# Theory of the filamentary instability in a collisional plasma

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We use kinetic considerations for a plasma with multiply ionized ions to discuss the region, intermediate as far as the scale of the inhomogeneities is concerned, between the usual collisionless region, where the ponderomotive Miller force is effective, and the usual collisional region, where the nonlinear action of radiation on the plasma corresponds to nonuniform Ohmic heating. We consider the consequences, in such an intermediate collisional region of the nonlinear action of radiation, of a theory of the effect of the usual convective filamentary instability and of the effect of the absolute double filamentary instability of an electromagnetic wave in the plasma.

1. One of the typical effects of the action of electromagnetic radiation on a plasma is the filamentation of beams of radiation propagating in the plasma. The simplest theoretical approach to such an effect consists in considering the filamentary instability (see, e.g., the review in Ref. 1). One then distinguishes two physically opposite limits, in one of which the plasma is considered to be collisionless and the nonlinear cause of the filamentation is the ponderomotive action of the radiation on the plasma. Such a mechanism for the filamentation effect turns out, for instance, to be the decisive one for laser radiation with a sufficiently long wavelength and for a plasma with a sufficiently high-temperature. Under the oppoiste conditions the cause of the filamentation is assumed to be for a collisional plasma the non-uniform heating of the plasma by the electromagnetic field, and the instability threshold is determined by the balance between heating and heat transfer.[1,2]

Under practical circumstances when laser radiation acts on a plasma the case of a plasma with a high degree of ionization of the ions occupies an important place. In our opinion the collisional theory of the action of electromagnetic radiation on such a plasma is to a considerable extent incomplete, and therefore the theory of the filamentation of radiation in a completely ionized collisional plasma is still not completed. We develop in what follows, in a manner suitable for the conditions of a linear filamentation theory, qualitatively new conditions for the display of the kinetics of charged particles in the description of the nonlinear action of radiation on a plasma with ions which are highly ionized. We then establish a new range of values of the characteristic scale of the field inhomogeneities the presence of which is caused by the high degree of ionization $Z$, and for which up to the present there was no theory.

In the theory of the action of strong high-frequency radiation on a plasma there are many papers which are devoted to the limit of not very strong fields for which the amplitude of the oscillatory velocity of an electron, $v_E = eE/m_e\omega_0$, is small compared to the thermal velocity of an electron, $(\varkappa T_e/m_e)^{1/2}$. In what follows we shall be interested in just such a case of relatively weak fields. The existing theory distinguishes between two opposite limiting cases: collisionless and collisional situations. In the collisional limit (see, e.g., Refs. 1 and 3) the nonlinear perturbation of the electron density by the spatially nonuniform electromagnetic field is, as to order of magnitude, determined by the formula

$$\delta n_{e,\,col} \sim n_e \frac{v_E^2}{v_{Te}^2}\frac{ƛ^2}{l_{ei}^2}, \tag{1.1}$$

where $n_e$ is the unperturbed electron density, $ƛ$ is a characteristic scale of the spatial inhomogeneity of the amplitude of the high-frequency electrical field strength, and $l_{ei}$ is the electron mean free path with respect to their collisions with the ions. Equation (1.1) is, according to Ref. 4, realized if the values of the inhomogeneity scale of the field are sufficiently large, when the inequality

$$ƛ \gg 10(l_{ee}l_{ei})^{1/2} = 10Z^{1/2}l_{ei}, \tag{1.2}$$

is realized, where $l_{ee} = Zl_{ei}$ is the electron mean free path with respect to their collisions with other electrons. In the opposite, collisionless limit we have for the perturbation of the electron density the following expression (see, e.g., Ref. 1)

$$\delta n_{e,\,str} \sim n_e \frac{v_E^2}{v_{Te}^2}. \tag{1.3}$$

One usually makes about this formula the statement that it is applicable when the condition $l_{ei} \gg ƛ$ is satisfied. This statement is exact for $Z \sim 1$ when $l_{ei} \sim l_{ee}$. The position is changed when we are dealing with a plasma with ions with a high degree of ionization. Since in that case $l_{ee} \gg l_{ei}$, the condition (1.2) alone suggests the existence of an intermediate range of values of the characteristic scale of the nonuniformity of the high-frequency electrical field where Eqs. (1.1) and (1.3) are inapplicable. The establishment of such an intermediate range requires a more precise definition of the applicability condition for Eq. (1.3). We show below that the collisionless limit (1.3) is realized when the inhomogeneity scale of the field is small and satisfies the inequality

$$ƛ \ll l_{ei}(l_{ei}/l_{ee})^{5/4} = l_{ei}Z^{-5/4}. \tag{1.4}$$

We show in the present paper that in the intermediate range of values of the inhomogeneity scale when we have

$$l_{ei}(l_{ei}/l_{ee})^{5/4} = l_{ei}^{9/4}\,l_{ee}^{-5/4} \ll ƛ \ll l_{ee}^{1/2}\,l_{ei}^{1/2} = l_{ei}(l_{ee}/l_{ei})^{1/2}, \tag{1.5}$$

the nonlinear electron-density perturbation caused by the inhomogeneous high-frequency electromagnetic field is given by a formula of the form:

$$\delta n_e \sim n_e \frac{v_E^2}{v_{Te}^2}\frac{l_{ee}^{5/7}}{l_{ei}^{9/7}}ƛ^{4/7} = n_e \frac{v_E^2}{v_{Te}^2} Z^{5/7}\left(\frac{ƛ}{l_{ei}}\right)^{4/7} \tag{1.6}$$

We expound in what follows a perturbation theory for the

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electron distribution by an electromagnetic field with relative short wavelengths when the inequality governing the strongly collisional limit is violated (see, e.g., Ref. 4). The expression found for the perturbation of the electron density is used, firstly, to describe the linear convective regime of the filamentary instability of the electromagnetic waves in a completely ionized plasma with ions with a high degree of ionization and, secondly, to describe the threshold of an absolute parametric instability such as the double filamentary one (see Refs. 5 and 6).

We note that for filamentary perturbations with a wave vector **k** we can, when we bear in mind the results of what is said below as well as an interpolation of Ref. 7, use the following general interpolatory expression for the perturbation of the electron density:

$$\delta n_e = -n_e \frac{v_E^2}{4v_{Te}^2}\left\{1 + C_0 \frac{Z^{5/7}}{(kl_{ei})^{4/7}} + \frac{1}{\alpha (kl_{ei})^2}\right\}, \qquad (1.7)$$

where we find for $Z \gg 1$ that we have $\alpha \approx 6.8$ and $C_0 \approx 1.73$.

One may assume that the experimental results of a study[8] of filamentation refer in accordance with the analysis of Ref. 9 to the intermediate range of wave vectors to which our theory below also applies.

