Does the probability for spontaneous emission depend on the density and the temperature?

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(Submitted 6 May 1992)
Zh. Eksp. Teor. Fiz. **103**, 345–373 (February 1993)

We discuss the problem of the dependence of the probability for spontaneous emission on the electron density and the temperature under conditions when the effect of the plasma environment is not important. Using kinetic Green's functions for nonequilibrium processes in the framework of the Keldysh formalism we derive generalized equations, valid for broad lines, for the transfer of resonance radiation and for the kinetics of excited particles. In thermodynamic equilibrium these equations give the correct expression for the Planck intensity. We perform numerical calculations of the spectral intensities of the spontaneous emission and of the intensities integrated over the frequency for transitions in the lithium-like carbon ion for which recently experimentally the total intensity has been observed to depend on the electron density. Our calculations show that even in a low-density plasma at low temperatures a significant difference between the intensity integrated over the spectrum and its vacuum value is possible. Spectral anomalies manifest themselves on the far wings of the lines which may turn out to be important for the problem of radiation transfer, in particular, for calculations of the mean free paths of photons in dense matter.

1. INTRODUCTION

In Ref. 1 Einstein was the first to introduce in his considerations the hypothesis of induced emission and also the concept of the probabilities for spontaneous and induced emission. In the same paper he gave a derivation of the Planck formula for the spectra density of equilibrium radiation from the conditions that the atom be in equilibrium with the thermal radiation. This paper essentially laid the foundations for the theory of the transfer of resonance radiation.

In the quantum theory of radiation the Einstein coefficient A_{mn}, which is equal to the probability for a spontaneous transition from an excited state m to a state n, is calculated using perturbation theory (see, e.g., Ref. 2). For a particle in vacuum the quantity A_{mn} is determined by the matrix element of the dipole moment operator and the transition frequency and is, of course, independent of the density (ρ) and the temperature (T). In a dense hot substance the emitting ion is in the environment of its neighbors and its wavefunctions and energy levels, determined in a number of models, depend on the density and the temperature,[3] which causes A_{mn} to depend on ρ and T. It is natural to assume that, as long as the size of the atomic orbits are small as compared to the distance between the particles or the screening radius of the Coulomb potential in the plasma, the ρ and T dependence of A_{mn} is not significant.

Experiments which have recently been carried out in a laser plasma produced by a pulsed CO_2 laser[4] and also by a ruby or Xe–Cl laser[5] with a moderate energy $\leqslant 10$ J and a long pulse length $\simeq 20$ to 150 ns have shown that in a plasma with a relatively low density (with an electron density $N_e \simeq 10^{18}$–10^{19} cm^{-3}) the ratio of the experimentally measured intensities for transitions from one and the same level in the visible ($J_{\rm vis}$) and the vacuum ultraviolet ($J_{\rm vuv}$) bands decreases with increasing N_e in this range by more than an order of magnitude as compared to the vacuum value of this quantity, determined by the ratio of the corresponding A coefficients. Ratios for CIV ions ($3p$–$3s$ transitions in the 5801–5812 Å and $3p$–$2s$ transitions in the 312 Å wavelength bands for lithium-like carbon), for CIII ions (5696 Å and 574 Å for $3d\ {}^1D$–$3p\ {}^1P$ and $3d\ {}^1D$–$2p\ {}^1P$ transitions, respectively, for beryllium-like carbon), and NV (4603–4620 Å and 209 Å for the $3p$–$3s$ and $3p$–$2s$ transitions in lithium-like nitrogen) were measured experimentally.

The authors of Refs. 4 and 5 gave a number of arguments showing that the observed decrease in $J_{\rm vis}/J_{\rm vuv}$ was not connected with the reabsorption of the resonance radiation and reached the conclusion that for electron densities $N_e \simeq 10^{19}$ cm^{-3} their frequent collisions with excited particles decrease the probability for spontaneous emission, $A_{\rm vis}$, for transitions in the visible wavelength band as compared to its "vacuum" value. Under those conditions the replacement of the Coulomb potential by the screened Debye potential does not make a significant difference in the magnitude of $A_{\rm vis}$.

The effects detected in Refs. 4 and 5 have more recently been studied in Refs. 6 and 7 in independent experiments and the existence of the effect was confirmed in Ref. 6, although the authors of Ref. 6 proposed reabsorption as the explanation of the effect while in Ref. 7 the effect of a decrease in $A_{\rm vis}$ with increasing N_e was not observed. The problem arises whether it is possible in principle that the probability for spontaneous emission can depend on the electron density and the temperature in conditions when the density effects[3] do not play an important role. We consider in the present paper the problem of the effect of a finite spectral line width on the intensity of the spontaneous emission and we show that even in a low-density plasma at low temperatures the emission intensity, integrated over the spectrum, can differ considerably from its vacuum value.

The paper is constructed as follows. In Sec. 2 we give a qualitative discussion of the problem considered here. In Sec. 3 we give a more formalized derivation of the equations for the transfer of resonance radiation and the kinetics of excited particles, valid in the case of broad lines. In Sec. 4 we give the results of numerical calculations of the spectral values and of the intensities of the spontaneous emission by an

1063-7761/93/020186-14$10.00

optically thin plasma integrated over frequency for the example of the transitions in the lithium-like carbon ion which were studied experimentally.[4,5] Although the results obtained do not explain the dependence of $J_{\text{vis}}/J_{\text{vuv}}$ on the electron density detected in Refs. 4 and 5, they show under what conditions the quantities J may depend on N_e.

2. QUALITATIVE CONSIDERATIONS

In the existing theory of the transfer of resonance radiation[8,9] in the approximation of complete redistribution over frequencies (CRF) of the absorbed and the emitted quanta the equation for the spectral density of the radiation is a kinetic equation for the photon frequency distribution function and can be written in the stationary case in the form (for simplicity we consider only two states: m is the upper and n the lower state)

$$(\mathbf{\Omega}, \nabla)I(\omega, \mathbf{\Omega}, \mathbf{r}) = -k_{nm}(\omega)I(\omega, \mathbf{\Omega}, \mathbf{r}) + \varepsilon_{mn}(\omega). \tag{1}$$

Here $\mathbf{\Omega}$ is a unit vector along the direction of flight of the photon. The quantity $k_{nm}(\omega)$ is the absorption coefficient of the resonance radiation defined in terms of the difference in populations.

$$k_{nm}(\omega) = \frac{\lambda^2}{4} A_{mn} \frac{g_m}{g_n} a_{mn}(\omega)(N_n - \frac{g_n}{g_m} N_m). \tag{2}$$

In (2) λ is the photon wavelength, the g_i are the statistical weights of the states i ($i = n,m$), $a_{nm}(\omega)$ is the profile of the spectral line determined by the various broadening mechanisms: radiative, collisional, Doppler, and Stark broadening (see Ref. 10), and N_i is the population of the ith state. The quantity $\varepsilon_{mn}(\omega)$ is the volume intensity of the spontaneous emission in the appropriate frequency and solid angle range which in the approximation of a complete redistribution over frequencies[11,12] we can write in the form

$$\varepsilon_{mn}(\omega) = \frac{N_m A_{mn}\hbar\omega}{4\pi} a_{mn}(\omega). \tag{3}$$

We can find the populations N_i from the balance equations which contain the rates for the excitation processes (including photo-excitation) and the decay of the corresponding states (see Refs. 8 and 9). Equations (2) and (3) have been written down in the resonance approximation when the deviation of the frequency from resonance is small compared to the frequency:

$$\Delta \equiv \omega - \omega_0 \ll \omega_0.$$

In thermodynamic equilibrium the populations satisfy the Boltzmann relation:[1]

$$\frac{N_m}{N_n} = \frac{g_m}{g_n} \exp\left(-\frac{\hbar\omega_0}{T}\right). \tag{4}$$

When the radiation is in equilibrium with matter the radiation intensity is determined by (1) with a right-hand side equal to zero and in accordance with Kirchhoff's law[10] is given by the expression

$$I(\omega) = \frac{\varepsilon_{mn}(\omega)}{k_{nm}(\omega)} = \frac{\hbar\omega^3}{4\pi^3 c^2 (g_m N_n / g_n N_m - 1)}. \tag{5}$$

Using (4) we get from (5) for the photon occupation numbers the Planck formula, as was shown for the first time in Ref. 1,

$$n(\omega) = \frac{1}{\exp(\hbar\omega_0/T) - 1}. \tag{6}$$

In fact, in the Planck formula the expression which is the analog of (6) contains not the transition frequency ω_0, but the running photon frequency ω. As long as the widths γ of the spectral lines are small compared to the temperature,

$$\gamma \ll T,$$

the difference between (6) and the true Planck formula is insignificant. However, one can pose the problem more formally as, is done, for instance, in Ref. 13 (see the controversy connected with this in Refs. 14 and 15) which is devoted to the Nyquist formula: which of the formulas is more accurate especially for widths which are not small? A similar problem had been discussed earlier in Ref. 16. The difference becomes more one of principle if we consider the problem about the radiation leaving an equilibrium plane layer (see Ref. 17):

$$I_{\text{layer}}(\omega, \mu) = \frac{\varepsilon_{mn}(\omega)}{k_{nm}(\omega)} \left[1 - \exp\left(-\frac{k_{nm}(\omega)L}{\mu}\right)\right]. \tag{7}$$

