Self-localized oscillations of domain walls of a uniaxial ferromagnet

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We use the reduced equations of magnetodynamics to study nonlinear small-amplitude excitations in the domain wall of a uniaxial magnet with a large anisotropy. We determine the "metastability" time of such solitons, analyze the collision processes of the envelope solitons with an isolated Bloch line, and we also elucidate the conditions for their parametric stabilization by an external magnetic field.

The nonlinear transformations of the structure of an oscillating domain wall (DW) and the solitonlike effects in its dynamics have been studied experimentally in cubic[1] and uniaxial[2] ferromagnets. The theoretical study of solitonlike effects have mainly been concerned with the dynamics of topological solitons—the Bloch lines (BL)[3,4]—although some numerical experiments with nonlinear small-amplitude excitations in unpinned DW were carried out in Ref. 4. One must say that the self-localized oscillations play a role in the dynamical "lifetime" of a DW as important as that of BL. They are essentially the seeds of a bound state of pairs of Bloch lines and accompany their creation and annihilation processes.

A study of the dynamics of topological solitons in pinned DW[2,3] has shown that they possess properties close to those of the electrodynamic solitons of a distributed Josephson junction which is described by a sine-Gordon equation with perturbations.[5] Due to the presence of a dynamic interaction of the BL with the DW there are not only similarities in this case, but also differences which are, for instance, connected with the possibility of Bloch-line clustering due to gyroscopic self-constriction.[3] It is interesting also to discuss the physical features of localized solitonlike oscillations of DW and to elucidate the nature of their interaction with BL.

The reduction of the magnetodynamic equations for the dynamics to a nonlinear Schrödinger equation[6] to describe small-amplitude DW oscillations shows that breather self-localized oscillations and envelope solitons can arise. However, in nonintegrable systems the problem of how fundamental such formations are, even in a nondissipative medium is unclear.[7–9] This is connected with the fact that localized DW oscillations which are periodic in time with a frequency below the activation gap are in general due to the nonlinearity itself, a source for the generation of oscillations at higher harmonics which are undamped at the periphery. These oscillations are exponentially small in the amplitude of the self-localized oscillation at the fundamental, $\varepsilon \ll 1$, i.e., they are proportional to $\exp(-1/\varepsilon)$.[7] In this connection it is of interest to estimate the possible "metastability" time of such solitons in DW and to discuss the problem of their stabilization.

In the present paper we use the methods of perturbation theory, assuming that in the system considered there are small parameters (small amplitude, weak dissipation and pumping), to analyze the phenomenon of the collision of envelope solitons with BL and the parametric stabilization of a self-localized DW oscillation in a dissipative medium.

1. INITIAL EQUATIONS AND CONSERVATION LAWS

To study spin wave excitations in a pinned DW we start from the dynamic equations obtained by Slonczewski[10] for magnetically uniaxial materials with a large quality factor $Q = K_u/2\pi M^2 \gg 1$, where K_u is the uniaxial anisotropy energy and M the magnetization. These equations can be put in the following form in normalized variables:[3]

$$\partial_t \psi - \partial_x^2 q + b^2 q = -\alpha \partial_t q + h_z, \tag{1}$$

$$\partial_t q + \partial_x^2 \psi - \frac{1}{2}\sin(2\psi) = \alpha \partial_t \psi + h_x \sin\psi, \tag{2}$$

where q is the coordinate of the center of the DW, normalized to the DW thickness $\delta_W = (A/K_u)^{1/2}$, reckoned along the y-axis which is normal to its plane; A is the exchange constant; ψ is the azimuthal angle at which the magnetization emerges from the plane of the DW, reckoned from the x-axis; we write $b^2 = H'\delta_W/4\pi M$, where H' is the gradient along the y-axis of the magnetic field parallel to the z-axis, stabilizing it in the position $q = 0$; h_z is the magnetic field along the z-axis, normalized to $4\pi M$; h_x is the magnetic field along the x-axis, normalized to $8M$; the x and z coordinates are measured in units of the BL width $\delta_L = (A/2\pi M^2)^{1/2}$; the time t is normalized to $(4\pi M\gamma)^{-1}$ where γ is the magnetogyric ratio; and α is the Hilbert damping parameter.

If there is no dissipation and there are no magnetic fields the set of Eqs. (1) and (2) has two integrals of motion. In fact, multiplying (1) by $\partial_t q$ and (2) by $\partial_t \psi$ and subtracting the one from the other we get after some transformations

$$\partial_t w + \partial_x p = -f + h_z \partial_t q + h_x \partial_t \cos\psi, \tag{3}$$

where

$$w = \frac{1}{2}\left[(\partial_x \psi)^2 + (\partial_x q)^2 + \sin^2\psi + b^2 q^2\right] \tag{3a}$$

is the spin wave energy density in the DW,

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$$p = -(\partial_x q \partial_t q + \partial_x \psi \partial_t \psi) \tag{3b}$$

is the energy flux density, and

$$f = \alpha[(\partial_t q)^2 + (\partial_t \psi)^2] \tag{3c}$$

is the dissipation energy density. Equation (3) describes the evolution of the energy density of a system in a dissipative medium. Similarly, by multiplying (1) by $\partial_x q$ and (2) by $\partial_x \psi$, subtracting and performing some transformations, we can find an equation for the evolution of the momentum:

$$\partial_t P + \partial_x \rho = -\alpha p + h_z \partial_x q + h_x \partial_x \cos\psi, \tag{4}$$

where

$$P = q\,\partial_x \psi \tag{4a}$$

is the spin wave momentum density and

$$\rho = q\,\partial_t \psi - \frac{1}{2}[(\partial_x q)^2 + (\partial_x \psi)^2 - b^2 q^2 - \sin^2\psi] \tag{4b}$$

is the momentum flux density.

