Theory of polarization processes which occur as a fast charged particle crosses a plasma–vacuum interface

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(Submitted 16 March 1992)
Zh. Eksp. Teor. Fiz. **102**, 841–851 (September 1992)

The crossing of a plasma–vacuum interface by a fast nonrelativistic charged particle is analyzed. The analysis is carried out for crossings in both directions. The temporal and spatial evolution of the potential and energy of the particle and of the surface charge and space charge which the particle induces is analyzed in the model of a cold plasma. The transformation of the image charge into a wakefield charge is studied.

As it passes through a material medium, a fast charged particle excites oscillations of the charge density behind itself.[1-3] These wakefields and the particle energy losses associated with their excitation have been studied widely for a variety of media, particularly plasmas.[3-10] Wakefields have recently reattracted interest because of the development of new methods for accelerating particles.[11-13]

In most studies of wakefields it has been assumed that the medium is unbounded. The wakefields are excited as the particle enters the medium, or they disappear when the particle leaves the medium, because of a number of transient polarization processes which occur near the interface. Among these processes, the excitation of surface oscillations[14,15] and the associated additional energy loss have been studied previously. In connection with the development of new particle acceleration methods, numerical calculations[16] have recently determined the distance from the sharp plasma boundary at which the amplitude of the wakefield excited by an ultrarelativistic particle reaches the same level as in an unbounded medium.

In the present study we have attempted a more comprehensive investigation of the polarization processes which arise as a fast but nonrelativistic charged particle crosses a plasma–vacuum interface, in both directions. A theory for the polarization processes is derived in the model of a cold electron plasma. Expressions are derived for the scalar potential, the surface charge density, the total charge on the surface, and the change in the energy of the particle which occurs as the interface is crossed. A wakefield space charge arises because of an "overflow" of induced surface charge. The latter oscillates at the plasma frequency and screens the wakefield of the particle outside the plasma. The effect of the interface on the energy of the particle is not simply the effect of the surface-oscillation field; it is instead determined by the effects of various near-surface fields. One result of this situation is that the particle does not lose energy but instead acquires energy as it crosses the interface from the vacuum into the plasma.

1. A nonrelativistic particle with a charge Q is moving at a velocity u along the z axis, which is directed perpendicular to the interface between two media. In the time interval $-\infty < t < 0$ the particle is in the first medium ($z<0$), while in the time interval $\infty > t > 0$ it is in the second medium ($z>0$). Each of the media has only a temporal dispersion; the respective dielectric constants are $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$. Working from the equation $\operatorname{div}\mathbf{D} = 4\pi\rho_0$, where $\rho_0 = Q\delta(z-ut)\delta(\mathbf{r}_\perp)$, and the continuity conditions at $z=0$ for the normal component of the magnetic induction and the tangential component of the electric field, we find the following expression for the potential φ ($\mathbf{E} = -\nabla\varphi$)

$$\varphi(\varkappa;\omega;z)=\begin{cases}\dfrac{Qu}{2\pi^2(\omega^2+\varkappa^2u^2)\varepsilon_1(\omega)}\left[\exp\left(\dfrac{i\omega z}{u}\right)+\dfrac{\varepsilon_1(\omega)-\varepsilon_2(\omega)}{\varepsilon_1(\omega)+\varepsilon_2(\omega)}\exp(\varkappa z)\right], & z<0\\[2ex] \dfrac{Qu}{2\pi^2(\omega^2+\varkappa^2u^2)\varepsilon_2(\omega)}\left[\exp\left(\dfrac{i\omega z}{u}\right)+\dfrac{\varepsilon_2(\omega)-\varepsilon_1(\omega)}{\varepsilon_2(\omega)+\varepsilon_1(\omega)}\exp(-\varkappa z)\right], & z>0\end{cases}\tag{1}$$

where $\varkappa$ and ω are variables which correspond to Fourier expansions in $\mathbf{r}_\perp$ and t. The first terms in square brackets in expressions (1) correspond to the potential of the particle in an unbounded medium, while the second terms correspond to the potential which arises because of the interface.

