Features of the Cherenkov emission of drift waves in hydrodynamics and in a plasma

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We calculate the radiative forces acting on (monopole and dipole) point sources when they move uniformly in a rotating fluid or an inhomogeneous plasma. These forces are caused by the emission of Rossby waves (in a fluid) or drift waves (in a plasma) through the Cherenkov mechanism. We show that owing to the anisotropic nature of both types of waves apart from the strong wave resistance force directed opposite to the velocity of the motion there exists also a force which is normal to the velocity of the source. On the basis of the results we give an interpretation of the properties of the well known Larichev–Reznik solution which describes stationary dipole vortices which can move without radiative losses in a well defined range of velocities. We discovered that the westward motion of such vortices in a nonuniformly rotating fluid is unstable under small changes in the angle at which the trajectory is inclined, whereas eastward motion is stable. A similar effect occurs for vortices in a plasma.

1. INTRODUCTION

It is well known that a fast moving source in any medium in which some kind of wave can exist can emit those waves through the Cherenkov mechanism, subject to satisfaction of the resonance condition[1]

$$W \cos \gamma = V_p, \tag{1}$$

where W is the absolute magnitude of the source velocity, V_p the phase velocity of the wave, and γ the angle between the directions of the motion of the source and that of the wave. It is clear from this formula that if the velocity of the source in an isotropic medium exceeds the wave speed, emission takes place in a cone with an apex angle which is the smaller the higher the velocity of the source.

The Cherenkov emission of waves in anisotropic media has a number of interesting features, part of which are considered in what follows by using as example Rossby waves in a nonuniformly rotating fluid (in the β-plane approximation[2]). It turns out, in particular, that when the source moves along a parallel, emission is possible only when its velocity is directed westwards. Thanks to the well known analogy between Rossby waves and drift waves in a plasma,[3] the results obtained here refer also to the plasma case. Moreover, it has been shown in Ref. 4 that in rotating gravitating astrophysical systems (galaxies) there exist wave processes the nature of which is the same as those occurring for Rossby waves or drift waves in a plasma. Hence, the conclusions of our paper also relate to those astrophysical objects.

2. EMISSION OF WAVES IN A BAROTROPIC MODEL

We consider a point source moving uniformly in a straight line at an angle α to the x-axis, which is directed eastwards along a parallel, in a nonuniformly rotating fluid in the β-plane (Fig. 1). The basic equation describing the wave field, which is expressed in terms of a current function $\psi(x,y,t)$, in the linear approximation then has the following form:[5]

$$\frac{\partial \Delta \psi}{\partial t} + \beta \frac{\partial \psi}{\partial x} = \frac{\partial f(x-Ut, y-Vt)}{\partial t}, \tag{2}$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the two-dimensional Laplace operator, $\beta > 0$ is a constant coefficient, U and V are the components of the source velocity $\mathbf{W}$, and the expression for the function $f(x,y)$ depends on the nature of the source. If the source is a monopole, we have

$$f(x, y) = \Gamma_m \delta(x) \delta(y),$$

where Γ_m is its intensity; for a dipole source with its moment perpendicular to the velocity vector of the source we have

$$f(x, y) = \Gamma_d \left(\sin \alpha \frac{\partial}{\partial x} - \cos \alpha \frac{\partial}{\partial y} \right) \delta(x) \delta(y).$$

One sees easily that the solution ψ_d of Eq. (2) for a dipole source can be obtained from the corresponding solution ψ_m for the monopole source; these solutions are connected with one another by the relation

$$\frac{\psi_d}{\Gamma_d} = \left(\sin \alpha \frac{\partial}{\partial x} - \cos \alpha \frac{\partial}{\partial y} \right) \frac{\psi_m}{\Gamma_m}. \tag{3}$$

This fact will be used by us in what follows.

A moving source emits Rossby waves with different wave vectors

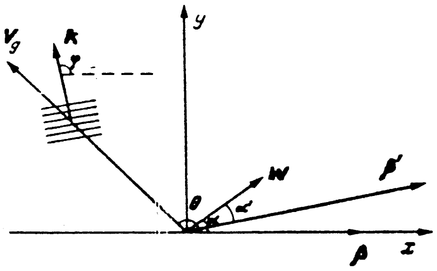

FIG. 1. A wave packet emitted by a moving source propagates with a group velocity $\mathbf{v}_g$ at an angle θ to the x-axis, whereas the phase velocity of the waves is directed at an angle φ to the x-axis. The meaning of the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\beta}'$ is explained in Sec. 3 of the text.