The integrals of motion make it possible for us to obtain evolution equations for adiabatically changing soliton parameters in a perturbed medium. This applies especially to the dynamics of an isolated Bloch line, the self-similar solution for which in an unperturbed DW is completely characterized by two free parameters—the velocity and the position of the center.[11] For a larger number of free parameters in a solitary wave the above-mentioned conservation laws are insufficient for a complete description of the evolution of the soliton in a perturbed medium. In that case we need use a singular perturbation theory which is the analog of the Bogolyubov–Mitropolskiĭ method.[12]

2. SMALL-AMPLITUDE LOCALIZED DOMAIN WALL OSCILLATIONS

We shall discuss the possibility that self-localized breather states exist near the DW ground state $\psi = q = 0$. We look for a solution which is periodic in time in the form of a Fourier series

$$\mathbf{a} = \begin{pmatrix}\psi\\ q\end{pmatrix} = \sum \mathbf{a}_n \exp(-in\theta) + \text{c.c.}, \tag{5}$$

where

$$\mathbf{a}_n = \begin{pmatrix}\psi_n\\ q_n\end{pmatrix},$$

while the phase θ satisfies the condition $\partial_t \theta = \omega = \text{const}$. It is clear that because of the specific form of the nonlinear term in (2) the expansion (10) will contain only odd harmonics, $n = 1,3,5,\ldots$. For small-amplitude oscillations $|\psi| \ll \pi$ we get from (1) and (2) an infinite chain of coupled equations:

$$-i\omega\psi_1 - \partial_x^2 q_1 + b^2 q_1 = 0, \tag{6}$$

$$-i\omega q_1 + \partial_x^2\psi_1 - \psi_1 + 2|\psi_1|^2\psi_1$$

$$+ (\psi_1^*)^2\psi_3 + 4|\psi_3|^3\psi_1 + \ldots = 0, \tag{7}$$

$$-3i\omega\psi_3 - \partial_x^2 q_3 + b^2 q_3 = 0, \tag{8}$$

$$-3i\omega q_3 + \partial_x^2\psi_3 - \psi_3 + \frac{2}{3}\psi_1^3 + 4|\psi_1|^2\psi_3 - 2|\psi_3|^2\psi_3 + \ldots = 0. \tag{9}$$

We now assume that the amplitudes of the harmonics in the expansion (5) are of increasing order of smallness, in fact, that $a_n \sim \varepsilon^n$ where $\varepsilon = \max(\psi_1) \ll \pi$. In that case, assuming that in the wave the size of the localization region is determined by its amplitude, i.e., that we have $\partial_x a_1 \sim a_1\varepsilon$, it follows from (6) and (7) that to within third-order terms

$$(1 + b^2)\partial_x^2\psi_1 + (\omega^2 - b^2)\psi_1 + 2b^2|\psi_1|^2\psi_1 = 0. \tag{10}$$

For $|x| \to \infty$ the solution of Eq. (10) with the boundary conditions $\psi_1 = \partial_x \psi_1 = 0$ has the well known form

$$\psi_1 = \frac{\psi_0}{2\text{ch}(\Delta x)}, \tag{11}$$

in which the size of the localization region is determined by the parameter

$$\Delta = \psi_0 b/(2(1 + b^2)^{1/2}).$$

Moreover, there is a nonlinear frequency shift given by the relation

$$\omega^2 = b^2(1 - \psi_0^2/4). \tag{12}$$

In the same approximation it follows from (7) that

$$q_1 = \frac{\psi_0}{2ib\,\text{ch}(\Delta x)} + O(\varepsilon^3). \tag{13}$$

Now, retaining the third-order terms in the second pair of equations, (8) and (9), we find that

$$\begin{cases} -3ib\psi_3 + (b^2 - \partial_x^2)q_3 = 0, \\ (1 - \partial_x^2)\psi_3 + 3ibq_3 = \dfrac{\psi_0}{12\text{ch}^3(\Delta x)}. \end{cases} \tag{14}$$

After a direct and an inverse Fourier transformation we find from this set

$$\psi_3 = \int_{-\infty}^{+\infty} \frac{\psi_0^3(k^2 + b^2)(\Delta^2 + k^2)\exp(ikx)}{48\Delta^3(k^2 + k_+^2)(k^2 - k_-^2)\text{ch}(\pi k/(2\Delta))}\,dk, \tag{15}$$

$$q_3 = \int_{-\infty}^{+\infty} \frac{\psi_0^3 ib(\Delta^2 + k^2)\exp(ikx)}{16\Delta^3(k^2 + k_+^2)(k^2 - k_-^2)\text{ch}(\pi k/(2\Delta))}\,dk, \tag{16}$$

where

$$k_\pm^2 = \frac{1}{2}[(1 + 34b^2 + b^4)^{1/2} \pm (1 + b^2)].$$

One can easily evaluate the integrals in (15) and (16), by using the theory of residues after the substitution $k = \hat{z} = k_x + ik_y$, if we choose an integration contour consisting of a section of the real axis $\hat{z} = k_x$ ($-R < k_x < +R$, $k_y = 0$) and the semicircle $\hat{z} = R\exp(i\phi)$, where $0 < \phi < \pi$ for $x > 0$ and $-\pi < \phi < 0$ for $x < 0$. To eliminate the singularities of the integration on the real axis we must put $k_- = \lim(k_- + i\beta)$ with $\beta \to 0$. The general solution of the system (14) can then be written in the form