Here L is the thickness of the layer and μ the cosine of the angle of the photon flight reckoned from the direction of the normal to the layer. The first factor in (7) is given by Eq. (5). Even if $\gamma \ll T$ holds, for large optical depths of the layer,

$$\tau = k_{nm}(\omega_0)L \gg 1$$

for mismatches Δ small compared to the equivalent line width $\Delta\omega_{\text{eq}}$, determined from the relation

$$k(\omega^*)L \simeq 1, \quad \Delta\omega_{\text{eq}} = \omega^* - \omega_0 \simeq \gamma\tau^{1/2}$$

(for a Lorentz profile) Eq. (7) can give a considerable difference from the Planck formula, if $\hbar\Delta\omega_{\text{eq}} \geqslant T$. According to (7), (5), and (6) the intensity of the emerging radiation is practically independent of the frequency for $\Delta \leqslant \Delta\omega_{\text{eq}}$ [ω_0 occurs in the exponent of Eq. (6)], whereas in the true Planck formula the intensity can vary considerably (by a factor $e \approx 2.7$ when $\hbar\Delta\omega_{\text{eq}} \simeq T$). Since there appears such a discrepancy we must admit that the formulation of the appropriate radiation theory expounded above is unsound. A qualitative solution of the emerging difficulties was proposed in Ref. 9. We note that the absorption coefficient (2) contains a correction for induced emission and in the case of thermodynamic equilibrium it can be written in the form

$$k_{nm} \quad (1 - \exp(-\hbar\omega_0/T)).$$

When the radiation transfer occurs in the continuous spectrum we obtain an expression for the corresponding correction factor[10]

$$k_{\text{cont. sp.}} (1 - \exp(-\hbar\omega/T)).$$

If in Eqs. (2) and (3) we replace formally the occupation of the upper level N_m by the quantity

$$N_m^* = N_m \exp(-\hbar(\omega - \omega_0)/T),$$

the correction for induced emission in the expression for the absorption coefficient in the line is at equilibrium the same as the similar one in the continuum and Eq. (5) gives the correct expression for the Planck intensity. The factor $\exp(-\hbar(\omega-\omega_0)/T)$ introduced here differs little from unity for $\gamma \ll T$. However, at low temperatures or for broad lines this modification of the transfer theory leads to considerable differences from the one expounded above.

We turn to the problem of the integral power of spontaneous emission which was measured in Refs. 4 and 5. If we use the approximation (3) and integrate this expression over the frequency we get

$$\int_0^\infty \varepsilon_{mn}(\omega)d\omega = \int_0^\infty \frac{N_m A_{mn}\hbar\omega}{4\pi} a(\omega)d\omega. \tag{8}$$

In the resonance approximation the frequency dependence of the probability for spontaneous emission,

$$A_{mn}(\omega) \simeq \omega^3 d_{mn}^2$$

(d_{mn} is the matrix element of the dipole moment operator) and of the factor $\hbar\omega$ in (8) can be neglected for narrow spectral lines. Removing the smooth frequency dependence proportional to ω^4 from under the integral at the point of resonance ($\omega = \omega_0$) we get

$$\int_0^\infty \varepsilon_{mn}(\omega)d\omega = \frac{N_m A_{mn}^0 \hbar\omega_0}{4\pi}, \tag{9}$$

since the profile $a(\omega)$ of the spectral line is normalized to unity. In Eq. (9)

$$A_{mn}^0 = A_{mn}(\omega_0)$$

is the vacuum value of the probability for spontaneous emission calculated using perturbation theory.[2] In actual fact, the integral (8) diverges if we do not use the resonance approximation, since in the wings of the line the profile contains a Lorentz tail,

$$a(\omega) \simeq \frac{\gamma}{2\pi(\omega-\omega_0)^2},$$

which does not guarantee the convergence of the integral (8):

$$\int_0^\infty \omega^4 a(\omega)d\omega = \infty.$$

If we use the modification of transfer theory proposed in Ref. 9, i.e., substitute in (8) instead of N_m the quantity N_m^* we obtain

$$\int_0^\infty \varepsilon_{mn}(\omega)d\omega = \int_0^\infty \frac{N_m A_{mn}\hbar\omega}{4\pi} \exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right] a(\omega)d\omega. \tag{10}$$

The integral (10) appearing here converges for large mismatches. At low temperatures or large linewidths the converging integral (10) may differ noticeably from expression (9) which was taken as the basis for the interpretation of the experiment.[4,5] The qualitative discussions given in the present section can be justified quite rigorously in the framework of L. V. Keldysh's theory using kinetic Green's functions for nonequilibrium processes.[18-24] In particular, a justification for the heuristic replacement in Ref. 9 of N_m by N_m^* is given in Refs. 23 and 24. More important is that in a rigorous treatment Eqs. (2) and (3) are also changed. In the next section we give a more rigorous formulation of the corresponding equations for the transfer of resonance radiation which is valid for broad spectral lines and low temperatures.

3. TRANSFER TO RESONANCE RADIATION IN BROAD LINES

We noted above that the deviation of the results of the standard theory for the transfer of resonance radiation [formulated using Eqs. (1) to (3)] from the true Planck formula in the equilibrium case is not very important if the linewidth γ, or the equivalent width of an optical dense system, $\Delta\omega_{\text{eq}}$, are small as compared to the temperature. If, however, this condition is not satisfied the theory must be generalized and it is convenient to formulate it using the method of kinetic Green's functions.[18-24]

If one takes into account the partial redistribution in frequency (PRF) of the absorbed and emitted photons,[9,21-24] besides the occupation numbers N_i of the atomic states the spectral densities $N_i(\omega)$, which are usually interpreted[9] as the probability distribution of excited particles that are able to emit a photon of frequency ω, also occur in the theory of the transfer of resonance radiation. In the CRF regime the $N_i(\omega)$ are connected with the corresponding occupation numbers N_i through the relation

$$N_i(\omega) = 2\pi N_i a_i(\omega), \tag{11}$$

$$\int N_i(\omega)\frac{d\omega}{2\pi} = N_i. \tag{12}$$

In Eq. (11) $a_i(\omega)$ is the spectral line profile. These relations assume that the transitions from the excited state $i = m$ take place to the ground state $i = n$ which has a zero width so that the corresponding quantity $N_n(\omega)$ is proportional to a δ-function of the frequency.

The spectral density of the excited particles can be expressed in terms of the kinetic Green's function[18,20-24] $G_i^{-+}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)$, defined as the quantum statistical average of the field operators $\hat\psi$ in the Heisenberg representation:

$$iG^{-+}(x,\xi;x',\xi') = -\langle\hat\psi^+(x',\xi')\hat\psi(x,\xi)\rangle. \tag{13}$$

For definiteness here we have assumed Fermi statistics for the particles, $x = \{\mathbf{r},t\}$ is the four-coordinate of the center of mass of the particle, and the ξ are variables characterizing the motion of the atomic electron described by a set of atomic wavefunctions $\varphi_i(\xi)$. The projections of the Green's function (13) on the basis atomic functions determine the function $G_i^{-+}(x,x')$:

$$G_i^{-+}(x,x') = \int \varphi_i^*(\xi)G^{-+}(x,\xi;x',\xi')\varphi_i(\xi')d\xi d\xi'. \tag{14}$$

We consider in what follows the interaction of an atom with a quantized electromagnetic field which does not contain coherent states so that the off-diagonal functions $G_{ik}^{\alpha\alpha'}$ ($i \neq k$, and the $\alpha = \pm$ are Keldysh variables characterizing the kind of chronological ordering of the field operators[18,20]) are assumed to be equal to zero.

The spectral distribution function of the excited particles can be expressed in terms of $G_i^{-+}(x,x')$ through the following relation:[23,24]

$$N_i(\mathbf{R}, \mathbf{p}, \omega, T) = -i\int G_i^{-+}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)\exp(i\omega\tau - i\mathbf{p}\boldsymbol{\rho})d\tau d\boldsymbol{\rho}. \tag{15}$$

Here we have

$$\tau = t_1 - t_2, \quad T = (t_1 + t_2)/2, \quad \boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2,$$

$$\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2.$$

The wavevector $\mathbf{p}$ is connected with the particle velocity $\mathbf{v}$:

$$\mathbf{v} = \hbar\mathbf{p}/M,$$

where M is the particle mass.

Equation (15) means that the spectral particle velocity and frequency distribution, a special case of which is the function $N_i(\omega)$ from (11) and (12), appears in general in the transfer theory for resonance radiation.

In the Keldysh technique[18,20] the information about the dynamics of the quantum system is described by the retarded G_i^R and advanced G_i^A Green's functions and its kinetics is described by the functions G^{-+} and G^{+-}, where these are connected through the relation

$$G_i^{+-} = G_i^R - G_i^A + G_i^{-+}. \tag{16}$$

For nondegenerate systems the last term can usually be dropped in an expansion in the parameter $N\lambda_T^3$, where λ_T is the thermal de Broglie wavelength for the particle concerned.