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$$\mathbf{k}=(k_x,\ k_y)=k(\cos\varphi,\ \sin\varphi)$$

and frequencies ω which are connected with one another by the well known dispersion relation (see, e.g., Ref. 2):

$$\omega=-\frac{\beta k_x}{k_x^2+k_y^2}=-\frac{\beta\cos\varphi}{k}. \tag{4}$$

Here φ is the angle between the wave vector of the emitted wave and the x axis. This relation together with the Cherenkov resonance condition (1) in which we have $\gamma=\varphi-\alpha$, determine the connection between the direction and the absolute magnitude of the wave vector of the emitted waves:

$$k^2=-\frac{\beta}{W}\frac{\cos\varphi}{\cos(\varphi-\alpha)}. \tag{5}$$

Hence it follows that the direction of the wave vector of the emitted waves can lie in the range of angles

$$\pi/2<\varphi<\pi/2+\alpha, \tag{6}$$

determined by the conditions $\omega>0$ and $k^2>0$.

In the frame of reference fixed to the moving source the wave field is clearly time-independent and its spatial distribution in the far field has, in polar coordinates (r,θ), the form

$$\psi(r,\ \theta)\sim D(\theta)\exp(ik_r r)/r^{1/2}, \tag{7}$$

where $k_r(\theta)$ is the radial wave number and $D(\theta)$ is a function characterizing the distribution in angle θ of the field intensity.[5] Far from the source at distances $r\gg k^{-1}$ the propagation of the wave packets occurs in the radial direction with the group velocity

$$\mathbf{V}_g=\left(-W\cos\alpha+\frac{\beta}{k^2}\cos(2\varphi),\ -W\sin\alpha+\frac{\beta}{k^2}\sin(2\varphi)\right). \tag{8}$$

The angle θ at which the group velocity is directed to the x-axis is connected with the angles α and φ through the relation, following from Eqs. (5) and (8):

$$\operatorname{tg}\theta=\frac{(\mathbf{V}_g)_y}{(\mathbf{V}_g)_x}=\frac{\sin\alpha\cos\varphi+\cos(\varphi-\alpha)\sin(2\varphi)}{\cos\alpha\cos\varphi+\cos(\varphi-\alpha)\cos(2\varphi)}. \tag{9}$$

This relation enables us to write the radial wave number k_r in the form

$$k_r=k\cos(\varphi-\theta)$$
$$=\left[-\frac{\beta}{W}\frac{\cos\varphi}{\cos(\varphi-\alpha)}\right]^{1/2}\frac{2\cos\varphi\cos(\varphi-\alpha)}{[\Phi(\varphi,\alpha)]^{1/2}}, \tag{10}$$

where

$$\Phi(\varphi,\ \alpha)=\cos^2\varphi+\cos^2(\varphi-\alpha)$$
$$+2\cos\varphi\cos(\varphi-\alpha)\cos(2\varphi-\alpha).$$

We introduce the momentum density for a wave with wavevector $\mathbf{k}$ in the far zone using the conventional definition

$$\mathbf{p}=\mathbf{k}N\sim\mathbf{k}\rho|\psi|^2,$$

where N is the quasiparticle number per unit volume, or wave action density, and ρ the fluid density. If we know the quasiparticle-number flux-density distribution

$$S(\varphi)=N|\mathbf{V}_g|$$

over different directions of the wave vector $\mathbf{k}$, we can find the momentum flux through a cylindrical surface of unit height emitted by the source in unit time:

$$\Pi=\int_{\pi/2}^{\pi/2+\alpha}\mathbf{k}S(\varphi)r\,d\varphi. \tag{11}$$

The emitted momentum flux determines the radiative reaction force:

$$\mathbf{F}=-\Pi.$$

Korotaev[5] has earlier found the longitudinal (with respect to the velocity) component of this force (the wave resistance force) for a monopole vortex source:

$$F_{\parallel}=-\int_{\pi/2}^{\pi/2+\alpha}k_{\parallel}S_m(\varphi)r\,d\varphi=-\frac{\rho\Gamma_m^2}{4\pi}\int_{\pi/2}^{\pi/2+\alpha}k_{\parallel}\,d\varphi, \tag{12}$$

where

$$k_{\parallel}=k\cos(\varphi-\alpha).$$

Hence it follows that the quasiparticle number flux density is equal to

$$S_m(\varphi)=\rho\Gamma_m^2/4\pi r. \tag{13}$$

We note in passing a fact which, in our opinion, is far from trivial: the quasiparticle number flux density turns out to be independent of the direction of the wave vector for the Cherenkov emission of Rossby waves by a monopole vortex source in the anisotropic medium considered.