$$a_3 = a_3^0 + a_3^{\sim}, \tag{17}$$

in which a_3^0 is that part of the solution of the system (14) determined by the residues on the imaginary axis in the points $\hat{z}_m^2 = -\Delta^2(2m+1)^2$, $m = 0,1,2,\ldots$. This part of the solution describes the localized wave and is damped as $|x| \to \infty$. It can be approximately found directly from (14) when one drops the spatial derivatives so that

$$\psi_3^0 \simeq -\frac{\psi_0^3}{48\text{ch}^3(\Delta x)} + O(\varepsilon^5),$$
$$q_3^0 \simeq -\frac{i\psi_0^3}{16b\,\text{ch}^3(\Delta x)} + O(\varepsilon^5). \tag{18}$$

The second part $a_3^{\sim}$ of the solution of the system (14) refers to the "emission" of spin waves at a frequency $3b(1-\psi_0^2/8)$. In the continuous spin-wave spectrum this frequency corresponds to a wavenumber $k = k_- \simeq 2\sqrt{2b}$ (for $b \ll 1$). The amplitude of the travelling wave is determined by the residue of the integrand in (15) and (16) at the point

$$\hat{z}_+ = \lim(k_- + i\beta)$$

for $\beta \to 0$, if $x > 0$ and

$$\hat{z}_- = \lim(k_- - i\beta)$$

for $\beta \to 0$, if $x < 0$. Therefore in the far wave zone, for instance for $x > 0$ and also taking into account that usually we have $b \ll 1$,

$$\psi_3^{\sim} = -\frac{i\pi\psi_0^3 k_-(b^2 + k_-^2)\exp(ik_- x)}{48\Delta^3(k_+^2 + k_-^2)\text{ch}(\pi k_-/(2\Delta))}, \tag{19a}$$

$$q_3^{\sim} = \frac{\pi\psi_0^3 b k_- \exp(ik_- x)}{16\Delta^3(k_+^2 + k_-^2)\text{ch}(\pi k/(2\Delta))}. \tag{19b}$$

Thus in the far wave zone undamped spin oscillations appear with an exponentially small amplitude,

$$a_3 \sim \exp(-\pi 2\sqrt{2}/\psi_0).$$

The emission of spin waves causes the original wavepacket to lose energy and hence to have a finite lifetime. From the energy conservation law (3), after averaging in time over a period of the soliton oscillations and integrating over the spatial variable x, taking into account the fluxes at infinity, we can obtain an equation for the temporal evolution of its amplitude, $\psi_0(t)$,

$$\partial_t\psi_0 = -\frac{24\pi^2\sqrt{2}b}{\text{ch}^2(2\pi\sqrt{2}/\psi_0)}. \tag{20}$$

The numerical solution of this equation is shown in Fig. 1. For large times it is approximately described by the formula

$$\psi_0 = \frac{4\pi\sqrt{2}}{\ln[6\pi b(t+t_0)]}, \tag{21}$$

where

$$t_0 = \frac{\exp[4\pi\sqrt{2}/\psi_0(0)]}{6\pi b} \tag{22}$$

may be called the "lifetime" of the small-amplitude soliton. This time increases without bound as $\psi_0 \to 0$. Therefore only in the limit of an infinitesimally small amplitude can we talk about the self-localization of a wavepacket of DW binding oscillations. The example of integrable systems such as the sine-Gordon system, shows, however, that taking higher terms of the expansion in ψ into account in the initial system may cause the excitations to be damped by interference in the far wave zone. It was shown in Ref. 7 by a numerical and qualitative analysis that for the occurrence of self-localized solutions in a system with a cubic nonlinearity one needs at least an additional term of fifth order with a self-consistent coefficient in the expansion in ψ.[1)] The time (22) can be considered to be a rough estimate for the possible metastability time of self-localized excitations in a DW.

It will be of interest in connection with what we have said above to give an estimate of the metastability time which we have found. Taking $\psi(0) = 0.5$ we find from (22) that $t_0 b = 1.44 \cdot 10^{14}$ periods of the oscillations. It is clear in this case that one can neglect the effect of the emission on the higher harmonics. If we put $\psi_0(0) = \pi$, we have $t_0 b = 15.5$, which indicates important possible changes in the structure of the initial wavepacket (11)–(13) after a few periods. For large amplitudes, however, the formulas for both the localized solution and for its lifetime are invalid.

We have therefore carried out numerical solutions of the Cauchy problem with an initial perturbation in the form (11)—(13). They showed that for small amplitudes, $|\psi_0| < 0.5$, the solution was observed to be solitonlike during

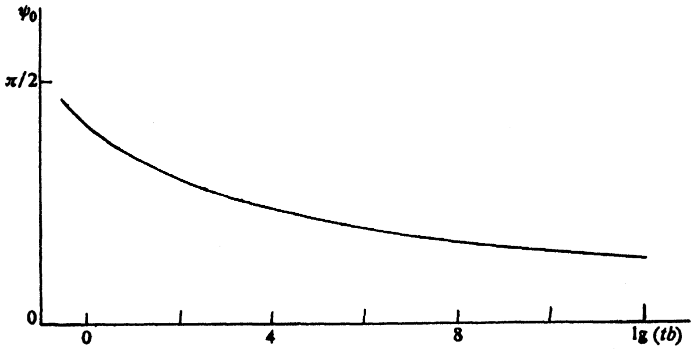

FIG. 1. Evolution according to perturbation theory of the amplitude of a self-localized spin oscillation in a pinned DW in a nondissipative medium.

