D^{(2)}(|1sulm\rangle \leftarrow |n_f l_f m_f n_t l_t m_t\rangle) \left\{ \frac{\pi}{v}(E_F - E_R) T^{(2)}(|n_f l_f m_f' n_t l_t m_t'(\Gamma)\rangle \leftarrow |n_f l_f m_f' 1s(K)\rangle \leftarrow |1s^2(I);\rho) \right.$$

$$\left. + \frac{\Gamma}{2} \int dq_{\parallel} \frac{P}{E_F - E_K - q_{\parallel} v} \tilde{T}^{(2)}_{q_{\parallel}}(|n_f l_f m_f' n_t l_t m_t'(\Gamma)\rangle \leftarrow \left\{ |n_f l_f m_f' 1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho) \right\} \right\}, \quad (39a)$$

and

$$\tilde{a}_2^R(1sulm;\rho) = \frac{Z^2 2\sqrt{2}i}{\pi^3 v[(E_F - E_R)^2 + \Gamma^2/4]}$$

$$\times \sum_{\substack{m_1,m_2,\\ m_1',m_2'}} C^{LM}_{l_1 m_1 l_2 m_2} C^{LM}_{l_1 m_1' l_2 m_2'} \sum_{j=1}^{2} \sum_{t\neq j} D^{(2)}(|1sulm\rangle \leftarrow |n_f l_f m_f n_t l_t m_t\rangle) \left\{ \frac{\pi}{v} \frac{\Gamma}{2} T^{(2)}(|n_f l_f m_f' n_t l_t m_t'(\Gamma)\rangle \leftarrow |n_f l_f m_f' 1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho) \right.$$

$$\left. - (E_F - E_R) \int dq_{\parallel} \frac{P}{E_F - E_K - q_{\parallel} v} \tilde{T}^{(2)}_{q_{\parallel}}(|n_f l_f m_f' n_t l_t m_t'(\Gamma)\rangle \leftarrow \left\{ \leftarrow |n_f l_f m_f' 1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho) \right\} \right\} \quad (39b)$$

is the amplitude of the resonant ionization of helium in the presence of one resonance. Just as in the case of the amplitude $\tilde{a}^{c2}$, each of the amplitudes a_1^R and a_2^R includes transitions that occur both on and off the energy shell.

By subdividing the amplitudes $\tilde{d}$, $\tilde{a}^{c1}$, $\tilde{a}^{c2}$ and $\tilde{a}^R$ each into sums of two terms we aimed to separate terms with equal powers of $\sqrt{-1}$. The difference between the degrees of imaginarity of individual terms in each of the amplitudes (35) and (37)–(39) is that they contain propagators due either to interaction of the incident ion with the helium electrons, or to interelectron interaction. Each of the propagators breaks up into a sum of two terms with different powers of the imaginary unity

$$\frac{1}{E-\tilde{E}\pm i0}=\frac{P}{E-\tilde{E}}\mp i\pi\delta(E-\tilde{E}). \tag{40}$$

The energy conservation law is satisfied for one of the terms of (40). Therefore the amplitudes $\tilde{d}$ and $\tilde{a}^{c1}$, which contain only one propagator each, terms with the same power of $\sqrt{-1}$ describe transitions either on or off the energy shell. On the other hand, in the case of the amplitudes $\tilde{a}^{c2}$ and a^R, which contain two propagators each, terms with equal power of $\sqrt{-1}$ describe already transitions both on and off the energy shell. The power of $\sqrt{-1}$ in amplitudes corresponding to different transitions is an important property, since it makes it possible to determine the phase differences between different amplitudes $\tilde{Q}$ and hence the possibility of interference of various transitions with one another.

The structure functions $T^{(1)}$, $T^{(2)}$, $\tilde{T}^{(2)}_{q_\parallel}$, $T^{(3)}$, and $\tilde{T}^{(3)}_{q_\parallel}$ describe electron transitions from a state contained in extreme right-hand position of the transition chains into a state in the extreme left position. Each of the transition-chain states contained in the structure functions is determined by the product of wave functions of individual electrons entering in some state of the helium atom, designated in parentheses. Antisymmetrization effects are taken into account either by a corresponding numerical factor or by the form of the actual expression for the structure functions. Each simple arrow ($\leftarrow$) in the transition chain corresponds to a transition under the action of an electron with the incident ion, while each double arrow ($\Leftarrow$) corresponds to a transition resulting from a correlated electron–electron interaction. Since V is a single-electron operator, its action can effect a transition of only one electron. The states indicated on the right and left of a simple arrow can therefore have different characteristics of only one electron. In the functions $T^{(1)}$, $T^{(2)}$, and $T^{(3)}$ the energy-conservation law is satisfied in each collision of the incident particle with the helium electrons, i.e., as many times as there are simple arrows in the chain of the structure-function transitions. The parallel component of the momentum transferred to the helium atom in each collision is determined by the energy difference of the atomic system before and after the collision. In the first collision act the parallel component of the momentum is determined by the difference of the energies before and after the first collision,while in the second it is determined by the atomic-system energy difference after the first and second collisions. The energy of the system in states resulting from configuration mixing does not enter in the energy conservation law, and is not taken into account in the determination of the parallel component of momentum transfer. These states are labeled in the equation by the letter Γ. For the functions $\tilde{T}^{(2)}_{q_\parallel}$ and $\tilde{T}^{(3)}_{q_\parallel}$ the energy conservation law is satisfied only for the entire collision process.

