The nonforce topological effect of an electromagnetic field on a Hubbard chain with attraction

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The equations of Bethe's ansatz describing the behavior of a Hubbard chain with attractive electrons at lattice sites in gauge nonforce electric and magnetic fields are derived. Mesoscopic corrections to the energy of the ground state, which determine the oscillations of persistent currents in the chain as the electromagnetic field flux varies, are found. Finally, equations that describe the same system at a nonzero temperature are studied.

As is known, the nonforce topological effect of electric and magnetic fields on charged particles with a magnetic moment manifests itself, for instance, in the quantum effects of Aharonov and Bohm[1] and Aharonov and Casher.[2] These effects are due to acquisition, by the wave functions of the particles with charges and magnetic moments, of phases proportional to the fluxes of the magnetic and electric fields, respectively. The gauge invariance of the electromagnetic field is the reason why only particles moving along closed trajectories exhibit this property. Systems of the greatest interest in this connection are those in which the nonforce topological effect of the electromagnetic field manifests itself along with quantum features related to the nature of the particle interaction in the systems, for instance, solid-state highly correlated electronic systems.

The nonforce effect of the electromagnetic field may reveal itself in a special manner in the low-temperature phases of such quantum systems, say, in superconductors with a finite order parameter this effect manifests itself in the quantization of the magnetic flux.[3] Another possible indication of the specific behavior of effects of the Aharonov-Bohm type in a solid may be the singularities of persistent-current oscillations in systems with Peierls period doubling.[4]

Quantum fluctuation effects are known to show themselves most vividly in low-dimension systems, where they are enhanced by singularities in the density of states. Study of low-dimension systems has been stimulated by the discovery of the quantum Hall effect[5] and high-T_c superconductivity,[6] in which the low dimension plays an important role. Unfortunately, the approximation methods used in the theoretical description of low-dimension systems, such as the mean-field approximation and its modifications and variational methods, may even yield qualitatively incorrect results. Hence, there is a particular interest in studying nonforce topological effects in exactly solvable quantum models of highly correlated electronic systems. In previous papers[7–9] effects of the Aharonov-Bohm and Aharonov-Casher types were studied in one-dimensional quantum models of the Hubbard type with repulsion of the electrons at the lattice sites. Here we will continue examining nonforce topological effects in a Hubbard chain[9] by considering the case of a coupling constant of the opposite sign, which corresponds to electron attraction.

The present paper investigates mesoscopic corrections resulting from gauge electromagnetic fields to the energy of a Hubbard chain in the form of a ring with attraction between the electrons at the lattice sites. The interest in this system lies in the fact that, being exactly solvable, it models the behavior of a "low-dimension superconductor." In addition, the special behavior of the oscillations of charge and spin persistent currents in the ring guarantees the existence of both charge states and spin states. We will derive the equations of Bethe's ansatz that describe the behavior of the system in the presence of nonforce external electromagnetic fields and in the ground state and at a finite temperature. It will be found that, depending on the magnitude of the magnetic field, the ground state may contain mesoscopic Aharonov–Bohm oscillations both with a "metallic flux quantum" $\Phi_0 = ch/e$ and with a "superconducting flux quantum" $\Phi_S = \Phi_0/2$, while Aharonov-Casher oscillations occur only with a "metallic flux quantum" $F_0 = ch/\mu$, with μ the magneton (see Ref. 9). Oscillations with "flux quanta" $\Phi_0/2n$ and $F_0/2n$, with n an integer, generally manifest themselves at a finite temperature. These are caused by the presence of bound states (strings of length n), which contribute to the thermodynamics of the system at a finite temperature and at any value of the magnetic field and the occupation numbers of the chain.