Using Eq. (13) we can also find the transverse component of the radiative reaction force acting on a monopole source:

$$F_{\perp}=-\int_{\pi/2}^{\pi/2+\alpha}k_{\perp}S_m(\varphi)r\,d\varphi=-\frac{\rho\Gamma_m^2}{4\pi}\int_{\pi/2}^{\pi/2+\alpha}k_{\parallel}\operatorname{tg}\varphi\,d\varphi. \tag{14}$$

Both components of the force acting on the monopole can be expressed in terms of elliptic integrals of the first and second order, $K(x)$ and $E(x)$ (we have already noted that the expression for $F_{\parallel}$ was first found in Ref. 5):

$$F_{\parallel}=-\frac{\rho\Gamma_m^2}{4\pi}\left(\frac{\beta}{W}\right)^{1/2}\int_{\pi/2}^{\pi/2+\alpha}[-\cos\varphi\cos(\varphi-\alpha)]^{1/2}d\varphi$$
$$=-\frac{\rho\Gamma_m^2}{4\pi}\left(\frac{\beta}{W}\right)^{1/2}\left[2E\left(\sin\frac{\alpha}{2}\right)-(1+\cos\alpha)K\left(\sin\frac{\alpha}{2}\right)\right], \tag{15}$$

$$F_{\perp}=-\frac{\rho\Gamma_m^2}{4\pi}\left(\frac{\beta}{W}\right)^{1/2}\int_{\pi/2}^{\pi/2+\alpha}[-\cos\varphi\cos(\varphi-\alpha)]^{1/2}\operatorname{tg}(\varphi-\alpha)\,d\varphi$$
$$=-\frac{\rho\Gamma_m^2}{4\pi}\left(\frac{\beta}{W}\right)^{1/2}\sin\alpha K\left(\sin\frac{\alpha}{2}\right). \tag{16}$$

We note that the radiation force decreases with increasing source velocity, $\propto W^{-1/2}$. We show in Fig. 2 the components of the radiative reaction force as functions of the angle α. There is no longitudinal component—wave resistance force—for eastward motion (for $\alpha = 0$). Indeed, such a source does not excite Rossby waves since their wavevector cannot have an easterly component according to the dispersion relation (4). When the angle α increases, the longitudinal force increases monotonically, reaching a maximum in the westerly direction ($\alpha = \pi$).

The transverse force reaches a maximum for a motion close to the meridional direction ($\alpha \approx 106°$) and tends to incline the trajectory of the source in the easterly direction. If, however, the source moves in a latitudinal direction along the x-axis eastward or westward, there is no transverse radiative force, owing to the symmetry of the emitted waves to the left and to the right from the axis of the motion. However, it appears immediately however little the direction of the source velocity is inclined to the meridian. When moving eastward at a small angle α to the parallel there arises a *restoring* transverse force

$$F_\perp \approx -\frac{\rho\Gamma_m^2}{8}\left(\frac{\beta}{W}\right)^{1/2}\alpha, \tag{17}$$

whereas westward motion at a small angle $\delta = \pi - \alpha$ to the latitude leads to the appearance of a *deflecting* force

$$F_\perp \approx -\frac{\rho\Gamma_m^2}{4\pi}\left(\frac{\beta}{W}\right)^{1/2}\delta\ln|\delta|. \tag{18}$$

As a result of this, westward motion of the source is unstable: the presence of a small meridional velocity component causes its further increase with time, nonlinearly (but not exponentially).

We can also find the radiative reaction force for a dipole source, using Eq. (3). The quasiparticle number flux density S_d is determined by the asymptotic behavior (7) of the field in the far zone where

$$S_d \sim |\psi|^2.$$

Bearing in mind that when we evaluate derivatives in Eq. (3) we must differentiate only the rapidly oscillating part of the solution (7), we obtain for a dipole source

$$S_d = \frac{\rho\Gamma_d^2}{4\pi r}k_r^2\sin^2(\theta-\alpha). \tag{19}$$

Substituting (19) into Eq. (11) we obtain the logitudinal and transverse components of the radiative reaction force acting on a point dipole:

$$F_\parallel = -\frac{\rho\Gamma_d^2}{4\pi}\int_{\pi/2}^{\pi/2+\alpha}k_\parallel k_r^2\sin^2(\theta-\alpha)\,d\varphi$$

$$= -\frac{\rho\Gamma_d^2}{4\pi}\left(\frac{\beta}{W}\right)^{3/2}\int_{\pi/2}^{\pi/2+\alpha}\left(\frac{-\cos\varphi}{\cos(\varphi-\alpha)}\right)^{3/2}$$

$$\frac{4\cos^2\varphi\cos^5(\varphi-\alpha)\sin^2(2\varphi-\alpha)}{[\Phi(\varphi,\alpha)]^2}\,d\varphi, \tag{20}$$

