Hydrodynamic fluctuations and scattering of electromagnetic waves in semiconductors in strong electric fields

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Hydrodynamic fluctuations in a nonequilibrium semiconductor electron gas are analyzed under conditions such that an electron temperature can be introduced. Such a gas has two fluctuation degrees of freedom, which correspond to excitations of stochastic waves of the electron density and of the electron temperature. The corresponding length and time scales are the Maxwellian relaxation time τ_M, the electron temperature τ_T, and the respective diffusion lengths L_M and L_T. Expressions are derived for the spectral densities of low-frequency, long-wave fluctuations for arbitrary relations among the fluctuation frequencies ω and the times τ_M and τ_T and also among the fluctuation wave vectors $\mathbf{q}$ and the lengths L_M and L_T. In general, the spectra are not Lorentzian. This is true in particular of thermodynamic equilibrium. The mutual correlation function, which determines the relationship between the fluctuations of the electron density and of the electron temperature, is a nonmonotonic, sign-varying function of the frequency. This behavior indicates the existence of regions of correlation and of anticorrelation. At thermodynamic equilibrium, the mutual effects of the fluctuations reduce to simply a redistribution of their intensity over the spectrum, with no change in the integral intensity. In a nonequilibrium gas, the mutual effects of the fluctuations in the electron density and temperature are directly related to an additional kinetic correlation. This kinetic correlation arises from an electron-electron interaction and leads to a violation of the Price fluctuation-diffusion relations. This additional kinetic correlation leads to a radical violation of the similarity of the equilibrium fluctuation spectra and the nonequilibrium spectra with wave vectors perpendicular to the external electric field. In the absence of an electron-electron interaction, this similarity prevails. A theory for the scattering of light by hydrodynamic fluctuations of hot electrons is derived in the same approximation. An expression is derived for the differential cross section for optical scattering. It is shown that the additional kinetic correlation may either intensify or suppress the scattering of light. It also leads to a qualitatively new effect: a correlation-induced shift of the peak of the spectral line of the scattered light. Possibilities for experimentally studying the scattering of light by long-wave, low-frequency electron fluctuations are discussed.

1. INTRODUCTION

Hydrodynamic fluctuations are long-wave, low-frequency stochastic excitations of a physical system against the background of its steady state. It is assumed here that there are some characteristic microscopic spatial and temporal relaxation parameters l and τ such the following conditions hold for the typical wave vectors $\mathbf{q}$ and frequencies ω of the fluctuations:

$$\omega\tau \ll 1, \quad ql \ll 1. \tag{1.1}$$

In this case the system behaves as a continuous medium and can be described by a system of macroscopic equations.[1]

This assertion does not, on the other hand, mean that the description of the fluctuations by such a system of macroscopic equations is exhaustive. The ultimate cause (or source) of any fluctuations consists of random microscopic processes, specifically, collisions of particles of the system with each other and with a heat reservoir. This circumstance leads to a two-step evolution of fluctuating excitations which arise under the influence of such random perturbations.[2]

In the first step, the initial perturbation undergoes a rapid relaxation to a locally steady-state distribution. This relaxation proceeds as a relaxation in momentum space and is controlled by Boltzmann-Langevin equations.[3] A local steady state is established over times on the order of the microscopic relaxation time τ. This process occurs independently in each element of coordinate space. After a time on the order of τ, the details of the original distribution in momentum space are forgotten, and this momentum-space distribution assumes a form corresponding to a steady state. In general, this distribution depends not only on the momenta of the particles but also on certain parameters which are macroscopic characteristics of the physical system. These macroscopic variables depend on the coordinates, since random fluxes which occur in momentum space in the first step of the relaxation process lead to local fluxes in coordinate space and thus to a coordinate dependence of the macroscopic properties of the system.

In the second step, the subsequent relaxation of the fluctuating excitation toward a final, spatially homogeneous, steady state is controlled by hydrodynamic equations for macroscopic variables. This process is an evolution of hydrodynamic fluctuations. We can already see that although it is possible to breakup the fluctuation process into two steps—a fast (kinetic) relaxation and a slower (hydrodynamic) one—these two steps are closely interrelated. The first step shapes the initial conditions for the second step of the space-time evolution of the fluctuation. The develop-

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ment of the fluctuation in this step is controlled by a Boltzmann–Langevin equation, from which a closed system of stochastic equations of the hydrodynamic type should follow. This system of equations describes the evolution of the fluctuation in the second step of the process. The second step determines the specific nature of the space-time behavior of the fluctuations of the macroscopic characteristics of the physical system which appear as parameters in the expressions describing the kinetic step of the fluctuation process. These macroscopic parameters play the role of integrals of motion in the expressions for the first step. A complete description of the hydrodynamic fluctuations thus generally also requires an analysis of the kinetic step of their evolution.

In the present paper we analyze hydrodynamic fluctuations in a nonequilibrium semiconductor electron gas. We assume that in the steady state the state of the gas is characterized by an electron temperature T different from that of the heat reservoir, T_0. We know that the following inequalities must therefore hold:

$$\tau_\mathbf{p}\ll\tau_{ee}\ll\tau_{\mathscr{E}}, \tag{1.2}$$

where $\tau_\mathbf{p}$ and $\tau_{\mathscr{E}}$ are the relaxation times of, respectively, the momentum $\mathbf{p}$ and the energy $\mathscr{E}$ of the electrons in the course of their scattering by the heat reservoir, and τ_{ee} is the time scale of electron-electron collisions.

Electron fluctuations in such systems have been studied in several places.[3-6]

Gantsevich *et al.*[3,4] found kinetic equations for the correlation functions of fluctuations of distribution functions. They took both electron-phonon scattering and electron-electron interaction into account. Their approach might be called the "moment method." Because of the intense interparticle scattering, an additional, collisional correlation arises in the fluctuations in the distributions function, and the Price fluctuation-diffusion relation is violated. That relation asserts that the spectral densities of spatially uniform fluctuations in the current and the diffusion coefficient are proportional. This is clearly a nonequilibrium effect. Using kinetic equations for the fluctuations, Gantsevich *et al.*[3,4] derived the basic equations of fluctuation hydrodynamics. They used those equations to study long-wave, low-frequency fluctuations about a spatially uniform, nonequilibrium steady state. The sole macroscopic variable, which ultimately determined the hydrodynamic fluctuations completely, was a fluctuation of the electron density, $\delta n(\mathbf{r},t)$, which depended on the coordinates $\mathbf{r}$, of the point, and the time t. Correspondingly, hydrodynamic fluctuations were described in Refs. 3 and 4 by only a single equation: a stochastic continuity equation.

Under conditions (1.2), however, the steady-state distribution of the electrons in a semiconductor is characterized by a macroscopic parameter in addition to the electron density: the electron temperature. In general, an electron gas thus has two fluctuation degrees of freedom. The hydrodynamic fluctuations are described in this case by a system of two stochastic equations. Still, there can be situations in which the intensity of concentration fluctuations is well above the intensity of temperature fluctuations, and the results of Ref. 3 and 4 are sufficient for describing hydrodynamic fluctuations. For the particular macroscopic parameters selected, the dynamics of this electron gas is characterized by two time scales: the Maxwellian relaxation time τ_M and the relaxation time τ_T of the electron temperature. Corresponding to these two time scales are two length scales: the Maxwellian relaxation length $L_M=(D\tau_M)^{1/2}$ and the electron cooling length $L_T=(D\tau_T)^{1/2}$, where D is the diffusion coefficient. Under the conditions

$$\tau_T\ll\tau_M,\ \omega\tau_T\ll1,\ q^2L_T{}^2\ll1 \tag{1.3}$$

fluctuations of the electron density thus play the leading role. This statement does not, however, mean that fluctuations of the electron temperature can be completely ignored. As we will see below, it is specifically in the case in which inequalities (1.3) hold that the contribution of the fluctuations in the electron temperature reduces, under nonequilibrium conditions in the system, to a renormalization of the kinetic coefficients in the dispersion relation for a hydrodynamic mode. In this case, the latter mode is an electron density wave.

A theory for kinetic and hydrodynamic fluctuations was derived in Refs. 5 and 6 by a Langevin approach. That approach starts with the introduction of background random fluxes in linearized kinetic equations for occupation-number fluctuations. The correlation function for the occupation numbers is ultimately expressed in terms of correlation functions of the random fluxes, which are calculated both for the interaction of the electrons with the heat reservoir and for the case of binary collisions. These results can be used to calculate the spectral densities of the fluctuations of any physical quantity whose average value can be found from the known steady-state electron distribution. The fluctuation spectra found by the moment method[3,4] and by the Langevin procedure[5,6] are identical, indicating that the two approaches are equivalent (that they are equivalent was shown in a general way in Ref. 7). A system of stochastic transport equations was derived in the electron-temperature approximation in Ref. 6 for the general case in which low-frequency, long-wave fluctuations are represented as fluctuations of the electron density and the electron temperature. The hydrodynamics of the fluctuations in that general case was not studied there, however. As in the situation discussed in Refs. 3 and 4, there may be conditions such that fluctuations of the electron temperature play the leading role, while fluctuations in the electron density simply renormalize the kinetic coefficients. This situation requires satisfaction of the inequalities

$$\tau_M\ll\tau_T,\ \omega\tau_M\ll1,\ q^2L_M{}^2\ll1. \tag{1.4}$$

Kogan and Shadrin[8] used this approximation in the general procedure developed in Refs. 5 and 6 to calculate the intensity of the fluctuations in the electron temperature, integrated over the spectrum. This intensity was then used to calculate the differential cross section for quasielastic scattering of unpolarized electromagnetic radiation in an electron plasma.

There is a wide range of experimental situations in which conditions (1.3) or (1.4) may prove rather restrictive. In addition, the relaxation parameters τ_M and τ_T may behave in different ways as an electron gas is heated by an external field. For example, when electrons are scattered by acoustic lattice vibrations, the Maxwellian time increases with increasing electric field, while the electron cooling time decreases. As a result, the sign of the first inequalities in (1.3) and (1.4) may change. If the properties of the semiconductor are instead such that τ_M and τ_T are comparable in magnitude, then the results of Refs. 3, 4, and 8 are inapplicable for studying hydrodynamic fluctuations, even at equilibrium. Furthermore, as is shown below, it is worthwhile to carry out a special study of the hydrodynamics of fluctuations under conditions such that inequalities (1.3) and (1.4) do not hold. In particular, as we have already mentioned, the Price fluctuation-diffusion relation is violated in a gas with an intense electron-electron interaction. A measure of the extent of this violation is the correlation tensor introduced in Ref. 3. In intermediate situations, that tensor can contribute substantially to measurable properties; this circumstance should be of assistance in experimentally observing this effect.

New features for fluctuation hydrodynamics in such situations are effects of a mutual correlation between the fluctuations in the electron density and those in the electron temperature. As we will see below, these effects contain additional information about microscopic processes in the systems of interest and about the nonequilibrium states of these systems.

Our purpose in the present paper is to study the fluctuations in a semiconductor electron gas for the general case in which restrictions (1.3) or (1.4) are not imposed.

We also derive a theory for the scattering of electromagnetic waves by low-frequency, long-wave fluctuations of a nonequilibrium electron gas in a semiconductor. We derive expressions for the differential cross section for the scattering of unpolarized electromagnetic radiation, for arbitrary relations between the space-time characteristics of the fluctuations (ω and $\mathbf{q}$) and the properties of the semiconductor (τ_M, τ_T, L_M, and L_T). As we will see, a study of the scattering of electromagnetic radiation under these conditions adds to the capabilities of fluctuation spectroscopy of semiconductors and of nonequilibrium states of the electron gas in semiconductors.

2. SPECTRUM OF LONG-WAVE LOW-FREQUENCY FLUCTUATIONS IN AN ELECTRON GAS WITH AN INTENSE INTERPARTICLE INTERACTION

To describe the electron fluctuations we introduce the correlation function

$$\langle \delta F_{\mathbf{p}}(\mathbf{r},t)\delta F_{\mathbf{p}_1}(\mathbf{r}_1,t_1)\rangle,$$

where

$$\delta F_{\mathbf{p}}(\mathbf{r},t)=F_{\mathbf{p}}(\mathbf{r},t)-\bar{F}_{\mathbf{p}}(\mathbf{r},t) \tag{2.1}$$

is a fluctuation in the one-particle electron distribution function in the state with momentum $\mathbf{p}$ at point $\mathbf{r}$ at time t. In other words, this is the deviation of the exact distribution function $F_{\mathbf{p}}(\mathbf{r},t)$ from the average function $\bar{F}_{\mathbf{p}}(\mathbf{r},t)$. In a spatially uniform steady state of the system, the correlation function

$$\langle \delta F_{\mathbf{p}}(\mathbf{r},t)\delta F_{\mathbf{p}_1}(\mathbf{r}_1,t_1)\rangle$$

depends on only the differences $\mathbf{r}-\mathbf{r}_1$ and $t-t_1$, and its Fourier transform gives, according to the Wiener–Khinchin theorem the spectral density of the fluctuation process

$$(\delta F_{\mathbf{p}}\delta F_{\mathbf{p}_1})_{\mathbf{q}\omega}=\frac{1}{V_0}\iint\langle \delta F_{\mathbf{p}}(\mathbf{r},t)\delta F_{\mathbf{p}_1}(\mathbf{r}_1,t_1)\rangle \times\exp[i\omega(t-t_1)-i\mathbf{q}(\mathbf{r}-\mathbf{r}_1)] \times d(t-t_1)\,d(\mathbf{r}-\mathbf{r}_1), \tag{2.2}$$

where V_0 is the volume of the crystal. One can work from the known spectral density of fluctuations in the occupation numbers, $(\delta F_{\mathbf{p}}\delta F_{\mathbf{p}_1})_{\mathbf{q}\omega}$, to calculate the spectrum of electron fluctuations in any physical quantity.