The Hamiltonian of the system in the form of a ring is

$$H=-\sum_{j,\sigma}\{a_{j,\sigma}^{+}a_{j+1,\sigma}\exp(i\alpha_\sigma/N_a)+\text{H.c.}\}-4U\sum_j n_{j,1}n_{j,-1} - (h_e/2)\sum_j (n_{j,1}-n_{j,-1})-A\sum_{j,\sigma} n_{j,\sigma}, \qquad (1)$$

where $U>0$ is the Hubbard attraction constant (the hopping constant is set equal to unity), h_e the external magnetic field, A the chemical potential, $a_{j,\sigma}^{+}$ ($a_{j,\sigma}$) the operator of creation (annihilation) of an electron with spin $\sigma(\sigma = \pm 1)$ at site j, N_a is the number of sites, $n_{j,\sigma} = a_{j,\sigma}^{+}a_{j,\sigma}$, and α_σ the phase acquired by the wave function of the system because of the nonforce effect of the magnetic flux Φ through the plane bounded by the ring (the Aharonov-Bohm effect) and/or the flux F of the radially directed electic field generated by a charged string in the middle of the ring (the Aharonov–Casher effect), with $F = 4\pi\tau$, where τ is the linear density of the charge on the string, and

$$\frac{4}{\pi}\frac{\Phi}{\Phi_0}=\alpha_1+\alpha_{-1}, \qquad \frac{4}{\pi}\frac{F}{F_0}=\alpha_1-\alpha_{-1}. \qquad (2)$$

The eigenfunctions in the coordinate representation are established by Bethe's ansatz method.[10] The quantum

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numbers (the rates k_j and λ_α) that parametrize the wave functions and the eigenvalues of Hamiltonian (1) can be found from the following system of equations:

$$N_a k_j = 2\pi (I_j + \alpha_1) + 2\sum_{\alpha=1}^{M} \operatorname{arctg}((\sin k_j - \lambda_\alpha)/U),$$

$$2N_a \operatorname{Re}(\arcsin(\lambda_\alpha - iU)) = 2\pi (J_\alpha - \alpha_1 - \alpha_{-1}) + 2\sum_{j=1}^{N-2M} \operatorname{arctg}((\lambda_\alpha - \sin k_j)/U) + 2\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M} \operatorname{arctg}((\lambda_\alpha - \lambda_\beta)/2U), \qquad (3)$$

where the I_j and J_α are integers or half-integers, N is the number of electrons, and $2M$ the number of electrons bound in local pairs (see Ref. 11). The energy of the respective state is

$$(E/N_a) = -\sum_{j=1}^{N-2M} [2\cos k_j + A + (h_e/2)] - \sum_{\alpha=1}^{M} [4\operatorname{Re}(1 - (\lambda_\alpha - iU)^2)^{1/2} + 2A]. \qquad (4)$$

Depending on the occupation number $\nu = N/N_a$ and the magnitude of the external magnetic field h_e, the ground state of the chain is formed by the free electrons, characterized by a quasimomentum k_j, and the bound states of two electrons at the sites (local pairs), characterized by the rates λ_α. Equations (3) clearly show that only the fractional parts $\{\{\alpha_1\}\}$ and $\{\{\alpha_1 + \alpha_{-1}\}\}$ of the Aharonov–Bohm and Aharonov–Casher phases are important, since the integral parts only renormalize the sets of numbers I_j and J_α. Thus, quantum nonforce effects in a Hubbard chain with attractive electrons at the sites are of a mesoscopic nature. The ground state of such a chain is formed by filling the "Dirac seas" of free electrons and local pairs.[11]

To calculate the finite-size corrections we use the method developed in Ref. 12. The correction, brought about by magnetic and electric fluxes, to the energy of the ground state of a Hubbard chain with attractive electrons at the sites is

$$\Delta E(\alpha_\sigma)/N_a = \frac{2\pi v_f}{N_a^2}\left[\left(\zeta_{ff}(k_0)\left\{\left\{\frac{\alpha_1}{2\pi}\right\}\right\} + \zeta_{fb}(\lambda_0)\left\{\left\{\frac{\alpha_S}{2\pi}\right\}\right\}\right)^2\right] + \frac{2\pi v_b}{N_a^2}\left[\left(\zeta_{bf}(\lambda_0)\left\{\left\{\frac{\alpha_1}{2\pi}\right\}\right\} + \zeta_{bb}(\lambda_0)\left\{\left\{\frac{\alpha_S}{2\pi}\right\}\right\}\right)^2\right], \qquad (5)$$