Each of the functions $T^{(3)}$ and $\tilde{T}^{(3)}_{q_\parallel}$ includes two chains of transitions, one determined by the sequence of the upper states of the chain (if only one state is located between the arrows, it is contained in both chains), and the other by the lower states. These structure functions are sums of two structure functions for each of the chains of the transition.

$$T^{(3)}(|n_1l_1m_1n_2l_2m_2(F)\rangle \leftarrow \begin{Bmatrix}|n'l'm'n_2l_2m_2(K)\rangle\\ |n''l''m''n_2l_2m_2(\Gamma)\rangle\end{Bmatrix} \Leftarrow \begin{Bmatrix}|n''l''m''1s(\Gamma)\rangle\\ |n'l'm'1s(K)\rangle\end{Bmatrix} \leftarrow$$

$$|1s^2(I)\rangle;\rho) = T^{(3)}(|n_1l_1m_1n_2l_2m_2(F)\rangle \leftarrow |n'l'm'n_2l_2m_2(K)\rangle \Leftarrow |n''l''m''1s(\Gamma)\rangle \leftarrow |1s^2(I)\rangle;\rho) + T^{(3)}(|n_1l_1m_1n_2l_2m_2(F)\rangle \leftarrow |n''l''m''n_2l_2m_2(\Gamma)\rangle \Leftarrow |n'l'm'1s(K)\rangle \leftarrow |1s^2(I)\rangle;\rho). \tag{41}$$

The $\pm$ sign preceding the functions $T^{(3)}$ and $\tilde{T}^{(3)}_{q_\parallel}$ containing two chains of transitions denotes that the corresponding structure function responsible for the upper chain enters in the equation with the upper sign, and that for the lower chain with the lower sign.

The expression for the structure function should be written as going from left to right along the transition chain and assigning the function $D^{(1)}$ to each simple arrow and $D^{(2)}$ to each dual one, with indication of the states of the active electrons, i.e., those electrons whose states can change on going from the start of the arrow to its end (the function $D^{(1)}$ has one such electron and the function $D^{(2)}$ has two). The sequence of the single-electron states from the left to the right of the double arrow in the function D^2 is firmly fixed, to ensure strict unambiguity in the treatment of electron scattering by one another, when both the direct and exchange processes are possible. It is thus understood that for the link $\gamma\nu\Leftarrow pq$ the transition of an electron from a state $p(q)$ can take place only in a state $\gamma(\nu)$. When the correlation interaction takes the system out of the initial state I and takes it into the final state F, the antisymmetrization is automatically taken into account in the expression for the function $\tilde{Q}(\tilde{n}_1l_1m_1\tilde{n}_2l_2m_2;\rho)$. In these cases a double arrow in the transition chain contained in a structure function is set in correspondence with one $D^{(2)}$ function. To take into account antisymmetrization of each double arrow designating, in a chain of structure-function transitions, a transfer of a system from a K state to a Γ state or conversely (this takes place only for the functions $T^{(3)}$ and $\tilde{T}^{(3)}_{q_\parallel}$), it is necessary to set in correspondence a sum of two $D^{(2)}$ functions corresponding to the direct and exchange processes. We write down expressions for some of the structure functions:

$$T^{(1)}(|nlm1s(F)\rangle \leftarrow |1s^2(I);\rho) = D^{(1)}(|nlm\rangle \leftarrow |1s\rangle\left|\frac{E_F-E_I}{v};\rho\right), \tag{42}$$

$$T^{(3)}(|n_1l_1m_1n_2l_2m_2(F)\rangle \leftarrow |n'l'm'n_2l_2m_2(K)\rangle \Leftarrow$$

$$|n''l''m''1s(\Gamma)\rangle \leftarrow$$

$$|1s^2(I)\rangle; \rho) = D^{(1)}(|n_1l_1m_1\rangle \leftarrow |n'l'm'\rangle \left|\frac{E_F - E_K}{v}; \rho\right)$$

$$\times [D^{(2)}(|n'l'm'n_2l_2m_2\rangle \Leftarrow |n''l''m''1s\rangle)$$

$$+ D^{(2)}(|n_2l_2m_2n'l'm'\rangle \Leftarrow |n''l''m''1s\rangle)]$$

$$\times D^{(1)}(|n''l''m''\rangle \leftarrow |1s\rangle \left|\frac{E_K - E_I}{v}; \rho\right), \quad (43)$$

$$\tilde{T}^{(3)}_{q_\parallel}(|n_1l_1m_1n_2l_2m_2(F)\rangle \Leftarrow |n''l''m''1s(\Gamma)\rangle \leftarrow$$

$$|n'l'm'1s(K)\rangle \leftarrow |1s^2(I)\rangle; \rho)$$

$$= D^{(2)}(|n_1l_1m_1n_2l_2m_2\rangle \Leftarrow |n''l''m''1s\rangle)$$

$$\times D^{(1)}(|n''l''m''\rangle \leftarrow |n'l'm'|q_\parallel; \rho)$$

$$\times D^{(1)}(|n'l'm'\rangle \leftarrow |1s\rangle \left|\frac{E_F - E_I - q_\parallel v}{v}; \rho\right). \quad (44)$$