The most direct way to find the spectral density $(\delta F_{\mathbf{p}}\delta F_{\mathbf{p}_1})_{\mathbf{q}\omega}$ is to work directly from an equation for this quantity, using the moment method.[3] In the case at hand, however, the specific realization of this approach turns out to be rather complicated. Furthermore, it is necessary first to calculate the simultaneous correlation function which appears as a source in the equation. It is thus more convenient to use the Langevin procedure[5,6] to calculate spectrum (2.2).

The Boltzmann–Langevin equation for the fluctuation $\delta F_{\mathbf{p}}(\mathbf{r},t)$ is[6]

$$\hat{\mathscr{B}}_{\mathbf{p}}(\mathbf{r},t)\delta F_{\mathbf{p}}(\mathbf{r},t)\equiv\left[\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\hat{L}_{\mathbf{p}}\right]\delta F_{\mathbf{p}}(\mathbf{r},t)+e\delta\mathbf{E}(\mathbf{r},t)\frac{\partial \bar{F}_{\mathbf{p}}}{\partial \mathbf{p}} =y_{\mathbf{p}}(\mathbf{r},t), \tag{2.3}$$

where $y_{\mathbf{p}}(\mathbf{r},t)$ is a random Langevin source; $\mathbf{v}$ is the electron velocity; $\hat{L}_{\mathbf{p}}$ is the linearized kinetic-equation operator, given by

$$\hat{L}_{\mathbf{p}}=e\mathbf{E}_0\frac{\partial}{\partial \mathbf{p}}+\hat{I}_{\mathbf{p}}{}^{th}+\hat{I}_{\mathbf{p}}{}^{ee}\{\bar{F}_{\mathbf{p}}\} \tag{2.4}$$

($\mathbf{E}_0$ is the external electric field, the operator $\hat{I}_{\mathbf{p}}^{th}$ represents the interaction of electrons with the reservoir, $\hat{I}_{\mathbf{p}}^{ee}\{\bar{F}_{\mathbf{p}}\}$ is the linearized operator representing electron-electron interactions), and $\delta\mathbf{E}(\mathbf{r},t)$ is the self-consistent fluctuation electric field, which is determined by a Poisson equation,

$$\operatorname{div}\delta\mathbf{E}(\mathbf{r},t)=\frac{4\pi e}{\varepsilon V_0}\sum_{\mathbf{p}'}\delta F_{\mathbf{p}'}(\mathbf{r},t), \tag{2.5}$$

where ε is the dielectric constant of the lattice.

It is convenient to take Fourier transforms in (2.3) and (2.5). As a result we find

$$\hat{\mathscr{B}}_{\mathbf{p}}(\mathbf{q},\omega)\delta F_{\mathbf{p}}(\mathbf{q},\omega)\equiv[-i\omega+i\mathbf{q}\mathbf{v}+\hat{L}_{\mathbf{p}}]\delta F_{\mathbf{p}}(\mathbf{q},\omega) -i\mathbf{q}U(\mathbf{q})\frac{\partial\bar{F}_{\mathbf{p}}}{\partial\mathbf{p}}\sum_{\mathbf{p}'}\delta F_{\mathbf{p}'}(\mathbf{q},\omega)=y_{\mathbf{p}}(\mathbf{q},\omega), \tag{2.6}$$

where

$$\delta\mathbf{E}(\mathbf{q},\omega)=-\frac{i}{e}\mathbf{q}U(\mathbf{q})\sum_{\mathbf{p}'}\delta F_{\mathbf{p}'}(\mathbf{q},\omega), \tag{2.7}$$

$$U(\mathbf{q})=\frac{4\pi e^2}{\varepsilon V_0 q^2}. \tag{2.8}$$

We seek a solution of Eq. (2.6) in the low-frequency, long-wave approximation (1.1), using the Chapman–Enskog procedure.[9] This method was used in Ref. 3, but it was modified through the incorporation of conditions (1.3) there. For that reason, it cannot be applied directly to the case at hand.

In the electron-temperature approximation, (1.2), the nonequilibrium steady-state distribution function is

$$\bar{F}_{\mathbf{p}}=\bar{F}_0(\mathscr{E}_{\mathbf{p}})+\bar{F}_1(\mathbf{p}), \tag{2.9}$$

$$\bar{F}_0(\mathscr{E}_{\mathbf{p}})=\frac{n_0}{N_c(T)}\exp(-\mathscr{E}_{\mathbf{p}}/T), \qquad \bar{F}_1(\mathbf{p})=-e\mathbf{E}_0\tau_{\mathbf{p}}\frac{\partial F_0(\mathscr{E}_{\mathbf{p}})}{\partial\mathbf{p}}, \tag{2.10}$$

where n_0 is the electron density, $\tau_{\mathbf{p}}$ is the momentum relaxation time, and $N_c(T)$ is the effective density of states, given by

$$N_c(T)=\int_0^\infty g(\mathscr{E})\exp\left(-\frac{\mathscr{E}}{T}\right)d\mathscr{E}=2\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2},$$

$$g(\mathscr{E})=\frac{m^{3/2}}{\pi^2\hbar^3}\mathscr{E}^{1/2}. \tag{2.11}$$

It can be seen from (2.9)–(2.10) that the function $\bar{F}_{\mathbf{p}}$ depends on the two parameters n_0 and T, which are external with respect to the kinetic-equation operator (in its nonlinearized form), since these parameters do not appear directly in this operator. They are instead determined from boundary conditions imposed on the distribution function $\bar{F}_{\mathbf{p}}$ (the normalization condition and energy balance). In other words, the parameters n_0 and T are integrals of motion for the kinetic operator. It follows that the linearized operator $\hat{L}_{\mathbf{p}}$ in (2.4) has two eigenfunctions which correspond to zero eigenvalues:

$$\hat{L}_{\mathbf{p}}\frac{\partial\bar{F}_{\mathbf{p}}}{\partial n_0}=0, \qquad \hat{L}_{\mathbf{p}}\frac{\partial\bar{F}_{\mathbf{p}}}{\partial T}=0. \tag{2.12}$$

That this is true can be verified directly by using the explicit expression for $\hat{L}_{\mathbf{p}}$ in (2.4) and the expression for the two-particle collision integral,

$$\hat{I}_{\mathbf{p}}^{ee}\{\bar{F},\bar{F}\}=\sum_{\mathbf{p}_1,\mathbf{p}',\mathbf{p}_1'}[W_{\mathbf{p}'\mathbf{p}_1'}\bar{F}_{\mathbf{p}}\bar{F}_{\mathbf{p}_1}-W_{\mathbf{p}\mathbf{p}_1}^{\mathbf{p}'\mathbf{p}_1'}\bar{F}_{\mathbf{p}'}\bar{F}_{\mathbf{p}_1'}]. \tag{2.13}$$

Here $W_{\mathbf{p}'\mathbf{p}_1'}^{\mathbf{p}\mathbf{p}_1}$ is the probability for a transition of the particles from states $\mathbf{p}$, $\mathbf{p}_1$ to states $\mathbf{p}'$, $\mathbf{p}_1'$ in the course of the electron-electron collision.

The "zeros" of the operator $\hat{L}_{\mathbf{p}}$ [see (2.12)] mean that the result of the application of this operator to an arbitrary function $\Psi_{\mathbf{p}}$ remains invariant with respect to the transformation

$$\hat{L}_{\mathbf{p}}\Psi_{\mathbf{p}}=\hat{L}_{\mathbf{p}}\left[\Psi_{\mathbf{p}}+C_1\frac{\partial\bar{F}_{\mathbf{p}}}{\partial n_0}+C_2\frac{\partial\bar{F}_{\mathbf{p}}}{\partial T}\right], \tag{2.14}$$

where C_1 and C_2 are arbitrary constants.

We now rewrite Eq. (2.6) in a form convenient for carrying out an iterative procedure. We use property (2.14) of the operator $\hat{L}_{\mathbf{p}}$:

$$\hat{L}_{\mathbf{p}}\left[\delta F_{\mathbf{p}}(\mathbf{q},\omega)-\frac{\delta n(\mathbf{q},\omega)}{n_0}\bar{F}_{\mathbf{p}}-\delta T(\mathbf{q},\omega)\frac{\partial\bar{F}_{\mathbf{p}}}{\partial T}\right]$$
$$=i[\omega-\mathbf{q}\mathbf{v}]\delta F_{\mathbf{p}}(\mathbf{q},\omega)+i\mathbf{q}U(\mathbf{q})\frac{\partial\bar{F}_{\mathbf{p}}}{\partial\mathbf{p}}\sum_{\mathbf{p}'}\delta F_{\mathbf{p}'}(\mathbf{q},\omega)+y_{\mathbf{p}}(\mathbf{q},\omega), \tag{2.15}$$

where we have introduced the new constants $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$. As we will see, these constants determine the fluctuations in the density and electron temperature. We have also made use of the circumstance that the equation

$$\frac{\partial\bar{F}_{\mathbf{p}}}{\partial n_0}=\frac{\bar{F}_{\mathbf{p}}}{n_0}.$$

holds for distribution function (2.9)–(2.10). We then write $\delta F_{\mathbf{p}}(\mathbf{q},\omega)$ as a power series in the parameters in (1.1):

$$\delta F_{\mathbf{p}}(\mathbf{q},\omega)=\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)+\delta F_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)+\dots \tag{2.16}$$

Since the operator $\hat{L}_{\mathbf{p}}$ is characterized in order of magnitude by the quantity $1/\tau_{\mathbf{p}}$, and since $l\approx\bar{v}\tau_{\mathbf{p}}$, the left side of Eq. (2.15) is predominant. In the first step of the iterative procedure we find

$$\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)=\frac{\delta n(\mathbf{q},\omega)}{n_0}\bar{F}_{\mathbf{p}}+\delta T(\mathbf{q},\omega)\frac{\partial\bar{F}_{\mathbf{p}}}{\partial T}. \tag{2.17}$$

The quantity $\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)$ includes both a spherically symmetric part and an antisymmetric part with respect to the variable $\mathbf{p}$.

Substituting (2.16), (2.17) into (2.15), we find an equation for $\delta F_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)$:

$$\hat{L}_{\mathbf{p}}\delta F_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)=Z_{\mathbf{p}}\{\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)\}, \tag{2.18}$$

where

$$Z_{\mathbf{p}}\{\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)\}=i[\omega-\mathbf{q}\mathbf{v}]\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)$$
$$+i\mathbf{q}U(\mathbf{q})V_0\,\delta n(\mathbf{q},\omega)\frac{\partial\bar{F}_{\mathbf{p}}}{\partial\mathbf{p}}+y_{\mathbf{p}}(\mathbf{q},\omega). \tag{2.19}$$

Since the property of the operator in (2.14) was taken into account in the first step of the iterative procedure, the function $\delta F_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)$ does not contain the "zeros" of the operator $\hat{L}_{\mathbf{p}}$ [and the same is true of all the following terms in series (2.16)]; furthermore, the function $\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)$ in (2.17) completely determines the spherically symmetric part of the solution of the original equation, (2.15). The latter conclusion follows from the circumstance that under approximation (1.2) the last term in the operator $\hat{L}_{\mathbf{p}}$ in (2.4) is predominant in the equations for the symmetric part of the solution in each step of the iterative procedure. That term yields as a result a spherically symmetric function

$$f_{\mathbf{p}}^{+}=a\bar{F}_0(\mathscr{E}_{\mathbf{p}})+b\frac{\partial\bar{F}_0(\mathscr{E}_{\mathbf{p}})}{\partial T}, \tag{2.20}$$

which has already been incorporated in (2.17). We are thus to determine from Eq. (2.18) the function which is antisymmetric with respect to $\mathbf{p}$. Using conditions (1.2), we find

$$\delta F_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)=\tau_{\mathbf{p}}Z_{\mathbf{p}}^{-}\{\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)\}. \tag{2.21}$$

On the right side of (2.21) we need to discard the term containing the frequency ω, along with the term containing the wave vector $\mathbf{q}$. For these two terms we find the following estimates from (2.19) and (2.17), using (2.9)–(2.10):

$$i[(\omega-\mathbf{q}\mathbf{v})\delta F_{\mathbf{p}}^{(0)}(\mathbf{q},\omega)]^{-}\sim i[\omega\bar{F}_{\mathbf{p}}^{1}-\mathbf{q}\mathbf{v}\bar{F}_0(\mathscr{E}_{\mathbf{p}})]$$
$$\approx i\left[\omega\tau_{\mathbf{p}}\frac{eE_0\tau_{\mathbf{p}}}{\bar{p}}-q\bar{v}\tau_{\mathbf{p}}\right]\frac{\bar{F}_0(\mathscr{E}_{\mathbf{p}})}{\tau_{\mathbf{p}}},$$

Since $eE_0\tau_{\mathbf{p}}/\bar{p}\ll 1$, this result justifies the discarding of the term containing ω. From (2.21) and (2.17) we then find the solution of Eq. (2.18) which we are seeking:

$$\delta F_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)=-i\mathbf{q}\frac{\delta n(\mathbf{q},\omega)}{n_0}\tau_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\bar{F}_0(\mathscr{E}_{\mathbf{p}}) -i\mathbf{q}\delta T(\mathbf{q},\omega)\tau_{\mathbf{p}}\mathbf{v}\frac{\partial \bar{F}_0(\mathscr{E}_{\mathbf{p}})}{\partial T} + i\mathbf{q}U(\mathbf{q})V_0\delta n(\mathbf{q},\omega)\tau_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\frac{\partial \bar{F}_0(\mathscr{E}_{\mathbf{p}})}{\partial \mathscr{E}_{\mathbf{p}}}+\tau_{\mathbf{p}}y_{\mathbf{p}}^{-}(\mathbf{q},\omega). \tag{2.22}$$

Expressions (2.16), (2.17), and (2.22) determine the solution of the original equation, (2.6), in the low-frequency, long-wave approximation. The constants $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$ which appear here should be found from the conditions under which Eq. (2.6) can be solved. These conditions are the continuity equation and the energy transport equation for the fluctuations, namely,

$$\sum_{\mathbf{p}}\hat{\mathscr{B}}_{\mathbf{p}}(\mathbf{q},\omega)\delta F_{\mathbf{p}}(\mathbf{q},\omega)=0, \tag{2.23}$$

$$\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}\hat{\mathscr{B}}_{\mathbf{p}}(\mathbf{q},\omega)\delta F_{\mathbf{p}}(\mathbf{q},\omega)=\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}y_{\mathbf{p}}(\mathbf{q},\omega). \tag{2.24}$$

In (2.23) we have used the following property of a random source:

$$\sum_{\mathbf{p}}y_{\mathbf{p}}(\mathbf{q},\omega)=0. \tag{2.25}$$

This property means that the number of particles is conserved in collisions.