$$F_\perp = -\frac{\rho\Gamma_d^2}{4\pi}\int_{\pi/2}^{\pi/2+\alpha}k_\perp k_r^2\sin^2(\theta-\alpha)\,d\varphi$$

$$= -\frac{\rho\Gamma_d^2}{4\pi}\left(\frac{\beta}{W}\right)^{3/2}\int_{\pi/2}^{\pi/2+\alpha}\left(\frac{-\cos\varphi}{\cos(\varphi-\alpha)}\right)^{3/2}\frac{\Psi(\varphi,\alpha)}{[\Phi(\varphi,\alpha)]^2}\,d\varphi, \tag{21}$$

where

$$\Psi(\varphi,\alpha) = 4\sin(\varphi-\alpha)\cos^2\varphi\cos^4(\varphi-\alpha)\sin^2(2\varphi-\alpha).$$

We did not succeed in expressing the components of the radiative force acting upon a dipole in terms of elementary or special functions, but the integrals in Eqs. (20) and (21) can easily be found numerically. In Fig. 3 we show $F_{\parallel,\perp}$ as functions of the angle α. As in the previous case of a monopole source, the longitudinal wave-resistance force increases monotonically with increasing α. The transverse force behaves qualitatively as in the previous case; its maximum is reached for $\alpha \approx 158°$ while it vanishes for $\alpha = 0$ and π because of the canceling of the radiative forces to the left and the right of the source. Small deviations from these directions again lead to *restoring* (if $\alpha \ll 1$) and *deflecting* (if $\delta \ll 1$) forces so that eastward motion is stable and westward motion unstable.

We note also that when the velocity W of the dipole source increases the radiative force decreases $\propto W^{-3/2}$. The unbounded increase in the radiative forces as $W \to 0$ both for monopole and for dipole sources is explained by the fact that

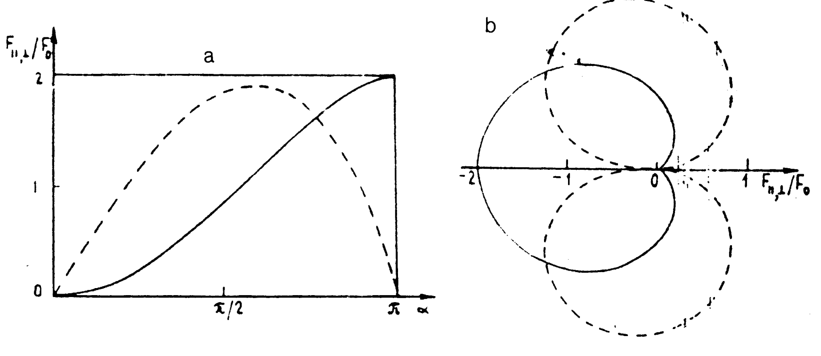

FIG. 2. The components of the radiative reaction force (a—in Cartesian, b—in polar coordinates), acting on a point monopole, as functions of the angle α in a barotropic model: the solid line is the longitudinal component $F_\parallel$ and the dashed line the transverse component $F_\perp$ [$F_0 = (\rho\Gamma_m^2/4\pi)(\beta/W)^{1/2}$].

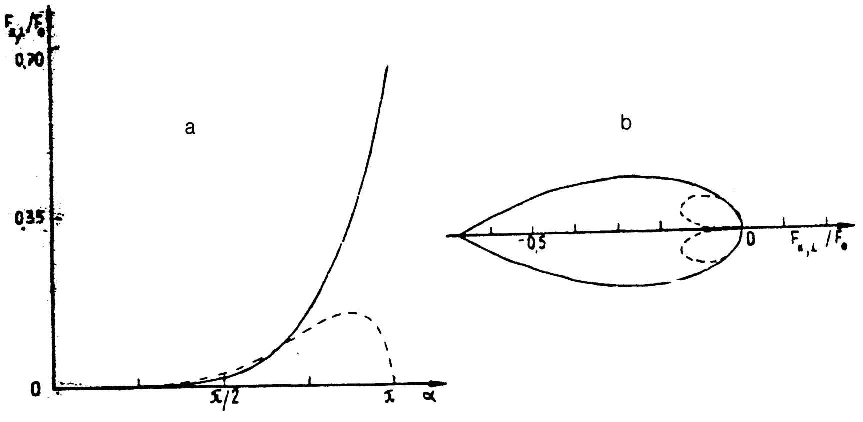

FIG. 3. The components of the radiative reaction force (a: in Cartesian, b: in polar coordinates), acting on a point dipole, as functions of the angle α in a barotropic model: the solid line is the longitudinal component $F_{\parallel}$ and the dashed line the transverse component $F_{\perp}$ $[F_0 = (\rho\Gamma_d^2/4\pi)(\beta/W)^{3/2}]$.