Not only the atomic Green's functions $G_i^{\alpha\alpha'}$ but also the photon Green's function $D_{ij}^{\alpha\alpha'}(x,x')$ defined in terms of the Heisenberg operators of the photon electric field strength appears in the set of equations describing the transfer of resonance radiation. For instance, for $D_{ij}^{-+}(x,x')$ we have[20]

$$iD_{ij}^{-+}(x, x') = \langle\hat{E}_i(x')\hat{E}_j(x)\rangle. \tag{17}$$

For a perfect photon gas in infinite space the function $D_{ij}^{-+}(\omega,\mathbf{k})$ has the form[20]

$$iD_{ij}^{-+}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)(2\pi)^2\hbar\omega_{\mathbf{k}}(n_{\mathbf{k}}\delta(\omega - \omega_{\mathbf{k}})$$

$$+ (1 + n_{-\mathbf{k}})\delta(\omega + \omega_{\mathbf{k}})). \tag{18}$$

The function $D_{ij}^{+-}(\omega,\mathbf{k})$ can be expressed in terms of D^{-+} by the relation[20]

$$D^{+-}(\omega, \mathbf{k}) = D^{-+}(-\omega, -\mathbf{k}). \tag{19}$$

In (18) we have $\omega_k = c|\mathbf{k}|$, c is the velocity of light, and the n_k are the photon occupation numbers connected with the spectral intensity density $I(\omega,\boldsymbol{\Omega})$ through the relation

$$n_k = \frac{\pi\lambda^2}{\hbar\omega_k}I(\omega_k, \boldsymbol{\Omega}). \tag{20}$$

The transport equation (1) is a particular case of the Dyson equation[18–20] for the function D_{ij}^{-+}:[23,24]

$$-2i(\omega\frac{\partial}{\partial t} + c\omega_k(\boldsymbol{\Omega}, \nabla))D_{ij}^{-+} = 4\pi\hbar\omega^2(\Pi^{+-}D_{ij}^{-+} - \Pi^{-+}D_{ij}^{+-}). \tag{21}$$

Using Eqs. (18) to (20) we can express the absorption coefficient $k_{mn}(\omega)$ and the volume source $\varepsilon_{mn}(\omega)$, which appear in (1), in terms of the polarization operators $\Pi^{\alpha\alpha'}$:

$$k_{nm}(\omega) = -\frac{2\pi i\hbar\omega_k}{c}(\Pi_{nm}^{+-} - \Pi_{mn}^{-+}), \tag{22}$$

$$\varepsilon_{mn}(\omega) = -\frac{2i\hbar^2\omega_k^2}{c\lambda^2}\Pi_{mn}^{-+}. \tag{23}$$

In the resonance approximation for the polarization operators $\Pi^{\alpha\alpha'}$ we can obtain the expressions[23,24] (for simplicity we assume that the states m and n are nondegenerate)

$$\Pi_{nm}^{+-}(\omega, \mathbf{k}) = i\frac{d_{nm}^2}{3}\int\frac{d\omega_p d\mathbf{p}}{(2\pi)^4}G_{mm}^{+-}(p + k)G_{nn}^{-+}(p), \tag{24}$$

$$\Pi_{mn}^{-+}(\omega, \mathbf{k}) = i\frac{d_{nm}^2}{3}\int\frac{d\omega_p d\mathbf{p}}{(2\pi)^4}G_{mm}^{-+}(p + k)G_{nn}^{+-}(p). \tag{25}$$

To find the atomic Green's functions $G_i^{\alpha\alpha'}$ we need expressions for the retarded G_i^R and advanced G_i^A functions. From the Dyson equation[18,20] we can find that

$$G_i^R(p) = G_i^R(\omega, \mathbf{p}) = \frac{1}{\hbar\omega - \hbar\omega_i - E(\mathbf{p}) + \mu - \Sigma_i^R(\omega, \mathbf{p})}. \tag{26}$$

Here $\hbar\omega_i$ is the energy of the ith state,

$$E(p) = p^2/2M$$

is the translational energy, and μ is the chemical potential. We can express the mass operator Σ_i^R as follows:[23,24]

$$\Sigma_i^R = \frac{1}{2}\left(\Sigma_i^{--} - \Sigma_i^{++}\right) - \frac{1}{2}\left(\Sigma_i^{+-} - \Sigma_i^{-+}\right). \tag{27}$$

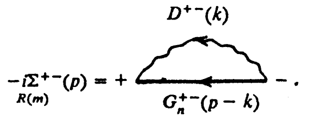
The function G^A is obtained from G^R by taking the complex conjugate. The first term in (27) characterizes the shift of the level due to the interaction with the photons (Lamb shift) and the electrons (collisional shift) and the second one characterizes the width of the state and by virtue of the assumption of a low atomic density ($N\lambda_T^3 \ll 1$) we have $\Sigma^{+-} \gg \Sigma^{-+}$. We thus have

$$\mathrm{Im}\,\Sigma \approx -\Sigma^{+-}/2.$$

Omitting the shift in the level (including it in ω_i) in the present case we write

$$\Sigma^R = -i\gamma/2.$$

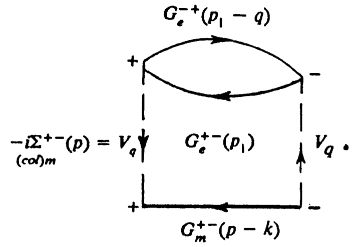
For the radiative decay from the state m to the state n in the resonance approximation the Feynman diagram[18,20] we have



(28)

The thick solid line here indicates the dressed atomic Green's function and the wavy line the photon Green's function. The vertex insertions in (28) are small in the resonance approximation.

For collisions with electrons we have in the Born approximation for $\Sigma_{(\mathrm{col})m}{}^{+-}(p)$ the Feynman diagram[23,24]



(29)

Free-electron Green's functions are here indicated by a thin solid line, and a dashed line corresponds to the interaction potential V_q with an electron (in the present case we consider elastic scattering). To calculate the values of Σ_i^{+-} in (28) and (29) we must know the Green's function G_i^{+-}. We can similarly write down an expression for the mass operators Σ_i^{-+}.

The kinetic equation for the Green's function G_i^{-+}, which is also the corresponding Dyson equation,[18-20] can be written in the form

$$\left(-i\hbar\frac{\partial}{\partial T} + i\frac{\hbar^2 \mathbf{p}}{M}\nabla_R\right)G_i^{-+}(\mathbf{R}, T, \mathbf{p}, \omega)$$

$$= -\Sigma_i^{-+}G_i^{+-} + \Sigma_i^{+-}G_i^{-+}. \qquad (30)$$

In what follows the quantity will be denoted by T to avoid confusion with the notation for the temperature of the system. Using Eqs. (28) and (29), and also Eqs. (26) and (16), we can obtain closed equations for G_i^{-+}. We note that the total occupations N_i of the states which appear in the CRF approximation are obtained from (15) by integrating over the frequencies and the momenta:

$$N_i(\mathbf{R}, T) = \int \frac{d\omega}{2\pi}\frac{d\mathbf{p}}{(2\pi)^3} N_i(\mathbf{R}, \mathbf{p}, \omega, T). \qquad (31)$$

The kinetic (in the present case balance) equations for the occupation numbers are obtained from (30) by integration over ω and $\mathbf{p}$ in accordance with (31).

We note that in thermodynamic equilibrium we can use the temperature Green's function technique.[25] The temperature Green's function has a form similar to (26) except that instead of the frequency ω the quantity $i\omega_n$ occurs, which for Fermi statistics runs through a series of discrete values

$$\omega_n = (2n+1)\pi/\beta,$$

where β is the reciprocal temperature of the system.

The momentum distribution function $N_i(\mathbf{p})$ can be obtained from the temperature Green's function by using the relation:[25-27]

$$N_i(\mathbf{p}) = \frac{1}{\beta}\sum_{\omega_n} G(\mathbf{p}, \omega_n). \qquad (32)$$

Using Eq. (26) we get in accordance with what we have said[26,27] (to simplify the notation we have put the quantity $\hbar$ equal to unity):

$$N_i(\mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega/2\pi}{e^{\beta\omega}+1} 2\pi a_i(\omega - \omega_i - E(\mathbf{p}) + \mu). \qquad (33)$$

Here $a_i(\varepsilon)$, with

$$\varepsilon = \omega - \omega_i - E(\mathbf{p}) + \mu,$$

has the shape of a Lorentz profile:

$$a_i(\varepsilon) = \frac{\gamma_i/2\pi}{\varepsilon^2 + (\gamma_i/2)^2}. \qquad (34)$$

From (33) there follows an expression for the equilibrium spectral density [or in the case of (15) for the Green's function $G_i^{-+}(p)$]:

$$N_i(\omega, \mathbf{p}) = -iG_i^{-+}(p) = \frac{2\pi a_i(\varepsilon)}{\exp((\varepsilon + \omega_i + E(\mathbf{p}) - \mu)/T) + 1}. \qquad (35)$$

We note that Eq. (35) for the equilibrium kinetic Green's function can also be obtained from the Dyson equation (30) by putting the right-hand side of that equation equal to zero (this corresponds to the CRF approximation):

$$G_i^{-+}\Sigma_i^{+-} = G_i^{+-}\Sigma_i^{-+}. \qquad (36)$$

If we use Eqs. (28) and (29) we can obtain two equations, one of which must be satisfied due to the condition that the atoms are in equilibrium with the radiation [taking for $\Sigma^{\alpha\alpha'}$ the quantity $\Sigma^{\alpha\alpha'}_{(R)i}$ from (28)], and the second due to the condition for equilibrium with the electrons [taking the quantity $\Sigma^{\alpha\alpha'}_{(\mathrm{col})i}$ from (29)].