Substituting the explicit expression for the operator $\hat{\mathscr{B}}_{\mathbf{p}}(\mathbf{q},\omega)$ in (2.6) into (2.23)–(2.24), we find the equations

$$-i\omega\delta n(\mathbf{q},\omega)+i\mathbf{q}\delta\mathbf{j}(\mathbf{q},\omega)=0, \tag{2.26}$$

$$-i\omega\delta\bar{\mathscr{E}}(\mathbf{q},\omega)+i\mathbf{q}\delta\mathbf{j}_{\mathscr{E}}(\mathbf{q},\omega)-e\mathbf{E}_0\delta\mathbf{j}(\mathbf{q},\omega)-en_0\mathbf{u}\delta\mathbf{E}(\mathbf{q},\omega)+\delta P(\mathbf{q},\omega)=\tilde{U}(\mathbf{q},\omega). \tag{2.27}$$

Here $\delta\mathbf{j}(\mathbf{q},\omega)$ and $\delta\mathbf{j}_{\mathscr{E}}(\mathbf{q},\omega)$ are fluctuations in the particle flux density and the energy flux density, respectively; $\mathbf{u}$ is the electron drift velocity: $\delta\bar{\mathscr{E}}(\mathbf{q},\omega)$ is a fluctuation in the average electron energy in a unit volume; $\delta P(\mathbf{q},\omega)$ is a fluctuation in the power transferred to the lattice by the electrons; and $\tilde{U}(\mathbf{q},\omega)$ is the Langevin source of energy fluctuations.

Using expressions (2.16), (2.17), and (2.22), we find the following expressions for these quantities:

$$\delta\mathbf{j}(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\delta F_{\mathbf{p}}(\mathbf{q},\omega)=\mathbf{W}\delta n(\mathbf{q},\omega) + n_0\frac{\partial\mathbf{W}_1}{\partial T}\delta T(\mathbf{q},\omega)+\tilde{\mathbf{I}}(\mathbf{q},\omega), \tag{2.28}$$

$$\delta\mathbf{j}_{\mathscr{E}}(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\delta F_{\mathbf{p}}(\mathbf{q},\omega)=\frac{B_T}{D_0}\mathbf{W}\delta n(\mathbf{q},\omega) + n_0\frac{\partial}{\partial T}\left[\frac{B_T}{D_0}\mathbf{W}_1\right]\delta T(\mathbf{q},\omega)+\tilde{\mathbf{Q}}(\mathbf{q},\omega). \tag{2.29}$$

Here we have introduced the following notation for the "complex velocities" of the excitations:

$$\mathbf{W}=\mathbf{W}_1-i\mathbf{q}\frac{N}{T}U(\mathbf{q})D_0, \tag{2.30}$$

$$\mathbf{W}_1=\mathbf{u}-i\mathbf{q}D_0 \tag{2.31}$$

and

$$D_0=\frac{1}{N}\sum_{\mathbf{p}}\frac{1}{3}v^2(\mathscr{E}_{\mathbf{p}})\tau_{\mathbf{p}}\bar{F}_0(\mathscr{E}_{\mathbf{p}}) \tag{2.32}$$

is the ordinary diffusion coefficient, $N=n_0V_0$ is the total number of particles, and the coefficient B_T is given by

$$B_T=\frac{1}{N}\sum_{\mathbf{p}}\frac{1}{3}v^2(\mathscr{E}_{\mathbf{p}})\mathscr{E}_{\mathbf{p}}\tau_{\mathbf{p}}\bar{F}_0(\mathscr{E}_{\mathbf{p}})=T^{1/2}\frac{\partial}{\partial T}[T^{3/2}D_0(T)]. \tag{2.33}$$

For the fluctuations $\delta\bar{\mathscr{E}}(\mathbf{q},\omega)$ and $\delta P(\mathbf{q},\omega)$ we have

$$\delta\bar{\mathscr{E}}(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}\delta F_{\mathbf{p}}(\mathbf{q},\omega)={}^3/_2T\delta n(\mathbf{q},\omega)+{}^3/_2n_0\delta T(\mathbf{q},\omega) \tag{2.34}$$

$$\delta P(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}\hat{I}_{\mathbf{p}}^{th}\delta F_{\mathbf{p}}(\mathbf{q},\omega)=P_0(T)\delta n(\mathbf{q},\omega) + n_0\frac{\partial P_0(T)}{\partial T}\delta T(\mathbf{q},\omega), \tag{2.35}$$

where

$$P_0(T)=\frac{1}{N}\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}\hat{I}_{\mathbf{p}}^{th}\bar{F}_0(\mathscr{E}_{\mathbf{p}}) \tag{2.36}$$

is the power transferred to the lattice by one electron in the steady state.

The Langevin sources of fluctuations of the energy $\tilde{U}(\mathbf{q},\omega)$, the particle flux $\tilde{\mathbf{I}}(\mathbf{q},\omega)$, and the energy flux $\tilde{\mathbf{Q}}(\mathbf{q},\omega)$ introduced above are given by[6]

$$\tilde{U}(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}y_{\mathbf{p}}(\mathbf{q},\omega), \tag{2.37}$$

$$\tilde{\mathbf{I}}(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}y_{\mathbf{p}}(\mathbf{q},\omega), \tag{2.38}$$

$$\tilde{\mathbf{Q}}(\mathbf{q},\omega)=\frac{1}{V_0}\sum_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\mathscr{E}_{\mathbf{p}}y_{\mathbf{p}}(\mathbf{q},\omega). \tag{2.39}$$

Although the spectral densities of the fluctuations in the hydrodynamic parameters of the electron gas, $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$, are the topics of primary interest below, it is worthwhile to briefly analyze system of equations (2.26)–(2.27) for the fluctuations $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$ themselves in certain limiting cases.

Let us assume that conditions (1.3) hold and that they are supplemented in the nonequilibrium situation by the inequality

$$qu\tau_T\ll 1. \tag{2.40}$$

Here τ_T is the relaxation time of the electron temperature, which arises in Eq. (2.27) in a natural way:

$$\frac{1}{\tau_T}=\frac{2}{3}\frac{\partial}{\partial T}[P_0(T)-e\mu_0(T)E_0^2] \tag{2.41}$$

(μ_0 is the electron mobility). In this case, we can ignore terms $\sim i\mathbf{q}\delta T(\mathbf{q},\omega)$ in the expressions for the fluctuations in the fluxes, (2.28)–(2.29), and also in (2.22). In other words, we can ignore effects of thermal diffusion and of the electron thermal conductivity. Combining Eqs. (2.26)–(2.27), we find an expression for $\delta T(\mathbf{q},\omega)$:

$$\delta T(\mathbf{q},\omega)=\frac{2\tau_T}{3n_0}\Big\{[\tilde{U}(\mathbf{q},\omega)+e\mathbf{E}_0\tilde{\mathbf{I}}(\mathbf{q},\omega)] -\mathbf{u}T\left[\frac{B_T}{D_0T}-\frac{1}{2}\right]i\mathbf{q}\delta n(\mathbf{q},\omega) -2\mathbf{u}NU(\mathbf{q})\,i\mathbf{q}\delta n(\mathbf{q},\omega)+T\left[\frac{3}{2}-\frac{B_T}{D_0T}\right]\frac{\delta n(\mathbf{q},\omega)}{\tau_M}\Big\}, \tag{2.42}$$

where

$$\frac{1}{\tau_M}=\frac{4\pi e n_0\mu_0(T)}{\varepsilon} \tag{2.43}$$

is the Maxwellian relaxation time.

The first term in braces in (2.42) is the leading term. It determines the fluctuations in the electron temperature in a spatially homogeneous nonequilibrium system, since Langevin sources do not have a space-time dispersion.[6] We also see that the size of the relative temperature fluctuations, $\delta T(\mathbf{q},\omega)/T$, is small in comparison with the size of the relative density fluctuations, $\delta n(\mathbf{q},\omega)/n_0$, because of inequalities (1.3) and (2.40). On the other hand, the dispersive part of the fluctuation, $\delta T(\mathbf{q},\omega)$ "tracks" the fluctuation $\delta n(\mathbf{q},\omega)$ exactly. However, $\delta T(\mathbf{q},\omega)$ cannot be ignored, although it is relatively small. Substitution of (2.42) and (2.28)–(2.29) leads to a renormalization of the kinetic coefficients $D_0(T)$, $\mu_0(T)$, etc., and also of the Langevin sources. For $\tilde{D}_{\alpha\beta}(T)$, for example we have

$$\tilde{D}_{\alpha\beta}(T)=D_0(T)\left[1+\frac{2}{3}\tau_T eE_0^2\frac{\partial\mu_0}{\partial T}\left(\frac{B_T}{D_0T}-\frac{1}{2}\right)\delta_{\alpha z}\right]\delta_{\alpha\beta}, \tag{2.44}$$

where we have assumed $\mathbf{E}_0=\mathbf{e}_zE_0$. It is important to note that this renormalized diffusion coefficient can be introduced only under conditions (1.3) and (2.40). The operator expression found in Ref. 3 for the "nonequilibrium" diffusion coefficient,

$$\tilde{D}_{\alpha\beta}(T)=\frac{1}{N}\sum_{\mathbf{p}}v_\alpha\hat{L}_{\mathbf{p}}^{-1}(v_\beta-u_\beta)\bar{F}_{\mathbf{p}}, \tag{2.45}$$

which gives us (2.44) when the operator is inverted, should thus also be supplemented with these conditions. A renormalization of the diffusion coefficient arises in a similar way in the problem of a drift electrical instability.[10]

Using (2.42), we find the following result for the density fluctuation from continuity equation (2.26):

$$\delta n(\mathbf{q},\omega)=-\frac{i\mathbf{q}\delta\tilde{\mathbf{i}}(\mathbf{q},\omega)}{1/\tilde{\tau}_M+\tilde{D}_{\alpha\beta}q_\alpha q_\beta-i(\omega-\mathbf{qu})}, \tag{2.46}$$

where

$$\delta\tilde{\mathbf{i}}(\mathbf{q},\omega)=\tilde{\mathbf{I}}(\mathbf{q},\omega)+\frac{2}{3}\tau_T\mathbf{E}_0[\tilde{U}(\mathbf{q},\omega)+e\mathbf{E}_0\tilde{\mathbf{I}}(\mathbf{q},\omega)]\frac{\partial\mu_0}{\partial T}, \tag{2.47}$$

$$\frac{1}{\tilde{\tau}_M}=\frac{1}{\tau_M}\left[1+\frac{4}{3}\tau_T eE_0^2\left(\frac{\mathbf{q}\mathbf{E}_0}{qE_0}\right)^2\frac{\partial\mu_0}{\partial T}\right]. \tag{2.48}$$

The last term in (2.42) did not contribute to the Doppler frequency shift of the fluctuation in terms of the parameter $\tau_T/\tau_M\ll 1$.

Expression (2.46) was derived in Refs. 3 and 4 by a modified Chapman–Enskog procedure. That procedure was based on a simplified version of transformation (2.14), in which only the first of Eqs. (2.12) was taken into account. Accordingly, it can be seen from the discussion above that the algorithm proposed in Refs. 3 and 4 for studying the hydrodynamic fluctuations implicitly assumes that conditions (1.3) and (2.40) hold. Those conditions determine the range of applicability of that algorithm. The physical meaning here is that effects of the electron thermal conductivity are being ignored, as are cross effects in the fluctuating fluxes of particles and energy.