The expressions for the remaining structure functions can be easily obtained by analogy. In the arguments of the function $D^{(1)}$ are indicated the values of the parallel component of the momentum transfer to the helium atom in the corresponding collision act. Apart from a factor that depends on the incident-ion velocity v', the function $D^{(1)}$ is the quasiclassical amplitude of the single-electron transition[32]

$$D^{(1)}(|\tilde{n}_1l_1m_1\rangle \leftarrow |\tilde{n}lm\rangle|q_\parallel; \rho)$$

$$\equiv \int \frac{d^2q_\perp}{q^2} \exp(-iq_\perp\rho)\langle\tilde{n}_1l_1m_1|\exp(iqr)|\tilde{n}lm\rangle$$

$$= 2\pi\sum_{l',m'} i^{m'}(2l' + 1)\left[\frac{(2l + 1)(l' - |m'|)!}{(2l_1 + 1)(l' + |m'|)!}\right]^{1/2} C^{l_1 0}_{l0l'0}C^{l_1m_1}_{lml'm'}$$

$$\times \int \frac{q_\perp dq_\perp}{q^2} J_{|m'|}(q_\perp\rho)P_{l'}^{|m'|}(\cos\theta_q)\int r^2drR^*_{n_1l_1}(r)\tilde{R}_{nl}(r)j_{l'}(qr), \quad (45)$$

and the function $D^{(2)}$ is a matrix element of the potential of the correlation interaction W^c

$$D^{(2)}(|\tilde{n}_1l_1m_1\tilde{n}_2l_2m_2\rangle \Leftarrow |\tilde{n}_1'l_1'm_1'\tilde{n}_2'l_2'm_2'\rangle)$$

$$\equiv \langle\tilde{n}_1l_1m_1\tilde{n}_2l_2m_2|W^c|\tilde{n}_1'l_1'm_1'\tilde{n}_2'l_2'm_2'\rangle$$

$$= \sum_{p,q}\left[\frac{(2l_1 + 1)(2l_2' + 1)}{2l_1' + 1)(2l_2 + 1)}\right]^{1/2} C^{l_1'0}_{l_10p0}C^{l_1'm_1'}_{l_1m_1pq}C^{l_20}_{l_20p0}C^{l_2'm_2'}_{l_2m_2pq}$$

$$\times \int r_1^2dr_1r_2^2dr_2\frac{r_<^p}{r_>^{p+1}}R^*_{n_1l_1}(r_1)R^*_{n_2l_2}(r_2)\tilde{R}_{n_1'l_1'}(r_1)\tilde{R}_{n_2'l_2'}(r_2), \quad (46)$$

where $J_m(x)$ is a Bessel function of order m, and $P_l^m(\cos\theta)$ are associated Legendre polynomials. The symbol

$$S_{\tilde{n}} = \sum_n + 4\pi(-1)^l\int du$$

denotes summation over the principal quantum number n for the states of a discrete spectrum and integration over the electron momentum $\varkappa$ in the continuum.

Using expressions (32)–(46) we obtain the amplitudes for all the considered inelastic transitions.

2.3. Cross sections of electronic transitions

We focus our attention on the causes of the charge asymmetry in the integral cross sections of inelastic transitions induced in helium by fast collisions with particles and antiparticles. The difference between the cross sections of inelastic transitions for particles and antiparticles is determined by the cross-section interference terms proportional to Z^3. We shall write these terms for each of the processes considered. To this end we must know the degree of imaginarity of each of the amplitudes (34)–(39). Using the results of the preceding section, we find that the amplitudes $\tilde{d}_{on}$ and $\tilde{a}_1^R$ are proportional to i^{m_1}, the amplitudes $\tilde{a}^{I1}$, $\tilde{d}_{off}$ and $\tilde{a}_2^R$ to i^{m_1+1}, the amplitudes $\tilde{a}_{on}^{c1}$, $\tilde{a}^{I2}$ and $\tilde{a}_2^{c2}$ to $i^{m_1+m_2}$, and the amplitudes $\tilde{a}_{off}^{c1}$ and $\tilde{a}_1^{c2}$ to $i^{m_1+m_2+1}$. The differences in the cross sections of one- and two-electron excitations and ionization of helium, for particles and antiparticles, will be determined by the following expressions:

1) Single-electron excitation:

$$\frac{\Delta\sigma}{2} = 2\sum_m\int d\rho\{a^{I1}(1snlm; \rho) + a_{off}^{c1}(1snlm; \rho)\}$$

$$\times\{d_{off}(1snlm; \rho) + a_1^{c2}(1snlm; \rho)\}^*. \quad (47)$$

Expression (47) shows that when account is taken, in the one-electron excitation amplitude (20) of terms up to second order of smallness in the potential V inclusive. The onset of charge asymmetry in the cross section of single-electron excitation of helium is due interference between, on the one hand, direct electron excitation and electron transitions to an excited state due to single scattering of an ion by helium and correlation interaction of the atomic electrons and, on the other hand, stepwise transitions of an electron into an excited state as a result of double scattering of an ion by helium and the correlation interaction of the atomic electrons. From among the transitions resulting from single scattering of an ion by helium and the correlation interaction of atomic electrons, and the stepped transitions, only transitions taking place off the energy shell take part in the interference.