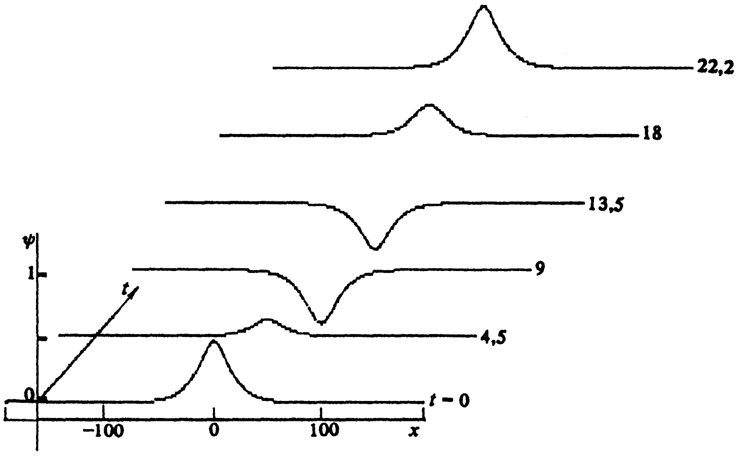

FIG. 2. Evolution of the amplitude of a self-localized spin oscillation with a small initial amplitude $\psi_0(0) = 0.5$ in a pinned DW. Numerical calculations using Eqs. (1) and (2) with $\alpha = h_x = h_z = 0$.

the whole of the calculation time $t_0 b \simeq 100$ (Fig. 2). Solutions with larger amplitudes, $|\psi_0| > 0.75$, rapidly disintegrated into dispersive large-amplitude spin waves[2)] as is shown in Fig. 3. Attempts to obtain a large-amplitude breather through annihilation of the kinks (Bloch lines) which were initially at rest also had to be concluded unsuccessfully due to the appearance of dispersive spin waves. Direct numerical experiments therefore confirm the metastable nature of the localized nonlinear DW oscillations.

3. INTERACTION OF ENVELOPE SOLITONS OF BENDING DOMAIN WALL OSCILLATIONS WITH BLOCH LINES

We now consider a moving solitary wave of localized DW oscillations when BL are present. We shall assume that we can neglect the effect of dissipation during the collision time, that the amplitude of the envelope soliton is sufficiently small, $|\psi| \ll \pi/2$, and that the emission at higher harmonics is negligibly small. We put

$$\mathbf{a} = \mathbf{a}_B + \mathbf{a}_B^* + \mathbf{a}_L,$$

where

$$\mathbf{a}_B = \begin{pmatrix} \psi_B \\ q_B \end{pmatrix}$$

describes the small-amplitude wave, while

$$\mathbf{a}_L = \begin{pmatrix} \psi_L \\ q_L \end{pmatrix}$$

is a Bloch line. Form Eqs. (1) and (2) for $\alpha = h_{z,x} = 0$ we can then find two coupled systems, one of which describes the small-amplitude DW oscillations in the presence of BL:

$$\begin{cases} \partial_t \psi_B - \partial_x^2 q_B + b^2 q_B = 0, \\ -\partial_t q_B - \partial_x^2 \psi_B + (1 - 2|\psi_B|^2)\psi_B = \frac{2}{3}\psi_B^3 + 2\psi_B \sin^2\psi_L. \end{cases} \tag{23}$$

There is also the system which is the complex conjugate of (23) for a_B^*. The other system of equations describes BL in the presence of small DW oscillations:

$$\begin{cases} \partial_t \psi_L - \partial_x^2 q_L + b^2 q_L = 0, \\ \partial_t q_L + \partial_x^2 \psi_L - \frac{1}{2}\sin(2\psi_L) = F, \end{cases} \tag{24}$$

where

$$F = 2|\psi_B|^2 \sin(2\psi_L) + (\psi_B^2 + \psi_B^{*2})\sin(2\psi_L).$$

Since according to (11) the size of the localization region of the self-localized wave for $|\psi_B| \ll \pi/2$ is much larger than the size of the core of the BL, which is $\simeq 1$, the right-

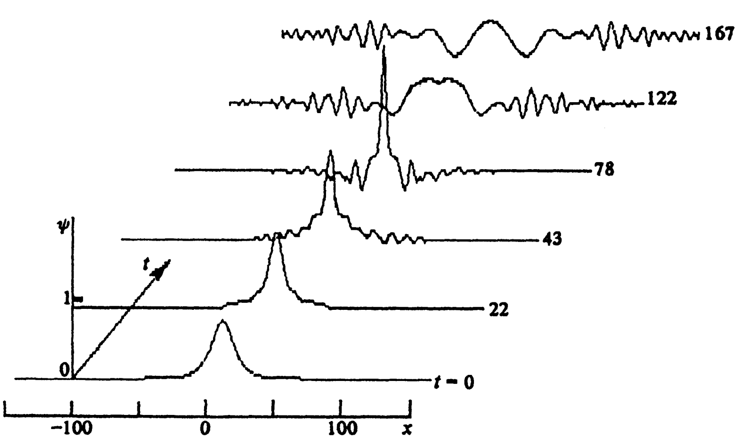

FIG. 3. Decay of a solitonlike spin oscillation with an initial amplitude $\psi_0(0) = 0.75$ in a pinned DW in a nondissipative medium ($\alpha = h_x = h_z = 0$). The chosen times corresponds to maximum amplitudes at the center.