In the opposite situation, in which conditions (1.4) hold, along with the inequality

$$qu\tau_M\ll 1, \tag{2.49}$$

we can completely ignore the density fluctuations, i.e., the terms $\sim\delta n(\mathbf{q},\omega)$, in all the expressions for $\delta F_{\mathbf{p}}(\mathbf{q},\omega)$ [(2.16), (2.17), and (2.22)], in balance equations (2.26) and (2.27), and in the expressions for $\delta\mathbf{j}(\mathbf{q},\omega)$, $\delta\mathbf{j}_{\mathscr{E}}(\mathbf{q},\omega)$, $\delta\overline{\mathscr{E}}(\mathbf{q},\omega)$ and $\delta P(\mathbf{q},\omega)$ which appear in them. The effect is to substantially simplify the problem. However, we cannot ignore in all the expressions, the contribution from fluctuations $\delta\mathbf{E}(\mathbf{q},\omega)$ of the self-consistent electric field, despite the fact that $\delta\mathbf{E}(\mathbf{q},\omega)$ and $\delta n(\mathbf{q},\omega)$ are proportional according to (2.7). The reason is that the contribution of the self-consistent terms to balance equation (2.27) is proportional to the ratio $\delta n(\mathbf{q},\omega)/\tau_M$. The fluctuation $\delta n(\mathbf{q},\omega)$ itself, found from continuity equation (2.26), is proportional to the small parameter τ_M, which cancels out upon substitution into (2.27). From (2.26) we find an expression for $\delta n(\mathbf{q},\omega)$:

$$\delta n(\mathbf{q},\omega)=-\tau_M\left[i\mathbf{q}\tilde{\mathbf{I}}(\mathbf{q},\omega)+i\mathbf{q}n_0\frac{\partial\mathbf{W}_1}{\partial T}\delta T(\mathbf{q},\omega)\right]. \tag{2.50}$$

We see that the relative size of the density fluctuation, $\delta n(\mathbf{q},\omega)/n_0$, is small, along with relative size of the fluctuation $\delta T(\mathbf{q},\omega)/T$, in terms of the parameters in (1.4) and (2.49). In contrast with the preceding case, the fluctuations in the density track the fluctuations in the temperature in this case. Using (2.50), we find the electron temperature fluctuation from energy balance equation (2.27):

$$\delta T(\mathbf{q},\omega)=\frac{\delta\tilde{e}(\mathbf{q},\omega)}{1/\tau_T+{}^2/_3\varkappa_0q^2+{}^4/_3(\mathbf{q}\mathbf{E}_0/qE_0)^2eE_0^2\partial\mu_0/\partial T-i(\omega-{}^2/_3\alpha_0\mathbf{qu})}, \tag{2.51}$$

where

$$\delta\tilde{e}(\mathbf{q},\omega)=\frac{2}{3n_0}\Big[\tilde{U}(\mathbf{q},\omega)+e\mathbf{E}_0\tilde{\mathbf{I}}(\mathbf{q},\omega) +i\mathbf{q}\left(\frac{B_T}{D_0}\tilde{\mathbf{I}}(\mathbf{q},\omega)-\tilde{\mathbf{Q}}(\mathbf{q},\omega)\right)-2e\left(\mathbf{E}_0\frac{\mathbf{q}}{q}\right)\left(\tilde{\mathbf{I}}(\mathbf{q},\omega)\frac{\mathbf{q}}{q}\right)\Big], \tag{2.52}$$

and $\varkappa_0$ is the electron thermal conductivity per particle. It can be written in the form[11]

$$\varkappa_0=D_0\left[\gamma_0+(\gamma_0-1)\frac{\partial\ln\gamma_0}{\partial\ln T}+\frac{3}{2}\right],\quad \gamma_0=\frac{\partial\ln D_0(T)}{\partial\ln T},\tag{2.53}$$

$$\alpha_0=\frac{\varkappa_0}{D_0}-\gamma_0.\tag{2.54}$$

Expressions (2.46) and (2.51) completely determine the hydrodynamic fluctuations in the two limiting situations defined by inequalities (1.3), (2.40) and (1.4), (2.49). Long-wave, low-frequency fluctuations of other physical quantities can be found under these approximations with the help of expressions (2.16)–(2.17) and (2.22) for $\delta F_p(\mathbf{q},\omega)$.

In general, for arbitrary ω and $\mathbf{q}$, crossover effects begin to play an important role in Eqs. (2.26)–(2.29). Correspondingly, a simple relationship between $\delta T(\mathbf{q},\omega)$ and $\delta n(\mathbf{q},\omega)$ as in (2.42) or (2.50) does not hold, and the results do not reduce to a renormalization of the kinetic coefficients in the expressions for the basic fluctuating quantities. In this case the fluctuation spectra have some qualitatively new features even under thermodynamically equilibrium conditions.

We make use of the known[12] relationship between the correlation function

$$\langle\delta A(\mathbf{q},\omega)\delta B^*(\mathbf{q}'.\ \omega')\rangle$$

of the fluctuations in the Fourier components of arbitrary quantities $\delta A(\mathbf{q},\omega)$ and $\delta B(\mathbf{q},\omega)$, on the one hand, and the spectral density $(\delta A\delta B)_{\mathbf{q}\omega}$ of the fluctuation process,

$$\langle\delta A(\mathbf{q},\omega)\delta B^*(\mathbf{q}',\omega')\rangle=\frac{(2\pi)^4}{V_0}(\delta A\delta B)_{\mathbf{q}\omega}\delta(\omega-\omega')\delta(\mathbf{q}-\mathbf{q}'),\tag{2.55}$$

on the other. We also note that the operations of taking an average and applying an arbitrary linear operator $\hat{T}(\mathbf{q},\omega)$ to the fluctuating quantities commute:

$$\begin{aligned}&\langle\hat{T}(\mathbf{q},\omega)\delta A(\mathbf{q},\omega)\hat{T}^*(\mathbf{q}',\omega')\delta B^*(\mathbf{q}',\omega')\rangle\\&=\hat{T}(\mathbf{q},\omega)\hat{T}^*(\mathbf{q}',\omega')\langle\delta A(\mathbf{q},\omega)\delta B^*(\mathbf{q}',\omega')\rangle,\end{aligned}\tag{2.56}$$

We can then immediately write equations for the spectral densities of hydrodynamic fluctuations. From (2.26)–(2.31) and (2.34)–(2.35) we find the system of equations

$$\begin{aligned}&(\delta\tilde{n}^2)_{\mathbf{q}\omega}[\omega_1^2+\nu_M^2]-2\,\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}[\omega_1q_1-\nu_Mq_2]\\&\quad+2\,\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}\\&\quad\times[\omega_1q_2+\nu_Mq_1]+(\delta\tilde{T}^2)_{\mathbf{q}\omega}[q_1^2+q_2^2]=(\mathscr{I}^2)_{\mathbf{q}\omega},\end{aligned}\tag{2.57}$$

$$\begin{aligned}&(\delta\tilde{n}^2)_{\mathbf{q}\omega}[q_3^2+q_4^2]+2\,\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}[\omega_2q_3-\nu_Tq_4]+2\,\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}\\&\quad\times[\omega_2q_4+\nu_Tq_3]+(\delta\tilde{T}^2)_{\mathbf{q}\omega}[\omega^2+\nu_T^2]=(\mathscr{P}^2)_{\mathbf{q}\omega},\end{aligned}\tag{2.58}$$

$$\begin{aligned}&(\delta\tilde{n}^2)_{\mathbf{q}\omega}[\omega_1q_3-\nu_Mq_4]+\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}[(\omega_1\omega_2+\nu_M\nu_T)\\&\quad-(q_1q_3+q_2q_4)]\\&\quad+\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}[(\omega_1\nu_T-\omega_2\nu_M)-(q_1q_4-q_2q_3)]\\&\qquad-(\delta\tilde{T}^2)_{\mathbf{q}\omega}[\omega_2q_1-\nu_Tq_2]=-(\mathscr{R}^2)_{\mathbf{q}\omega},\end{aligned}\tag{2.59}$$

$$\begin{aligned}&(\delta\tilde{n}^2)_{\mathbf{q}\omega}[\omega_1q_4+\nu_Mq_3]-\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}[(\omega_1\nu_T-\omega_2\nu_M)\\&\quad+(q_1q_4-q_2q_3)]\\&\quad+\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}[(\omega_1\omega_2+\nu_M\nu_T)+(q_1q_3+q_2q_4)]\\&\qquad+(\delta\tilde{T}^2)_{\mathbf{q}\omega}[\omega_2q_2+\nu_Tq_1]=-(\mathscr{S}^2)_{\mathbf{q}\omega}.\end{aligned}\tag{2.60}$$

Here we have introduced

$$\delta\tilde{n}(\mathbf{q},\omega)=\frac{\delta n(\mathbf{q},\omega)}{n_0},\quad \delta\tilde{T}(\mathbf{q},\omega)=\frac{\delta T(\mathbf{q},\omega)}{T},\tag{2.61}$$

$$\omega_1=\omega-\mathbf{qu},\quad \omega_2=\omega-\frac{2}{3}\left(\frac{\varkappa_0}{D_0}+\gamma_0^2\right)\mathbf{qu},\tag{2.62}$$

$$q_1=(\gamma_0-1)\mathbf{qu},\quad q_2=\gamma_0D_0q^2,\tag{2.63}$$

$$q_3=-\frac{2}{3}\left[\gamma_0+1+2\frac{N}{T}U(\mathbf{q})\right]\mathbf{qu},\quad q_4=-\frac{2}{3}\gamma_0\nu_M,\tag{2.64}$$

$$\nu_M=\frac{1}{\tau_M}+D_0q^2,\quad \nu_T=\frac{1}{\tau_T}+\frac{2}{3}\left[\frac{\varkappa_0}{D_0}+\gamma_0^2\right]D_0q^2.\tag{2.65}$$

The spectral densities of the Langevin sources on the right sides of Eqs. (2.57)–(2.60) are linear combinations of the spectral densities of the original sources, (2.37)–(2.39). The latter can be found easily from the correlation functions which have been found for them.[6] As a result we find

$$(\mathscr{I}^2)_{\mathbf{q}\omega}=\frac{2}{N}D_0q^2,\tag{2.66}$$

$$(\mathscr{P}^2)_{\mathbf{q}\omega}=\frac{4}{3N}\nu_{\mathscr{E}},\tag{2.67}$$

$$(\mathscr{R}^2)_{\mathbf{q}\omega}=\frac{4}{3N}\gamma_0D_0q^2,\tag{2.68}$$

$$(\mathscr{S}^2)_{\mathbf{q}\omega}=\frac{4}{3N}\mathbf{qu},\tag{2.69}$$

where

$$\nu_{\mathscr{E}}=\nu_T+\frac{1}{\tau_{\mathscr{E}}}-\frac{1}{\tau_T},\quad \frac{1}{\tau_{\mathscr{E}}}=\frac{2}{3}\frac{P_0(T)}{T-T_0},\tag{2.70}$$

and T_0 is the reservoir temperature.

We first note that the "crossover" spectral density of the fluctuations, $(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}$, is a complex quantity. Its real and imaginary parts can be found by making use of the symmetry properties of the correlation functions.[12] We find

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=\frac{1}{2}[(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}+(\delta\tilde{T}\delta\tilde{n})_{\mathbf{q}\omega}],\tag{2.71}$$

$$\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=-\frac{i}{2}[(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}-(\delta\tilde{T}\delta\tilde{n})_{\mathbf{q}\omega}].\tag{2.72}$$

Equations (2.57)–(2.60) are solved in the Appendix. We move on to an examination of the solutions found there.

3. THERMODYNAMIC EQUILIBRIUM

Setting $\mathbf{E}_0=0$ and $T=T_0$, we find $\tilde{D}_{\mathbf{q}}=D_0$, $\Sigma_{\mathbf{q}}(0)=0$, from (A1)–(A12), since we have $\tau_T=\tau_{\mathscr{E}}$ and $R(T)=0$. For this spectral densities of the fluctuations in the electron density and temperature we find

$$(\delta\tilde{n}^2)_{\mathbf{q}\omega}=\frac{2}{N}D_0q^2\frac{\omega^2+\nu_T\nu_T^0}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2},\tag{3.1}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}\omega}=\frac{4}{3N}\frac{\omega^2\nu_T+\nu_M^2\nu_T^0}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2},\tag{3.2}$$

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=\frac{4}{3N}\gamma_0D_0q^2\frac{\omega^2-\nu_M\nu_T^0}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2},\tag{3.3}$$

$$\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=0,\tag{3.4}$$

where

$$\nu_T^0=\nu_T-{}^2/{}_3\gamma_0D_0q^2.\tag{3.5}$$

The first conclusion which we draw from (3.1)–(3.4) is that incorporating the cross correlation of the fluctuations

$\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$ leads to spectra of a non-Lorentzian shape. In this sense, γ_0 is a critical parameter. If the diffusion coefficient D_0 is independent of the temperature, we have $\gamma_0 = 0$, and the spectra are

$$(\delta\tilde{n}^2)_{\mathbf{q}\omega} = \frac{2}{N} D_0 q^2 \frac{1}{\omega^2+\nu_M^2}, \tag{3.6}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}\omega} = \frac{4}{3N} \nu_T^0 \frac{1}{\omega^2+(\nu_T^0)^2}, \tag{3.7}$$

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=0, \quad \mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=0. \tag{3.8}$$

According to (3.8), the condition $\gamma_0 = 0$ is thus equivalent to the absence of a cross correlation. It is also interesting to note that spectra (3.6) and (3.7) are found from general expressions (3.1)–(3.2), respectively, in limiting cases (1.3) and (1.4).

In the spatially homogeneous situation ($\mathbf{q}\to 0$), there are no fluctuations of the density, and the only nonzero spectral density is that of the temperature fluctuations:

$$(\delta\tilde{T}^2)_{\omega} = \frac{4}{3N} \frac{\frac{1}{\tau_T}}{\omega^2+1/\tau_T^2}. \tag{3.9}$$

Yet another interesting feature of spectra (3.1)–(3.4) follows from the frequency sum rules which these spectra satisfy. The intensity of the fluctuations of some arbitrary physical quantity, integrated over the frequency spectrum, $(\delta A^2)_{\mathbf{q}}$, is given by

$$(\delta A^2)_{\mathbf{q}} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (\delta A^2)_{\mathbf{q}\omega} d\omega. \tag{3.10}$$

From this expression we find

$$(\delta\tilde{n}^2)_{\mathbf{q}} = \frac{1}{N} \frac{D_0 q^2}{1/\tau_M + D_0 q^2}, \tag{3.11}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}} = \frac{2}{3N}, \tag{3.12}$$

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}}=0, \quad \mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}}=0. \tag{3.13}$$

The results in (3.11)–(3.13) do not depend on whether the condition $\gamma_0 = 0$ holds, nor do they depend on the specific relations among the parameters τ_T, τ_M, ω and L_T, L_M, and $\mathbf{q}$. They also hold for the spatially homogeneous situation (with $\mathbf{q} = 0$). Under equilibrium conditions, the effect of the cross correlation of the fluctuations in the electron density and temperature thus reduces to one of simply redistributing the fluctuation intensity over the spectrum, i.e., changing the frequency dependence of the intensity. The integral intensities (which are numerically equal to the areas under the spectral curves), in contrast, remain the same and are independent of the space-time parameters τ_T and L_T, which determine the energy dynamics of the electron system.