2) Two-electron excitation:

$$\frac{\Delta\sigma}{2} = 2\sum_M\sum_{\substack{m_1,m_1',\\ m_2,m_2'}} C^{LM}_{l_1m_1l_2m_2}C^{LM}_{l_1m_1'l_2m_2'}$$

$$\times\int d\rho\{[a^{I2}(n_1l_1m_1n_2l_2m_2; \rho) + a_2^{c2}(n_1l_1m_1n_2l_2m_2; \rho)]$$

$$\times a_{on}^{c1*}(n_1l_1m_1'n_2l_2m_2'; \rho) + a_1^{c2}(n_1l_1m_1n_2l_2m_2; \rho)$$

$$\times a_{off}^{c1*}(n_1l_1m_1'n_2l_2m_2'; \rho)\}. \quad (48)$$

In the case of two-electron excitation of helium the difference between the cross sections for particles and quasiparti-

cles is determined by interference of the transitions resulting from single scattering of an ion by helium and from the correlation interaction of atomic electrons, with transitions resulting from double scattering of an ion by helium and the correlation interaction of atomic electrons, as well as interference of independent transitions of two atomic electrons as a result of interaction of each of them with an incident ion with transitions of the electrons into excited states in which the electron ejected as a result of interaction with incident ion is scattered by another helium electron.

3) Single-electron ionization:

$$\frac{\Delta\sigma}{2} = 2\int du \int d\rho\{a^{I1}(1su;\rho)d^{*}_{off}(1su;\rho) + a^{I1}(1su;\rho)a_1^{c2*}(1su;\rho) + a^{I1}(1su;\rho)a_2^{R*}(1su;\rho) + a^{c1}_{off}(1su;\rho)d^{*}_{off}(1su;\rho) + a^{c1}_{off}(1su;\rho)a_2^{R*}(1su;\rho) + a^{c1}_{on}(1su;\rho)d^{*}_{on}(1su;\rho) + a^{c1}_{on}(1su;\rho)a_1^{R*}(1su;\rho)\} + 2\sum_{n,l,m}\int du\int d\rho\{a^{c1}_{off}(nlmu;\rho)a_1^{c2*}(nlmu;\rho) + a^{c1}_{on}(nlmu;\rho)a^{I2*}(nlmu;\rho) + a^{c1}_{on}(nlmu;\rho)a_2^{c2*}(nlmu;\rho)\}. \quad (49)$$

Single-electron ionization of helium as the features of both one-electron transitions (one electron goes into the continuum and the state of the other is unchanged) and two-electron transitions (one electron goes into the continuum and the other into an excited discrete state). Therefore the difference between the cross sections of one-electron ionization of helium for particles and antiparticles is determined by interference of the transitions already listed in the two preceding subsections, and also by interference of the transitions of one electron into the continuum via resonant states with direct transitions of one electron into the continuum and with transitions, resulting from single scattering of the ion by the helium and correlated interaction of the atomic electrons, and interference of the steplike transitions of each electrons into the continuum, taking place on the energy shell, and transitions of one electron into the continuum such that the electron ejected as a result of the interaction with the incident ion is scattered by another helium electron.

4) Two-electron ionization:

$$\frac{\Delta\sigma}{2} = 2\int du_1\int du_2\int d\rho\{[a^{I2}(u_1u_2;\rho) + a_2^{c2}(u_1u_2;\rho)]a^{c1*}_{on}(u_1u_2;\rho) + a_1^{c2}(u_1u_2;\rho)a^{c1*}_{off}(u_1u_2;\rho)\}. \quad (50)$$

As seen from (48) and (50), the charge asymmetry in the case of two-electron ionization of helium is of the same type as in the case of two-electron excitation.

3. DISCUSSION OF RESULTS

We analyze in this section, from the standpoint of our present theoretical approach, the results of the published experimental research and theoretical calculations of inelastic scattering of charged particles by helium. We begin with the excitation of nS, nP, and nD states of helium by protons and antiprotons. The main contribution to the difference between the excitation cross sections for protons and antiprotons will be made by interference of direct excitation and stepwise transitions of one electron, which take place off the energy shell. In the case of excitation of nS states, a direct s–s transition is optically forbidden and is suppressed compared with an s–p transition. Therefore stepwise s–p–s transitions will make a substantial contribution. The cross-section interference term responsible for the charge asymmetry can have a large weight and the differences between the cross sections of nS excitation for protons and antiprotons can be appreciable. A similar situation obtains also for excitation of nD states, when the stepwise s–p–d transitions also make a large contribution. The situation for nP excitation, however, is different. A direct s–p transition is optically allowed, while stepwise s–p–p and s–s–p transitions include one optically forbidden transition and are therefore small. The relative contribution of the interference term to the cross section is consequently small compared with the contribution from the direct excitation, and the difference between the excitation cross sections of the nP states should therefore be large for the protons and antiprotons should not be large.