Introducing the notation

$$G_i^{+-}(p) = (G_i^R - G_i^A)(1 - \widetilde{N}_i(p)), \qquad (37)$$

$$G_i^{-+}(p) = -(G_i^R - G_i^A)\widetilde{N}_i(p) \qquad (38)$$

and using the definition

$$G_i^R - G_i^A = -2\pi i a_i(\varepsilon), \tag{39}$$

we can obtain Eq. (35) from (36) with the quantity $\widetilde{N}_i(p)$ equal to[23,24]

$$\widetilde{N}_i(p) = \left\{ \exp\left[\frac{\varepsilon + \omega_i + E(\mathrm{p}) - \mu}{T}\right] + 1 \right\}^{-1}. \tag{40}$$

If the line width γ_i is small compared to the temperature

$$\gamma_i/T \ll 1,$$

we can neglect the quantity ε in the exponential in (35) and (40), and from (35) for a nondegenerate gas of particles ($N\lambda_T^3 \ll 1$) we get:

$$N_i(\omega, \mathrm{p}) \approx 2\pi a_i(\varepsilon) \exp\left(-\frac{\omega_i + E(\mathrm{p}) - \mu}{T}\right). \tag{41}$$

Integrating Eq. (41) over the momenta we get

$$N_i(\omega) = 2\pi a_i(\varepsilon) N_i \tag{42}$$

[cf. (11) and (12)] where N_i is the equilibrium density of excited atoms in the ith state:

$$N_i = \frac{\exp(-\omega_i/T)}{\lambda_T^3} \exp(\mu/T). \tag{43}$$

In the case of a degenerate state an extra factor g_i, equal to the statistical weight of the state occurs in (43);

$$\lambda_T = (2\pi/MT)^{1/2}$$

is the thermal (de Broglie) wavelength of the atom. For an arbitrary ratio of the width to the temperature Eqs. (41) and (42) are not satisfied and one must use Eq. (35).

We note that we can obtain a more general solution than (35) from the condition (36) that the rates of direct and inverse collisional processes are equal [with $\Sigma^{\alpha\alpha'}$ equal to $\Sigma^{\alpha\alpha'}_{(\mathrm{col})}$ from (28)]. We introduce the quantity

$$\widetilde{N}_i = \frac{1}{\lambda_T^3} \exp(-\omega_i/T_i) \exp(\mu/T), \tag{44}$$

which differs from (43) in that the distribution over the levels is characterized by a running temperature T_i rather than by the electron temperature T. This equation generalizes expressions such as (41) and (42) to the case of a non-Boltzmann distribution of the atomic particles over the levels. We get for the spectral density of the excitations an expression which is valid for any ratio of the width and the temperature:

$$N_i(\omega, \mathrm{p}) = \frac{2\pi a_i(\varepsilon) \widetilde{N}_i \lambda_T^3 \exp[-(\varepsilon + E(\mathrm{p}))/T]}{1 + \widetilde{N}_i \lambda_T^3 \exp[-(\varepsilon + E(\mathrm{p}))/T]}. \tag{45}$$

The density of atoms in the state i is given by the expression

$$N_i = \widetilde{N}_i \lambda_T^3 \int_{-\infty}^{+\infty}\!\!\int \frac{d\varepsilon d\mathrm{p}}{(2\pi\hbar)^3} \frac{a_i(\varepsilon) \exp[-(\varepsilon + E(\mathrm{p}))/T]}{1 + \widetilde{N}_i \lambda_T^3 \exp[-(\varepsilon + E(\mathrm{p}))/T]}. \tag{46}$$

For narrow lines ($\gamma \ll T$) when we can neglect the quantity ε/T in (46) and use the fact that the parameter $\widetilde{N}_i \lambda_T^3 \ll 1$ is small, we find that the auxiliary quantity $\widetilde{N}_i$ introduced in (44) is the same as the true occupation number. In the case of equilibrium populations of the atomic states ($T_i = T$) the quantities $\widetilde{N}_i$ satisfy the Boltzmann relations (4). However, the "true" populations (46) do not satisfy these relations in the case of broad lines, not even in equilibrium.

Using Eqs. (37), (38), and (45) we can obtain the values of the appropriate polarization operators (24) and (25), which by virtue of (22) and (23) determine the absorption coefficient and the volume source in the equations for the transfer of resonance radiation. For instance, we thus get for the emission spectrum

$$\varepsilon_{mn}(\omega) = \frac{4}{3} \frac{d_{mn}^2}{\hbar c^3} \frac{\omega^3}{4\pi} \widetilde{N}_m \exp\left[-\frac{\omega - \omega_0}{T}\right] \varphi(\omega) \hbar\omega. \tag{47}$$

We have written here ($\omega_0 = \omega_m - \omega_n$)

$$\varphi(\omega) = \lambda_T^3 \int_{-\infty}^{+\infty} d\varepsilon \int \frac{d^3\mathrm{p}}{(2\pi\hbar)^3} \frac{a_m(\varepsilon + \omega - \omega_0 - \mathrm{kv}) a_n(\varepsilon) \exp[-(\varepsilon + F(\mathrm{p}))/T]}{\{1 + \widetilde{N}_m \lambda_T^3 \exp[-(\varepsilon + \omega - \omega_0 + E(\mathrm{p}))/T]\}\{1 + \widetilde{N}_n \lambda_T^3 \exp[-(\varepsilon + E(\mathrm{p}))/T]\}}. \tag{48}$$

For narrow lines ($\gamma \simeq \varepsilon \ll T$) and a nondegenerate gas ($\widetilde{N}_i \lambda_T^3 \ll 1$) Eq. (48) gives

$$\varphi(\omega) = \lambda_T^3 \int_{-\infty}^{+\infty} d\varepsilon \int \frac{d^3\mathrm{p}}{(2\pi\hbar)^3} a_m(\varepsilon + \omega - \omega_0 - \mathrm{kv}) \times a_n(\varepsilon) \exp\left[-\frac{E(\mathrm{p})}{T}\right], \tag{49}$$

i.e., the usual convolution of profiles, taking into account the Doppler shift of the line. Similarly we get for the absorption coefficient

$$k_{nm} = \frac{\lambda^2}{4} \frac{4}{3} \frac{\omega^3 d_{nm}^2}{\hbar c^3} \varphi(\omega) \left(\widetilde{N}_n - \widetilde{N}_m \exp\left[-\frac{\omega - \omega_0}{T}\right]\right) \tag{50}$$

[cf. (2)]. The generalization to the degenerate case is obvious.

Equations (47) and (50) thus generalize the standard theory for the tranfer of resonance radiation in the CRF approximation to the case of broad spectral lines which leads, firstly, to replacing the line profile by the expression $\varphi(\omega)$ which depends on the densities $\widetilde{N}_i$ of the atomic particles and, secondly, justifies the above replacement of N_m by N^*_m, which is reflected in (47) and (50) by the factor $\exp[-(\omega-\omega_0)/T]$. In equilibrium the correct expression for the Planck intensity,

$$I(\omega)=\frac{\varepsilon_{mn}(\omega)}{k_{nm}(\omega)}=\frac{\hbar\omega^3}{4\pi^3c^2(e^{\omega/T}-1)}\tag{51}$$

follows from (47) and (50).

We note that here the equations for the kinetics of the excited particle densities are formulated for the quantities $\widetilde{N}_i$. In particular, the rate of spontaneous decay of the state m is given by the expression

$$\frac{4}{3}\frac{d^2_{nm}}{\hbar c^3}\widetilde{N}_m\int_0^\infty d\omega_k\omega_k^3\exp\left[-\frac{\omega_k-\omega_0}{T}\right]\varphi(\omega_k),\tag{52}$$

and the rate of $n\to m$ photoexcitation can be written in the form

$$\frac{4}{3}\frac{d^2_{nm}}{\hbar c^3}\int d\Omega d\omega_k\omega_k^3n_k\left[\widetilde{N}_n-\widetilde{N}_m\exp\left[-\frac{\omega_k-\omega_0}{T}\right]\right]\varphi(\omega_k).\tag{53}$$

We can similarly write down the expressions St_{n-m} for transitions between the states n and m under the action of collisions with electrons:

$$St_{n-m}=W_{nm}\left[\widetilde{N}_n-\widetilde{N}_m\exp\left[-\frac{\omega_n-\omega_m}{T}\right]\right],\tag{54}$$

where the probability W_{nm} for a collisional transition now depends nonlinearly on the densities $\widetilde{N}_i$ [cf. (29)]:

$$\begin{aligned}W_{nm}&\simeq\lambda_T^3\int d\varepsilon d\mathbf{p}d\mathbf{p}_1d\mathbf{q}|V_{nm}(\mathbf{q})|^2a_m(\varepsilon)a_n\\&\times\{\varepsilon+E(\mathbf{p})+\varepsilon^{(e)}(\mathbf{p}_1)-\omega_{nm}\\&-E(\mathbf{p}-\mathbf{q})-\varepsilon^{(e)}(\mathbf{p}_1+\mathbf{q})\}\exp(-(\varepsilon+E(\mathbf{p})+\varepsilon^{(e)}(\mathbf{p}_1))/T)\\&\times\ F(\varepsilon,\mathbf{p},\mathbf{p}_1,\mathbf{q}).\end{aligned}\tag{55}$$

The function $F(\varepsilon,\mathbf{p},\mathbf{p}_1,\mathbf{q})$ is given by the expression

$$\begin{aligned}F(\varepsilon,\mathbf{p},\mathbf{p}_1,\mathbf{q})&=\left\{\left[1+\widetilde{N}_n\lambda_T^3\exp\left(-\frac{\varepsilon+E(\mathbf{p})}{T}\right)\right]\right.\\&\times\left[1+\widetilde{N}_n\lambda_T^3\exp\left(-\frac{\varepsilon+E(\mathbf{p})+\varepsilon^{(e)}(\mathbf{p}_1)-\omega_{nm}-\varepsilon^{(e)}(\mathbf{p}_1+\mathbf{q})}{T}\right)\right]\\&\times\left[1+\exp\left(-\frac{\varepsilon^{(e)}(\mathbf{p}_1)-\mu_e}{T}\right)\right]\\&\times\left.\left[1+\exp\left(-\frac{\varepsilon^{(e)}(\mathbf{p}_1+\mathbf{q})-\mu_e}{T}\right)\right]\right\}^{-1}.\end{aligned}\tag{56}$$

In Eqs. (55) and (56) $\varepsilon^{(e)}(\mathbf{p})$ is the electron kinetic energy and μ_e their chemical potential.