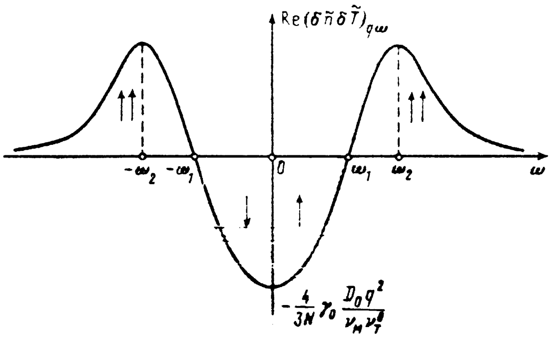

FIG. 1. Schematic diagram of the real part of the "crossover" spectral density of fluctuations in the electron density and in the electron temperature at thermodynamic equilibrium.

We also note some aspects of the crossover spectral densities of the fluctuations in (3.3)–(3.4). It follows from (3.3) that the real part of the spectral density is (first) an alternating-sign function of the frequency and (second) nonmonotonic. Figure 1 shows a rough sketch of this spectrum. The basic parameters of the spectral curve are

$$\omega_1=(\nu_M\nu_T^0)^{1/2}, \tag{3.14}$$

$$\omega_2=\{\nu_M\nu_T^0+[\nu_M\nu_T^0(\nu_M^2+\nu_T^2+2\nu_M\nu_T^0)]^{1/2}\}^{1/2}. \tag{3.15}$$

If $\gamma_0 < 0$, the curve in Fig. 1 should be the mirror image in the frequency axis. In the frequency interval $-\omega_1 < \omega < \omega_1$ the spectral density is negative, while outside this interval it is positive. The meaning here is that there exist regions of correlation and anticorrelation of the fluctuations $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$. In the first region, the signs of the fluctuations $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$ are the same, on the average, while in the second region they are opposite. The situation is shown schematically by the arrows in Fig. 1. According to the first of conditions (3.13), the area under the curve in the region of negative values of the spectral density is exactly equal to the area under the curve in the region of positive values.

4. SPECTRAL DENSITY OF TRANSVERSE FLUCTUATIONS

Let us examine the spectra of nonequilibrium hydrodynamic fluctuations with wave vectors $\mathbf{q}$ transverse to the pump electric field: $\mathbf{q}\perp\mathbf{E}_0$. In this case, general expressions (A3)–(A7) simplify substantially, and we find from (A1)–(A2)

$$(\delta\tilde{n}^2)_{\mathbf{q}\omega}^{\perp} = \frac{2}{N} D_0 q^2 \frac{\omega^2+\nu_M\nu_T^0-{}^2/_9\gamma_0^2 D_0 q^2 R(T)/NT^2}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2}, \tag{4.1}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}\omega}^{\perp} = \frac{4}{3N} \frac{\omega^2\nu_T+\nu_M^2\nu_T^0-(R(T)/3NT^2)(\omega^2+\nu_M^2)}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2}, \tag{4.2}$$

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}^{\perp} = \frac{4}{3N}\gamma_0 D_0 q^2 \frac{\omega^2-\nu_M\nu_T^0-(R(T)/3NT^2)\nu_M}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2}, \tag{4.3}$$

$$\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}^{\perp} = \frac{4}{3N}\gamma_0 D_0 q^2 \frac{(R(T)/3NT^2)\omega}{(\omega^2-\nu_M\nu_T^0)^2+\omega^2(\nu_M+\nu_T)^2}. \tag{4.4}$$

For fluctuations of this symmetry, there is usually a similarity between the nonequilibrium spectra with $\mathbf{q}\perp\mathbf{E}_0$ and the equilibrium spectra (with an arbitrary orientation of $\mathbf{q}$). In the case at hand, in contrast, the nonequilibrium nature of this system leads to a dramatic violation of this similarity, as can be seen from (4.1)–(4.4). The reason for this violation is an additional kinetic correlation, caused by the electron-electron interaction. Under equilibrium conditions, we would have $R(T) = 0$, and this additional correla-

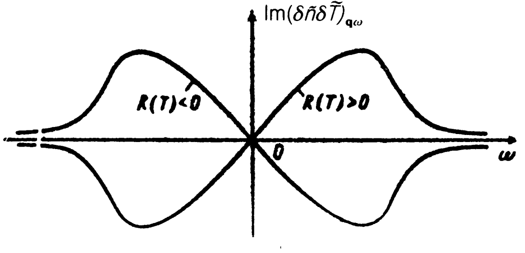

FIG. 2. Frequency dependence of the imaginary part of the "cross" spectral density under nonequilibrium condition, for fluctuations with a wave vector $\mathbf{q}\perp\mathbf{E}_0$.

tion would not exist. Depending on the sign of $R(T)$, this additional correlation may either increase the fluctuation intensity [if $R(T)<0$] or reduce it [if $R(T)>0$]. It can be shown[3] that the sign of $R(T)$ is given by

$$\operatorname{sign}[R(T)]=\operatorname{sign}\left[\frac{(T-T_0)/T_0}{\partial\ln T/\partial\ln E_0^2}-1\right]. \tag{4.5}$$

When the sign of $R(T)$ is changed, the imaginary part of the cross spectral density in (4.4) also changes sign. Figure 2 shows a rough sketch of the frequency dependence of this imaginary part.

The additional correlation also affects the integral intensity of the fluctuations. From (3.10) and (4.1)–(4.4) we find the following expressions for the integral intensities:

$$(\delta\tilde{n}^2)_{\mathbf{q}}{}^{\perp}=\frac{1}{N}\frac{D_0q^2}{1/\tau_M+D_0q^2}\left[1-\frac{2}{9}\gamma_0{}^2\frac{R(T)}{NT^2}\frac{D_0q^2}{\nu_T{}^0(\nu_M+\nu_T)}\right], \tag{4.6}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}}{}^{\perp}=\frac{2}{3N}\left[\left(1+\frac{2}{9}\gamma_0{}^2\frac{R(T)}{NT^2}.\frac{D_0q^2}{\nu_T{}^0(\nu_M+\nu_T)}\right)-\frac{R(T)}{3NT^2}\frac{1}{\nu_T{}^0}\right] \tag{4.7}$$

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}}{}^{\perp}=-\frac{2}{9N}\frac{R(T)}{NT^2}\gamma_0D_0q^2\frac{1}{\nu_T{}^0(\nu_M+\nu_T)}, \tag{4.8}$$

$$\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}}^{\perp}=0. \tag{4.9}$$

Comparison of (4.6) with (3.11) shows that the additional correlation leads to a renormalization of the integral intensity of the fluctuations in the electron density. It follows from (4.7) and (3.12) that the renormalization is accompanied by the appearance of a spatial dispersion of the integral intensity of the fluctuations in the electron temperature; there is no such dispersion in the equilibrium case. The last term in (4.7) also determines the contribution of the additional correlation in the spatially homogeneous situation (with $\mathbf{q}=0$):

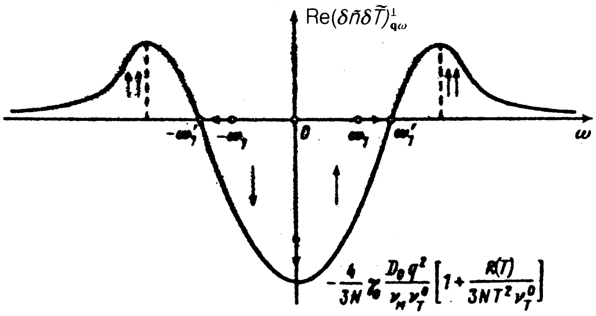

FIG. 3. Sketch of the nonequilibrium spectrum $\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})^{\perp}_{\mathbf{q}\omega}$ for $R(T)>0$.

$$(\delta\tilde{T}^2)_{\mathbf{q}=0}=\frac{2}{3N}\left[1-\frac{R(T)}{3NT^2}\tau_T\right]. \tag{4.10}$$

The most noticeable change is in the cross spectral densities. We see from (4.3) and (4.8) that there is a shift of the frequency ω_1 in the $\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})^{\perp}_{\mathbf{q}\omega}$ spectrum; this frequency is now

$$\omega_1'=\omega_1\left(1+\frac{R(T)}{3\nu_T{}^0\nu_MNT^2}\right)^{1/2}. \tag{4.11}$$

There is also a change in the extreme value of the intensity at $\omega=0$. The situation is shown qualitatively in Fig. 3.

For the spectrum $\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})^{\perp}_{\mathbf{q}\omega}$ the integral intensity is again zero, as under equilibrium conditions. For the nonequilibrium system, this assertion is valid only for fluctuations with $\mathbf{q}\perp\mathbf{E}_0$, the only fluctuations which we have been discussing in this section of the paper.

5. CASES IN WHICH FLUCTUATIONS IN EITHER THE ELECTRON DENSITY OR THE TEMPERATURE ARE PREDOMINANT

General expressions (A1)–(A7) take a particularly simple form in the limiting cases defined by inequalities (1.3), (2.40), and (1.4), (2.49). These are the cases in which the fluctuations in the density and the electron temperature, respectively, are predominant. These cases were studied in Refs. 3, 4 and 5, 6; we will cite the final expressions here. A comparison of these expressions with the general results leads to a better understanding of intermediate cases.

For the case defined by inequalities (1.3), (2.40) we have

$$(\delta\tilde{n}^2)_{\mathbf{q}\omega}=\frac{2}{N}\frac{[\tilde{D}_{\mathbf{q}}-\Sigma_{\mathbf{q}}(0)]q^2}{(\omega-\mathbf{qu})^2+(1/\tilde{\tau}_M+\tilde{D}_{\mathbf{q}}q^2)^2}, \tag{5.1}$$

$$(\delta\tilde{n}^2)_{\mathbf{q}}=\frac{1}{N}\frac{[\tilde{D}_{\mathbf{q}}-\Sigma_{\mathbf{q}}(0)]q^2}{1/\tilde{\tau}_M+\tilde{D}_{\mathbf{q}}q^2}. \tag{5.2}$$

These expressions can also be found directly from (2.46) by using (2.55). It can be seen from (5.1)–(5.2) that with $\mathbf{q}\perp\mathbf{E}_0$ the nonequilibrium spectra in this limiting case have exactly the same form as the equilibrium spectra for $\mathbf{E}_0=0$ and are Lorentzian. This is not true in the general case, as was shown above.

For the situation defined by conditions (1.4), (2.49), we find from the general expressions, or directly from (2.51)–(2.52),

$$(\delta\tilde{T}^2)_{\mathbf{q}\omega}=\frac{4}{3N}$$
$$\times\frac{\nu_T{}^0-R(T)/3NT^2}{(\omega-{}^2/{}_3\alpha_0\mathbf{qu})^2+[\nu_T{}^0+{}^4/{}_3(\gamma_0-1)(\mathbf{qE}_0/qE_0)^2e\mu_0E_0{}^2/T]^2}, \tag{5.3}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}}=\frac{2}{3N}\frac{\nu_T{}^0-R(T)/3NT^2}{\nu_T{}^0+{}^4/{}_3(\gamma_0-1)(\mathbf{qE}_0/qE_0)^2e\mu_0E_0{}^2/T}. \tag{5.4}$$

An additional correlation arises in the spatially homogeneous spectrum (with $\mathbf{q}=0$):

$$(\delta \tilde{T}^2)_\omega = \frac{4}{3N} \frac{1/\tau_T - R(T)/3NT^2}{\omega^2 + 1/\tau_T^2}. \qquad (5.5)$$

The integral intensity in this case is the same as (4.10). The spatially homogeneous spectrum is Lorentzian and is given by (5.5), regardless of the relations among τ_T, τ_M, and ω.

In summary, in the first case, after the kinetic stage of the evolution of the fluctuations has given way to the hydrodynamic stage, there is initially a rapid relaxation to thermal equilibrium ("rapid" at the hydrodynamic time scale). Then comes a slow stage of a diffusion-Maxwellian relaxation of the perturbation of the electron density, $\delta \tilde{n}(\mathbf{q},\omega)$. In the expressions for the fluctuation of the distribution function, (2.16)–(2.17) and (2.22), we can ignore terms $\sim i\mathbf{q}\delta T(\mathbf{q},\omega)$. This process is a stochastic electron-density wave, as we have already mentioned, and the spectrum of its fluctuations is described by (5.1) and (5.2).

In the second case, in contrast, the hydrodynamic stage of the evolution of the fluctuations begins with a rapid relaxation to a state which is homogeneous in terms of the electron density. Then comes a slow relaxation of the electron temperature, which constitutes a stochastic temperature wave. The spectral density of the fluctuations is described by expressions (5.3) and (5.4). In the expressions for the fluctuation of the distribution function, $\delta F_\mathbf{p}(\mathbf{q},\omega)$, we can ignore terms $\sim \delta n(\mathbf{q},\omega)$ except for the term associated with the Maxwellian relaxation, as was pointed out earlier.