hand sides of Eqs. (23) and (24) can be considered to be perturbations which, without changing the self-similarity of the soliton solutions, lead to an adiabatic change in its parameters. In that case one can consider the interaction of the solitons using perturbation theory methods.[13]

We start the discusson of the properties of an envelope soliton by considering the unperturbed system (23) with zero right-hand side. As in the previous case we shall assume that because of the weak localization of the DW oscillations we have $\partial_x \mathbf{a}_B \simeq \varepsilon \mathbf{a}_B$, where $\varepsilon \simeq \max(\psi_b) \ll 1$. For simplicity we also assume that $\partial_x \mathbf{a}_B \ll b\mathbf{a}_B$. Moreover, we shall neglect the effect of emission at higher harmonics and thus drop small terms of order $\simeq \varepsilon^3$ which correspond to its generation. In that case, after eliminating q_B from Eqs. (23) we get the following equation for the complex amplitude ψ_B of the angle at which the DW emerges from the plane:

$$(\partial_t^2 + \hat{L})\mathbf{C} = \mathbf{d} = \begin{pmatrix} d \\ d^* \end{pmatrix}, \tag{25}$$

where

$$\mathbf{C} = \begin{pmatrix} \psi_B \\ \psi_B^* \end{pmatrix}, \quad d = 2b^2\psi_B \sin^2\psi_L,$$

while the operator $\hat{L}$ has the form

$$\hat{L} = (b^2 - \partial_x^2)(1 - 2|\psi_B^2| - \partial_x^2) \simeq b^2 - (1 + b^2)\partial_x^2 - 2b^2|\psi_B^2|. \tag{26}$$

The unperturbed homogeneous Eq. (25) has a self-similar solution describing an envelope soliton

$$\psi_B = \frac{\psi_0 \exp[i(kx - \theta)]}{2\mathrm{ch}[\Delta(x - X_B)]}, \tag{27}$$

in which k is the wavenumber,

$$X_B = x_0 + \int_0^t v_B dt$$

is the position of the center of the soliton, $v_B = (1 + b^2)k/b$ is the group velocity of the wave (for $k \ll b$),

$$\theta(t) = \int_0^t \omega dt + \beta$$

is the phase of the soliton, β is the phase shift, and ω is the frequency which satisfies the dispersion relation

$$\omega^2 = b^2 + (1 + b^2)k^2 - b^2\psi_0^2/4. \tag{28}$$

According to (23), in this approximation the amplitude of the DW oscillations is then given by the formula

$$q_B = \frac{\psi_0 \exp[i(kx - \theta)]}{2ib\,\mathrm{ch}[\Delta(x - X_B)]}. \tag{29}$$

We now assume that the presence of a perturbation $\mathbf{d} \neq 0$ leads to a slow (adiabatic) change with time in the soliton parameters, i.e., $r_j\,(j = 1,2,3,4) = \psi_0, x_0, v_B, \beta = r_j(\varepsilon t)$, where $\varepsilon \ll 1$. We can write the solution of the perturbed system in the form $\mathbf{C} = \mathbf{C}_0 + \mathbf{g}$ where

$$\mathbf{C}_0 = \begin{pmatrix} \psi_B \\ \psi_B^* \end{pmatrix}$$

is a solution of unperturbed system (25) which; determined by Eq. 28, and

$$\mathbf{g} = \begin{pmatrix} g_B \\ g_B^* \end{pmatrix}$$

is a small correction ($|g_B| \ll |\psi_B|$) to the main solution which satisfies the linearized equation

$$(\partial_t^2 + \hat{L}_g)\mathbf{g} = \begin{pmatrix} d_g \\ d_g^* \end{pmatrix}, \tag{30}$$

in which we have

$$d_g = -2ib\sum_j \partial_t r_j\, \partial_{r_j} \psi_B + \frac{2b^2\psi_B}{\mathrm{ch}^2(x - X_L)},$$

X_L is the position of the center of the BL, and the operator $\hat{L}_g$ has the following form:

$$\hat{L}_g = \begin{vmatrix} b^2 - (1 + b^2)\partial_x^2 - 4b^2|\psi_B^2| & -2b^2\psi_B^2 \\ -2b^2\psi_B^{*2} & b^2 - (1 + b^2)\partial_x^2 - 4b^2|\psi_B^2| \end{vmatrix}. \tag{31}$$