we have considered point models; taking into account the finite size of the source leads to more complex velocity dependence of the forces in the region of small W.[6]

3. EMISSION OF WAVES IN A BAROCLINIC MODEL

A study of Cherenkov emission in the framework of a baroclinic model (such as a barotropic model with a free surface) reduces, in spite of the apparent complexity, to the barotropic case. Indeed, we can write down the basic equation for the current function:

$$\frac{\partial}{\partial t}(\Delta\psi-\psi/\mathrm{Ro}^2)+\beta\frac{\partial\psi}{\partial x}=\frac{\partial f(x-Ut,y-Vt)}{\partial t}, \tag{22}$$

where Ro is the so-called Rossby–Obukhov deformation radius.

We change to a frame of reference fixed to the moving source, using the change of variables

$$\mathbf{r}'=\mathbf{r}-\mathbf{W}t.$$

We find then from (22) an equation for the emitted wave field:

$$-(\mathbf{W},\nabla)\Delta\psi+(\boldsymbol{\beta}',\nabla)\psi=-(\mathbf{W},\nabla)f(\mathbf{r}'), \tag{23}$$

where we have introduced the vector

$$\boldsymbol{\beta}'=\beta\mathbf{x}_0+\mathbf{W}\,\mathrm{Ro}^{-2},$$

and where $\mathbf{x}_0$ is a unit vector in the direction of the x-axis.

The equation for the baroclinic wave field, written in the form (23), has the same form as for the barotropic case.[1)] Therefore all solutions obtained in the previous section can be used for baroclinic waves, if we replace in the corresponding formulas the parameter β by

$$\beta'=\{[\beta\sin\alpha]^2+[\beta\cos\alpha+W\,\mathrm{Ro}^{-2}]^2\}^{1/2},$$

and the angles α, φ, and θ by the effective angles

$$\alpha\to\alpha',\quad \varphi\to\varphi',\quad \theta\to\theta',$$

taken in polar coordinates from the semiaxis determined by the vector β' (Fig. 1). Figures 2 and 3 retain their shape if we understand by α, α'. However, in the original x and y coordinates the radiative forces look different as we must, when changing to those coordinates, take into account the relation between the angles α and α'. From the expression for β' follow the relations (cf. Ref. 5)

$$\cos\alpha'=\frac{\cos\alpha+W/V_R}{[(\cos\alpha+W/V_R)^2+\sin^2\alpha]^{1/2}},$$
$$\sin\alpha'=\frac{\sin\alpha}{[(\cos\alpha+W/V_R)^2+\sin^2\alpha]^{1/2}}. \tag{24}$$

Here $V_R=\beta/\mathrm{Ro}^2$ is the Rossby velocity.

The quantities α' and α behave differently as functions of the parameter W/V_R (Fig. 4).

For $W<W_R$ the effective angle α' changes monotonically with increasing α—in this case the qualitative nature of the emission diagram and the α-dependence of the forces $F_{\parallel,\perp}$ remain the same as for barotropic waves. However, the quantitative characteristics change in this case, especially the way the radiative force depends on the source velocity is no longer proportional to $W^{-1/2}$ but is more complicated.

For $W>V_R$ there is a change in the qualitative nature of the α-dependence of the radiative force which is formally connected with the nonmonotonic nature of the $\alpha'(\alpha)$ curve in this case (see Fig. 4). In particular, as $\alpha\to\pi$ (motion in westward direction) the effective angle $\alpha'\to 0$. Hence it follows that the emission of Rossby waves with nubmer n vanishes not only when the source moves to the east ($\alpha=0$) but also when it moves to the west.

We show in Fig. 5 plots of $F_{\parallel}$ and $F_{\perp}$ as functions of α for three values of the velocity of a monopole source. When W approaches the threshold velocity V_R of the given mode the position of the maximum of the $F_{\perp}$ force shifts to π and its value approaches

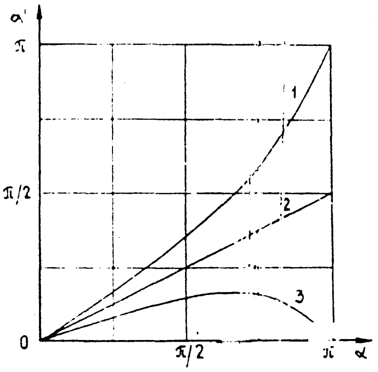

FIG. 4. The effective angle α' as function of the true angle α for different source velociites: $W/V_R=0.5$ (*1*), 1 (*2*), 2 (*3*).