It follows from this discussion that in each of the limiting cases considered here the electron subsystem has a single important degree of freedom in the stage of the hydrodynamic evolution of the fluctuations. The fluctuation processes associated with the second degree of freedom adiabatically track the processes associated with this independent degree of freedom. This subordination of the degrees of freedom leads to (first) a renormalization of the relaxation parameters and (second) a Lorentzian shape of the spectral densities of the hydrodynamic fluctuations. This is always the case for linear systems with a single degree of freedom.

6. SCATTERING OF LIGHT BY HYDRODYNAMIC FLUCTUATIONS

The theory of low-frequency, long-wave fluctuations derived here can be used to study the scattering of light by a nonequilibrium semiconductor electron gas. We have in mind a scattering which is controlled by electron collisions,[4] in which the light is scattered by macroscopic fluctuations of the electron density and the electron temperature, which in turn determine a fluctuation of the electron component of the dielectric constant.

This problem is the subject of an extensive literature, because of both the fundamental and applied importance of the phenomenon. The scattering of light by condensed media has been studied most comprehensively in the well-known series of monographs in Refs. 14–18, where various mechanisms and sources for the electron scattering of light were studied. An approach based on the Chapman–Enskog procedure, similar to the procedure of Ref. 4, was used in Ref. 19. A theory was derived there for quasielastic electron scattering of light in a heavily doped semiconductor by fluctuations of the energy density and momentum of the carriers and by fluctuations of the spin density. In each of these cases, however, only one of the listed mechanisms was important, so the situation which prevailed was one in which there was a single fluctuation degree of freedom. The collision-controlled scattering of light by free electrons in semiconductors, in which fluctuations of the electron density play a leading role, was studied theoretically in Ref. 4. This scattering was observed experimentally in Ref. 20. There has been no previous study of electron scattering of light in semiconductors under conditions such that fluctuations in both the electron density and the electron temperature play an important role.

A quantitative characteristic of the scattering properties of a medium is the differential scattering cross section

$$d^2\sigma(\mathbf{k}_s, \Omega_s)/dOd\Omega_s,$$

which is defined as the ratio of $d^2\mathscr{E}_\mathbf{n}(\Omega_s)$, which is the fraction of the energy which is scattered by the system per unit time into a solid angle dO in the frequency interval $d\Omega_s$ along the direction of the inner vector $\mathbf{n} = \mathbf{k}_s|k_s$, to the time average of the Poynting vector of the incident wave, $\mathbf{S}$ (Ref. 21):

$$\frac{d^2\sigma(\mathbf{k}_s, \Omega_s)}{dO\, d\Omega_s} = \frac{d^2\mathscr{E}_\mathbf{n}(\Omega_s)}{|\langle \mathbf{S}\rangle|\, dO\, d\Omega_s}, \qquad (6.1)$$

where $\mathbf{k}_s$ and Ω_s are the wave vector and frequency of the scattered electromagnetic wave.

If a monochromatic electromagnetic plane wave of frequency Ω_i and wave vector $\mathbf{k}_i$:

$$\vec{\mathscr{E}}_i(\mathbf{r}, t) = \vec{\mathscr{E}}_0 \exp[-i(\Omega_i t - \mathbf{k}_i\mathbf{r})], \qquad (6.2)$$

is incident on the scattering medium, then we have

$$|\langle \mathbf{S}\rangle| = \frac{c}{8\pi}\mathscr{E}_0^2, \qquad (6.3)$$

where c is the velocity of light.

The energy $d^2\mathscr{E}_\mathbf{n}(\Omega_s)$ can be calculated by the method of retarded potentials.[21] In an electron gas with a fluctuating electron density $\delta n(\mathbf{r},t)$ and a fluctuating temperature $\delta T(\mathbf{r},t)$, the field of the incident wave in (6.2) gives rise to a fluctuating current

$$\delta \mathbf{j}_i(\mathbf{r}, t) = \left[\frac{\partial\sigma(\Omega_i)}{\partial n_0}\delta n(\mathbf{r}, t) + \frac{\partial\sigma(\Omega_i)}{\partial T}\, \delta T(\mathbf{r}, t)\right]\vec{\mathscr{E}}_i(\mathbf{r}, t), \qquad (6.4)$$

where $\sigma(\Omega_i)$ is the high-frequency conductivity of the nonequilibrium electron gas. It is assumed here that the following conditions hold:

$$\mathscr{E}_0 \ll E_0, \quad \omega \ll \Omega_i,\ \Omega_s. \qquad (6.5)$$

The fluctuation response in (6.4) serves as a source in the wave equation and causes a fluctuation in the vector potential of the scattered wave, $\delta\mathbf{A}_s$. The magnitude of this fluctuation in the wave zone is[21]

$$\delta\mathbf{A}_s(R_0, t) = \frac{1}{R_0 c}\int \delta\mathbf{j}_i\left(\mathbf{r}, t - \frac{R_0}{c} + \frac{\mathbf{rn}}{c}\right) d\mathbf{r}. \qquad (6.6)$$

Here R_0 is the distance from the scattering volume to the point at which the radiation is detected. The wave-zone approximation requires that the distance R_0 be far greater than the characteristic dimensions of the scattering volume and also far greater than the wavelength of the scattered light.

Expression (6.6) can be used to establish the relationship between the monochromatic harmonics of the fluctuations of the vector potential and the electron density and temperature. Multiplying this expression by $\exp(i\Omega_s t)$, and integrating over time, we find

$$\delta\mathbf{A}_s(R_0,\Omega_s)=\frac{V_0}{R_0 c}\vec{\mathscr{E}}_0 \times\exp(ik_sR_0)\left[\frac{\partial\sigma(\Omega_i)}{\partial n_0}\delta n(\mathbf{q},\omega)+\frac{\partial\sigma(\Omega_i)}{\partial T}\delta T(\mathbf{q},\omega)\right], \tag{6.7}$$

where we have used Fourier transforms as in (2.2), and where we have introduced

$$\omega=\Omega_s-\Omega_i,\quad \mathbf{q}=\mathbf{k}_s-\mathbf{k}_i, \tag{6.8}$$

$$k_s=\frac{\Omega_s}{c},\quad \mathbf{k}_s=k_s\mathbf{n}. \tag{6.9}$$

Equations (6.8) relate the frequencies and wave vectors of the incident and scattered waves and the frequency and wave vector of the fluctuations of the electron density and temperature.

Using (6.7), we can find the Fourier components of the electric and magnetic components for the field of the scattered wave in the wave zone:[21]

$$\delta\mathbf{H}_s(R_0,\Omega_s)=-i\frac{\Omega_s}{c}[\delta\mathbf{A}_s(R_0,\Omega_s)\mathbf{n}], \tag{6.10}$$

$$\delta\mathbf{E}_s(R_0,t)=i\frac{\Omega_s}{c}[[\delta\mathbf{A}_s(R_0,\Omega_s)\mathbf{n}]\mathbf{n}]. \tag{6.11}$$

Making use of the properties of complex random quantities,[22] we can write the average value of the Poynting vector of the scattered wave as

$$\langle\mathbf{S}_s(R_0,t)\rangle=\frac{c}{8\pi}\operatorname{Re}\langle[\delta\mathbf{E}_s(R_0,t)\delta\mathbf{H}_s^*(R_0,t)]\rangle. \tag{6.12}$$

Using spectral expansions for $\delta\mathbf{E}_s$ and $\delta\mathbf{H}_s$, we can rewrite it as

$$\langle\mathbf{S}_s(R_0,t)\rangle=\mathbf{n}\frac{c}{8\pi}\int_{-\infty}^{+\infty}(\delta\mathbf{E}_s\delta\mathbf{H}_s)_{\Omega_s}\frac{d\Omega_s}{2\pi}, \tag{6.13}$$

where $(\delta\mathbf{E}_s\delta\mathbf{H}_s)_{\Omega_s}$ is the spectral density, given by

$$\langle\delta\mathbf{E}_s(R_0,\Omega_s)\delta\mathbf{H}_s^*(R_0,\Omega_s')\rangle=2\pi(\delta\mathbf{E}_s\delta\mathbf{H}_s)_{\Omega_s}\delta(\Omega_s-\Omega_s'). \tag{6.14}$$

We then find the energy fraction $d^2\mathscr{E}_\mathbf{n}(\Omega_s)$ to be

$$d^2\mathscr{E}_\mathbf{n}(\Omega_s)=\frac{c}{8\pi}(\delta\mathbf{E}_s\delta\mathbf{H}_s)_{\Omega_s}R_0^2dO\frac{d\Omega_s}{2\pi} \tag{6.15}$$

The differential scattering cross section is

$$\frac{d^2\sigma(\mathbf{k}_s,\Omega_s)}{dOd\Omega_s}=\frac{1}{2\pi}\frac{(\delta\mathbf{E}_s\delta\mathbf{H}_s)_{\Omega_s}}{\mathscr{E}_0^2}R_0^2. \tag{6.16}$$

Substituting (6.7), (6.10), and (6.11) into (6.14), and using (6.8) and (6.9), we find the cross section for the scattering of the light, which is related to the fluctuations of the electron density and the electron temperature:

$$\begin{aligned}\frac{d^2\sigma(\mathbf{q},\omega)}{dO\,d\omega}=\frac{V_0^2}{2\pi}\frac{\Omega_i^2}{c^4}&\\ \times\sin^2\theta\Bigg[&\left|\frac{\partial\sigma(\Omega_i)}{\partial n_0}\right|^2(\delta n^2)_{\mathbf{q}\omega}+\left|\frac{\partial\sigma(\Omega_i)}{\partial T}\right|^2(\delta T^2)_{\mathbf{q}\omega}\\ &+2\operatorname{Re}\left[\frac{\partial\sigma(\Omega_i)}{\partial n_0}\frac{\partial\sigma^*(\Omega_i)}{\partial T}\right]\operatorname{Re}(\delta n\delta T)_{\mathbf{q}\omega}\\ &-2\operatorname{Im}\left[\frac{\partial\sigma(\Omega_i)}{\partial n_0}\frac{\partial\sigma^*(\Omega_i)}{\partial T}\right]\operatorname{Im}(\delta n\delta T)_{\mathbf{q}\omega}\Bigg].\end{aligned} \tag{6.17}$$

Here θ is the angle between the vectors $\mathbf{n}$ and $\vec{\mathscr{E}}_0$ [for unpolarized electromagnetic radiation, an average over all polarizations of the incident wave yields $\langle\sin^2\theta\rangle=\frac{1}{2}(1+\cos^2\vartheta)$, where ϑ is the angle between the wave vectors $\mathbf{k}_i$ and $\mathbf{k}_s$]. In addition, in (6.1) we have used the second of inequalities (6.5), and we have set $\Omega_s\approx\Omega_i$. We have also used (6.8), from which we find $d\Omega_s=d\omega$.

We can also write an expression for the high-frequency conductivity; in the limit $\Omega_i\tau_\mathrm{p}\gg1$, it is

$$\sigma(\Omega_i)=\frac{e^2n_0}{m\Omega_i^2}\left(\frac{1}{\tau}+i\Omega_i\right), \tag{6.18}$$

where

$$\frac{1}{\tau}=-\frac{2}{3}\int_0^\infty\frac{1}{\tau_\mathrm{p}(\mathscr{E})}\mathscr{E}^{3/2}\frac{d\bar{F}_0(\mathscr{E})}{d\mathscr{E}}d\mathscr{E}\Big/\int_0^\infty\mathscr{E}^{1/2}\bar{F}_0(\mathscr{E})d\mathscr{E}. \tag{6.19}$$

In this case the differential scattering cross section in (6.17) becomes

$$\begin{aligned}\frac{d^2\sigma(\mathbf{q},\omega)}{dOd\omega}=\frac{V_0^2}{2\pi}\left(\frac{e^2}{mc^2}\right)^2&\\ \times\sin^2\theta\Bigg[&(\delta n^2)_{\mathbf{q}\omega}+\frac{n_0^2}{T^2}\left(\frac{d\ln\tau}{d\ln T}\right)^2\frac{1}{(\Omega_i\tau)^2}(\delta T^2)_{\mathbf{q}\omega}\\ &+2\frac{n_0}{T}\frac{d\ln\tau}{d\ln T}\frac{1}{(\Omega_i\tau)^2}\operatorname{Re}(\delta n\delta T)_{\mathbf{q}\omega}\\ &+2\frac{n_0}{T}\frac{d\ln\tau}{d\ln T}\frac{1}{\Omega_i\tau}\operatorname{Im}(\delta n\delta T)_{\mathbf{q}\omega}\Bigg].\end{aligned} \tag{6.20}$$

By way of comparison, in the collisionless regime (as $\tau\to\infty$) the scattering cross section is determined exclusively by fluctuations of the electron density:

$$\frac{d^2\sigma(\mathbf{q},\omega)}{dO\,d\omega}=\frac{V_0^2}{2\pi}\left(\frac{e^2}{mc^2}\right)^2\sin^2\theta(\delta n^2)_{\mathbf{q}\omega}. \tag{6.21}$$

This conclusion does not mean, however, that there are no fluctuations of the electron temperature in this electron gas or that there is no mutual correlation of these fluctuations with the fluctuations in the electron density.