This situation is in fact observed in calculations made in the framework of the multichannel diffraction approximation[21] and in the method of strong coupling of the channels.[20] Since the excitation cross sections for stepped transitions decreases with increase of the ion energy E like $1/E^2$, and the cross section for direct excitation decreases like $(A + B\ln E)/E$, the difference (both absolute and relative) in the excitation cross sections for fast protons and antiprotons should decrease with increase of the collision energy. This is in fact confirmed by calculations.[21]

As seen from Eq. (48), the main contribution to the difference of the excitation cross sections of autoionization constants of helium for particles and antiparticles will be determined by the interference between the excitation of the electrons by interaction of each of them with an incident ion, and transitions in which the electron is scattered by another gallium electron, after which both electrons are in excited states. These transitions take place on the energy surface, and the excitation of two helium electrons with participation of correlative interaction is via the continuum, and results from the interaction of the configurations in the final state (see Ref. 19). The interference term (48) will have a maximum in the collision-energy region where the contributions from the above two excitation mechanisms to the total cross section for excitation of autoionized states are comparable. Its value in this energy region can be comparable with the cross section. A considerable difference should be observed here between the cross sections of two-electron excitation of helium for particles and antiparticles.

Let us analyze the published experimental and theoretical cross sections of two-electron excitation. Tables I and II list the experimental and theoretical excitation cross sections for $2s^2{}^1S$, $2p^2{}^1D$ and $2s2p{}^1P$ auto-ionization states of helium and the total cross section for excitation of the $2p^2{}^1D$ and $2s2p{}^1P$ states for protons, antiprotons, and electrons. All the theoretical calculations of the cross sections of excitation of the auto-ionization states of helium by protons and antiprotons[16,18,20,23,26] can be subdivided into three groups in accordance with the extent to which they take charge asymmetry into account.

1. Calculations[16,18,26] with account taken of only dis-

TABLE I. Experimental and theoretical cross sections for the excitation of the $2s^2\,{}^1S$, $2p^2\,{}^1D$ and $2s2p\,{}^1P$ auto-ionization states of helium, in units of 10^{-20} cm^2.

Energy, MeV/ nucleon	Ion	$2s^2\,{}^1S$		$2p^2\,{}^1D$		$2s2p\,{}^1P$		Reference
		Experiment	Theory	Experiment	Theory	Experiment	Theory	
1,5	e^-	0,0816	—	—	—	1,17	—	[17]
	p^+	0,0318	—	1,84	—	0,608	—	[17]
	p^+	—	0,74	—	0,48	—	3,0	[16]
	p^-	—	0,73	—	0,27	—	3,3	[16]
	p^+ and p^-	—	0,14	—	—	—	—	[20]
6,0	p^+ and p^-	—	0,20	—	—	—	—	[16]

crete states. In all such calculations, using both the method of strong coupling of the channels[16,26] and the second Born approximation,[18,26] the difference between the excitation cross sections for protons and antiprotons is determined by the term $a_1^{c2}(a_{\text{off}}^{c1})^*$, which does not make the main contribution to (48) and from which, furthermore, all the transitions via the continuum, have been discarded (the principal term $a^{I2}(a_{\text{on}}^{c1})^*$ is not taken into account, since the amplitude a_{on}^{c1} describes transitions that go only through the continuum). The cross sections for excitation of auto-ionization states for protons, calculated in each of Refs. 16, 18, and 26, is therefore not significant (except for the excitation of the $2p^2\,{}^1D$ state at an ion energy $E = 1.5$ MeV) differ from the corresponding cross sections calculated for antiprotons in the same paper (see Tables I and II). The difference between the indicated calculations is attributed to the choice of different wave-function bases. The calculation in Ref. 18 differs from that in Ref. 16 and 26 also because the amplitude a_1^{c2} of Ref. 18 takes into account only transitions off the energy surface.

2. In the calculations[20] based on the method of strong coupling of the channels, use is made of the Sturm wave functions, which take into account states of both the discrete spectrum and the continuum. The interference term (48) should include here all the terms. Nonetheless, the calculated cross sections for excitation of the autoionized states of helium by protons do not differ from the corresponding cross sections for antiprotons (see Table I). The apparent reason is the incomplete allowance for the continuum states, particularly those with energy equal to that of the doubly excited state. Yet only such intermediate states which contribute to the amplitude a_{on}^{c1}.

3. Calculations[23] in second-order perturbation theory in the interaction potential of the incident ion with the helium electrons, with account taken of only transitions on the energy shell. In these calculations the discrepancy of the excitation cross sections of autoionization states for protons and antiprotons was determined only by the principal term of the interference term (48). The large excess (by up to six times) of the excitation cross sections for antiprotons over the corresponding cross sections for protons in the ion-energy interval $E = 0.1$–10 MeV can be attributed to exclusion of the contribution to the excitation cross section from transitions that take place outside the energy shell.

The available data,[13,17] assuming that the cross sections for excitation of helium autoionization states are equal for fast electrons and antiprotons of like velocity, agree with the approximate conclusions of the present paper.

In the case of single-electron ionization, a whole spectrum is observed of terms responsible for the asymmetry in the cross section. However, just as in the case of single-electron excitation, the largest contribution should come from interference of a direct transition of an electron to the continuum with the stepwise transitions, off the energy shell, of one electron into the continuum. Just as in the case of one-electron excitation, the difference between the ionization cross sections of helium for particles and antiparticles should decrease with increase of the collision energy, as is indeed observed in experiment[2,33–35] and in calculations[14] in the Monte Carlo method of classical trajectories. Since the main contribution to the electronic ionization is made by transitions of an electron into p and s states of the continuum,[36] the ratio of the cross sections for helium ionization by particles to ionization by the corresponding antiparticles should be contained between the analogous cross-section ratios for excitations of nS and nP states of helium.