We note that the width γ_i is the sum of the radiative and collisional widths given by Eqs. (26) and (29). For instance, we have for the radiative width of the state m decaying to the state n [cf. (22)]

$$\begin{aligned}\gamma_m^{(R)}(\varepsilon)&=\frac{4}{3}\\&\times\frac{d^2_{mn}}{\hbar c^3}\int_0^\infty\frac{d\omega_{\mathbf{q}}\omega_{\mathbf{q}}^3a_n(\varepsilon-\omega_{\mathbf{q}}+\omega_0+\mathbf{qv})}{1+\widetilde{N}_n\lambda_T^3\exp[-(\varepsilon-\omega_{\mathbf{q}}+\omega_0+\mathbf{qv}+E(\mathbf{p}-\mathbf{q}))/T]}.\end{aligned}\tag{57}$$

If the state n is the ground state, the profile $a_n(\varepsilon)$ is a δ-function: $a_n(\varepsilon)=\delta(\varepsilon)$ and the radiative width (57) is given by the equation

$$\gamma_m^{(R)}(\varepsilon)=A^0\frac{[1+(\varepsilon+\mathbf{qv})/\omega_0]^3\theta(\omega_0+\varepsilon+\mathbf{qv})}{1+\widetilde{N}_n\lambda_T^3\exp(-E(\mathbf{p}-\mathbf{q})/T)}.\tag{58}$$

Here A^0 is the classical probability for spontaneous emission:

$$A^0=4d^2\omega_0^3/\hbar c^3,$$

and θ is the Heaviside step function.

We note that the integral over the frequencies in Eq. (57) for the width converges in the general case because of the presence of the denominator. If the profile of the state n is broadened, for instance, by collisions, the radiative width depends, in general, on N_e and T. In the case of narrow lines we have

$$\gamma_m^{(R)}\approx A.$$

We can also obtain an expression for the collisional width in the elastic collision approximation [cf. (28)]:

$$\gamma_m^{(col)}(\varepsilon) = 4\pi^2\int \frac{d\mathbf{p}'d\mathbf{q}}{(2\pi)^6}\,|V_\mathbf{q}|^2 a_m(\varepsilon + E(\mathbf{p}) + \varepsilon^{(e)}(\mathbf{p}') - \varepsilon^{(e)}(\mathbf{p}'+\mathbf{q}) - E(\mathbf{p}-\mathbf{q})$$

$$\times\frac{n_e(\mathbf{p}')[1-n_e(\mathbf{p}'+\mathbf{q})]}{1+\widetilde{N}_m\lambda_T^3\exp[-(\varepsilon+E(\mathbf{p})+\varepsilon^{(e)}(\mathbf{p}')-\varepsilon^{(e)}(\mathbf{p}'+\mathbf{q})+\omega_m-\mu)/T]}. \qquad (59)$$

For the nondegenerate case ($N\lambda_T^3 \ll 1$ and $N_e\lambda_e^3 \ll 1$), if we neglect the recoil of the atom in collisions with the electrons we get from (59)

$$\gamma_m^{(col)}(\varepsilon) = 4\pi^2\int \frac{d\mathbf{p}'d\mathbf{q}}{(2\pi)^6}\,|V_\mathbf{q}|^2 n_e(\mathbf{p}')a_m(\varepsilon+\varepsilon^{(e)}(\mathbf{p}')$$

$$-\varepsilon^{(e)}(\mathbf{p}'+\mathbf{q})). \qquad (60)$$

Since the line profile $a_m(\varepsilon)$ itself is determined by the quantity $\gamma_m(\varepsilon)$ Eq. (60) is essentially an integral equation for the width. If we solve Eq. (60) by iteration, taking the appropriate δ-function as the zeroth approximation instead of $a_m(\varepsilon)$ we obtain the normally used definition for the collisional width[10] in the Born approximation with one difference, however, that the mismatch ε, which characterizes the fact that a collision with an excited atom with electrons leads to a redistribution of the excitations along the spectrum, occurs in the energy conservation law [the argument of the profile in (60)]. Taking the mismatch into account in (60) leads to an ε-dependence of γ_m in first approximation (cf. Ref. 28):

$$\gamma_m^{(col)}(\varepsilon) = \gamma^0\begin{cases} e^{-|\varepsilon|/T}, & \varepsilon<0, \\ 1, & \varepsilon>0, \end{cases} \qquad (61)$$

where γ^0 is the width calculated in the framework of the existing theory.[10]

Note that, in the light of what has been said here, the probability for spontaneous emission occurs in different problems with different meanings: 1) as the width $\gamma_m^{(R)}(\varepsilon)$ of the state [see (57) and (58)] occurring in the expression for the Lorentz profile (34); 2) as the total decay rate of the state [see (52)] occurring in the nonlinear balance equation for the "populations" $\widetilde{N}_i$; 3) as a characteristic of the integral emission [see (47) and (8) to (10)]. For narrow lines all these definitions give the same expression—A^0, calculated in the framework of standard perturbation theory,[2] independent of temperature and density. Since experimentally[4-7] the integral emission rate is determined, in what follows we shall study just that quantity.

In the spectral source $\varepsilon_{mn}(\omega)$ of (47) which occurs in Eq. (1) for the transfer of resonance radiation there appears the factor $\exp[-(\omega-\omega_0)/T]$, so that for broad lines ($\gamma \simeq T$) we can use the diffusion approximation which is inapplicable in the standard Biberman–Holstein theory[11,12] because in the far wing of the line the photon mean free path becomes commensurate with the size of the system and the possibility for emission decreases insufficiently fast with the mismatch from the resonance (proportional to the spectral line profile).

Note also that from the expressions given here there follow unexpected predictions for the far wings of the lines. At low temperatures ($T \ll \omega_0$) it follows from Eq. (47) that in the emission spectrum apart from the usual resonance peak near $\omega \simeq \omega_0$ which is connected with the function $\varphi(\omega)$ [see (47) to (50)] in the "red" wing, for $\omega \ll \omega_0$, one will observe a growth in intensity, due to the presence of the factor $\exp[-(\omega-\omega_0)/T]$, leading to an additional strongly nonresonance wavelength maximum (at $\omega \simeq T \ll \omega_0$). At high temperatures ($T \gg \omega_0$) there follows from the same expressions also the existence of an additional "blue" maximum for $\omega \simeq T \gg \omega_0$ in addition to the maximum near $\omega \simeq \omega_0$ caused by the function $\varphi(\omega)$.

On the quantitative front, we used in the expressions given above the resonance approximation which is represented by single-loop diagrams (without vertex insertions) for the mass Σ^{+-} [see (28) and (29)] and polarization $\Pi^{\alpha\alpha'}$ [(24),(25)] operators. Therefore a rigorous consideration of large mismatches from resonance requires going beyond the framework of the resonance approximation just as one must obviously go beyond the framework of the two-level atom. Estimates indicate that taking vertex insertions into account in the polarization operators does not qualitatively change the general conclusions about the possibility of additional red (for $T \ll \omega_0$) and blue (for $T \gg \omega_0$) maxima in the emission spectrum.

One should also mention that above we used Fermi statistics for the field operators of the atom. Strictly speaking the concept of a field operator for the atom is an approximate one since the atom as a bound state of electrons and a nucleus is an object with complicated statistical properties. Since above we considered mainly the nondegenerate case ($N\lambda_T^3 \ll 1$) the main conclusions of the present paper will, apparently, be retained also in a more rigorous approach.

4. RESULTS OF NUMERICAL CALCULATIONS

The spectral distribution of the intensity of the line emission for the $m \rightarrow n$ transition can be written in the form [see (47) and (48)]

$$a_{mn}(\omega) = \left(\frac{\omega}{\omega_0}\right)^3\int_{-\infty}^{+\infty} d\varepsilon\,\frac{a_m(\varepsilon+\omega-\omega_0)a_n(\varepsilon)\exp[-(\varepsilon+\omega-\omega_0)/T]}{\{1+N_0\lambda_T^3\exp[-(\varepsilon+\omega-\omega_0+\omega_m)/T]\}\{1+N_0\lambda_T^3\exp[-(\varepsilon+\omega_n)/T]\}}. \qquad (62)$$

In the line profile (62) we have included the frequency dependence $\omega^3\exp[-(\omega-\omega_0)/T]$ contained in Eq. (47); the "populations" (44) are assumed to be equilibrium ones; N_0 is the density of atoms (ions) in the ground state; we neglect the thermal motion of the emitters (which is valid for low temperatures and high densities).