The situation can be seen most simply by using the moment method.[3] In the collisionless regime, the following conditions hold:

$$\omega\tau_\mathrm{p}\gg1,\quad q\bar{v}\tau_\mathrm{p}\gg1. \tag{6.22}$$

The first two terms are the leading terms in the relaxation operator $\hat{\mathscr{B}}_\mathrm{p}(\mathbf{q},\omega)$ in (2.6); the specific form of the operator $\hat{L}_\mathrm{p}$ is unimportant, since the limit $\nu/(\omega-\mathbf{qv})\to0$ should be taken in the final expressions, where $\nu \approx \widehat{L}_\mathbf{p}$ characterizes the order of magnitude of the operator $\widehat{L}_\mathbf{p}$. The fluctuation spectrum in the collisionless regime is then determined by the system of equations

$$[-i(\omega-\mathbf{qv})+\nu](\delta F_\mathbf{p}\delta F_{\mathbf{p}_1})_{\mathbf{q}\omega}{}^{+}=\bar{F}_\mathbf{p}\delta_{\mathbf{pp}_1}+\Phi_{\mathbf{pp}_1}(\mathbf{q}), \tag{6.23}$$

$$[i\mathbf{q}(\mathbf{v}-\mathbf{v}_1)+2\nu]\Phi_{\mathbf{pp}_1}(\mathbf{q})=0. \tag{6.24}$$

We then find the following expression for the spectral density of the fluctuations in the distribution function over the entire frequency range:

$$(\delta F_\mathbf{p}\delta F_{\mathbf{p}_1})_{\mathbf{q}\omega}=(\delta F_\mathbf{p}\delta F_{\mathbf{p}_1})^{+}_{\mathbf{q}\omega}+(\delta F_{\mathbf{p}_1}\delta F_\mathbf{p})^{+}_{-\mathbf{q}-\omega}=2\pi\bar{F}_\mathbf{p}\delta_{\mathbf{pp}_1}\delta(\omega-\mathbf{qv}). \tag{6.25}$$

The spectral densities $(\delta n^2)_{\mathbf{q}\omega}$, $(\delta T^2)_{\mathbf{q}\omega}$, and $(\delta n\delta T)_{\mathbf{q}\omega}$ and the integral intensities of these fluctuations in the collisionless regime are

$$(\delta\tilde{n}^2)_{\mathbf{q}\omega}=\frac{1}{N}\left(\frac{2\pi m}{T}\right)^{1/2}\frac{1}{q}\exp\left(-\frac{m\omega^2}{2Tq^2}\right), \quad (\delta\tilde{n}^2)_\mathbf{q}=\frac{1}{N}, \tag{6.26}$$

$$(\delta\tilde{T}^2)_{\mathbf{q}\omega}=\frac{1}{9}\frac{1}{N}\left(\frac{2\pi m}{T}\right)^{1/2}\frac{1}{q}\exp\left(-\frac{m\omega^2}{2Tq^2}\right)\times\left[\frac{m^2\omega^4}{T^2q^4}-2\frac{m\omega^2}{Tq^2}+5\right], \tag{6.27}$$

$$(\delta\tilde{T}^2)_\mathbf{q}=\frac{2}{3N},$$

$$(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=\frac{1}{3}\frac{1}{N}\left(\frac{2\pi m}{T}\right)^{1/2}\frac{1}{q}\exp\left(-\frac{m\omega^2}{2Tq^2}\right)\left[\frac{m\omega^2}{Tq^2}-1\right],$$

$$(\delta\tilde{n}\delta\tilde{T})_\mathbf{q}=0. \tag{6.28}$$

Under conditions (6.22), in a gas with an electron temperature, the spectra of the electron-density and temperature fluctuations are therefore Gaussian. The change from a Lorentzian spectrum to a Gaussian spectrum upon transition from the collisional regime to the collisionless regime was pointed out theoretically in Ref. 23. It has been studied experimentally in gallium arsenide.[24] The scattering of light in the collisionless regime, however, is due exclusively to the fluctuations in the electron density. In the opposite case of frequent collisions, there are components in the scattering cross section from both fluctuations in the electron density and fluctuations in the electron temperature, as well as their mutual correlation functions. In the high-frequency limit, the latter quantities contain small factors $(\Omega_i\tau)^{-1}\ll 1$ and $(\Omega_i\tau)^{-2}\ll 1$ but may nevertheless make a contribution comparable to that of the first term in (6.20); their contribution may in fact be predominant.[8] In a nonequilibrium situation, the scattering cross section in (6.20) depends on not only the magnitude of the wave vector $\mathbf{q}$ but also its orientation with respect to the external electric field $\mathbf{E}_0$. In general, this anisotropy of the scattering cross section is rather complex. It does not reduce to simply a Doppler frequency shift by an amount on the order of $\mathbf{qu}$; it is determined by expressions (A3)–(A7). In the particular case $\mathbf{q}\perp\mathbf{E}_0$, the spectrum of cross section (6.20) is determined by (4.1)–(4.4). Since the imaginary part of the mutual correlation function in (4.4), $\mathrm{Im}(\delta\tilde{n}\delta\tilde{T})^{\perp}_{\mathbf{q}\omega}$, is an odd function of the frequency, the peak of the spectral line in (6.20) is shifted away from the point $\omega = 0$ in this case. The sign and magnitude of this shift are determined by an additional kinetic correlation which is described by the function $R(T)$. This correlation-induced shift of the peak of the spectral line is a qualitatively new effect in the scattering of light by a semiconductor electron gas with an intense interparticle interaction. The experimental observation of this new effect might serve as reliable proof of the additional correlation in such a gas. It might also give us the value of the correlation tensor $\Delta_{\alpha\beta}(\omega)$ in (A9). We should stress once again that this shift is not due to a drift of the electron system (that drift is proportional to $\mathbf{qu}$), since it occurs in the case $\mathbf{q}\perp\mathbf{E}_0$. If the condition $\omega\gg\mathbf{qu}$ holds [using $q\approx k_s - k_i\approx k_i\approx 2\pi/\lambda_i = \Omega_i/c$, we can rewrite this condition as $u\ll c(\omega/\Omega_i)$], then the correlation-induced shift of the spectral line of the scattering light will be predominant for fluctuations with arbitrary wave vectors $\mathbf{q}$, provided that all the terms in (6.20) are of the same order of magnitude.

7. CONCLUSION

We have studied hydrodynamic fluctuations in a nonequilibrium semiconductor electron gas under conditions such that an electron temperature can be introduced. Fluctuating excitations in such a gas constitute coupled stochastic waves of the electron density and temperature waves. The coupling arises from the mutuals effect of the fluctuations in the electron density, $\delta n(\mathbf{q},\omega)$, and those in the electron temperature, $\delta T(\mathbf{q},\omega)$, as a result of the thermal-diffusion contribution to the particle transport equation and the diffusion-drift contribution to the energy-transport equation.

The spectra of the fluctuations in a collision-controlled regime are non-Lorentzian for arbitrary relations among the fluctuation frequency ω, the Maxwellian relaxation time τ_M, and the electron temperature τ_T and also for arbitrary relations among the fluctuation wave vector $\mathbf{q}$, the screening length L_M, and the electron cooling length L_T. The mutual spectral density $(\delta n\delta T)_{\mathbf{q}\omega}$ is on the same order of magnitude as the spectral densities of the fluctuations, $(\delta n^2)_{\mathbf{q}\omega}$ and $(\delta T^2)_{\mathbf{q}\omega}$.

It is also important to note that the spectra of hydrodynamic fluctuations in an equilibrium electron gas with intense interparticle scattering are radically different from those for a gas in which the rate of electron-electron collision is small in comparison with other kinetic relaxation frequencies. On the other hand, there are no qualitative differences between the two cases in terms of steady-state kinetic effects under approximately equilibrium conditions. Consequently, a study of the spectra of hydrodynamic fluctuations under equilibrium conditions can provide important information on the microscopic processes which occur in a semiconductor electron gas and on the rates of these processes.

Under nonequilibrium conditions, the spectra of hydrodynamic fluctuations have a component which is peculiar to nonequilibrium systems with an intense electron-electron interaction. This component leads to numerous qualitatively new features in the fluctuation spectra. The additional kinetic correlation which rises in an electron gas of this sort may either strengthen or suppress the intensity of hydrodynamic fluctuations. Because of this correlation, the similarity between the equilibrium fluctuation spectra and the nonequilibrium spectra with wave vectors $\mathbf{q} \perp \mathbf{E}_0$ is disrupted.

Either at or away from equilibrium, there is a frequency region in which the fluctuations $\delta n(\mathbf{q},\omega)$ and $\delta T(\mathbf{q},\omega)$ have the same sign, on the average (this is a correlation region), and the mutual spectral density of the fluctuations is positive $(\delta n \delta T)_{\mathbf{q}\omega}$, and there is also a region in which these fluctuations have opposite signs (this is an anticorrelation region), and we have $(\delta n \delta T)_{\mathbf{q}\omega} < 0$.

Hydrodynamic fluctuations in semiconductors have been studied previously in the limiting cases defined by inequalities (1.3) (Refs. 3 and 4) and (1.4) (Refs. 5, 6, and 8). In each of these limiting cases, fluctuations of a single type—either fluctuations in the electron density or fluctuations in the electron temperature—play the leading role. Results for each of these limiting cases can be found from the general expressions derived above. It is also worthwhile to examine hydrodynamic fluctuations in situations which are intermediate between those defined by inequalities (1.3) and (1.4). Under these conditions, the contribution of the mutual spectral densities $(\delta n \delta T)_{\mathbf{q}\omega}$ to the intensity of electron fluctuations is significant in magnitude. These mutual spectral densities in turn contain direct information on the magnitude of the additional kinetic correlation. In addition, the actual properties of the semiconductor may be such that a transition from inequalities (1.4) to inequalities (1.3) will occur with increasing external electric field $\mathbf{E}_0$. The reason would be a difference in the field dependence of the times τ_M and τ_T. Under these conditions, the only way to find a satisfactory description of the fluctuation spectra is to work from the expressions derived here.

The approach developed here for studying fluctuations in systems with several degrees of freedom can be extended with essentially no substantial changes to hydrodynamic fluctuations in an electron gas with a reduced dimensionality. Reggiani[25] has made a first attempt to do this for the particular case of a single degree of freedom, for a $2D$ gas of electrons. In real structures, however, the density of $2D$ electrons is usually fairly high, and the electron-temperature approximation is good. A situation requiring the use of the general approach would arise more easily in this case than in $3D$ systems. The scattering of light, including the scattering by free electrons, is also being studied actively in structures of this type.[18]

Experiments on the scattering of electromagnetic waves are one method for studying solid-state plasmas. In particular, these waves may be scattered by low-frequency, long-wave fluctuational excitations of an electron gas. The efficiency with which the electromagnetic radiation is scattered and the spatial-temporal dispersion are described by a differential scattering cross section. In this paper we have derived an expression for the differential cross section for the scattering of light by fluctuations in the electron density and the electron temperature. In the collisionless regime specified by conditions (6.22), the scattering cross section is determined exclusively by the electron-density fluctuations. A study of this cross section provides information on the electron temperature, according to (6.26), and it also provides direct information on the electron distribution function. The reason is that under conditions (6.22) each particle scatters light independently, and the resultant Gaussian one-particle spectrum reflects the properties of the motion of individual particles. In the other (opposite) case, the so-called collisional regime, defined by (1.1), the scattered light contains information on collective excitations in the electron gas, and one can work from the scattering cross section to find the long-wave, low-frequency binary-correlation functions, which are important characteristics of a many-body system. A qualitatively new effect in the scattering of light by a nonequilibrium electron gas is the appearance of a correlation-induced shift of the peak of the spectral line. This shift is unrelated to the drift of electrons. It is determined by the existence of an additional correlation in a gas with an intense electron-electron interaction. This additional correlation will either suppress or intensify the light scattering, depending on the sign of the function $R(T)$.

We conclude with some numerical estimates of the properties of semiconductors and the conditions under which the cross section for the scattering of light is described by expression (6.20). We are interested in a situation in which all the terms in (6.20) make contributions on the same order of magnitude. First, in order to introduce an electron temperature, we must satisfy inequalities (1.2). We thus need fairly high carrier densities; high carrier densities in turn mean small values of the Maxwellian time τ_M. However, the inequality $\tau_{\mathrm{p}} \ll \tau_M$ must hold by virtue of the very nature of the hydrodynamic approach. In order to realize a collisional regime, we must therefore select semiconductors with a low mobility. Furthermore, experiments on scattering should be carried out in such a way that the incident electromagnetic radiation passes through the plasma without undergoing any substantial absorption. Accordingly, under the inequality $\Omega_i \tau_{\mathrm{p}} \gg 1$, the conditions $\Omega_{\mathrm{p}} \ll \Omega_i < \mathscr{E}_g/\hbar$ must also be satisfied, where $\mathscr{E}_g$ is the bandgap, and $\Omega_{\mathrm{p}} = (4\pi e^2 n_0/m\varepsilon)^{1/2}$ is the plasma frequency. Specifically, we consider a semiconductor with the properties of n-type GaP: $m = 0.13 m_0$, $T_0 = 77$ K, $n_0 = 3 \cdot 10^{16}$ cm^{-3}, $\mu_{77} = 600$ cm^2/(V·s), $\mathscr{E}_g = 2.4$ eV, $\varepsilon = 10$, $\varepsilon_\infty = 8$, and $s = 5 \cdot 10^5$ cm/s. Here m_0 is the mass of a free electron, ε_∞ is the high-frequency dielectric constant, and s is the sound velocity. For pronounced heating we choose $E_0 = 5 \cdot 10^3$ V/cm; we then have $T = 300$ K, $\mu_{300} = 150$ cm^2/(V·s), $\tau_{\mathrm{p}} = 1.1 \cdot 10^{-14}$ s, $\tau_{ee} = \varepsilon^2 m^{1/2} \cdot T^{3/2}/\sqrt{2\pi} n_0 e^4 L_k = 10^{-13}$ s, $\tau_T \approx \tau_{\mathrm{p}} T_0/2ms^2 = 2 \cdot 10^{-12}$ s, $\tau_M = \varepsilon/4\pi e n_0 \mu_{300} = 1.2 \cdot 10^{-12}$ s, and $\Omega_{\mathrm{p}} = 8.6 \cdot 10^{-12}$ rad/s. The mean free path is $l_p = \bar{v} \tau_{\mathrm{p}} = 3.5 \cdot 10^{-7}$ cm, the Maxwellian length is $L_M = \sqrt{D_0 \tau_M} = 2.2 \cdot 10^{-6}$ cm, and the electron cooling length is $L_T = \sqrt{D_0 \tau_T} = 2.8 \cdot 10^{-6}$ cm (the electron diffusion coefficient here is $D_0 = 3.88$ cm^2/s). A suitable source of the electromagnetic radiation would be a He–Ne laser with a vacuum frequency $\Omega_i = 6.3 \cdot 10^{14}$ rad/s ($\hbar\Omega_i = 0.41$ eV) and a vacuum wavelength $\lambda_i = 3$ μm. We would then have $k_i = 2.1 \cdot 10^4$ cm^{-1}, and the characteristic wave vectors of the fluctuational excitations in the semiconductor would be $q \approx 2k_i \sqrt{\varepsilon_\infty} = 1.2 \cdot 10^5$ cm^{-1}. The incident light will be absorbed somewhat, because of the generation of optical phonons ($\hbar\Omega_0 = 51 \cdot 10^{-3}$ eV). Accordingly, the properties which we have selected for the semiconductor, for the light being scattered, and the nonequilibrium state of the electron gas make it possible to satisfy all the necessary conditions for observing the collective regime of scattered light and for making all the terms in expression (6.20) equally significant. Other semiconductors with mobilities which are not too high, e.g., CdS, AlP, AlSb, GaN, and p-type semiconductors, might also be suitable. In high-mobility semiconductors such as *n*-type InSb and *n*-type GaAs, a collective regime of light scattering is again possible, but the primary contribution will come from fluctuations of the electron temperature [the second term in (6.20)]. When the scattering cross section is determined in part by fluctuations in the electron temperature and in the electron density the orientational dependence of the light scattering cross section becomes significant under nonequilibrium conditions.