To derive an expression for d_g we take into account that the core of the BL is described by the formula[11]

$$\psi_L = 2\mathrm{arctg}[\exp(x - X_L)]. \tag{32}$$

For the set of Eqs. (30) to be soluble the vector $\mathbf{d}_g$ must be orthogonal to the eigensolutions of the homogeneous set of equations conjugate to (30). By virtue of the self-adjointness of the operator $\hat{L}_g$ the vectors $\mathbf{C}_g^j = \partial \mathbf{C}_0^* / \partial r_j$ $(j = 1,...,4)$ are such solutions. From the orthogonality condition $(\mathbf{C}_g^j \mathbf{d}_g) = 0$ after integration, using (27) and (29), we get the required evolution equations which can be written in the following form:

$$\begin{cases} \partial_t X_B = v_B, \\ m_B \partial_t v_B + \dfrac{\partial U_{LB}}{\partial X_B} = 0, \\ \partial_t \beta = \dfrac{2b\Delta}{\mathrm{ch}^2[\Delta(X_L - X_B)]}\{X_L \Delta \mathrm{th}[\Delta(X_L - X_B)] - 1\}, \\ \partial_t \psi_0 = 0, \end{cases} \tag{33}$$

where

$$U_{LB} = -\frac{\psi_0^2}{\mathrm{ch}^2[\Delta(X_L - X_B)]} \tag{34}$$

is the potential energy of the interaction of the envelope soliton with the BL, and

$$m_B = \frac{2|\psi_0|}{b(1 + b^2)^{1/2}}$$

is the mass of the breather. Similarly, using perturbation theory for the BL,[10] and taking the form (24) as the approximate self-similar reference solution of Eq. (32) for ψ_L, and also taking

$$q_L = \frac{\pi v_L}{2b} \exp(-b|x - X_L|) \tag{35}$$

as the self-similar solution for bending DW, we can show that the equations for the BL position X_L and velocity v_L, averaged over a period $\tau = 2\pi/b$ of the oscillations of the small-amplitude soliton, have the form

$$\begin{cases} \partial_t X_L = v_L, \\ m_L v_L + \dfrac{\partial U_{LB}}{\partial X_L} = 0, \end{cases} \tag{36}$$

where $m_L = \pi^2/2b$ is the mass of the BL (the dimensionality of the breather and BL masses is $[m] = (2\pi\gamma^2 Q^{1/2})^{-1}$).

It follows from Eqs. (33) that in a collision between a BL and an envelope soliton the amplitude of the latter does not change, i.e., $\psi_0 = \text{const}$. The change in the phase shift $\beta(t)$ depends solely on the positions $X_L(t)$ and $X_B(t)$ of the solitons. As can be seen from (33) and (36) the positions of the solitons can be described by a Lagrangian of two interacting material particles with an attractive potential U_{LB} and masses m_L and m_B. One can obtain this result also from the conservation laws (3) and (4), if one assumes that the amplitude ψ_0 is constant.

The analysis of the equations obtained shows that a collision between attracting solitons with a bounded interaction potential can, depending on the initial kinetic energy of the solitons,

$$T = m_L v_L^2/2 + m_B v_B^2/2$$

lead to the capture of a small-amplitude soliton by a Bloch line or, if the interaction potential energy is larger than max U_{LB}, they pass through one another causing a shift in the initial phase or position. For instance, when the velocity of the self-localized wave is close to the critical one a BL at rest shifted in the opposite direction from the envelope soliton by an amount of order

$$\delta X_L \sim m_B/m_L \Delta.$$

For large velocities v_B this shift decreases, being inversely proportional to the square of velocity of the incident soliton.

4. STABILIZATION OF THE SPATIALLY LOCALIZED DOMAIN WALL OSCILLATIONS BY A MAGNETIC FIELD

In a dissipative medium ($\alpha \neq 0$) one needs an external energy input to sustain the damped oscillations. It may be produced directly by the parametric pumping either of a magnetic field h_x, or h_z. Such methods are well known in the theory of the stabilization of NLS solitons.[14] The conditions for the stabilization of self-localized DW oscillations by a field

$$h_z \sim \exp(-i\omega t)$$

with a frequency of the order of the activation gap, $\omega \sim b$, were considered in Ref. 6. Here we analyze the conditions for parametric stablization by a field

$$h_x(t) = h_x \exp(-i\omega t) + \text{c.c.}$$

at a frequency $\omega \sim 2b$.

It is clear that a uniform pump can stabilize only a standing wave of DW oscillations with $k = 0$. Taking as the standard solution of the unperturbed system (1), (2) the self-similar solution (27) to (29) with $k, X_B = 0$ and following the procedure, described above, of eliminating secular terms in the perturbed Eqs. (1), (2) with $\alpha, h_x \neq 0$ ($h_z = 0$), for the amplitude $\psi_0(t)$ and the frequency mismatch

$$\chi(t) = \int_0^t [\omega_p - 2b(1 - \psi_0^2/8)]dt - \beta(t)$$

we can obtain the following equation:

$$\begin{cases} \partial_t \psi = -\alpha(1 + b^2)\psi + bh_x \psi \sin\chi, \\ \partial_t \chi = \omega_p - 2b(1 - \psi^2/8) + h_x b \cos\chi. \end{cases} \tag{37}$$

The phase plane (ψ,χ) of the equation obtained here is a generating cylinder ($-\infty < \psi < +\infty, 0 < \chi < 2\pi$) with a phase portrait which is symmetric with respect to the line $\psi = 0$. The bifurcation diagram of Eq. (37) is shown in Fig. 4. It is determined by two parameters—the normalized amplitude of the pump field,

$$h = h_x/h_p,$$

where

$$h_p = \alpha(1 + b^2)/b,$$

and the normalized frequency mismatch

$$\Omega = (\omega_p - 2b)/[\alpha(1 + b^2)].$$

In this diagram in the $\Omega < 0$, $h < 1$ and $\Omega > 0$, $h < \sqrt{1 + \Omega^2}$ regions there are no equilibrium positions of the set (37) which are determined by the condition $\partial_t \psi = \partial_t \chi = 0$ with $\psi \neq 0$. In this case it is impossible to parametrically stabilize the soliton. The required equilibrium points occur only for $h > 1$.