where $\alpha_S = \alpha_1 + \alpha_{-1}$, v_f and v_{fb} are the Fermi velocities of the free electrons and local pairs, and ζ_{ik} $(i,k = f,b)$ are the excitation charges "dressed" because of the interaction (electrons and pairs). Clearly, such a mesoscopic correction is present only in the case of excitations without a gap in the spectrum. The ground-state energy corrections due to the nonforce topological effect of electromagnetic fields on the considered Hubbard chain and resulting from the gaps $\Delta_{f,b}$ in the excitations spectrum are proportional to $\exp(-N_a\Delta_{f,b}/v_{f,b})$, that is, are extremely small and thus will be ignored. The fractional parts $\{\{\alpha_1\}\}$ and $\{\{\alpha_1 + \alpha_{-1}\}\}$ determine, as Eqs. (3) demonstrate, the fraction of quantum numbers that transfer, owing to the gauge fields, from one edge of the Fermi bands of electrons and local pairs to the other edge, that is, they are related to the number of virtual excitations of the free-electron and local-pair types existing above the ground state because of the nonforce effect of magnetic fluxes (the Aharonov–Bohm effect) and electric fluxes (the Aharonov–Casher effect).

In the ground state the "dressed" charges are determined by the following equations:

$$\zeta_{ff}(k) = 1 - (1/2\pi)\int_{-\lambda_0}^{\lambda_0} d\lambda K_1(\lambda - \sin k)\zeta_{bf}(\lambda), \qquad (6)$$

$$\zeta_{fb}(k) = -(1/2\pi)\int_{-\lambda_0}^{\lambda_0} d\lambda' K_1(\sin k - \lambda')\zeta_{bb}(\lambda'), \qquad (7)$$

$$\zeta_{bf}(\lambda) = -(1/2\pi)\int_{-k_0}^{k_0} dk K_1(\lambda - \sin k)\cos k\,\zeta_{ff}(k) - (1/2\pi)\int_{-\lambda_0}^{\lambda_0} d\lambda' K_2(\lambda - \lambda')\zeta_{bf}(\lambda'), \qquad (8)$$

$$\zeta_{bb}(\lambda) = 1 - (1/2\pi)\int_{-k_0}^{k_0} dk K_1(\lambda - \sin k)\cos k\,\zeta_{fb}(k) - (1/2\pi)\int_{-\lambda_0}^{\lambda_0} d\lambda' K_2(\lambda - \lambda')\zeta_{bb}(\lambda'). \qquad (9)$$

The kernels $K_{1,2}(x)$ are specified as follows:

$$K_n(x) = 2Un/[(Un)^2 + x^2]. \qquad (10)$$

Note that the system of equations for the "dressed" charges of the simplest excitations of the Hubbard chain with repulsion is obtained from Eqs. (6)–(9) by reversing the sign of the integral terms. The Fermi velocities are given by

$$v_f = \varepsilon_f'(k_0)/\rho(k_0), \quad v_b = \varepsilon_b'(\lambda_0)/\sigma(\lambda_0). \qquad (11)$$

and the excitation energies $\varepsilon_f(k)$ and $\varepsilon_b(\lambda)$ "dressed" because of the interaction can be found from

$$\varepsilon_f(k) = \varepsilon_f^0(k) - (1/2\pi)\int_{-\lambda_0}^{\lambda_0} d\lambda K_1(\lambda - \sin k)\varepsilon_b(\lambda), \qquad (12)$$

$$\varepsilon_b(\lambda) = \varepsilon_b^0(\lambda) - (1/2\pi)\int_{-k_0}^{k_0} dk K_1(\lambda - \sin k)\cos k\,\varepsilon_f(k) + (1/2\pi)\int_{-\lambda_0}^{\lambda_0} d\lambda' K_2(\lambda - \lambda')\varepsilon_b(\lambda'). \qquad (13)$$

The limits of integration are specified by the conditions

$$\varepsilon_f(k_0) = 0, \quad \varepsilon_b(\lambda_0) = 0, \qquad (14)$$

and the "free" energies $\varepsilon_f^0(k)$ and $\varepsilon_b^0(\lambda)$ by

$$\varepsilon_f{}^0(k)=-[2\cos k+A+(h_e/2)],$$
$$\varepsilon_b{}^0(\lambda)=-[4\,\mathrm{Re}(1-(\lambda-iU)^2)^{1/2}+2A]. \qquad (15)$$