APPENDIX

The spectral densities of the long-frequency, low-wave fluctuations in the electron density and temperature are determined completely by Eqs. (2.57)–(2.60). Although we have to deal with fourth-order determinants in order to solve this system of equations, the resulting solution can be put in a fairly simple form, convenient for analysis, by carrying out certain transformations:

$$(\delta\tilde{n}^2)_{\mathbf{q}\omega}=\frac{\Delta_{n^2}(\mathbf{q},\omega)}{\Delta(\mathbf{q},\omega)},\quad (\delta\tilde{T}^2)_{\mathbf{q}\omega}=\frac{\Delta_{T^2}(\mathbf{q},\omega)}{\Delta(\mathbf{q},\omega)},\tag{A1}$$

$$\mathrm{Re}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=\frac{\Delta_{\mathrm{Re}}(\mathbf{q},\omega)}{\Delta(\mathbf{q},\omega)},\quad \mathrm{Im}(\delta\tilde{n}\delta\tilde{T})_{\mathbf{q}\omega}=\frac{\Delta_{\mathrm{Im}}(\mathbf{q},\omega)}{\Delta(\mathbf{q},\omega)}.\tag{A2}$$

The corresponding determinants of the system are

$$\Delta(\mathbf{q},\omega)=\Big[\omega\Big[\omega-\mathbf{qu}\Big(1+\frac{2}{3}\Big(\frac{\varkappa_0}{D_0}+\gamma_0^2\Big)\Big)\Big]$$
$$-\nu_M\Big(\frac{1}{\tau_T}+\frac{2}{3}\varkappa_0q^2\Big)$$
$$+\frac{2}{3}\Big(\frac{\mathbf{qu}}{qu}\Big)u^2\Big[\Big(1+\frac{\varkappa_0}{D_0}\Big)q^2-2\frac{\gamma_0-1}{D_0\tau_M}\Big]\Big]^2$$
$$+\Big[\omega(\nu_M+\nu_T)-\mathbf{qu}\Big[\frac{1}{\tau_T}+\frac{2}{3}\Big(\frac{\varkappa_0}{D_0}-\gamma_0\Big)\frac{1}{\tau_M}+\frac{4}{3}\varkappa_0q^2\Big]\Big]^2,\tag{A3}$$

$$\Delta_{n^2}(\mathbf{q},\omega)=\frac{2}{N}D_0q^2\Big[\nu_T^2\Big(1+\frac{\tilde{D}_{\mathbf{q}}-D_0}{\nu_T\tau_T}-\frac{\Sigma_{\mathbf{q}}(0)/2D_0}{\nu_T^2\tau_T^2}\Big)$$
$$-\frac{2}{3}\gamma_0^2D_0q^2\Big(\nu_T+\frac{R(T)}{3NT^2}\Big)+\Big[\omega-\frac{2}{3}\Big(\frac{\varkappa_0}{D_0}+\gamma_0^2\Big)\mathbf{qu}\Big]$$
$$\times\Big[\omega-\frac{2}{3}\Big(\frac{\varkappa_0}{D_0}-\gamma_0^2\Big)\mathbf{qu}\Big]\Big],\tag{A4}$$

$$\Delta_{T^2}(\mathbf{q},\omega)=\frac{4}{3N}\Big[(\omega^2+\nu_M^2)\Big(\frac{1}{\tau_T}+\frac{2}{3}\varkappa_0q^2-\frac{R(T)}{3NT^2}\Big)$$
$$+\frac{2}{3}\gamma_0^2\omega^2D_0q^2-2\omega\mathbf{qu}\Big(\frac{1}{\tau_{\mathscr{E}}}-\frac{2}{3}\gamma_0\frac{1}{\tau_M}+\frac{2}{3}\varkappa_0q^2\Big)$$
$$+(\mathbf{qu})^2\Big[\frac{1}{\tau_{\mathscr{E}}}-\frac{4}{3}\frac{1}{\tau_M}+\frac{2}{3}(\varkappa_0-D_0)q^2\Big]\Big],\tag{A5}$$

$$\Delta_{\mathrm{Re}}(\mathbf{q},\omega)=\frac{4}{3N}\Big[\gamma_0D_0q^2\Big[\omega^2-\nu_M\Big(\frac{1}{\tau_T}+\frac{2}{3}\varkappa_0q^2+\frac{R(T)}{3NT^2}\Big)\Big]$$
$$+\omega\mathbf{qu}\Big(\frac{1}{\tau_M}+\frac{1}{\tau_T}+\frac{\gamma_0-1}{\tau_{\mathscr{E}}}\Big)-(\mathbf{qu})^2\Big[\frac{1}{\tau_T}-\frac{2}{3}\gamma_0\frac{1}{\tau_M}$$
$$+(\gamma_0-1)\frac{1}{\tau_{\mathscr{E}}}+\frac{2}{3}\gamma_0(\varkappa_0-D_0)q^2\Big]\Big],\tag{A6}$$

$$\Delta_{\mathrm{Im}}(\mathbf{q},\omega)=\frac{4}{3N}\Big[\gamma_0D_0q^2\frac{R(T)}{3NT^2}\omega$$
$$-\mathbf{qu}\Big[\omega^2-D_0q^2\Big(\frac{1}{\tau_{\mathscr{E}}}+\frac{2}{3}\varkappa_0q^2\Big)$$

$$-\frac{1}{\tau_M}\Big(\frac{1}{\tau_T}-\frac{\gamma_0-1}{\tau_{\mathscr{E}}}-\frac{2}{3}(\gamma_0-2)\varkappa_0q^2$$
$$+\frac{4}{3}(\gamma_0-1)\Big(\frac{\mathbf{qu}}{qu}\Big)^2\frac{u^2}{D_0}\Big)\Big]$$
$$+(\mathbf{qu})^2\Big[1+\frac{2}{3}\Big(\frac{\varkappa_0}{D_0}+\gamma_0^2\Big)\Big]\omega-\frac{2}{3}\Big(1+\frac{\varkappa_0}{D_0}\Big)(\mathbf{qu})^3\Big].\tag{A7}$$

Here we have introduced the following notation for the component of an arbitrary vector $B_{\alpha\beta}$ along the vector **q**:

$$B_{\mathbf{q}}=\frac{B_{\alpha\beta}q_\alpha q_\beta}{q^2}.\tag{A8}$$

The quantity $\Sigma_{\alpha\beta}(0)$ is the low-frequency limit of the correlation tensor $\Delta_{\alpha\beta}(\omega)$ introduced in Ref. 3:

$$\Sigma_{\alpha\beta}(0)\equiv\Delta_{\alpha\beta}(\omega)|_{\omega\to0}=\frac{2}{3}(\gamma_0-1)^2\frac{u^2\tau_T^2}{NT^2}R(T)\delta_{\alpha z}\delta_{\alpha\beta}.\tag{A9}$$

It violates the Price fluctuation-diffusion relation:

$$(\delta j_\alpha\delta j_\beta)_\omega=\frac{n_0}{V_0}[D_{\alpha\beta}(\omega)+D_{\beta\alpha}(-\omega)-\Delta_{\alpha\beta}(\omega)-\Delta_{\beta\alpha}(-\omega)].\tag{A10}$$

The reason for this violation may be the additional kinetic correlation which stems from the electron-electron interaction. A measure of the additional correlation is the quantity $R(T)$, which was derived in Refs. 4 and 13 and which can be written in the form

$$R(T)=\sum_{\mathbf{pp}_1}\mathscr{E}_{\mathbf{p}}\mathscr{E}_{\mathbf{p}_1}\hat{I}^{ee}_{\mathbf{pp}_1}\{\bar{F},\bar{F}\}=3NT^2\Big[\frac{1}{\tau_T}-\frac{1}{\tau_{\mathscr{E}}}\Big].\tag{A11}$$

The "correlation flux" $\hat{I}^{ee}_{\mathbf{pp}_1}\{\overline{F,F}\}$ is determined by the operator in (2.13) without the one summation over $\mathbf{p}_1$.

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[1] E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics*, Part 2 (Pergamon Press, Oxford, 1980).

[2] S. V. Gantsevich, V. L. Gurevich, and R. Katilius, Phys. Cond. Matter **18**, 165 (1974).

[3] S. V. Gantsevich, V. L. Gurevich, and R. Katilius, Nuovo Cim. **2**, 1 (1979).

[4] S. V. Gantsevich, V. L. Gurevich, and R. Katilyus, Zh. Eksp. Teor. Fiz. **57**, 503 (1969) [Sov. Phys. JETP **30**, 276 (1969)].

[5] Sh. M. Kogan and A. Ya. Shul'man, Zh. Eksp. Teor. Fiz. **56**, 862 (1969) [*sic*].

[6] A. Ya. Shul'man and Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **57**, 2112 (1969) [Sov. Phys. JETP **30**, 1146 (1969)].

[7] S. V. Gantsevich, V. L. Gurevich, and R. Katilyus, Zh. Eksp. Teor. Fiz. **59**, 533 (1970) [Sov. Phys. JETP **32**, 291 (1970)].

[8] Sh. M. Kogan and V. D. Shadrin, Fiz. Tekh. Poluprovodn. **5**, 2224 (1971) [Sov. Phys. Semicond. **5**, 1942 (1971)].

[9] S. Chapman and T. G. Cowling, *Mathematical Theory of Non-Uniform Gases*, Cambridge, Oxford, 1953.

[10] Yu. V. Gulyaev, Fiz. Tekh. Poluprovodn. **3**, 246 (1969) [Sov. Phys. Semicond. **3**, 203 (1969)].

[11] A. V. Volkov and Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **52**, 1647 (1967) [Sov. Phys. JETP **25**, 1095 (1967)].

[12] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Part I, Nauka (Pergamon, New York, 1980).

[13] A. Ya. Shul'man, Fiz. Tverd. Tela (Leningrad) **12**, 1181 (1970) [Sov. Phys. Solid State **12**, 922 (1970)].

[14] M. Cardona (editor), *Light Scattering in Solids I: Introduction to Con-*

cepts, 2nd edition, Springer, Berlin, 1982.

[15] M. Cardona and G. Gruntherodt (editors), *Light Scattering in Solids II: Basic Concepts and Introduction*, Springer, Berlin, 1982.

[16] M. Cardona and G. Gruntherodt (editors), *Light Scattering in Solids III: Recent Results*, Springer, Berlin, 1982.

[17] M. Cardona and G. Gruntherodt (editors), *Light Scattering in Solids IV: Electronic Scattering, Spin Effects, SERC, and Morphic Effects*, Springer, Berlin, 1984.

[18] M. Cardona and G. Gruntherodt (editors), *Light Scattering in Solids V: Superlattices and Other Microstructures*, Springer, Berlin, 1989.

[19] V. A. Voĭtenko and I. P. Ipatova, Zh. Eksp. Teor. Fiz. **97**, 224 (1990) [Sov. Phys. JETP **70**, 125 (1990)].

[20] L. L. Akatov, S. V. Gantsevich, R. Katilyus *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 633 (1978) [JETP Lett. **27**, 598 (1978)].

[21] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Nauka, Moscow, 1988, Ch. IX (earlier editions have been published in English translation by Pergamon, New York).

[22] S. M. Rytov, *Introduction to Statistical Radiophysics. Part I*, Nauka, Moscow, 1976, Ch. VI.

[23] P. Hertel and J. Appel, Phys. Rev. B **26**, 5730 (1982).

[24] D. Abramson, K. Tsen, and R. Bray, Bull. Am. Phys. Soc. **27**, 336 (1982).

[25] L. Reggiani, *The Physics of Submicron Structures* (eds. H. L. Grubin, D. K. Ferry, and C. Jacoboni), Plenum, New York, 1986, p. 323.

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