Note that $h = 1$ (or $h_x = h_p$) is the condition for the threshold of the uniform parametric generation of zero-amplitude spin waves at the frequency $\omega = 2b$. This follows directly from an analysis of the linearized initial equations (1), (2). The condition $h > 1$ is necessary, but not sufficient, for the soliton stabilization.

The required nonzero equilibrium position exists for $\Omega < 0, h > 1$ and for $\Omega > 0, h > \sqrt{1 + \Omega^2}$ in the following point of the phase plane:

$$\psi_0 = 2[\alpha(1 + b^2)(\sqrt{h^2 - 1} - \Omega)]^{1/2}/\sqrt{b}$$

$$= 2[\alpha(1 + b^2)\sqrt{h^2 - 1} + 2b - \omega_p]^{1/2}/\sqrt{b},$$

$$\chi_0 = \pi - \text{Arcsin}\left(\frac{1}{h}\right). \tag{38}$$

Linearizing Eqs. (37) near this equilibrium position shows that the point (38) is an attractive focus when

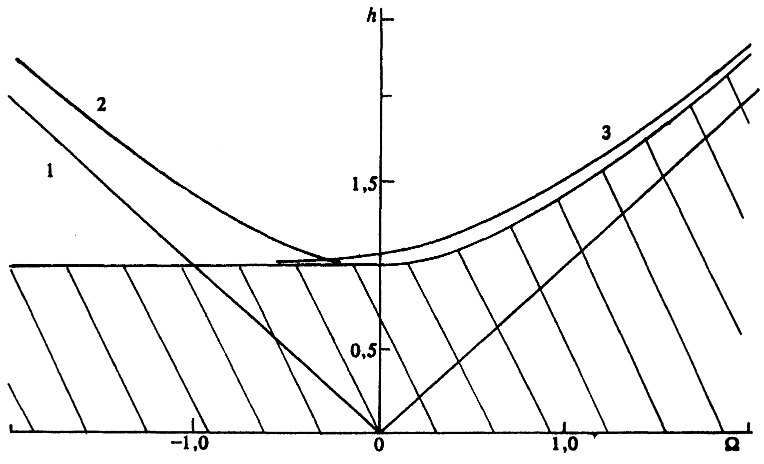

FIG. 4. Bifurcation diagram of the set of Eqs. (37). The area where there are no equilibrium positions with nonzero amplitude ($\psi_0 \neq 0$) is hatched: 1—the line $h = |\Omega|$; 2—$h\sqrt{1+\Omega^2}$; 3—$h = [1 + (\Omega/2 + \sqrt{\Omega^2/4 + 1/8})^2]^{1/2}$.

$$h > h_1 = \left[1 + \left(\frac{\Omega}{2} + \left(\frac{\Omega^2}{4} + \frac{1}{8}\right)^{1/2}\right)^2\right]^{1/2} \qquad (39)$$

and a node in the opposite case ($h < h_1$).

There is also the singular point

$$\psi_0 = 2[\alpha(1 + b^2)(|\Omega| - \sqrt{h^2 - 1})]^{1/2}/\sqrt{b}, \quad \chi_0 = \text{Arcsin}\left(\frac{1}{h}\right),$$

corresponding to an unstable saddle-point equilibrium position in the region $\Omega < 0$, $1 < h < \sqrt{1 + \Omega^2}$. Moreover, there are also two equilibrium points on the line $\psi = 0$ with $\cos \chi_0 = -\Omega/h$ for $|\Omega| < h$. In that case, they are saddle points when $h > \sqrt{1 + \Omega^2}$ holds and one of them becomes an attractive node, i.e., corresponds to stable equilibrium, for $h < \sqrt{1 + \Omega^2}$. In this connection the stabilization of the soliton for $1 < h < \sqrt{1 + \Omega^2}$ depends on the matching of the initial amplitude and phase of the self-localized oscillation and of the pump field. This is clear from the approximate form of the phase portrait of Eq. (37) for this case shown in Fig. 5a. It is shown in Fig. 5b that the stabilization is independent of the initial conditions for $h > \sqrt{1 + \Omega^2}$.

Note that for $h_x > 0$ the DW ground state ($\psi = 0$) is totally unstable. For parametric stabilization to be possible one must thus satisfy the condition

$$h_p = \alpha(1 + b^2)/b < 1,$$

which restricts the degree of bending "rigidity" of the DW and the magnitude of the damping in the system.

Moreover, it is clear from (38) that for small damping, $\alpha \ll 1 (\alpha < b)$, the amplitude of the stablized oscillation is basically determined by the frequency mismatch $\delta\omega = \omega_p - 2b$ and depends only weakly on the magnetic field mismatch $\delta h = h_x - h_p$. For small oscillation amplitudes, $\psi_0 \ll 1$, the requirements on the admissible range of frequency mismatch, $\delta\omega/\omega_p < \psi_0^2/8 \ll 1$, are rather rigid. When one goes beyond this range the self-similar approximation assumed here will be violated and the DW oscillations may become unstable. At any rate the conclusions of perturbation theory cease to be valid.

We give some numerical estimates determining the possibilities for observing parametric stabilization of small-amplitude solitons in DW in a uniaxial ferrite-garnet. Taking $4\pi M = 200$ G, $b = 0.2$, $\alpha = 0.02$, $\gamma = 2.8 \cdot 10^6$ MHz/Oe, and $\psi = 0.5$, we find $H_p = 8M\alpha(1 + b^2)/b = 12.7$ Oe, $f_p = \omega_p/2\pi = 4M\gamma b = 229$ MHz, and $\delta\omega/\omega_p = \psi_0^2/8 = 3\%$.