The quasimomentum and rate densities in the ground state, $\rho(k)$ and $\sigma(\lambda)$, satisfy the following system of equations:

$$\rho(k)=-\cos k(1/2\pi)\int_{-\lambda_0}^{\lambda_0}d\lambda K_1(\lambda-\sin k)\sigma(\lambda)-(1/2\pi), \qquad (16)$$

$$\sigma(\lambda)=-(1/2\pi)\int_{-k_0}^{k_0}dkK_1(\lambda-\sin k)\cos k\rho(k)$$
$$-(1/2\pi)\int_{-\lambda_0}^{\lambda_0}d\lambda' K_2(\lambda-\lambda')\sigma(\lambda')+(1/\pi)\mathrm{Re}(1-(\lambda-iU)^2)^{-1/2}. \qquad (17)$$

The principal term in the expansion of the ground-state energy in powers of N_a^{-1} is

$$(E/N_a)=-(1/2\pi)\int_{-k_0}^{k_0}dk\varepsilon_f(k)$$
$$-(1/\pi)\int_{-\lambda_0}^{\lambda_0}d\lambda'\varepsilon_b(\lambda')\mathrm{Re}(1-(\lambda-iU)^2)^{-1/2}. \qquad (18)$$

The numbers of free electrons and local pairs in the ground state are, respectively,

$$\frac{N-2M}{N_a}=\int_{-k_0}^{k_0}dk\rho(k),\qquad \frac{M}{N_a}=\int_{-\lambda_0}^{\lambda_0}d\lambda\sigma(\lambda). \qquad (19)$$

Depending on the magnitude of the external field, the ground state of the system is formed by different states. For instance, if $h_e\leqslant h_c$, where

$$2+A+h_c/2=(1/2\pi)\int_{-\lambda_0}^{\lambda_0}d\lambda K_1(\lambda)\varepsilon_b(\lambda).$$

the ground state is formed only by local pairs, while excitations of the free-electron type have a gap in the spectrum. If $h_e\geqslant h_S$, where

$$h_S=(2/\pi)\cos(\pi\nu)[2\pi-2\,\mathrm{arctg}(\sin(\pi\nu)/U)]+[2(1+U^2)^{1/2}]^{-1}$$
$$+(4U/\pi)[U^{-1}(1+U^2)^{1/2}\mathrm{arctg}(U(1+U^2)^{-1/2}\mathrm{ctg}(\pi\nu))+\pi\nu],$$

the ground state is "ferromagnetic," the chain's magnetic moment is at its maximum, and excitations of the local-pair and free-electron types have a gap in the spectrum when $h_e>h_S$. For $h_c\leqslant h_e\leqslant h_S$, excitations of the local-pair and free-electron types are gapless and an intermediate state sets in. The situation is similar to that in type II superconductors. Note that since the system is one-dimensional, the critical temperature is zero. It must also be stressed that in our case of a Hubbard chain with attraction between electrons at the lattice sites the order parameter is zero even at absolute zero; the superconductivity correlators decrease in a power-like manner as the distance increases,[13,14] though more slowly than the spin correlators and free-electron correlators do.

Hence, because of a nonforce topological electromagnetic field, even at absolute zero the system manifests mesoscopic oscillations of the charge and spin current, rather than quantization of the external field fluxes, as in superconductors.

Let us consider some limiting cases of Eq. (5). Obviously, at $h_e=h_S$ there are no local pairs. The energy correction term is

$$\frac{\Delta E_h(\alpha_\sigma)}{N_a}=\frac{8\pi^2\sin(\pi\nu)}{N_a^2}\left\{\left\{\frac{\Phi_h}{\Phi_0}+\frac{F}{F_0}\right\}\right\}^2 \qquad (20)$$