In the case of two-electron ionization of helium, just as for two-electron excitation, the main contribution to the interference term (50) will be made by interference between electron transitions to the continuum as a result of interaction of each of them with the incident ion and transitions in which the electron ejected through interaction with the incident ion is scattered by another helium electron and both electrons go into the continuum. At ion energies such that the contributions to the double-ionization cross section from

TABLE II. Experimental and theoretical total cross section for the excitation of $2p^2\,{}^1D + 2s2p\,{}^1P$ for helium by electrons, protons, and antiprotons at energies 1.5 MeV/nucleon in units of 10^{-20} cm.^{2}

Ion	Experiment	Theory	Reference
e^-	3,05	—	[17]
e^-	1,92	—	[17]
p^+	2,45	—	[17]
p^+	2,75	—	[17]
p^+	—	3,48	[16]
p^-	—	3,57	[16]

each of the indicated mechanisms have close values, this term has a maximum and is comparable with the double-ionization cross section. Experimental investigations of double ionization of helium by protons and antiprotons[2,3] have shown that the cross sections for double ionization of helium by antiprotons is practically double that for protons at ion energies from 2 to 5 MeV.

Note that in the region where the interference term responsible for the charge asymmetry is a maximum the relative contribution of the interference term to the total cross section for double ionization should be larger in the case of two-electron ionization of the helium than in the case of two-electron excitation. The reason is that in the latter case the ratio of the modulus a_{on}^{c1} of the amplitude describing the transitions to the energy surface to the modulus a_{off}^{c1} of the amplitude describing transitions off the energy surface should be higher than in the case of two-electron ionization, since the energy denominator in the amplitude a_{off}^{c1} [see Eqs. (37)] is smaller for two-electron excitation than for two-electron ionization. In double helium ionization the relative difference between the cross sections for particles and antiparticles should be larger than in double excitation.

4. CONCLUSION

The present investigation of inelastic transitions excited in helium by collisions with fast charged particles has revealed the causes of the charge asymmetry in the total cross sections of single- and two-electron excitation and ionization. It is shown that in fast ion–atom collisions, when polarization of the helium-atom electron shell by the incident ion and the influence of the competing process, charge exchange is scattering of a positive ion by helium can be neglected, the difference between the cross sections of the inelastic transitions in question for particles and antiparticles can be attributed to interference between different inelastic-transition mechanisms. The specific processes responsible for the charge asymmetry in the integral scattering cross section are determined for each of the considered inelastic transitions, and those making the decisive contribution are identified.

The published theoretical and experimental data on the cross section of single- and two-electron excitation and ionization are analyzed in light of the presented approach. In the case of excitation of auto-ionization states of the helium atom, a relation between the results of different calculations is established from the standpoint of the physics of the process considered.

It is shown that in the cases of single-electron excitation and single-electron ionization of helium the integral cross-section interference terms responsible for the charge asymmetry differ substantially, whereas in the case of two-electron transitions the corresponding interference terms in the two-electron excitation and ionization cross section are determined by the same transition variety.

One of the important results of the present investigation is the confirmation that a difference between the particle and antiparticle excitation and ionization cross sections is possible in the case two-electron transitions only for a correlation interaction of atomic electrons [see Eqs. (48) and (50)], whereas in the case of one-electron transitions the correlation interaction determines only corrections to the principal terms in the scattering cross section interference terms responsible for the charge asymmetry [Eqs. (47) and (49)].

APPENDIX

Let us show that the amplitude of a two-electron transition in a helium atom, determined in the independent-electron approximation, does not contain a non-energy part in second order of perturbation theory. According to (18), $a^{I2}(F \leftarrow I;\rho)$ takes the form

$$a^{I2}(F \leftarrow I;\rho) = -\Big(\sum_K + \int dK\Big)\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \exp[i(E_F - E_K)t_1]$$
$$\times \exp[i(E_K - E_I)t_2]\langle\Psi_F|\sum_{l=1}^{2}\frac{-Z}{|\mathbf{R}(t_1) - \mathbf{r}_l|}|\Psi_K\rangle$$
$$\times\langle\Psi_K|\sum_{l=1}^{2}\frac{-Z}{|\mathbf{R}(t_2) - \mathbf{r}_l|}|\Psi_I\rangle. \qquad \text{(A1)}$$