Finally, we discuss the relation of our study to Ref. 9 which is devoted to the kinetic theory of laser filamentation in a plasma. That paper was based upon an interpolatory numerical approximation of the effective thermal conductivity coefficient $\varkappa_{\rm eff}$:

$$\varkappa_{eff} = \frac{\varkappa_{SH}}{1 + (30k\lambda_e)^{4/3}}, \qquad (1.8)$$

where $\varkappa_{\rm SH}$ is the Spitzer–Härm thermal conductivity coefficient while the quantity $\lambda_e$ is connected as follows with the electron-ion mean free path $l_{ei}$: $\lambda_e = 2^{1/2} l_{ei} Z / 3\pi^{1/2}(Z+1)^{1/2}$. One should note that this formula differs from the one proposed in Ref. 10:

$$\varkappa_{eff} = \frac{\varkappa_{SH}}{1 + (30k\lambda_e)^2}. \qquad (1.9)$$

An analytical solution of the kinetic equation in our paper gives for $Z \gg 1$ in the notation of Ref. 9:

$$\varkappa_{eff} = \frac{\varkappa_{SH}}{1 + (21{,}1k\lambda_e)^{10/7}}. \qquad (1.10)$$

A comparison of this function with the curve given in Fig. 1 of Ref. 9 shows that it is practically the same as the functions (1.8) and (1.10) in the restricted range of $k\lambda_e$ values used there. Therefore, in addition to the theory of filamentation constructed by us in the present paper, we present the qualitatively new expression (1.10) for the effective electron thermal conductivity in the range of sufficiently large wave vectors. Finally, we comment regarding condition (1.5) for the filamentation instability, when at the instability threshold according to Eqs. (3.10) and (4.19) we have

$$\lambda\!\!\!^{-}_{th} \sim (cL/2\pi\omega_0)^{1/2},$$

where $\omega_0$ is the frequency of the pump field, $c$ is the velocity of light, and $L$ is the characteristic size of the uniform part of the plasma slab. Accordingly, the inequality (1.5) can be written in the form of the following condition on the length $L$:

$$L_0 Z^{-9/7} \ll L \ll L_0 Z^{-1}, \qquad (1.11)$$

where

$$L_0 = 2\pi(\omega_0/c) l_{ee}^2.$$

When the plasma is irradiated with the second harmonic of a neodymium laser

$$2\pi c/\omega_0 = 0{,}53\ \mu\text{m}$$

and for a plasma with values of the electron density $n_e \sim 10^{21}$ cm$^{-3}$ and electron temperature $T_e \sim 700$ eV, typically found in experiments, for $l_{ee} \sim 10^{-3}$ cm, we have $L_0 \sim 1$ cm. For, e.g., $Z = 5$ condition (1.11) therefore takes the form

$$10^{-3}\,\text{cm} \ll L \ll 0{,}2\ \text{cm},$$

which implies that it is satisfied in the corona of the target plasma in experiments on inertial confinement fusion and also in those involving the action of laser radiation on a previously produced plasma.

2. For a plasma in a high-frequency electromagnetic field $\boldsymbol{\varepsilon} = \frac{1}{2}\mathbf{E}\exp(-i\omega_0 t) + \text{c.c.}$ we obtain an expression for the correction to the electron distribution function which varies slowly with time and which is caused by the h.f. field. We write the slowly changing electron distribution function in the form $f = f_M + \delta f$ where $f_M$ is the Maxwellian distribution function while $\delta f$ is a small correction satisfying according to Eq. (2.3) of Ref. 4 the following equation:

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\frac{\partial \delta f}{\partial \mathbf{r}} + \frac{e}{m_e}\delta\mathbf{E}_0\frac{\partial f_M}{\partial \mathbf{v}} - J_{ei}[\delta f] - J_{ee}[\delta f]$$
$$= \frac{e^2}{4m_e^2\omega_0^2}\left\{\frac{\partial |\mathbf{E}|^2}{\partial \mathbf{r}}\frac{\partial f_M}{\partial \mathbf{v}} + \frac{1}{2}\frac{\partial^2 f_M}{\partial v_i \partial v_j}\mathbf{v}\frac{\partial}{\partial \mathbf{r}}(E_iE_j^* + E_i^*E_j)\right.$$
$$\left. - (E_iE_j^* + E_i^*E_j)\frac{\partial}{\partial v_i}J_{ei}\left[\frac{\partial f_M}{\partial v_j}\right]\right\}, \qquad (2.1)$$

where $\delta\mathbf{E}_0$ is the quasistatic electric field, we have

$$J_{ei}[\delta f] = 3\left(\frac{\pi}{8}\right)^{1/2}\nu_{ei}\frac{v_{Te}^3}{v^3}\frac{\partial}{\partial v_k}\left[(v^2\delta_{kj} - v_kv_j)\frac{\partial \delta f}{\partial v_j}\right], \qquad (2.2)$$

$$J_{ee}[\delta f] = \frac{3(2\pi)^{1/2}}{4Z}\frac{\nu_{ei}v_{Te}^3}{n_e}\frac{\partial}{\partial v_k}$$
$$\times\left\{\int d\mathbf{v}'\frac{(\mathbf{v}-\mathbf{v}')^2\delta_{kj} - (\mathbf{v}-\mathbf{v}')_k(\mathbf{v}-\mathbf{v}')_j}{|\mathbf{v}-\mathbf{v}'|^3}\right.$$
$$\left.\times\left(\frac{\partial}{\partial v_j} - \frac{\partial}{\partial v_j'}\right)[f_M(v)\delta f(\mathbf{v}') + f_M(v')\delta f(\mathbf{v})]\right\}, \qquad (2.3)$$

$$\nu_{ei} = \frac{v_{Te}}{l_{ei}} = \frac{4(2\pi)^{1/2}e^2e_i^2n_i\Lambda}{3m_e^2v_{Te}^3}, \qquad (2.4)$$

$n_i$ is the ion density, and $\Lambda$ is the Coulomb logarithm. Equation (2.1) is especially suitable when the characteristic spatial scale of the changes in $f_M$ is large compared to the corresponding scale of changes in $\delta f$, $\delta\mathbf{E}_0$, and the quadratic combinations $E_iE_j^* + E_i^*E_j$ of the amplitudes of the h.f. field.

In what follows we restrict ourselves to a case such that we can neglect $\partial\delta f/\partial t$ and asume that $\delta\mathbf{E}_0$ is a potential field, $\delta\mathbf{E}_0 = -\nabla\delta\varphi$. It is then convenient to write $\delta f$ in the form (we write $I = e^2|\mathbf{E}|^2/4m_e^2\omega_0^2v_{Te}^2$)

$$\delta f = f_M\left\{\left(\frac{v^2}{3v_{Te}^2}-2\right)I-\frac{e\delta\varphi}{\varkappa T_e}\right\}+\delta\tilde{f},$$

$$I=\frac{e^2|\mathbf{E}|^2}{4m_e^2\omega_0^2 v_{Te}^2}. \tag{2.5}$$

We take the spatial dependence of $\delta\tilde{f}$ and of the combinations $E_iE_j^* + E_i^*E_j$ in the form $\exp(i\mathbf{kr})$. The expression for $\delta\tilde{f}$ then takes the form

$$i(\mathbf{kv})\delta\tilde{f}-J_{ei}[\delta\tilde{f}]-J_{ee}[\delta\tilde{f}]$$
$$=\frac{e^2}{4m_e^2\omega_0^2v_{Te}^2}\left\{-(2\pi)^{1/2}v_{Te}^3\nu_{ei}|\mathbf{E}|^2\frac{\partial}{\partial v_k}\left(\frac{v_k}{v^3}f_M\right)\right.$$
$$\left.+M_E\left[3\left(\frac{\pi}{2}\right)^{1/2}\nu_{ei}\frac{v_{Te}^2}{v^5}\left(3+\frac{v^2}{v_{Te}^2}\right)+i(\mathbf{kv})\frac{1}{2v_{Te}^2}\right]f_M\right\},$$

where

$$M_E=(E_iE_j^*+E_i^*E_j-{}^2/_3\delta_{ij}|\mathbf{E}|^2)(v_iv_j-{}^1/_3\delta_{ij}v^2). \tag{2.6}$$