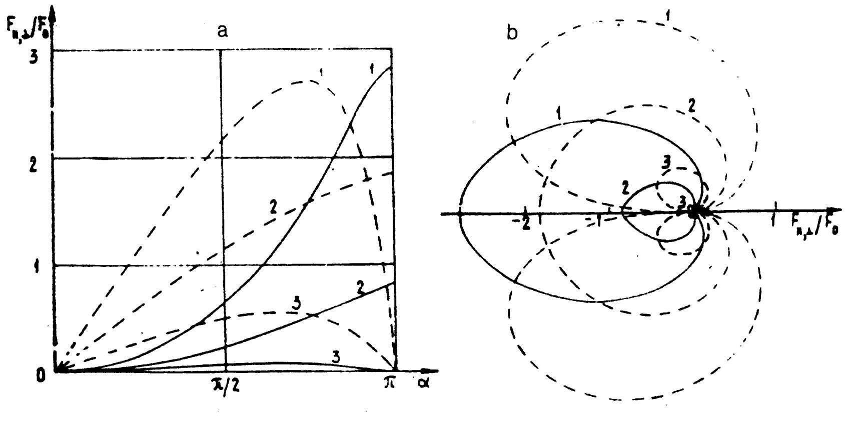

FIG. 5. The components of the radiative force as functions of the true angle α for different velocities of a monopole source in a baroclinic model (a: in Cartesian, b: in polar coordinates): $W/V_R = 0.5$ (*1*), 1 (*2*), 2 (*3*) $[F_0 = (\rho\Gamma_m^2/4\pi)(\beta/V_R)^{1/2}]$.

$$7{,}5\,\frac{\pi}{\Gamma_m^2}\left(\frac{V_R}{\beta}\right)^{1/2}.$$

The magnitude of the $F_\perp$ force is always equal to zero in the point Π itself. When the velocity W increases further the position of the maximum of the function $F_\perp(\alpha)$ moves away from the point π in the direction of smaller values of α. We show in Fig. 6 the same quantities as functions of the source velocity for three different angles. For a comparison we show by a dashed line on those figures the barotropic velocity-dependence of the forces

$$F \propto W^{-1/2}.$$

It is clear from the figures that this dependence is asymptotically satisfied in the $W \ll V_R$ region. For large source velocities the components of the radiative force depend again on W according to a power law with an index which depends on the angle α.

We show in Fig. 7 the α-dependence of $F_{\parallel,\perp}$ for three values of the velocity $W > V_R$ for a dipole source (for $W < V_R$ the qualitative picture is the same as in the barotropic case (Fig. 3) and is not of special interest). Finally, we show in Fig. 8 the way the components of the radiative force depend on the velocity of a dipole source. Here attention is drawn to the very steep decrease of the forces when the source velocity increases. In the $W \ll V_R$ region the curves approach the barotropic $F \propto W^{-3/2}$ law while in the $W \gg V_R$ region again all curves reach a power law with an index depending on the angle α.

The vanishing of the radiative forces as $\alpha \to 0, \pi$ explains the existence of exact solutions describing dipole vortices in the β-plane[7] which can shift either eastwards with any velocity or westwards with a velocity $W > V_R$.[2)] Indeed, only in those cases is there a complete absence of radiative forces.

When the source moves with an arbitrary velocity in a direction differing from the latitudinal and also when it moves strictly westwards with a velocity $W < V_R$ the emission of Rossby waves leads to a nonstationary evolution of the vortex accompanied by the emission of Rossby waves.

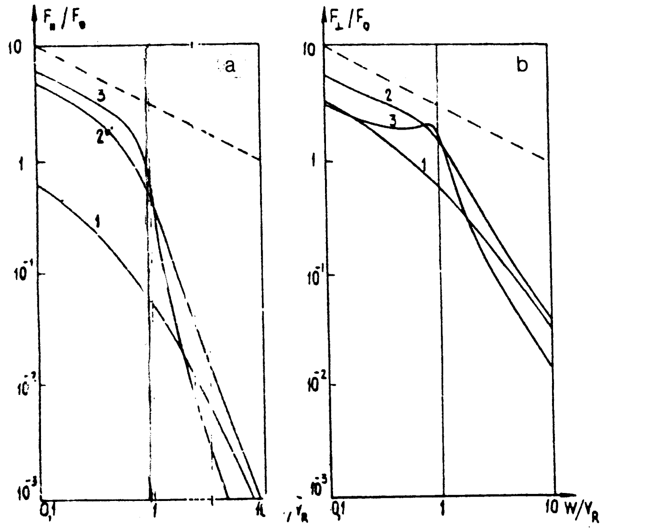

FIG. 6. The components of the radiative force (a—$F_\parallel$, b—$F_\perp$) as functions of the velocity of a monopole source in a baroclinic model for three different angles: $\alpha = \pi/4$ (*1*), $3\pi/4$ (*2*), $11\pi/12$ (*3*). The dashed line corresponds to a $W^{-1/2}$ dependence $[F_0 = (\rho\Gamma_m^2/4\pi)(\beta/V_R)^{1/2}]$.