For transitions to the ground state $a_n(\varepsilon)$ is a δ-function and Eq. (62) can be simplified:

$$a_{mn}(\omega)=\left(\frac{\omega}{\omega_0}\right)^3 \times\frac{a_m(\omega-\omega_0)\exp[-(\omega-\omega_0)/T]}{\{1+N_0\lambda_T^3\exp[-(\omega-\omega_0+\omega_m)/T]\}(1+N_0\lambda_T^3)}. \tag{63}$$

For narrow lines ($\gamma\ll T$) and a nondegenerate gas ($N\lambda_T^3\ll 1$) the profiles (62) and (63) have the standard normalization:

$$J_{mn}=\int a_{mn}(\omega)d\omega=1. \tag{64}$$

The calculations were carried out for the lines corresponding to the $3p\to 2s$ ($\lambda_{\rm vuv}=312$ Å) and $3p\to 3s$($\lambda_{\rm vis}=5801$–5812 Å) transitions of the lithiumlike carbon ion CIV which were considered in Refs. 4 and 5. The level scheme is shown in Fig. 1. The energies of the levels were taken from Refs. 29 and 30. The widths γ_i occurring in Eq. (34) for the spectral function of the level $a_i(\varepsilon)$ depend on the actual broadening mechanisms. For the lines considered at temperatures of the order of a few eV one can restrict the discussion to the collisional and radiative mechanisms over a wide range of densities ($10^{16}<N_e<10^{23}$ cm^{-3}):

$$\gamma=\gamma^{(col)}+\gamma^{(R)}, \tag{65}$$

where the widths $\gamma^{(\rm col)}$ and $\gamma^{(R)}$ are calculated from Eqs. (61) and (58). In Fig. 2 we show a comparison of the collisional ($\gamma^{(\rm col)}$) and Stark ($\gamma^{(\rm St)}$) widths for the $3p$ level of the CIV ion. It is clear that over the wide range of densities $10^{16}\leqslant N_e\leqslant 10^{23}$ cm^{-3} for temperatures $1\leqslant T\leqslant 10$ eV the collisional broadening mechanism dominates: $\gamma^{(\rm col)}\gg\gamma^{(\rm St)}$. (The Doppler broadening can also be neglected at the temperatures considered.[4,5]) We note that in the density range

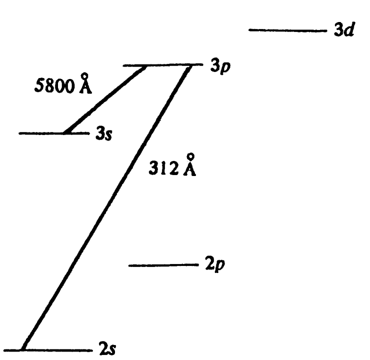


FIG. 1. Level scheme for the CIV ion.

$10^{18}\leqslant N_e\leqslant 10^{19}$ cm^{-3}, studied in Refs. 4 and 5, the density dependence of the width of the CIV 5801.5 Å line corresponding to the $3p\to 3s$ transition is well reproduced by the $\gamma^{(\rm col)}(N_e,T)$ curves: $\gamma^{(\rm exp)}[\rm eV]=1.7\times 10^{-15}\times(N_e[\rm cm^{-3}])^{2/3}$ obtained in Refs. 4 and 5 on the basis of the quasistatic approximation[31] including the experimental data on Ref. 32. This agreement is caused by the weak temperature dependence of $\gamma^{(\rm col)}$: in the range of plasma parameters $T\simeq 1$–10 eV and $N_e\simeq 10^{18}$–10^{19} cm^{-3} the decrease in the collisional width due to the factor $T^{-1/2}$ [see (66)] when the temperature increases is balanced by the growth in $\ln\Lambda$.

In connection with the experimental data obtained in Refs. 4 and 5 one should note one fact which is related to the measurement of the profile of the CIV 312 Å line (the $3p\to 2s$ transition). According to our estimates for densities $10^{18}\leqslant N_e\leqslant 10^{19}$ cm^{-3} the width of that line must be $\Delta\lambda_{\rm vuv}\approx 0.01$–$0.1$ Å, whereas according to the data from Ref. 5, $\Delta\lambda_{\rm vuv}\approx 1$ Å holds (see Fig. 3 in Ref. 5) when the spectrograph has a resolving power of 0.8–3 Å. In this connection there arise additional problems for the interpretation of the experimental data of Refs. 4 and 5.

The probabilities for the radiative $3p\to 2s$ and $3p\to 3s$ transitions are given in Refs. 4 and 5. The missing constants (oscillator strengths) were calculated using Cowan's program,[33] which gives good agreement with known data.[4,5]

The collisional (electron) widths $\gamma^0=\langle\sigma v\rangle N_e$ [see (61)] were calculated in the "hydrogen" approximation (with the logarithm).[31,34] In this case the cross-section for electron collisions can be written in the form (Born type cross-section)

$$\sigma_{ij}=8\pi a_0^2\frac{\rm Ry}{E}\frac{\rm Ry}{\Delta E_{ij}}f_{ij}\ln\Lambda, \tag{66}$$

where a_0 is the Bohr radius, E is the electron energy, ΔE_{ij} is the transition energy, Ry $=13.6$ eV, f_{ij} is the oscillator strength, and $\Lambda=\rho_{\rm max}/\rho_{\rm min}$. The upper ($\rho_{\rm max}$) and lower ($\rho_{\rm min}$) impact parameters were defined as follows:

$$\rho_{max}=\min(\rho_D,v/\omega_{ij}),\quad \rho_{min}=\max(\lambda_e,\rho_C,a,\rho_W),$$

where

$$\rho_D=(T/4\pi e^2N_e)^{1/2}$$

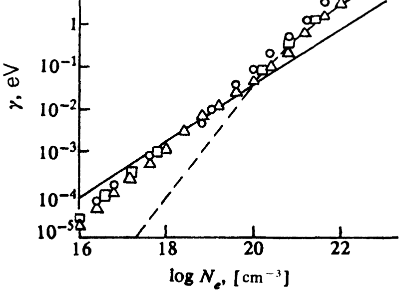


FIG. 2. Width of the $3p$ level of the CIV ion: full drawn curve: approximation of Refs. 4 and 5; dashed curve: Stark width; collisional width γ^0 for $T=1$ (○), 4 (□), and 8 (△) eV.

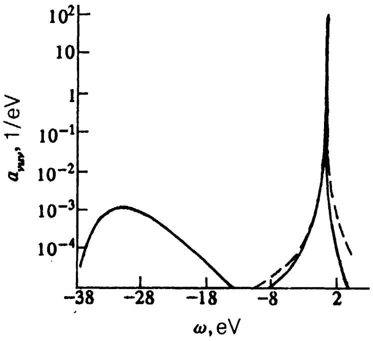


FIG. 3. Line profile $a_{\rm vuv}(\omega)$ as function of the mismatch $\omega - \omega_0$ for $N_e = 10^{19}$ cm^{-3}, $T = 1.3$ eV (the dashed line shows the Lorentz profile with width γ^0).

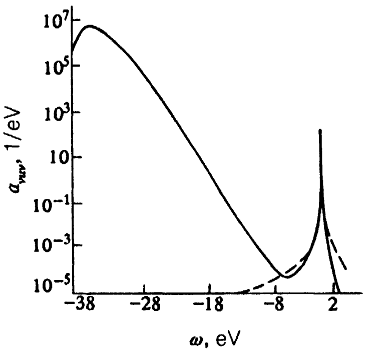


FIG. 4. The same as Fig. 3 for $N_e = 10^{19}$ cm^{-3}, $T = 0.7$ eV.

is the Debye radius, v is the electron velocity,

$$\omega_{ij} = \Delta E_{ij}/\hbar$$

is the transition frequency, λ_e is the electron de Broglie wavelength,

$$\rho_C = (Z - 1)e^2/mv^2$$

is the Coulomb radius, Z is the spectroscopic symbol of the ion,

$$a = \max(a_i, a_j)$$

is the ion radius,

$$a_k \equiv a_{nl} = (3n^2 - l(l + 1))a_0/2Z,$$

and ρ_W is the Weisskopf radius:

$$\rho_W^{(2)} = \pi e d_{ij}/\hbar v$$

for the linear Stark effect,

$$\rho_W^{(4)} = (\pi e^2 d_{ij}^2/2\hbar v \Delta E_{ij})^{1/3}$$

for the quadratic Stark effect. Here d_{ij} is the dipole moment of the transition. If it turns out that $\ln \Lambda < 1$ we put $\ln \Lambda = 1$.

For a Maxwellian distribution the averaging of the cross-section (66) over the velocities gives

$$\langle \sigma v \rangle_{ij} = 16\pi^{1/2} a_0^2 v_0 \left(\frac{\mathrm{Ry}}{\Delta E_{ij}}\right)^{3/2} f_{ij} \beta^{1/2} e^{-\beta} \overline{\ln \Lambda}, \qquad (67)$$

where v_0 is the electron velocity in the first Bohr orbit, we have $\beta = \Delta E_{ij}/T$, and the bar over the logarithm indicates that in the expressions for $\rho_{\max}$ and $\rho_{\min}$ we have substituted the average (thermal) electron velocity. Equation (67) gives the excitation rate by electron collisions. For the inverse process (deexcitation) we have

$$g_j \langle \sigma v \rangle_{ji} = g_i \langle \sigma v \rangle_{ij} \exp(\Delta E_{ij}/T), \qquad (68)$$

where g_i and g_j are the statistical weights of the levels i and j.