In the limit of a high degree of ionization, $Z\gg1$, in which we are interested we can neglect the electron-electron collision integral of the anisotropic part $\delta\tilde{f}$. This makes it possible to write

$$\delta\tilde{f}=\frac{e^2}{8m_e^2\omega_0^2v_{Te}^4}M_Ef_M+\delta f_c \tag{2.7}$$

and to use for the function $\delta f_c$ a somewhat simpler expression:

$$i(\mathbf{kv})\delta f_c-J_{ei}[\delta f_c]-J_{ee}[\delta f_c]=\frac{e^2\nu_{ei}}{4m_e^2\omega_0^2v_{Te}^2}\left\{-(2\pi)^{1/2}v_{Te}^3\cdot\right.$$
$$\left.\times|\mathbf{E}|^2\frac{\partial}{\partial v_k}\left(\frac{v_k}{v^3}f_M\right)+3\left(\frac{\pi}{2}\right)^{1/2}\frac{v_{Te}^3}{v^5}\left(3-\frac{v^2}{2v_{Te}^2}\right)M_Ef_M\right\}. \tag{2.8}$$

We solve this equation by writing the function $\delta f_c$ as a sum of an isotropic, $\delta f_0=\langle\delta f_c\rangle$, and an anisotropic, $\delta f_a=\delta f_c-\delta f_0$, part where $\langle\ \rangle$ indicates averaging over the angles of the vector $\mathbf{v}$. We can then write according to (2.8):

$$i\langle\mathbf{kv}\delta f_a\rangle=J_{ee}[\delta f_0]-\frac{e^2\nu_{ei}}{4m_e^2\omega_0^2v_{Te}^2}$$
$$\times\left\{(2\pi)^{1/2}v_{Te}^3|\mathbf{E}|^2\frac{\partial}{\partial v_k}\left(\frac{v_k}{v^3}f_M\right)\right\}, \tag{2.9}$$

$$i\mathbf{kv}\delta f_0+i(\mathbf{kv}\delta f_a-\langle\mathbf{kv}\delta f_a\rangle)=J_{ei}[\delta f_a]+\frac{e^2\nu_{ei}}{4m_e^2\omega_0^2v_{Te}^2}$$
$$\times\left\{3\left(\frac{\pi}{2}\right)^{1/2}\frac{v_{Te}^3}{v^5}\left(3-\frac{v^2}{2v_{Te}^2}\right)M_Ef_M\right\}. \tag{2.10}$$

We continue the solution by assuming that the terms on the left-hand side of (2.10) containing $\delta f_a$ are small. This is possible if the conditions

$$|\mathbf{kv}|\,|\delta f_a|\ll\max\left\{kv|\delta f_0|;\ \nu_{ei}If_M\frac{v_{Te}^3}{v^5}\left|3-\frac{v^2}{2v_{Te}^2}\right|\right\} \tag{2.11}$$

are satisfied. If these conditions are satisfied we can write the solution of Eq. (2.10) in the form:

$$\delta f_a=\frac{1}{2\nu(v)}\left\{-i(\mathbf{kv})\delta f_0+\frac{e^2\nu_{ei}}{4m_e^2\omega_0^2v_{Te}^2}\left(\frac{\pi}{2}\right)^{1/2}\right.$$
$$\left.\times\frac{v_{Te}^3}{v^5}\left(3-\frac{v^2}{2v_{Te}^2}\right)M_Ef_M\right\}, \tag{2.12}$$

where $\nu(v)=3\sqrt{\pi/8}\,\nu_{ei}v_{Te}^3/v^3$. Substituting (2.12) into (2.11) shows that conditions (2.11) are satisfied if we assume that

$$\frac{kv}{\nu(v)}\ll1, \tag{2.13}$$

which we must consider for different values of the electron velocities.

For velocities of the order of the thermal velocity, $v\sim v_{Te}$, condition (2.13) has the simple form:

$$kl_{ei}\ll1. \tag{2.14}$$

We can rewrite Eq. (2.13) in the form:

$$\lambda\!\!\!^{-}=\frac{1}{k}\gg\frac{v}{\nu(v)}\sim\frac{v^4}{v_{Te}^4}l_{ei}. \tag{2.15}$$

Comparing this inequality with inequalilty (1.2) we can show that Eq. (1.1) is defined for relatively large values of the electron velocities up to

$$v\sim v_{Te}Z^{1/8}. \tag{2.16}$$

At the same time it is understandable that under conditions when the density perturbation is formed by the electron distribution in the low velocity region, Eq. (2.13) [or (2.15)] can correspond to relatively small values of $\lambda\!\!\!^{-}$.

Assuming (2.13) to be satisfied we get after substituting (2.12) into (2.19) the following equation for the function $\delta f_0$:

$$\frac{k^2v^2}{6\nu_{ei}\nu(v)}\delta f_0=\frac{1}{\nu_{ei}}J_{ee}[\delta f_0]-(2\pi)^{1/2}Iv_{Te}^3\frac{\partial}{\partial v_k}\left(\frac{v_k}{v^3}f_M\right). \tag{2.17}$$

If we write $\delta f_0$ in the form:

$$\delta f_0(v)=\frac{9\pi}{8}\frac{I}{k^2l_{ei}^2}f_M(v)\Phi\left(\frac{v^2}{2v_{Te}^2}\right), \tag{2.18}$$

we can formulate for the function $\Phi(x)$ the following boundary value problem, where the function $\Phi(x)$ is defined by the following equation:

$$\frac{1}{N}\left\{\Phi''(X)\cdot\frac{3}{2}\int_0^X dX'(X')^{1/2}\exp(-X')\right.$$
$$+X^{1/2}\left(\frac{3}{2}-X\right)\int_X^\infty dX'\exp(-X')$$
$$\times[\Phi'(X)-\Phi'(X')]-\int_0^X dX'(X')^{3/2}$$
$$\left.\times\exp(-X')[\Phi'(X)-\Phi'(X')]\right\}$$
$$-X^3\Phi(X)=-1 \tag{2.19}$$

and the boundary condition:

$$\lim_{X\to0}[X^{3/2}\Phi'(X)]=N, \tag{2.20}$$

where

$$N=({}^4/_9\pi^{1/2})Zk^2l_{ei}^2. \tag{2.21}$$

We shall now assume that in what follows $N \gg 1$. If the condition

$$N^{2/7}=0{,}67Z^{2/7}(kl_{ei})^{4/7}\sim[(l_{ee}l_{ei})^{1/2}\lambda\!\!\!^{-}{}^{-1}]^{4/7}\gg 1 \qquad (2.22)$$