The subscript "h" indicates that the magnitude of the external magnetic field is fixed. For $h_e<h_c$, only the local pairs with $k_0=v_f=\zeta_{bf}(\lambda_0)=0$ are gapless. For small occupation numbers ν, the energy correction resulting from the magnetic flux is

$$\frac{\Delta E_h(\alpha_0)}{N_a}=\frac{v_b}{N_a^2}\zeta_{bb}{}^2(\lambda_0)\left\{\left\{\frac{\Phi}{\Phi_S}\right\}\right\}^2 \qquad (21)$$

where $\zeta^2_{bb}(\lambda_0)=[1+(\nu/2)U^{-1}(1+U^2)^{1/2}]^{-1}$. At half-occupancy we have

$$\frac{\Delta E_h(\alpha_\sigma)}{N_a}=\frac{2\pi I_0(\pi/2U)}{N_a^2 I_1(\pi/2U)}\left\{\left\{\frac{\Phi}{\Phi_S}\right\}\right\}^2 \qquad (22)$$

Finally, at an occupancy close to half-occupancy the "dressed" charge is

$$\zeta_{bb}{}^2(\lambda_0)=[2-\ln^{-1}((8/\pi e)^{1/2}I_0(\pi/2U)/(1-\nu))]^{-1},$$

where $I_n(x)$ is the modified Bessel function, and e is the base of natural logarithms.

Equations (5) and (20)–(22) show that for $h_e\leqslant h_c$ a Hubbard chain with attractive electrons at the sites exhibits, in the ground state, mesoscopic oscillations of the charge persistent current and the related diamagnetic moment with a "superconducting flux quantum" Φ_S. At $h_e=h_S$ the spin current in the chain oscillates with the electric flux F, with the period of oscillations being "metallic," F_0 (see Ref. 9). In the intermediate state with $h_c\leqslant h_e\leqslant h_S$, there may occur both oscillations of the spin current with the "metallic" period and oscillations of the charge current with the period Φ_S (these are due to the combination of "superconducting" and "metallic" oscillations with periods Φ_S and Φ_0, respectively). The amplitudes of the other oscillations in the ground state ("metallic" oscillations of the charge and spin currents for $h_e<h_c$ and all oscillations for $h_e>h_S$) are much smaller and are proportional to $\exp(-N_a)$.

At finite temperatures the eigenfunctions can be classified in the following way (see Ref. 15): $N-2M'$ real-valued quasimomenta k_j corresponding to free electrons; complex-valued quasimomenta k_j corresponding to electron pairs or bound states of pairs such that $\sin k^l_{\alpha,n}=\lambda'^l_{\alpha,n}=\pm iU$, where

$$\lambda_{\alpha,n}'^l=\lambda'_{\alpha,n}-iU(n+1-2l),$$

with $l=1,2,\dots,n$, form a string of length n with a real part $\lambda'_{\alpha,n}$ characterizing the bound state of n pairs (α is an index labeling M'_n strings of length n, and $M' = \Sigma_{n=1}^{\infty} nM'_n$ since the total number of values is $k - N$); and complex-valued λ, corresponding to bound spin states, equal to $\lambda^{l}_{\alpha,n} = \lambda_{\alpha,n} - iU(n+1-2l)$, where $l = 1,2,...,n$, and α is an index labeling M_n strings of length n ($N_1 = \Sigma_{n=1}^{\infty} nM_n$, and $N_1 + M' = M$ is the total number of electrons with spin "down"). Note that in the case of repulsion the imaginary part of k, corresponding to bound states, changes its sign. Takahaski[16] has lately proved that these solutions comprise a complete set for the Hubbard Hamiltonian. The same paper shows that the raising operators of the projections of angular momentum and quasimomentum (see Refs. 17 and 18) are actually creation operators for excitations of the spin and charge (pair) bound-state types. To study the nonforce topological effect on the Hubbard chain it has proved more convenient to consider not the system of equations (3) but the following system:

$$N_a k_j = 2\pi(I_j+\alpha_1) + 2\sum_{n=1}^{\infty}\Big[\sum_{\alpha=1}^{M_n} \mathrm{arctg}((\sin k_j-\lambda_{\alpha,n})/nU) + \sum_{\alpha=1}^{M_n'} \mathrm{arctg}((\sin k_j-\lambda'_{\alpha,n})/nU)\Big],$$

$$2N_a \,\mathrm{Re}(\arcsin(\lambda'_{\alpha,n}-niU)) = 2\pi(J'_{\alpha,n}-n(\alpha_1+\alpha_{-1})) + 2\sum_{j=1}^{N-2M'} \mathrm{arctg}((\lambda'_{\alpha,n}-\sin k_j)/nU) + 2\sum_{m=1}^{\infty}\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M_n} \Theta_{n,m}((\lambda'_{\alpha,n}-\lambda'_{\beta,m})/U),$$