CONCLUSION

The studies given here have thus shown that in a pinned domain wall of a uniaxial ferromagnetic with a large anisot-

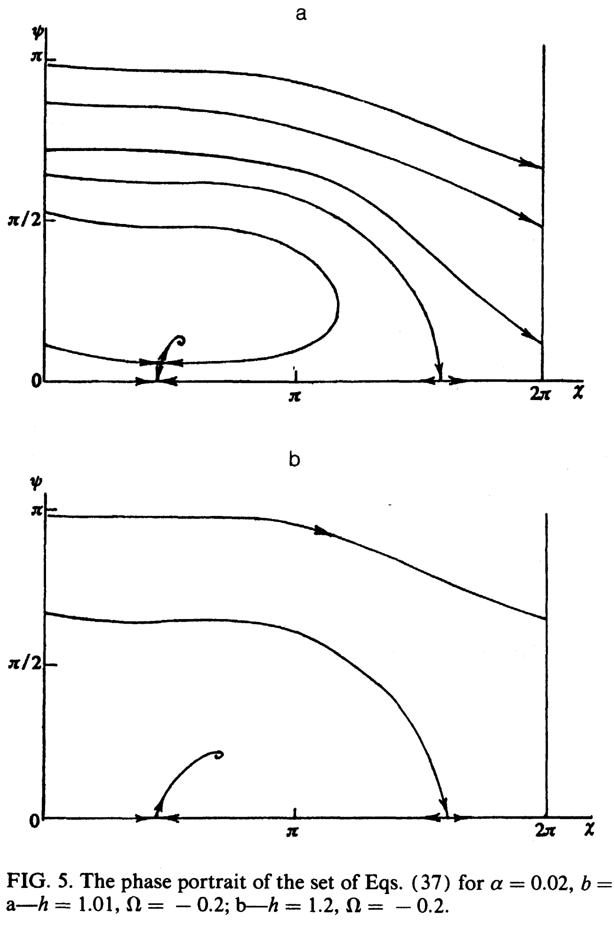

FIG. 5. The phase portrait of the set of Eqs. (37) for $\alpha = 0.02$, $b = 0.2$: a—$h = 1.01$, $\Omega = -0.2$; b—$h = 1.2$, $\Omega = -0.2$.

ropy there may exist small-amplitude self-localized spin wave excitations which have the properties of NLS solitons. Like the breathers of the plasma oscillations in a distributed Josephson junction which are attracted to the fluxons, the self-localized DW oscillations interact with the BL. Because of the small amplitude of the solitonlike DW oscillations their interaction with BL is described by a single-well attractive potential which is proportional to the square of the amplitude of the oscillations. It is clear [as can be seen from the expression for the perturbing term in Eq. (24)] that in the case of a cluster containing several BL, the interaction with a breather increases proportional to the number of BL.

Together with the phenomenon of the trapping of the envelope solitons of the bending oscillations of unmoving BL and their clusters the opposite shift of a BL in which envelope solitons pass through it when their kinetic energy is larger than the attractive potential is also possible. This last effect, connected with the momentum conservation law of countermoving solitons in the absence of emission, is characteristic for nonreflecting solitons in integrable systems.[15] Taking into account the emission connected with the partial scattering of small-amplitude solitonlike DW oscillations by BL reduces this effect.

In contrast to the breathers of the sine-Gordon equation the localized solitonlike DW excitations are metastable; this is connected with the generation of spin waves with a complex spectrum at frequencies which are multiples of the ground-state frequency. Direct numerical calculations show that when one increases the amplitude of the breather-type solitons they do not change into a bound state of Bloch lines but disintegrate into dispersing spin waves along the DW. An alternating magnetic field may stabilize the self-localized DW oscillations when the synchronization conditions for direct[6] or parametric pumping are satisfied, similar to the NLS solitons.[14] The stablization effect has a threshold in a dissipative medium. For a small pumping amplitude near the threshold value the stabilization effect depends on the initial conditions. If the initial conditions are such that the oscillation frequency is not trapped, one will observe periodic oscillations of the soliton amplitude in the case of the direct pumping method, while in the case of parametric pumping the soliton is damped. When the pumping amplitude increases the stablization of the soliton ceases to depend on the initial conditions. In that case the restrictions on the admissible range for the mismatch of the pumping frequency are rather rigid for small-amplitude oscillations because the nonlinear frequency shift is small. For large amplitudes of the pumping field perturbation theory becomes inapplicable. Here one may expect that as in a distributed Josephson junction[16] parametric transformations of the small-amplitude solitons will occur and chaotic and dissipative structures will form. However, this requires a separate consideration.

In conclusion one should note that diffraction methods for visualizing subdomain structures[18] may turn out to be efficient for the observation of the interaction of the nonlinear DW oscillations with BL in addition to the methods for direct magneto-optic visualization of the DW bending in conjunction with high-speed photography.[2,17]

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[1)]The problem of the existence of self-localized solutions in a system with a ψ^4 nonlinearity was discussed in Ref. 9.

[2)]An additional feature of the numerical solution of the set of Eqs. (1), (2) was the nonlinear distortion of the shape of the small-amplitude soliton (11)–(13). When the amplitude is increased additional self-constriction occurs which is apparently connected with the effect of the leading derivative in the initial system of equations.

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