is satisfied, the solution of the boundary value problem (2.19) and (2.20) is, according to Appendix 1, given by Eq. (A1.15):

$$\Phi(X)=-\frac{4}{\pi}\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)\sin\frac{\pi}{7}N^{8/7}\xi^{-1/4}K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)$$
$$+\frac{4}{7}N^{8/7}\xi^{-1/4}\left\{I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_\xi^\infty dyy^{-1/4}K_{1/7}\left(\frac{4}{7}y^{7/4}\right)\right.$$
$$\left.+K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_0^\xi dyy^{-1/4}I_{1/7}\left(\frac{4}{7}y^{7/4}\right)\right\}, \qquad (2.23)$$

where $\xi = XN^{2/7}$. Inequality (2.22) makes it possible to speak about a range of values of the spatial nonuniformity scales which is an additional range characterized by inequality (1.2). At the same time inequality (2.22) together with (2.15) imposes the following condition on the range of velocities,

$$\frac{v^4}{v_{Te}^4}\ll\sqrt{Z} \qquad (2.24)$$

for which, according to (2.15), the solution (2.23) is suitable. We shall see in what follows that this condition is not of great importance.

According to Eq. (2.5) we have:

$$\delta n_e=-n_eI-n_e\frac{e\delta\varphi}{\varkappa T_e}+\delta n_{ec}, \qquad (2.25)$$

where

$$\delta n_{ec}=\int d\mathbf{v}\delta f_0=\frac{9\pi^{1/2}n_eI}{4k^2l_{ei}^2}N^{-8/7}\int_0^\infty duu^{1/2}\Phi(uN^{-2/7})\exp(-uN^{-2/7}). \qquad (2.26)$$

The main contribution to the density perturbation (2.26) is in the limit (2.22) given by the first term on the right-hand side of (2.23). We then have:

$$\delta n_{ec}=-\frac{9}{\pi^{1/2}}\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)\sin\frac{\pi}{7}$$
$$\times N^{8/7}\frac{n_eI}{k^2l_{ei}^2}\int_0^\infty duu^{1/4}K_{1/7}\left(\frac{4}{7}u^{7/4}\right)$$
$$=-C_0n_eI\frac{Z^{5/7}}{(kl_{ei})^{4/7}}, \qquad (2.27)$$

where

$$C_0=(5184\pi/343)^{1/7}\Gamma(^2/_7)\Gamma(^3/_7)/\Gamma(^1/_7)\approx 1{,}73.$$

Since the main contribution to the integral in Eq. (2.27) arises from the $u \sim 1$ region, this means that the important range of velocities determining $\delta n_{ec}$ correspond to low values of the velocity:

$$v^4/v_{Te}^4\sim N^{-4/7}\sim(Zk^2l_{ei}^2)^{-4/7}. \qquad (2.28)$$

According to this the condition (2.13) for the discussion given here has the form:

$$Z^{-4/7}\ll(kl_{ei})^{1/7}. \qquad (2.29)$$

This condition is automatically satisfied for $Z \gg 1$ and $N \gg 1$ [see (2.22)] which corresponds to the condition which is the opposite of inequality (1.2):

$$\lambda\!\!\!^{-}\ll(l_{ee}l_{ei})^{1/2}=Z^{1/2}l_{ei}. \qquad (2.30)$$

We show in Appendix 2 that under the condition $ZT_e \gg T_i$ the effect of the potential $\delta\varphi$ of the quasistatic field, caused by the high-frequency electromagnetic field, on the perturbation of the electron density is negligibly small. We see therefore, comparing $n_eI$ and $\delta n_{ec}$ in Eq. (2.25), that the usual result (1.3) of the collisionless discussion is realized under condition (1.4). In contrast we have for the density perturbation, under condition (1.5) ($\lambda\!\!\!^{-} = k^{-1}$) and neglecting $\delta\varphi$:

$$\delta n_e=-C_0n_e\frac{Z^{5/7}}{(kl_{ei})^{4/7}}\frac{e^2|\mathbf{E}|^2}{4m_e^2\omega_0^2v_{Te}^2}. \qquad (2.31)$$

This formula is the final result of this section.

3. We use the expression obtained in the previous section for the perturbation of the electron density by a high-frequency electromagnetic field and apply it to the description of the filamentation effect of a beam of electromagnetic radiation. The structure of the radiation field describing the filamentary instability has the form:

$$\mathbf{E}=[E_{01}+E_{11}(z)\exp(iky)+E_{-11}(z)\exp(-iky)]\exp(ik_0z)\mathbf{e}_x, \qquad (3.1)$$

where $E_{01}$ is the electric field strength of the pump field and the $E_{\pm 11}$ are the amplitudes of the filamentary perturbations. Up to terms which are linear in such amplitudes we have

$$|E|^2=|E_{01}|^2+(E_{11}E_{01}^*+E_{01}E_{-11}^*)\exp(iky)$$
$$+(E_{01}^*E_{-11}+E_{01}E_{11}^*)\exp(-iky). \qquad (3.2)$$

The linearized reduced Maxwell equations can be written in the form (see, e.g., Ref. 3):

$$\left(2ik_0\frac{d}{dz}-k^2\right)\frac{E_{11}(z)}{E_{01}}=\frac{\omega_0^2}{c^2}\frac{\delta n_e}{n_c}$$
$$=\left(-2ik_0\frac{d}{dz}-k^2\right)\frac{E_{-11}^*(z)}{E_{01}^*}. \qquad (3.3)$$

The pump field is here assumed to be independent of the coordinates and the characteristic spatial scale for changes in the $E_{\pm 11}$ amplitudes as function of the $z$ coordinate is taken to be large compared to $k^{-1}$. Here we have $n_c = m_e\omega_0^2/4\pi e^2$; $k_0 = (\omega_0/c)\sqrt{1-n_e/n_c}$.

In accordance with the previous section and Eq. (3.2) we have in Eqs. (3.3):

$$\delta n_e=-\frac{C_0Z^{5/7}}{(kl_{ei})^{4/7}}\frac{n_e}{16\pi n_c\varkappa T_e}(E_{11}E_{01}^*+E_{01}E_{-11}^*). \qquad (3.4)$$

Equations (3.3) and (3.4) make it possible to assume that $E_{11}$ and $E^*_{-11}$ are proportional to $\exp(Gz)$ and to write for the coefficient $G$ of the spatial amplification of the filament the following expression

$$G(k)=\left\{\frac{k^2}{k_0^2}\left[q^2(k)-\frac{k^2}{4}\right]\right\}^{1/2}, \qquad (3.5)$$

where

$$q^2(k)=\frac{C_0 Z^{5/7} n_e \omega_0^2 |E_{01}|^2}{(kl_{ei})^{4/7}\cdot 32\pi n_c^2 \varkappa T_e c^2}. \qquad (3.6)$$