The expression for the electric field vector which follows from (1) is the same as that given in Refs. 17 and 18 if we take the limit $u \ll c$ in the latter. The expression for the electric field corresponding to (1) was analyzed for the case without dispersion in Ref. 19.

2. We can use (1) to study the temporal and spatial evolution of the potential only if the functions $\varepsilon_{1,2}(\omega)$ are known. In the case of a trial particle whose velocity u is large in comparison with the average electron velocities of the medium, v_T, it is common to use the plasma dielectric constant[4]

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\nu)},\tag{2}$$

where ω_p is the plasma frequency, and ν is the effective electron collision rate. (For applications to continuous media, it is also assumed, in our macroscopic approach, that the length of the plasma wave is much greater than the interatomic distances.)

Let us take a brief look at the known results for an infinite medium with dielectric constant (2). In the limit $\nu \ll \omega_p$, the potential set up by the particle is[4] ($r = |\mathbf{r}_\perp|$)

$$\Phi_0(r;z;t)\equiv\frac{\varphi_0}{Qk_p}=\frac{1}{k_p(r^2+\xi^2)^{1/2}}-W_2(k_p|\xi|;k_pr)+2\theta(-\xi)\exp(\gamma\xi)\sin(k_p\xi)K_0(k_pr),\tag{3}$$

where $\xi = z - ut$, $k_p = \omega_p/u$, $\gamma = \nu/2u$, $K_0(x)$ is the modified Bessel function, $\theta(x)$ is the unit step function $[\theta(0) = \frac{1}{2}]$,

$$W_2(a;\, b) = \int_0^\infty \frac{dx}{x^2+1} \exp(-ax) J_0(bx), \tag{4}$$

and $J_0(bx)$ is a Bessel function.

The function W_2 falls off monotonically with increasing $|\xi|$ and also with increasing r.

Its properties are examined in more detail in Appendix 1.

Figure 1 shows equipotentials calculated from (3) and (4). The results of corresponding calculations[6-9] for a plasma with electron thermal motion in the limit $u \gg v_T$ agree well with the results in Fig. 1.

The first term in (3) corresponds to the Coulomb potential. The meaning of the other terms can be seen easily by examining the induced charge density:

$$\rho' = -\rho_0 - \Delta\varphi/4\pi = Qk_p\theta(-\xi)\delta(\mathbf{r}_\perp)\exp(\gamma\xi)\sin(k_p\xi). \tag{5}$$

We see that a charge arises only behind the particle, and only on its wake. Since the charge density in (5) vanishes at $\xi = 0$, fringing potentials [the second term on the right in (3)] arise near the particle ($k_p|\xi| < 1$, $|k_p r < 1$). Far behind the particle ($k_p\xi < -1$), v a weakfield potential [the last term in (3)] corresponds to the charge density in (5).

There are two points to note here. First, an induced charge arises only on the line $\mathbf{r}_\perp = 0$ in our model as the trial particle moves. This result means that the electron fluid away from this line behaves as an incompressible fluid ($\operatorname{div} \mathbf{v} = 0$, where $\mathbf{v}$ is the fluid flow velocity). When the thermal motion is taken into account, the charge density is smeared over a scale $\sim v_T/\omega_p$. Second, the total wakefield charge induced on the line $\mathbf{r}_\perp = 0$ is equal to the charge of the particle but with the opposite sign ($-Q$). It follows that as a particle enters a neutral medium a charge equal to the charge of the particle arises away from the wake of the particle. In particular, some calculations which have been carried out show that as a particle moves along the axis of a plasma cylinder an induced charge Q moves along the surface of the cylinder.