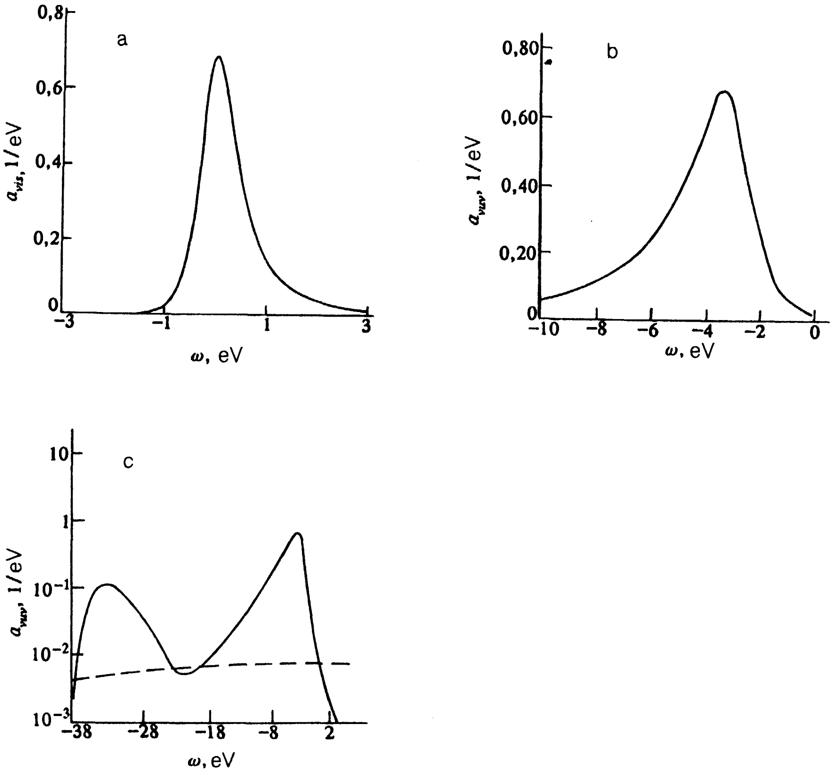


FIG. 5. a: Line profile $a_{\rm vis}(\omega)$ for $N_e = 10^{21}$ cm^{-3}, $T = 1$ eV; b,c: the same as Fig. 3 for $N_e = 10^{23}$ cm^{-3}, $T = 1.1$ eV.

We note that the cross-sections and electron excitation rates used, (66) and (67), agree quite well (with an accuracy $\leqslant 10\%$) with the data of recently performed calculations in the tight binding approximation.[35]

In Figs. 3—5 and the Table we show the results of the calculations of the profiles $a_{\rm vis}(\omega)$ and $a_{\rm vuv}(\omega)$ [see Eqs. (62) and (63)] and the corresponding integral intensities

$$J = \int a(\omega) d\omega$$

of the $\lambda_{\rm vis} = 5801–5812$ Å and $\lambda_{\rm vuv} = 312$ Å lines in the temperature and density ranges $0.5 \leqslant T \leqslant 5$ eV, $10^{18} \leqslant N_e \leqslant 10^{23}$ cm^{-3}. The total intensities are normalized to the Einstein probability so that for $J = 1$ the emission probability is the same as the usual (vacuum) probability. For low densities, $N_e \leqslant 10^{19}$ cm^{-3}, and not too low temperatures, $T \geqslant 1$ eV, the profiles of both lines have the usual Lorentz shape. For instance, for $N_e = 10^{19}$ cm^{-3}, $T = 1.3$ eV the $a_{\rm vis}(\omega)$ and $a_{\rm vuv}(\omega)$ profiles are practically the same and have very similar widths: $\gamma_{\rm vis} = 4.6 \times 10^{-3}$ eV, $\gamma_{\rm vuv} = 4.1 \times 10^{-3}$ eV which differ little from the collisional width of the $3p$ level: $\gamma^0 = 4.0 \times 10^{-3}$ eV. No shift of the lines was observed since the collisional shift was not taken into account in the calculations. However, in the low-frequency range an additional maximum for the 312 Å line appears, which can be seen clearly in the logarithmic scale of Fig. 3. However, its contribution to the total intensity is unimportant in this case. The areas under the profiles are equal, $J_{\rm vis} \approx J_{\rm vuv} \approx 1$ [cf. (64)].

For the same density, $N_e = 10^{19}$ cm^{-3}, but at a lower temperature, $T_e = 0.7$ eV, the $a_{\rm vis}(\omega)$ profile still retains its Lorentz shape ($\gamma_{\rm vis} = 4.9 \times 10^{-3}$ eV for a collisional width $\gamma^0_{3p} = 4.6 \times 10^{-3}$ eV) whereas the low-frequency peak at the 312 Å line increases steeply (Fig. 4c) and gives the main contribution to the total intensity $J^{\rm (tot)}_{\rm vuv} = 2.55 \times 10^7$. On the other hand, the area under the profile in the region of the central frequency is, as before, close to unity $J^{(\omega_0)}_{\rm vuv} = 0.99$.

The intensity of the low-frequency peak for the $3p \to 2s$ line can be approximately estimated using a formula which follows from the general expression (63) for the profile at low temperatures ($T \ll \omega_0$):

$$\int a_{vuv}(\omega) d\omega \approx 8(6!/\pi)(\gamma^0_{3p}/\omega_0)(T/\omega_0)^7 \exp(\omega_0/T), \quad (69)$$

where $\omega_0 = 39.67$ eV is the frequency of the $3p \to 2s$ transition.

In the case considered this formula gives an area under the low-frequency peak equal to 2×10^7 (cf. the result 2.55×10^7 of the numerical integration). When the temperature is lowered the accuracy of Eq. (69) increases. For instance, for $N_e = 10^{19}$ cm^{-3} and $T = 0.5$ eV we get from Eq. (69) 1.3×10^{16} and the exact value is 1.56×10^{16} (see Table).

We show in Fig. 5 the profiles $a_{\rm vis}(\omega)$ for $N_e = 10^{21}$ cm^{-3}, $T = 1$ eV and $a_{\rm vuv}(\omega)$ for $N_e = 10^{23}$ cm^{-3}, $T = 1.1$ eV. Here both profiles differ considerably from the usual

TABLE I. Integral intensity $J = \int a(\omega) d\omega$ of the lines of the carbon CIV ion in the vacuum ultraviolet (vuv) and the visible (vis) regions of the spectrum ($\lambda_{\rm vuv} = 312$ Å, $\lambda_{\rm vis} = 5801–5812$ Å) for different electron densities and temperatures (the contribution from the central peak of the $\lambda_{\rm vuv}$ line is given in brackets).

N_e, cm^{-3}	T, eV	J_{vuv}		J_{vis}
10^{23}	0,5	$1{,}56 \cdot 10^{16}$	(6,01)	*
	0,7	$2{,}55 \cdot 10^{7}$	(4,29)	*
	0,8	$5{,}70 \cdot 10^{4}$	(3,69)	*
	0,9	554,0	(3,27)	*
	1,0	17,6	(2,90)	*
	1,05	5,97	(2,74)	*
	1,1	3,43	(2,60)	*
	1,2	2,44	(2,36)	*
	1,3	2,16	(2,15)	*
	1,5	1,81	(1,81)	*
	2,0	1,30	(1,30)	*
	3,0	0,836	(0,836)	*
	4,0	0,637	(0,637)	*
	5,0	0,534	(0,534)	*
10^{19}	0,5	$1{,}56 \cdot 10^{16}$	(0,987)	22,21
	0,7	$2{,}55 \cdot 10^{7}$	(0,990)	0,986
	0,8	$5{,}70 \cdot 10^{4}$	(0,992)	0,987
	0,9	552,0	(0,991)	0,988
	1,0	15,7	(0,992)	0,998
	1,1	1,81	(0,994)	0,990
	1,2	1,07	(0,995)	0,990
	1,3	1,006	(0,995)	0,991
10^{18}	1	15,72	(0,990)	0,994
10^{19}		15,71	(0,992)	0,998
10^{20}		15,68	(0,960)	0,950
10^{21}		15,57	(0,868)	0,773
10^{22}		15,76	(1,040)	*
$3 \cdot 10^{22}$		16,29	(1,566)	*
10^{23}		17,64	(2,901)	*

*The data for $J_{\rm vis}$ have a meaning only for $N_e < 3.3 \times 10^{21}$ cm^{-3}, since for higher densities the radiation with $\lambda = 5800$ Å cannot propagate in the plasma.

Lorentzian. Firstly, both are asymmetric. The halfwidths at half-height (γ^r is the "red" and γ^b the "blue" wing) are, respectively: $\gamma^r_{\text{vis}} = 0.36$ eV, $\gamma^b_{\text{vis}} = 0.52$ eV; $\gamma^r_{\text{vuv}} = 1.4$ eV, $\gamma^b_{\text{vuv}} = 0.82$ eV, while for the $\lambda = 5800$ Å line the short-wavelength wing decreases exponentially:

$$a^b_{vis} \approx \exp(-|\omega - \omega_0|/0{,}67\,\text{eV}),$$

and for the $\lambda = 312$ Å line the "red" wing decreases exponentially:

$$a^r_{vuv}(\omega) \approx \exp(-|\omega - \omega_0|/2{,}2\,\text{eV}).$$

Note that for the 312 Å line these widths are significantly smaller than the "zeroth" collisional width γ^0, which for such high densities is very large: $\gamma^0 = 41.4$ eV. A particular kind of narrowing of the spectral lines is thus observed. Moreover, one should note that the center of the 312 Å line is shifted: $\Delta_{\text{vuv}} = -3.4$ eV. Under these conditions the total intensity is:

$$J_{vis} = 0{,}773, \quad J_{vuv} = 3{,}43.$$

The probability for the $3p \to 3s$ transition is thus little changed, but the probability for the $3p \to 2s$ transition is three and a half times larger than the Einstein probability. If we take into account only the peak near the central frequency for the 312 Å line, it gives a contribution to J_{vuv} equal to 2.6. The whole variety of possibilities in differences between the probabilities for the radiative $3p \to 3s,2s$ transitions from the "classical" (Einstein) values for different densities and temperatures is illustrated in the Table.