The maximum value of the spatial amplification coefficient,

$$G_m\equiv G(k_m)=k_0 C_g \frac{Z^{5/9}}{(k_0 l_{ei})^{4/9}}\left[\frac{C_0 n_e \omega_0^2 |E_{01}|^2}{32\pi n_c^2 \varkappa T_e k_0^2 c^2}\right]^{7/9}. \qquad (3.7)$$

(where $C_g = 2^{-2/9}\cdot 3\cdot 7^{-7/9}\cdot 5^{5/18}\approx 0.89$), is realized for the following value of the wavevector of the filament

$$k_m=k_0\frac{Z^{5/18}}{(k_0 l_{ei})^{2/9}}\left[\frac{5C_0 n_e \omega_0^2 |E_{01}|^2}{112\pi n_c^2 \varkappa T_e k_0^2 c^2}\right]^{7/18}. \qquad (3.8)$$

A comparison of Eqs. (3.7) and (3.8) enables us to see that the inequality $k_m \gg G_m$ is satisfied. Of course, the wavelength of the filamentary perturbations, $\lambda = k^{-1}$, must satisfy conditions (1.5) in order that Eqs. (3.5), (3.7), and (3.8) be applicable.

If we use for an estimate of the filamentation threshold $G_m L = 2\pi$, where $L$ is the thickness of the plasma layer, we have according to (3.7)

$$|E_{01}|_{th}^2=C_g^{-9/7}\frac{32\pi n_c^2 \varkappa T_e k_0^2 c^2}{C_0 n_e \omega_0^2}\frac{(k_0 l_{ei})^{4/7}}{Z^{5/7}}\left(\frac{2\pi}{k_0 L}\right)^{9/7}. \qquad (3.9)$$

An estimate for the wave vector at threshold then gives

$$k_{m,\,th}=C_k k_0\left(\frac{2\pi}{k_0 L}\right)^{1/2}, \qquad (3.10)$$

where $C_k = (20/9)^{1/4}\approx 1.22$.

4. Equation (2.31) suffices also for a description of the threshold of the double filamentary instability effect which occurs when there is in a plasma a surface reflecting the electromagnetic radiation, in which case the interference between the incident and the reflected pump waves, acting on the plasma, leads to an absolute parametric instability. The theory of the double filamentary instability under the action of short-wavelength radiation is given in Ref. 5 where only the usual mechanism of the ponderomotive action of radiation on a plasma is taken into account. The results of a theory of the double filamentation effect under the conditions (1.2) is given in Ref. 6. Here we consider the threshold for the double filamentation instability under the conditions (1.5).

The structure of the electromagnetic field corresponding to the double filamentary instability has the form

$$\mathbf{E}=\sum_{\mu,\sigma=\pm 1}(E_{0\sigma}+E_{\mu\sigma}(z)\exp(i\mu k y))\exp(i\sigma k_0 z)\mathbf{e}_x, \qquad (4.1)$$

where the $E_{0\pm 1}$ are the amplitudes of the incident and the reflected pump waves, the $E_{\pm 11}$ are the amplitudes of the filamentary components excited in the usual way by the incident pump wave (see section 3 above), and the $E_{\pm 1-1}$ are the amplitudes of the filamentary components which would be excited by the reflected pump wave if we neglected interference. Assuming the plasma to be uniform and occupying a layer of thickness $L$ we use for the field amplitudes the usual boundary conditions (compare Ref. 6):

$$E_{01}(z=0)=E_0,\quad E_{\mu 1}(z=0)=0,$$
$$E_{0-1}(z=L)=rE_{01}(z=L),\quad E_{\mu-1}(z=L)=rE_{\mu 1}(z=L) \qquad (4.2)$$

($r$ is the coefficient for reflection from the rear boundary of the layer) and the following set of reduced equations:

$$\frac{1}{E_{0\sigma}}\left(2i\sigma k_0\frac{d}{dz}-k^2\right)E_{1\sigma}=\frac{\omega_0^2}{c^2}\frac{\delta n_e}{n_c}$$
$$=\frac{1}{E_{0\sigma}^*}\left(-2i\sigma k_0\frac{d}{dz}-k^2\right)E_{-1\sigma}^*. \qquad (4.3)$$

Since we have in the linear approximation in the amplitudes

$$|\mathbf{E}|^2=\sum_{\sigma=\pm 1}\{E_{0\sigma}E_{0\sigma}^*+[(E_{0\sigma}E_{-1\sigma}^*+E_{1\sigma}E_{0\sigma}^*)$$
$$\times\exp(iky)+\text{c.c.}]+\ldots\}, \qquad (4.4)$$

we can now in accordance with Eq. (2.31) write for the perturbation of the electron density which occurs in (4.3)

$$\delta n_e=-\frac{C_0 Z^{5/7} n_e}{(kl_{ei})^{4/7}16\pi n_c \varkappa T_e}\left\{\sum_{\sigma=\pm 1}(E_{0\sigma}E_{-1\sigma}^*+E_{1\sigma}E_{0\sigma}^*)\right\}. \qquad (4.5)$$

As a result the set of reduced field equations takes the form:

$$\left(2ik_0\frac{d}{dz}-k^2\right)\frac{E_{11}(z)}{E_{01}}=\left(-2ik_0\frac{d}{dz}-k^2\right)\frac{E_{-11}^*(z)}{E_{01}^*}$$
$$=\left(-2ik_0\frac{d}{dz}-k^2\right)\frac{E_{1-1}(z)}{rE_{01}}=\left(2ik_0\frac{d}{dz}-k^2\right)\frac{E_{-1-1}^*(z)}{r^*E_{01}^*}$$
$$=-2q^2\left(\frac{E_{-11}^*(z)}{E_{01}^*}+\frac{E_{11}(z)}{E_{01}}+r\frac{E_{-1-1}^*(z)}{E_{01}^*}+r^*\frac{E_{1-1}(z)}{E_{01}}\right). \qquad (4.6)$$

It follows, in particular, from the set of Eqs. (4.6) that:

$$\frac{E_{11}(z)}{E_{01}}+\frac{E_{-11}^*(z)}{E_{01}^*}+r^*\frac{E_{1-1}(z)}{E_{01}}+r\frac{E_{-1-1}^*(z)}{E_{01}^*}$$
$$=A_+\exp(\tilde{G}z)+A_-\exp(-\tilde{G}z), \qquad (4.7)$$

where $\tilde{G}(k)=\sqrt{(k^2/k_0^2)(q^2(k)(1+|r|^2)-k^2/4}$. We then have according to the set (4.6)

$$\frac{E_{11}(z)}{E_{01}}=i\frac{q^2}{k_0}\exp\left(-\frac{ik^2z}{2k_0}\right)$$
$$\times\left\{\frac{A_+}{G_+}[\exp(G_+z)-1]-\frac{A_-}{G_-}[\exp(-G_-z)-1]\right\}, \qquad (4.8)$$

$$\frac{E^{*}_{-11}(z)}{E^{*}_{01}}=-i\frac{q^2}{k_0}\exp\left(\frac{ik^2z}{2k_0}\right)$$
$$\times\left\{\frac{A_+}{G_-}[\exp(G_-z)-1]-\frac{A_-}{G_+}[\exp(-G_+z)-1]\right\},\tag{4.9}$$