Without loss of generality, we can use expression (9b) for the functions Ψ_F, Ψ_K, and Ψ_I. Using the Bethe integral (19) and replacing Ψ_F, Ψ_K, and Ψ_I by their specific values, we obtain for the amplitude a^{I2}

$$a^{I2}(F \leftarrow I;\rho) = \frac{-Z^2}{\pi^4 2\sqrt{2}}\int\frac{d^3q}{q^2}\exp(-i\mathbf{q}_\perp\rho)\int\frac{d^3p}{p^2}\exp(-i\mathbf{p}_\perp\rho)$$
$$\times\int_{-\infty}^{\infty} dt_1\int_{-\infty}^{t_1} dt_2 \exp[i(E_F - E_K - q_\parallel v)t_1]$$
$$\times\exp[i(E_K - E_I - p_\parallel v)t_2](\langle f_1|e^{i\mathbf{qr}}|i_1\rangle$$
$$\times\langle f_2|e^{i\mathbf{pr}}|i_2\rangle + \langle f_2|e^{i\mathbf{qr}}|i_2\rangle\langle f_1|e^{i\mathbf{pr}}|i_1\rangle$$
$$+ \langle f_2|e^{i\mathbf{qr}}|i_1\rangle\langle f_1|e^{i\mathbf{pr}}|i_2\rangle + \langle f_1|e^{i\mathbf{qr}}|i_2\rangle\langle f_2|e^{i\mathbf{pr}}|i_1\rangle). \qquad \text{(A2)}$$

Consider the expression

$$T_1 = \int_{-\infty}^{\infty} dt_1\int_{-\infty}^{t_1} dt_2 \exp[i(E_F - E_K - q_\parallel v)t_1]$$
$$\times\exp[i(E_K - E_I - p_\parallel v)t_2]$$
$$\times(\langle f_1|e^{i\mathbf{qr}}|i_1\rangle\langle f_2|e^{i\mathbf{pr}}|i_2\rangle + \langle f_2|e^{i\mathbf{qr}}|i_2\rangle\langle f_1|e^{i\mathbf{pr}}|i_1\rangle). \qquad \text{(A3)}$$

The energy of the helium-atom electrons in the intermediate state is $E_K = \varepsilon_{f_2} + \varepsilon_{i_1}$, for the first term and $E_K = \varepsilon_{f_1} + \varepsilon_{i_2}$ for the second. We have then for the first term $E_F - E_K = \varepsilon_{f_1} - \varepsilon_{i_1}$ and $E_K - E_I = \varepsilon_{f_2} - \varepsilon_{i_2}$ and analogously for the second $E_F - E_K = \varepsilon_{f_2} - \varepsilon_{i_2}$ and $E_K - E_I = \varepsilon_{f_1} - \varepsilon_{i_1}$. Using the last relations, we write for T_1

$$T_1 = \int_{-\infty}^{\infty} dt_1\int_{-\infty}^{t_1} dt_2 \exp[i(\varepsilon_{f_1} - \varepsilon_{i_1} - q_\parallel v)t_1]$$
$$\times\exp[i(\varepsilon_{f_2} - \varepsilon_{i_2} - p_\parallel v)t_2]$$
$$\times\langle f_1|e^{i\mathbf{qr}}|i_1\rangle\langle f_2|e^{i\mathbf{pr}}|i_2\rangle + \int_{-\infty}^{\infty} dt_1\int_{-\infty}^{t_1} dt_2 \exp[i(\varepsilon_{f_2} - \varepsilon_{i_2}$$
$$- q_\parallel v)t_1]\exp[i(\varepsilon_{f_1} - \varepsilon_{i_1} - p_\parallel v)t_2]\langle f_2|e^{i\mathbf{qr}}|i_2\rangle\langle f_1|e^{i\mathbf{pr}}|i_1\rangle. \qquad \text{(A4)}$$

The change of variables $t_1 \rightleftarrows t_2$ in the second term of (A4) yields

$$T_1 = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \exp[i(\varepsilon_{f_1} - \varepsilon_{i_1} - q_\parallel v)t_1]$$
$$\times \exp[i(\varepsilon_{f_2} - \varepsilon_{i_2} - p_\parallel v)t_2]$$
$$\times \langle f_1 | e^{i\mathbf{qr}} | i_1 \rangle \langle f_2 | e^{i\mathbf{pr}} | i_2 \rangle + \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{t_2} dt_1 \exp[i(\varepsilon_{f_2} - \varepsilon_{i_2} - q_\parallel v)t_2]$$
$$\times \exp[i(\varepsilon_{f_1} - \varepsilon_{i_1} - p_\parallel v)t_1] \langle f_2 | e^{i\mathbf{qr}} | i_2 \rangle \langle f_1 | e^{i\mathbf{pr}} | i_1 \rangle. \quad (A5)$$

We now change the order of the integration in the second term make simultaneously the change of variables $\mathbf{q} \rightleftarrows \mathbf{p}$. The result for T_1 is

$$T_1 = \int_{-\infty}^{\infty} dt_1 \exp[i(\varepsilon_{f_1} - \varepsilon_{i_1} - q_\parallel v)t_1] \langle f_1 | e^{i\mathbf{qr}} | i_1 \rangle$$
$$\times \int_{-\infty}^{\infty} dt_2 \exp[i(\varepsilon_{f_2} - \varepsilon_{i_2} - p_\parallel v)t_2] \langle f_2 | e^{i\mathbf{pr}} | i_2 \rangle. \quad (A6)$$