4. CONCLUSION

The analysis given here of the radiative forces acting on moving sources in a nonuniformly rotating fluid in the β-plane approximation shows that owing to the anisotropy of the medium there appears, apart from the wave-resistance force, also a transverse force which does not conserve work and which therefore cannot be evaluated from energy considerations (that method of calculation is widespread: see, e.g., Refs. 5 and 6). The magnitude of the transverse force is comparable to the wave resistance force. Knowing both components of the radiative force makes it possible to calculate the trajectory of a freely moving source in the β-plane and, especially, that of a monopole or dipole vortex.

The equations of motion for a rigidly rotating vortex were given in Ref. 5, taking into account the Coriolis and Magnus (Zhukovskiĭ, to use the terminology of Ref. 5) forces, but not the radiative force. That paper also obtained their approximate solutions. Subsequently a somewhat more complex model of the motion of a vortex with oscillations was considered in Ref. 6. As a result of this there appears a variable component of the wave resistance force which, according to Ref. 6, is proportional to the derivative of the wave resistance force for uniform motion. A feature of such a vortex motion is that the amplitude of its oscillations may

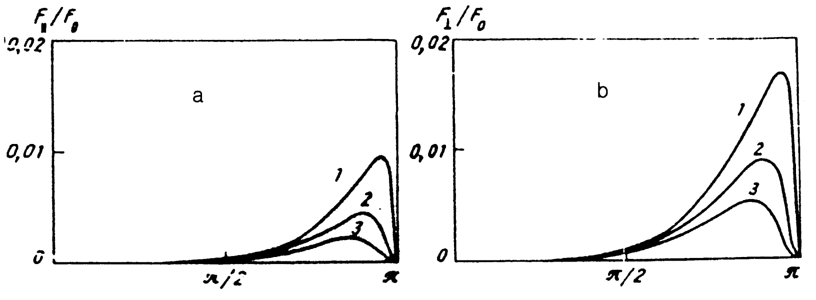

FIG. 7. The components of the radiative force (a—$F_\parallel$, b—$F_\perp$) as functions of the true angle α for different velocities of a dipole source in a baroclinic model: $W/V_R = 1.01$ (*1*), 1.05 (*2*), 1.1 (*3*) $[F_0 = (\rho\Gamma_d^2/4\pi)(\beta/V_R)^{3/2}]$.

grow exponentially in the linear approximation (radiative instability).

However, the conclusions of Refs. 5 and 6 are apparently only qualitative in nature since estimates show that the radiative forces that have been neglected may not only be comparable with the above mentioned forces, but also may exceed them appreciably. To confirm this we compare the Coriolis force acting on a moving monopole vortex with the radiative force. We shall assume that the vortex is a cylinder of radius R, rotating with a constant angular velocity ω_0. We then have for the parameter Γ_m

$$\Gamma_m = \pi R^2 \omega_0.$$

The magnitude of the Coriolis force is

$$F_{\rm Cor} \sim M\Omega W = \pi R^2 \rho \Omega W,$$

where $\Omega \approx 5\times 10^{-5}$ s^{-1} is the Coriolis parameter which is equal to the product of the angular rotational frequency of the earth and the sine of the latitude. For the radiative force we have the estimate [see Eqs. (16) and (17)]

$$F_{\rm rad} \sim \frac{\rho\Gamma_m^2}{4\pi}\left(\frac{\beta}{W}\right)^{1/2},$$

up to an angular factor of order unity (provided α is not too small). We choose the following values of the parameters, corresponding to actually observed vortices in the ocean:[5,6] $\omega_0 \approx 2\times 10^{-5}$ s^{-1}, $R \approx 10^5$ m (100 km), and we also put $\beta \approx 2\times 10^{-11}$ m^{-1} s^{-1} (Ref. 2). Hence we find $\Gamma_m \approx 6.3\times 10^5$ m^2 s^{-1}. The ratio $F_{\rm Cor}/F_{\rm rad} \sim 10\cdot W^{3/2}$. According to available data[5,6] the velocity of the vortices in the ocean is 1 to 15 cm·s^{-1} so that $F_{\rm Cor}/F_{\rm rad}$ lies within the limits 10^{-2} to 5×10^{-1}. The effect of other forces (such as the Magnus force) turns out to be even smaller.