$$\frac{r^{*}E_{1-1}(z)}{E_{01}}=\frac{|r|^2E_{11}(L)}{E_{01}}\exp\left(\frac{ik^2(z-L)}{2k_0}\right)$$
$$-i\frac{q^{-}|r|^2}{2k_0}\exp\left(\frac{ik^2z}{k_0}\right)$$
$$\times\left\{\frac{A_+}{G_-}[\exp(G_-z)-\exp(G_-L)]\right.$$
$$\left.-\frac{A_-}{G_+}[\exp(-G_+z)-\exp(-G_+L)]\right\}.\tag{4.10}$$

$$\frac{rE^{*}_{-1-1}(z)}{E^{*}_{01}}=\frac{|r|^2E^{*}_{-11}(L)}{E^{*}_{01}}\exp\left(-\frac{ik^2(z-L)}{2k_0}\right)$$
$$+i\frac{q^2|r|^2}{k_0}\exp\left(-\frac{ik^2z}{2k_0}\right)$$
$$\times\left\{\frac{A_-}{G_-}[\exp(G_+z)-\exp(G_-L)]\right.$$
$$\left.-\frac{A_-}{G_-}[\exp(-G_-z)-\exp(-G_-L)]\right\},\tag{4.11}$$

where

$$G_\pm=\tilde{G}\pm ik^2/2k_0.$$

When we wrote down Eqs. (4.10) and (4.11) we used the last boundary condition from (4.2).

When we take the boundary conditions (4.2) into account, substitution of the values $z=0$ and $z=L$ into (4.7) gives the following two equations

$$\frac{rE^{*}_{-1-1}(0)}{E^{*}_{01}}+\frac{r^{*}E_{1-1}(0)}{E_{01}}=A_++A_-,\tag{4.12}$$

$$\left[\frac{E_{11}(L)}{E_{01}}+\frac{E^{*}_{-11}(L)}{E^{*}_{01}}\right](1+|r|^2)$$
$$=A_+\exp(\tilde{G}L)+A_-\exp(-\tilde{G}L).\tag{4.13}$$

Substituting Eqs. (4.8) and (4.9) into (4.13) for $z=L$ we find:

$$A_+[Q\sin\eta+\eta\cos\eta]+A_-[-Q\sin\eta+\eta\cos\eta]=0,\tag{4.14}$$

where $Q=\tilde{G}L$ and $\eta=k^2L/2k_0$. The same substitution into (4.10) and (4.11) and the subsequent substitution of Eqs. (4.10) and (4.11) into (4.12) for $z=0$ gives:

$$A_+\left\{Q[\exp(Q)-\cos\eta]+\eta\left[1-\frac{1+|r|^2}{2|r|^2}\operatorname{cosec}\eta\right]\right\}$$
$$+A_-\left\{-Q[\exp(-Q)-\cos\eta]+\eta\left[1-\frac{1+|r|^2}{2|r|^2}\operatorname{cosec}\eta\right]\right\}=0.\tag{4.15}$$

Equations (4.8) to (4.11) and (4.14) determine, apart from a constant, the coordinate dependence of the amplitudes of the field excited in the double filamentary instability. Finally, the condition that Eqs. (4.14) and (4.15) can be solved gives the following dispersion equation

$$(Q/\eta)\operatorname{sh}Q\sin\eta-\operatorname{ch}Q\cos\eta=(1-|r|^2)/2|r|^2,\tag{4.16}$$

written in the form which was used before in the theory of the double filamentation (compare Refs. 5 and 6). The fact that $Q$ depends here on the mean free path and on the wavevector is qualitatively new.

Equation (4.16) determines the limiting value of the pump field strength as function of the wavevector of the excited filament. Using (4.16) to find $Q=Q(\eta=k^2L/2k_0)$ we obtain

$$|E_{01}|^2_{\lim}(\eta)=\frac{32\pi n_c^2\varkappa T_ek_0^2c^2(k_0l_{ei})^{4/7}}{(1+|r|^2)n_e\omega_0^2C_0Z^{5/7}(k_0L)^{9/7}}\frac{Q^2(\eta)+\eta^2}{(2\eta)^{5/7}}.\tag{4.17}$$

As an illustration we consider the consequence of Eq. (4.16) for $|r|^2=1$ on the threshold for the absolute filamentary instability. It is clear in this case that the value $Q=0$, which is the smallest in absolute magnitude, is realized for $\cot\eta=0$, i.e., for $\eta=\pi n-\pi/2$, $n=1,2,\ldots$. Values $Q\gg1$ are realized in the approximation $Q=\eta\cot\eta$, i.e., for $\eta=\pi n$, when $Q'(\eta)\to-\infty$. Finally, for $\eta=0$ the function $Q(\eta)$ has a maximum $Q=Q_0$ where $Q_0$ is determined by the equation $Q_0\tanh Q_0=1$, i.e., $Q_0\approx1.2$. We show in Fig. 1 the function

$$F(\eta)=[Q^2(\eta)+\eta^2]\eta^{-5/7},\tag{4.18}$$

which according to (4.17) describes the boundary of the double filamentary instability. We note that there exists a solution $Q(\eta)$ of (4.16) and, hence, that the function $F(\eta)$ is defined, for all values of $\eta$. We show in Fig. 1 that part of the region where $F(\eta)$ is defined, namely the range of values of $\eta$ from 0 to $\eta_c=\pi/2$, in which the function $F(\eta)$ reaches

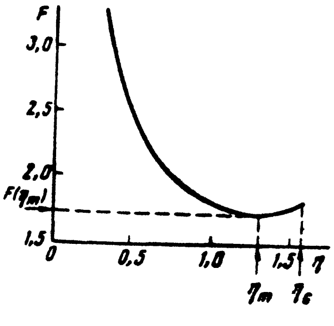


FIG. 1. The function $F(\eta)$ for the $|r|^2=1$ case.

its minimum value, equal to 1.74 (the minimum occurs for $\eta_m \approx 1.3$). Thus, at the threshold of the double filamentary instability filaments are created with wavevector:

$$k_d = k_0 (2\eta_m / k_0 L)^{1/2}. \qquad (4.19)$$

A comparison of this expression with (3.10) gives: $k_{m,\mathrm{th}} / k_d = \sqrt{\pi/\eta_m}\, C_k \approx 1.9$.

The ratio of the threshold of the double filamentary instability to the threshold of the ordinary filamentary instability is given by the formula:

$$\frac{|E_{01}|^2_{\mathrm{lim}}(\eta_m)}{|E_{01}|^2_{\mathrm{th}}} = \frac{C_g^{9/7}}{4\pi^{9/7}} \frac{F(\eta_m)}{(1+|r|^2)} \approx 0{,}049 \frac{F(\eta_m)}{1+|r|^2}. \qquad (4.20)$$

This ratio is for $|r|^2 = 1$ approximately equal to $\approx 0.04$. In other words, the threshold of the double instability lies about 25 times lower than the threshold for the ordinary filamentary instability. We show in Fig. 2 the function $F(\eta)$ of (4.18) obtained by solving Eq. (4.16) for $|r|^2 = 0.1$ for that part of the region where it is defined, namely the region of $\eta$ values from 0 to $\eta_c \approx 3.8$, where the function $F(\eta)$ reaches its minimum value, equal to 4.63 (the minimum occurs for $\eta_m \approx 1.6$). In that case we have therefore $k_{m,\mathrm{th}}/k_d \approx 1.7$, and the ratio (4.20) equals 0.2. The threshold of the absolute double instability is therefore also in this case 5 times lower than the threshold of the convective instability. This corresponds to the usual excess of the threshold of the convective instability over the threshold of the absolute double instability.