Using similar transformations also for T_2

$$T_2 = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \exp[i(E_F - E_K - q_\parallel v)t_1]$$
$$\times \exp[i(E_K - E_I - p_\parallel v)t_2]$$
$$\times (\langle f_2 | e^{i\mathbf{qr}} | i_1 \rangle \langle f_1 | e^{i\mathbf{pr}} | i_2 \rangle + \langle f_1 | e^{i\mathbf{qr}} | i_2 \rangle \langle f_2 | e^{i\mathbf{pr}} | i_1 \rangle) \quad (A7)$$

and substituting the resultant expressions for T_1 and T_2 in the amplitude a^{I2}, we obtain ultimately

$$a^{I2}(F \leftarrow I; \rho) = \frac{-Z^2\sqrt{2}}{\pi^2} \sum_{j=1}^{2} \sum_{t \neq j} \int \frac{d^3q}{q^2} \exp(-i\mathbf{q}_\perp \rho)$$
$$\times \langle f_j | e^{i\mathbf{qr}} | i_1 \rangle \delta(\varepsilon_{f_j} - \varepsilon_{i_1} - q_\parallel v) \int \frac{d^3p}{p^2} \exp(-i\mathbf{p}_\perp \rho)$$
$$\times \langle f_t | e^{i\mathbf{pr}} | i_2 \rangle \delta(\varepsilon_{f_t} - \varepsilon_{i_2} - p_\parallel v), \quad (A8)$$

Q. E. D.

[1] J. H. McGuire, Phys. Rev. Lett. **49**, 1153 (1982).
[2] L. H. Andersen, P. Hvelplund, H. Knudsen *et al.*, *ibid.*, **57**, 2147 (1986).
[3] L. H. Andersen, P. Hvelplund, H. Knudsen *et al.*, Phys. Rev. A **36**, 3612 (1987).
[4] J. F. Reading and A. L. Ford, Phys. Rev. Lett. **58**, 545 (1987).
[5] J. F. Reading and A. L. Ford, J. Phys. B **30**, 5747 (1987).
[6] R. E. Olson, Phys. Rev. A **36**, 1519 (1987).
[7] L. Vegh, *ibid.* **37**, 992 (1988).
[8] H. Charlton, L. H. Andersen, L. Brun-Nielsen *et al.*, Phys. Rev. B **21**, L545 (1988).
[9] A. L. Ford and J. F. Reading, J. Phys. B **21**, L685 (1988).
[10] P. D. Fainstein, V. H. Ponce, and R. D. Rivarola, *ibid.* **21**, 2989 (1988).
[11] M. Charlton, L. Brun-Nielsen, B. I. Deutch *et al.*, *ibid.* **22**, 2779 (1989).
[12] T. J. Gay and R. E. Olsen, Nucl. Instrum. Meth. Phys. Res. B **40/41**, 362 (1989).
[13] J. O. P. Pedersen and P. Hvelplund, Phys. Rev. Lett. **62**, 2373 (1989).
[14] D. R. Schultz, Phys. Rev. A **40**, 2330 (1989).
[15] A. L. Ford and J. F. Reading, Nucl. Instrum. Meth. Phys. Res. B **40/41**, 362 (1989).
[16] W. Fritsch and C. D. Lin, Phys. Rev. A **41**, 4776 (1990).
[17] J. P. Giese, M. Schulz, and J. K. Swenson, *ibid.* **41**, 1231 (1990).
[18] J. H. McGuire and J. C. Straton, *ibid.* **43**, 5184 (1991).
[19] V. A. Sidorovich, Phys. Lett. A **152**, 53 (1991).
[20] T. G. Winter, Phys. Rev. A **43**, 4727 (1991).
[21] V. V. Balashov, M. V. Gorelenkova, and A. I. Nagunov, Yad. Fiz. **53**, 663 (1991) [Sov. Phys. Nucl. Phys. **43**, 415 (1991)].
[22] V. A. Sidorovich, Phys. Lett. A **159**, 257 (1991).
[23] N. V. Novikov, Candidate's dissertation, Inst. Phys. Problems, Moscow State Univ. (1991).
[24] D. R. Schultz, R. E. Olson and C. O. Reinhold, J. Phys. B **24**, 521 (1991).
[25] N. Stolterfoht, Nucl. Instrum. Meth. Phys. Res. B **53**, 477 (1991).
[26] K. Miribayashi, Hino Ken-ichi, and M. Matsuzawa, Phys. Rev. A **44**, 7234 (1991).
[27] H. Knudsen and J. F. Reading, Phys. Rep. 212, 107 (1992).
[28] V. A. Sidorovich, V. S. Nikolaev, and J. H. McGuire, Phys. Rev. A **31**, 2193 (1985).
[29] V. V. Balashov, S. I. Grishanova, N. M. Kruglova *et al.*, Opt. Spektrosk. **18**, 859 (1970).
[30] H. Bethe, Ann. Phys. Lpz. **3**, 325 (1930).
[31] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics, Nonrelativistic Theory*, Pergamon, 1984.
[32] V. A. Sidorovich and V. S. Nikolaev, Opt. Spektrosk. **5**, 54 (1983).
[33] D. Fromme, G. Kruse, W. Raith *et al.*, Phys. Rev. Lett. **57**, 3031 (1986).
[34] M. E. Rudd, Y.-K. Kim and D. H. Madison, Rev. Mod. Phys. **57**, 965 (1985).
[35] R. G. Montague, M. F. A. Harrison, and A. C. H. Smith, J. Phys. B **17**, 3295 (1984).
[36] O. Aashamar and L. Kocbach, *ibid.* **10**, 869 (1977).

Translated by J. G. Adashko