It is clear from the data given that under the experimental conditions of Refs. 4 and 5 for $T > 1$ eV, $N_e \approx 10^{18}$–10^{19} cm^{-3} in the framework of the proposed theory the effect of the temperature and the density on the probability for spontaneous transitions in the ultraviolet and visible regions and on the ratio of their intensities is insignificant (see Table). Such effects may occur either for the given densities at temperatures $T < 1$ eV or at high temperatures ($T > 1$ eV) in a high density plasma. The effects observed in Refs. 4 and 5 are possibly connected with the fact that in a denser and colder plasma the density of lithiumlike ions is small, whereas their density increases when one goes away from the target and reabsorption of the radiation may occur for a transition to the ground state ($3p \to 2s$). However, at sufficiently large distances the matter density decreases and the effect of the reabsorption again diminishes. In such a case the ratio $J_{\text{vis}}/J_{\text{vuv}}$ of the intensities, as a function of the coordinate, may have a maximum value.

We note that the anomalies indicated above on the "red" wing of the line at low temperatures can be observed only under nonequilibrium conditions since the equilibrium densities of lithiumlike carbon ions is exponentially small at such temperatures; they may be observed, for instance, when one "injects" ions into a cold plasma.

It was noted in Sec. 3 that similar anomalies must be observed in the short-wavelength wing of the line in the case of high temperatures. We show in Fig. 6 the line profile $a_{\text{vuv}}(\omega)$ at a temperature $T = 1500$ eV and a density $N_e = 10^{23}$ cm^{-3}. Figure 6a shows the complete spectrum together with a Lorentz profile constructed with the width γ^0, which for the parameters considered is equal to $\gamma^0 = 2.47$ eV. In Fig. 6b we show the "blue" wing in the range $0.5 \leqslant \omega \leqslant 2.5$ keV. One clearly sees a maximum at a frequency $\omega \approx T$.

Under well defined conditions the spectral features of the spontaneous transitions between atomic and ionic levels noted here can turn out to be important for radiative transfer problems. In particular, the anomalous behavior of the "blue" wing of a broad line may lead to a noticeable lowering of the magnitude of the Rosseland mean free path:[36,37]

$$l_R = \frac{15}{4\pi^4} \int \frac{du}{k^{tot}_\omega} \frac{u^4 e^{-u}}{(1 - e^{-u})^3}, \tag{70}$$

where $u = \hbar\omega/T$ and k^{tot}_ω is the total absorption coefficient:

$$k^{tot}_\omega = k^l_\omega + k^{br}_\omega + k^{ph}_\omega + k^c_\omega. \tag{71}$$

Here k^l_ω, k^{br}_ω, k^{ph}_ω, and k^c_ω are the line, bremsstrahlung, photoionization, and Compton absorption coefficients, respectively (without the correction factor $[1 - \exp(-\hbar\omega/T)]$, taking into account the induced emission, which is included in the denominator of the weight function in the integrand).

We have carried out preliminary estimates of the Rosseland mean free paths for a hydrogen plasma at high temperatures and densities, taking into account the contribution of the high-frequency wing of the L_α line.

In accordance with Eq. (50) the absorption coefficient in the line is given by the expression

$$k^l_{n_1 n_2}(\omega) = \frac{2\pi^2 e^2}{mc} (gf)_{n_1 n_2} \frac{\omega}{\omega_0} \varphi(\omega) \frac{\lambda_e^3}{2} N_i N_e e^{I/T},$$

where gf is the oscillator strength multiplied by the statistical weight,

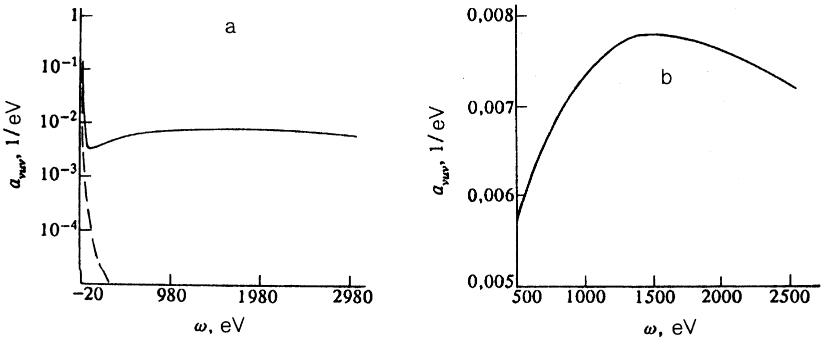


FIG. 6. The same as Fig. 3 for $N_e = 10^{23}$ cm^{-3}, $T = 1500$ eV.

$$\lambda_e = 2\pi^{1/2} a_0 (\mathrm{Ry}/T)^{1/2}$$

is the electron de Broglie wavelength, I is the ionization potential from the ground state (in the present case $I = \mathrm{Ry}$), and N_i and N_e are the ion and electron densities. For the wing of the line the function $\varphi(\omega)$ was taken in the form[34]

$$\varphi(\omega) \approx \frac{3}{2}\frac{(B\mathcal{E}_0)^{3/2}}{(\omega - \omega_0)^{5/2}} + \frac{1}{2\pi}\frac{\gamma^0}{(\omega - \omega_0)^2},$$

where

$$B = (3/8)^{2/3}(\hbar/me)(n_2^2 - n_1^2)$$

is the Stark constant for the $n_2 \to n_1$ transition, and

$$\mathcal{E}_0 = 2{,}603 e N_i^{2/3}$$

is the normal Holtsmark micropole strength.

The (bremsstrahlung and photoionization) absorption coefficient in the continuous spectrum was calculated in the Kramers approximation[10] (neglecting the Gaunt factor):

$$k_\omega^{br} + k_\omega^{ph} = \frac{2^8 \pi^{5/2}}{3^{3/2}} \alpha a_0^5 N_i N_e \left(\frac{T}{\hbar\omega}\right)^3 \left(\frac{\mathrm{Ry}}{T}\right)^{7/2} \left(1 + \frac{2\mathrm{Ry}}{T}\right).$$

The absorption due to the Compton effect was estimated using the Thomson cross-section $\sigma^T = (8\pi/3)(e^2/mc^2)^2 = 6.65 \times 10^{-25}$ cm^2:

$$k_\omega^c = N_e \sigma^T.$$

In Fig. 7 we show the density dependence of the ratio of the magnitude of the Rosseland mean free path l_R calculated taking into account the contribution of the wing of the L_α line [see (71)] to the magnitude of the Rosseland mean free path l'_R calculated with the absorption coefficient

$$k_\omega^{tot} = k_\omega^{br} + k_\omega^{ph} + k_\omega^c,$$

(i.e., neglecting the line absorption). It can be seen from the figure that for densities $\rho > 10^{-3}$ g·cm^{-3} (for a temperature $T = 100$ eV) the absorption in the far wing considerably decreases the photon mean free path. This effect is usually neglected in the calculations.[36,37]

We note that the estimates for large densities have a qualitative nature. Here to obtain a quantitatively reliable result we need refinements taking into account the fact that the plasma is not perfect (leading, in particular, to a diminution of the bremsstrahlung absorption when the electrons are degenerate, ionization of the hydrogen atoms in the plasma micropoles, overlap of Stark broadened spectral lines, and so on).

We must note also that there are also other causes leading to a density dependence of the probability for spontaneous emission which go beyond the confines of the present paper: 1) for densities above the critial one when the plasma frequency is higher than the radiation frequency (e.g., for visible radiation with $\lambda = 5800$ Å for $N_e > 3.3 \times 10^{23}$ cm^{-3}), the corresponding radiation does not propagate in the plasma; 2) cooperative spontaneous emission for $N_a \lambda^3 \gg 1$ (Dieke superradiation[38]) which occurs in the case of inversion in the corresponding transition and leads to a power of the radiation proportional to the square of the atomic density, N_a^2. These physical effects do not explain the dependence of the Einstein probability on the electron density observed in Refs. 4 and 5.

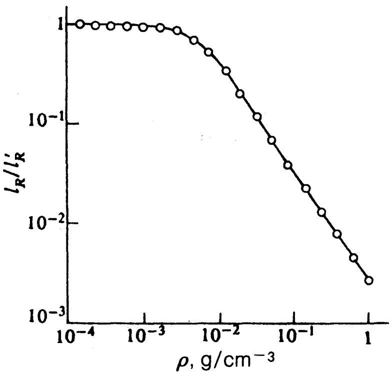


FIG. 7. Density dependence of the ratio of the magnitude of the Rosseland mean free path l_R, calculated taking the absorption in the wing of the L_α line of the hydrogen atom into account, to the magnitude of the Rosseland mean free path l'_R, calculated neglecting the line absorption, for $T = 100$ eV.

Evidence which, in our opinion, is persuasive was given in Ref. 39 that the Einstein coefficients are independent of N_e under conditions close to those realized in the experiments of Refs. 4 and 5, although Ref. 40 contains a number of objections against the arguments of the authors of Ref. 39.

The authors express their gratitude to A. L. Dogunov, A. V. Demure, V. A. Makhrov, A. Yu. Sechin, and A. E. Stepanov for useful discussions and help with carrying out the numerical calculations and also to H. Griem and S. Suckewer for their interest in this work and for stimulating discussions.

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Translated by D. ter Haar