## APPENDIX 1

We give here the approximate solution of the boundary-value problem (2.19) and (2.20) in the asymptotic $N \gg 1$ limit. We note first of all that the boundary condition (2.20) reduces to the simpler one: as $X \to 0$ we have

$$\Phi'(X) \to N/X^{3/2} \quad \text{for } X \to 0. \qquad (A1.1)$$

Since for small values of $X$ the function $\Phi'(X)$ is proportional to the large parameter $N$ one may expect that the region of small $X$ values may be very important. It is in this connection advisable to understand in what region the asymptotic formula (A1.1) is applicable. To do this we use Eq. (2.19) to obtain corrections to Eq. (A1.1):

$$\Phi'(X) = \frac{N}{X^{3/2}}\left[1 - \frac{4}{7} N X^{7/2} - \frac{4}{3\cdot 7\cdot 7} N^2 X^7 + \ldots\right]. \qquad (A1.2)$$

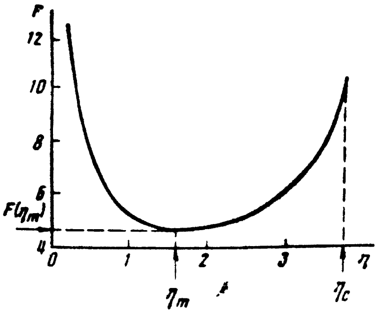


FIG. 2. The function $F(\eta)$ for the $|r|^2 = 0.1$ case.

This expansion is applicable for

$$N X^{7/2} \ll 1, \qquad (A1.3)$$

i.e., for very small values of the velocity as compared to the thermal one.

In the asymptotic limit $X \gg 1$ it follows from Eq. (2.19) that

$$\Phi(X) = \left[1 + \frac{C}{N}\right]\left(\frac{1}{X^3} + \ldots\right), \qquad (A1.4)$$

where

$$C = \int_0^\infty dX X^{3/2} \exp(-X) \Phi'(X). \qquad (A1.5)$$

Below we shall verify that

$$C \ll N. \qquad (A1.6)$$

A comparison of Eqs. (A1.2) and (A1.4) when there is a large parameter $N$ present enables us to rely on the applicability of the asymptotic formula (A1.4) also for small, but not too small $X$ values. Indeed, for $X \ll 1$ there follows from Eq. (2.19) the following asymptotic expansion in powers of $N^{-1}$:

$$\Phi(X) = \left[1 + \frac{C}{N}\right]\frac{1}{X^3}\left(1 + \frac{12}{N X^{7/2}} + \frac{3\cdot 13\cdot 15}{N^2 X^7} + \ldots\right). \qquad (A1.7)$$

This expansion is applicable for

$$N \gg N X^{7/2} \gg 10. \qquad (A1.8)$$

Since the asymptotic formula (A1.4) holds in a wide range of $X$ values (including those for which $X \ll 1$) it is clear here that for the determination of $\Phi(X)$ it is sufficient to consider the consequences of Eq. (2.19) only for $X \ll 1$ when it has the following form

$$\frac{1}{N}\left\{\Phi''(X) X^{5/2} + \frac{3}{2}\Phi'(X) X^{1/2} - \frac{3}{2} X^{1/2} \int_X^\infty dX' \exp(-X') \Phi'(X') + \int_0^X dX' (X')^{3/2} \Phi'(X')\right\} - X^3 \Phi(X) = -1. \qquad (A1.9)$$

We solve this equation, neglecting the integral terms, which corresponds for $X \ll 1$ to the assumptions:

$$\left| \int_X^\infty dX' \exp(-X') \Phi'(X') \right| \ll |\Phi'(X)|, \qquad (A1.10)$$

$$\left| \int_0^X dX' (X')^{3/2} \Phi'(X') \right| \ll \max\{|X^{5/2}\Phi'(X)|; N\}. \qquad (A1.11)$$

The solution then has the form

$$\Phi(X)=[XN^{2/7}]^{-1/4}\Big\{C_1 I_{1/7}(4/7[XN^{2/7}]^{7/4})+C_2 K_{1/7}(4/7[XN^{2/7}]^{7/4})$$
$$+4/7 N^{15/14}\Big\{K_{1/7}(4/7[XN^{2/7}]^{7/4})\int_0^X dy\,y^{-1/4} I_{1/7}(4/7[yN^{2/7}]^{7/4})$$
$$+I_{1/7}(4/7[XN^{2/7}]^{7/4})\int_X^\infty dy\,y^{-1/4}K_{1/7}(4/7[yN^{2/7}]^{7/4})\Big\}\Big\}, \tag{A1.12}$$

where $I_\nu$ and $K_\nu$ are Bessel functions of imaginary argument and $C_1$ and $C_2$ are constants which we shall now determine. From a comparison of this expression with the asymptotic expansion (A1.7) it follows that $C_1 = 0$. Moreover, for $X\to 0$ we have

$$\Phi(X)\to C_2\frac{1}{X^{1/2}}\frac{\pi}{2}\left(\frac{7}{2}\right)^{1/7}\Big/\Gamma\left(\frac{6}{7}\right)\sin\left(\frac{\pi}{7}\right)N^{1/7}. \tag{A1.13}$$

Comparing this expression with the asymptotic formula (A1.1) we find

$$C_2=-\frac{4}{\pi}\Gamma\left(\frac{6}{7}\right)\left(\frac{2}{7}\right)^{1/7}\sin\left(\frac{\pi}{7}\right)N^{6/7}. \tag{A1.14}$$

We have thus determined (A1.12).