The results can be used, in particular, for an interpretation and prediction of the motion of atmospheric cyclones and tropical hurricanes and also of synoptic vortices in the ocean.

In conclusion we note that an attempt was made in Refs. 8 and 9 to use integral invariants to prove the stability of Larichev–Reznik dipole vortices. However, this attempt turned out to be unsuccessful since, as was shown in Ref. 10, the proof was based on an erroneous premise. The problem of the stability of dipole vortices remained up to this moment an open one. It follows from our results that the westward motion of Larichev–Reznik dipole vortices is unstable; under the action of small fluctuations the vortices will turn to the east losing during their motion part of their energy to the emission of Rossby waves. The trajectories along which they move can, when all forces are taken into account, be rather complicated; some results in that direction were obtained through direct numerical computer calculations in the framework of the nonlinear Charney–Obukhov equation[2] or, in plasma terminology, the Hasegawa–Mima equation[3] (see Ref. 11). These results agree with the above-indicated qualitative considerations about the nature of the vortex motion.

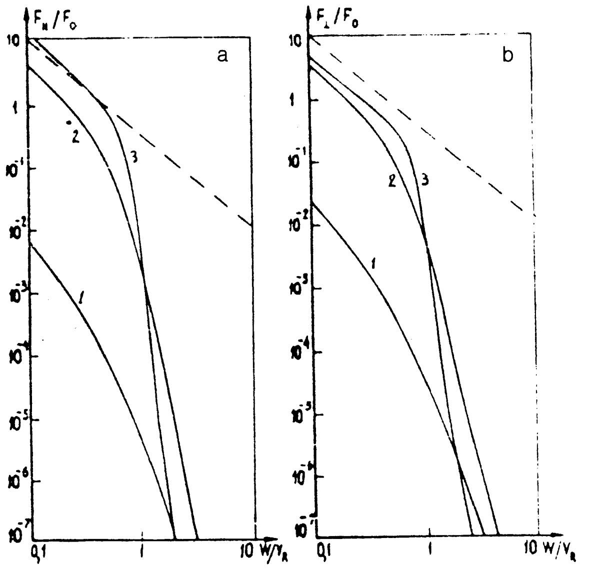

FIG. 8. The components of the radiative force (a—$F_\parallel$, b—$F_\perp$) as functions of the velocity of a dipole source in a baroclinic model for three different angles: $\alpha = \pi/4$ (*1*), $3\pi/4$ (*2*), $11\pi/12$ (*3*). The dashed line corresponds to a $W^{-3/2}$ dependence $[F_0 = (\rho\Gamma_d^2/4\pi)(\beta/V_R)^{3/2}]$.

[1)]One must, however, bear in mind that the effective strength of a baroclinic source (characterized in the barotropic case by the parameters Γ_m or Γ_d) must be determined with allowance for the transverse structure of the baroclinic mode.[5] We note also that the magnitude of the Rossby–Obukhov deformation radius Ro takes on different values for modes with different numbers n.

[2)]Barotropic vortices in a rotating fluid with a free surface and with a "solid cover" were considered in Ref. 7.

[1]B. B. Kadomtsev, *Collective Effects in a Plasma* [in Russian], Nauka, Moscow (1976).

[2]J. Pedlosky, *Geophysical Fluid Dynamics*, Springer (1979).

[3]V. I. Petviashvili and O. A. Pokhotelov, *Solitary Waves in a Plasma and in the Atmosphere* [in Russian], Energoatomizdat, Moscow (1989).
[4]V. V. Dolotin and A. M. Fridman, Zh. Eksp. Teor. Fiz. **99**, 3 (1991) [Sov. Phys. JETP **72**, 1 (1991)].
[5]G. K. Korotaev, *Theoretical Simulation of the Synoptic Variability of the Ocean* [in Russian], Nauk. Dumka, Kiev (1988).
[6]L. A. Ostrovskiĭ and I. S. Dolina, Mor. Gidrofiz. Zh. No. 3, 3 (1989).
[7]V. D. Larichev and G. M. Reznik, Dokl. Akad. Nauk SSSR **231**, 1077 (1976).
[8]E. W. Laedke and K. H. Spatschek, Phys. Fluids **29**, 134 (1986).
[9]E. W. Laedke and K. H. Spatschek, Phys. Fluids **31**, 1492 (1988).
[10]S. V. Muzylev and G. M. Reznik, Izv. Ross. Akad. Nauk, FAO **28**, 522 (1992).
[11]M. Makino, T. Kamimura, and T. Taniuti, J. Phys. Soc. Jpn. **50**, 980 (1981).

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