$$0 = 2\pi(J_{\alpha,n}-n(\alpha_1-\alpha_{-1})) + 2\sum_{j=1}^{N-2M'} \mathrm{arctg}((\lambda_{\alpha,n}-\sin k_j)/nU) + 2\sum_{m=1}^{\infty}\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M} \Theta_{n,m}((\lambda_{\alpha,n}-\lambda_{\beta,m})/U). \quad (23)$$

where I_j, $J_{\alpha,n}$, and $J'_{\alpha,n}$ are integers or half-integers, and

$$\Theta_{n,m}(x) = 2\,\mathrm{arctg}(x/|n-m|) + 4\,\mathrm{arctg}(x/(|n-m|+2)) + \ldots + 4\,\mathrm{arctg}(x/(n+m-2)) + 2\,\mathrm{arctg}(x/(n+m)),$$

for $n \neq m$ and

$$\Theta_{n,m}(x) = 4\,\mathrm{arctg}(x/2) + \ldots + 4\,\mathrm{arctg}(x/(2n-2)) + 2\,\mathrm{arctg}(x/(2n)),$$

at $n = m$. The equations describing a Hubbard chain with repulsive electrons at the lattice sites are obtained from (23) by changing the signs in the first system of equations (for k) in front of the sum over n.

The energy of a state with given N, N_1, and M' is

$$E/N_a = -\sum_{j=1}^{N-2M'} [2\cos k_j + A + h_e/2] - \sum_{n=1}^{\infty}\Big[\sum_{\alpha=1}^{M_n'} [4\,\mathrm{Re}(1-(\lambda'_{\alpha,n}-inU)^2)^{1/2} + 2A] + h_e n M_n\Big]. \quad (24)$$

The thermodynamics of a Hubbard chain with attraction and repulsion is constructed by a method similar to the one used in Refs. 15 and 19. The form of Eqs. (23) implies that just as in the ground state the response of a Hubbard chain to a nonforce topological effect is determined by the fractional parts of the corresponding phases (see also Ref. 9), at finite temperatures this role is played by the fractional parts of the phases, $\{\{\alpha_1\}\}$, $\{\{n(\alpha_1+\alpha_{-1}\}\}$, and $\{\{n(\alpha_1-\alpha_{-1}\}\}$. This means that generally, at a nonzero temperature, there may be oscillations of the "metallic" type with flux quanta Φ_0 and F_0, "superconducting" charge oscillations with a flux quantum $\Phi_0/2$ (as in the ground state of a Hubbard chain with attractive electrons), "superconducting" spin oscillations with a flux quantum $F_0/2$ (as in the ground state of a Hubbard chain with repulsive electrons[9]), and, finally, oscillations corresponding to virtual excitations of bounds states (strings) and generated by the nonforce gauge effect of the electromagnetic field, both charge oscillations with a period of $\Phi_0/2n$ and spin oscillations with a period of $F_0/2n$. The latter, naturally, are caused by the interaction and disappear at $U = 0$. Note that the lower the temperature the smaller the probability of excitation of strings with large n. Also note that for a Hubbard chain with attraction of the electrons at the lattice sites at low temperatures the oscillations of the "superconducting" spin type have a low probability, while for a chain with repulsion this is true of oscillations of the "superconducting" charge type.

Thus, we have set up the exact equations describing the behavior of a Hubbard chain with attractive electrons at the lattice sites in gauge electromagnetic fields, which act on the electrons in a nonforce manner. We have established that in the ground state, depending on the magnitude of the magnetic field and the band occupancy, there may be quantum mesoscopic oscillations both with "metallic" periods and with a "superconducting" period of oscillations with a magnetic flux related to virtual creation of local pairs. At nonzero temperatures there may be oscillations corresponding to virtual creation of strings with charges $2ne$ and magnetic moments $2n\mu$ under the action of electric and magnetic fluxes. These oscillations result from the interaction in the system and are absent in systems of noninteracting electrons. At low temperatures there should be oscillations with small values of n, which corresponds to a small probability of virtual creation of long strings.

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