To give estimates, which enable us to establish that the inequalities (A1.6), (A1.10), and (A1.11) which we assumed earlier are satisfied, it is convenient to write (A1.12) in the form

$$\Phi(X)=\tilde\Phi(u=XN^{2/7})=-\frac{4}{\pi}\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)$$
$$\times\sin\frac{\pi}{7}N^{8/7}u^{-1/4}K_{1/7}\left(\frac{4}{7}u^{7/4}\right)$$
$$+\frac{4}{7}N^{8/7}u^{-1/4}\left\{I_{1/7}\left(\frac{4}{7}u^{7/4}\right)\int_u^\infty dy\,y^{-1/4}K_{1/7}\left(\frac{4}{7}y^{7/4}\right)\right. . \tag{A1.15}$$

For $u\sim 1$ the first term on the right-hand side of (A1.15) is of the order of magnitude of $N^{8/7}$ whereas the second term which is proportional to $\sim N^{6/7}$ is small as compared to the first one. In the asymptotic $u\gg 1$ limit we have

$$\tilde\Phi(u)=-\frac{4}{\pi}\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)\sin\frac{\pi}{7}\left(\frac{7\pi}{8}\right)^{1/2}$$
$$\times N^{8/7}u^{-9/8}\exp\left(-\frac{4}{7}u^{7/4}\right)+\frac{N^{6/7}}{u^3}. \tag{A1.16}$$

We now discuss whether the assumptions under which we obtained the solution (A1.15) are satisfied. First of all we estimate the magnitude of (A1.5) which can be done in a natural way by using Eq. (A1.15). However, the same estimate also occurs when we use the asymptotic formula (A1.16). The largest contribution to the estimate of the magnitude of (A1.5) then comes from the first term of Eq. (A1.16) which gives $C\sim N^{5/7}$. Inequality (A1.6) therefore holds for

$$N^{2/7}\gg 1. \tag{A1.17}$$

We now turn to a discussion of inequality (A1.10). To do this we majorize the right-hand side,

$$\left|\int_X^\infty dX'\exp(-X')\Phi'(X')\right|<\left|\int_X^\infty dX'\Phi'(X')\right|=|\Phi(X)|.$$

Inequality (A1.10) will thus be satisfied if the inequality $|\Phi'(X)|\gg|\Phi(X)|$ is realized for $X\ll 1$. This last inequality is clearly satisfied in the region where the asymptotic expressions (A1.1) and (A1.7) hold. In the intermediate region between these two asymptotic expressions, when $u\sim 1$, we have $|\Phi(X)|\sim N^{8/7}$ according to (A1.15) and $\Phi'(X)=N^{2/7}\tilde\Phi'(u)\sim N^{2/7}N^{8/7}$. Inequality (A1.10) is thus satisfied, if inequality (A1.17) is satisfied. Finally, we consider inequality (A1.11). It is obvious that it is satisfied in the region where the asymptotic formula (A1.1) holds. In the $u\sim 1$ region inequality (A1.11) reduces to $N^{4/7}\gg 1$ which is a weaker condition than (A1.17). We see thus that the solution (A1.15) can be used when we calculate corrections to the electron distribution function $\Phi(X)$.

## APPENDIX 2

Bearing in mind the connection between the electron-ion, $l_{ei}$, and the ion-ion, $l_{ii}$, mean free paths, $l_{ei}=l_{ii}/2^{1/2})(ZT_e/T_i)^2$ we can write the left-hand side of inequality (1.5) in the form:

$$Z^{3/4}(T_e/T_i)^2 l_{ii}\ll\lambda. \tag{A2.1}$$

In the case, in which we are interested, of a high degree of ionization of the ions ($Z\gg 1$) and a ratio $T_e/T_i$ which is fairly large as compared to unity the left-hand side of inequality (A2.1) is large compared to the ion-ion mean free path. We are therefore interested in the strongly collisional ion limit. Neglecting the direct action of the electromagnetic field on the ions and also using in the ion-electron collision integral an expansion in powers of the ion to the electron velocity ratio we can write the ion kinetic equation in the following form:

$$i(\mathbf{k}\mathbf{v}_i)\delta f_i-i\mathbf{k}\frac{e_i}{m_i}\delta\varphi\frac{\partial f_{Mi}}{\partial\mathbf{v}_i}-J_{ii}[\delta f_i]$$
$$=\frac{4\pi e^2e_i^2\Lambda}{m_e m_i}\frac{\partial f_{Mi}}{\partial\mathbf{v}_i}\int d\mathbf{v}\frac{\mathbf{v}}{v^3}\delta f(\mathbf{v}), \tag{A2.2}$$

where $J_{ii}$ is the ion-ion collision integral, while we have taken the ion distribution function to be of the form $f_i=f_{Mi}+\delta f_i$ where $\delta f_i$ is a small perturbation as compared to the Maxwellian ion distribution $f_{Mi}$.

In accordance with Eqs. (2.5), (2.7), and (2.12) we have:

$$\int d\mathbf{v}\frac{\mathbf{v}}{v^3}\delta f(\mathbf{v})=\int d\mathbf{v}\frac{\mathbf{v}}{v^3}\delta f_a(\mathbf{v})=-i\mathbf{k}\int d\mathbf{v}\frac{\delta f_0(v)}{6v\nu(v)}$$
$$=-i\mathbf{k}\left(\frac{2}{\pi}\right)^{1/2}\frac{\beta}{3v_{Te}\nu_{ei}}, \tag{A2.3}$$

$$\beta=\left(\frac{\pi}{8}\right)^{1/2}v_{Te}\nu_{ei}\int d\mathbf{v}\frac{\delta f_0(v)}{v\nu(v)}=\int d\mathbf{v}\frac{v^2}{3v_{Te}^2}\delta f_0(v).$$

We can thus write the kinetic equation (A2.2) in the form:

$$i(\mathbf{kv}_i)\delta f_i - i\frac{\mathbf{k}}{m_i}\frac{\partial f_{Mi}}{\partial \mathbf{v}_i}\left(e_i\delta\varphi - \frac{\varkappa T_e}{n_i}\beta\right) = J_{ii}[\delta f_i]. \quad \text{(A2.4)}$$

The obvious solution of this equation has the form

$$\delta f_i(\mathbf{v}_i) = \left(\beta\frac{T_e}{T_i n_i} - \frac{e_i\delta\varphi}{\varkappa T_i}\right) f_{Mi}. \quad \text{(A2.5)}$$

Hence it follows, in particular, that the perturbation $\delta n_i$ of the ion density is

$$\delta n_i = \int d\mathbf{v}_i \delta f_i(\mathbf{v}_i) = \beta\frac{T_e}{T_i} - \frac{e_i n_i \delta\varphi}{\varkappa T_i}. \quad \text{(A2.6)}$$

Using this equation, Eq. (2.25), and the electroneutrality relation, which is realized under the conditions $kr_{De} \ll 1$ in which we are interested, we can determine the contribution of the electric field potential to (2.25) as follows:

$$-\frac{en_e\delta\varphi}{\varkappa T_e} = \left(1 + \frac{ZT_e}{T_i}\right)^{-1}\left(n_e I - \delta n_{ec} - \beta\frac{ZT_e}{T_i}\right). \quad \text{(A2.7)}$$

Since we have

$$\beta = \frac{n_e I}{4k^2 l_{ei}^2}\left(\frac{\pi}{2}\right)^{1/2}\int_0^\infty dX X^{1/4}\exp(-X)\Phi(X)$$

$$\sim \frac{n_e I Z^{3/7}}{(kl_{ei})^{8/7}} \sim n_e I\frac{N^{3/7}}{k^2 l_{ei}^2},$$

we can, using (2.27) and bearing in mind that $ZT_e \gg T_i$, state that by virtue of the latter inequality the first two terms on the right-hand side of (2.25) are large compared to the analogous contribution to (A2.1) while the contribution of the term in (A2.7) which contains $\beta$ is small compared to $\delta n_c$ in (2.25) in as far as the inequality $N^{2/7} \gg 1$ is satisfied.


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