3. We now consider the process by which a particle approaches the plane interface of a medium with a dielectric constant ε_2 as given by (2) from the vacuum side ($\varepsilon_1 = 1$). For the potential from (1) we find, for $t < 0$,

$$\Phi(r;\, z;\, t) \equiv \frac{\varphi}{Qk_p}$$

$$= \begin{cases} \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - \dfrac{1}{2^{1/2}} W_2(k_0\eta;\, k_0 r), & z<0 \\[2ex] \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - \dfrac{1}{2^{1/2}} W_2(k_0\xi;\, k_0 r), & z>0 \end{cases}, \tag{6}$$

where $\eta = |z| + u|t|$, $k_0 = \omega_0/u$, and $\omega_0 = \omega_p/2^{1/2}$ is the frequency of the surface oscillations of the electrons. Expression (6) contains an image potential along with the Coulomb potential. Although this potential is not a Coulomb potential in vacuum ($z < 0$), as it would be in the electrostatic case,[10] it does depend on the variable η, which specifies the distance from the observation point to a point in the medium ($z > 0$) at the same distance as the particle from the interface.

Figure 2 shows equipotentials $\Phi = \text{const}$ for the case of a particle at a distance $z = k_p^{-1}$ from the interface. The image potential is set up by a surface charge with a density

$$\sigma(r;\, t) = \frac{1}{4\pi}\left\{\frac{\partial\varphi}{\partial z}\bigg|_{z=-0} - \frac{\partial\varphi}{\partial z}\bigg|_{z=+0}\right\}$$

$$= -\frac{Qk_0^2}{2\pi} W_1(\omega_0|t|;\, k_0 r), \tag{7}$$

where $W_1(a;b) = -\partial W_2(a;b)/\partial a$. It is easy to see that the total charge induced at the surface is independent of the time and is equal to $-Q$. The surface density of this charge evolves in the following way according to (7). If the particle is far from the surface ($u|t| \gg k_0^{-1}$), then we have $\sigma/Qk_0^2 \approx -u|t|/2\pi k_0^2(u^2t^2 + r^2)^{3/2}$. This formula is the same as the known expression for the charge density in the electrostatic case.[10] The potential is also close to that in the electrostatic case. If $u|t| < k_0^{-1}$, and if the particle is near the interface, then the charge density distribution in (7) differs from the electrostatic distribution. In particular, at the time at which the particle crosses the interface ($t = 0$) we have $\sigma/Qk_0^2 = -K_0(k_0 r)/2\pi$ according to (7), and the surface charge density is concentrated in a region with a

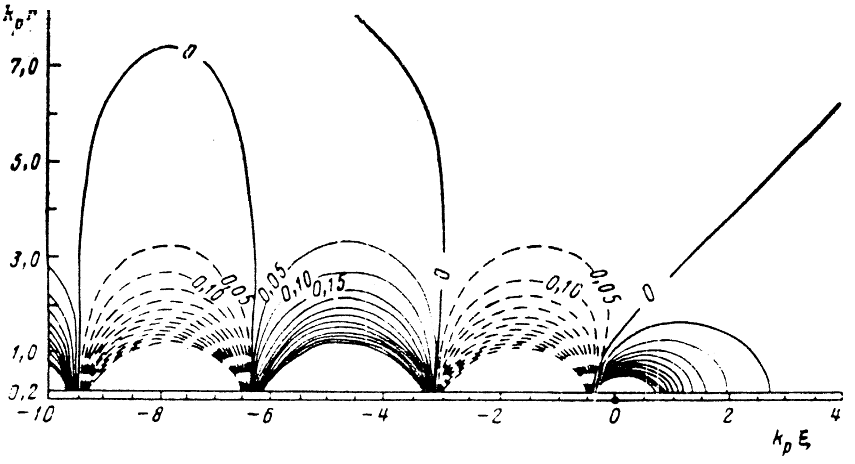

FIG. 1. Contour curves of the function Φ_0 in the r, ξ plane. The particle is moving from left to right. The interval between contour lines is 0.05. Solid lines—$\Phi_0 \geq 0$; dashed lines—$\Phi_0 < 0$. Along the scale of the variable r, the calculation begins at the value 0.2. Here $\gamma = 0$.

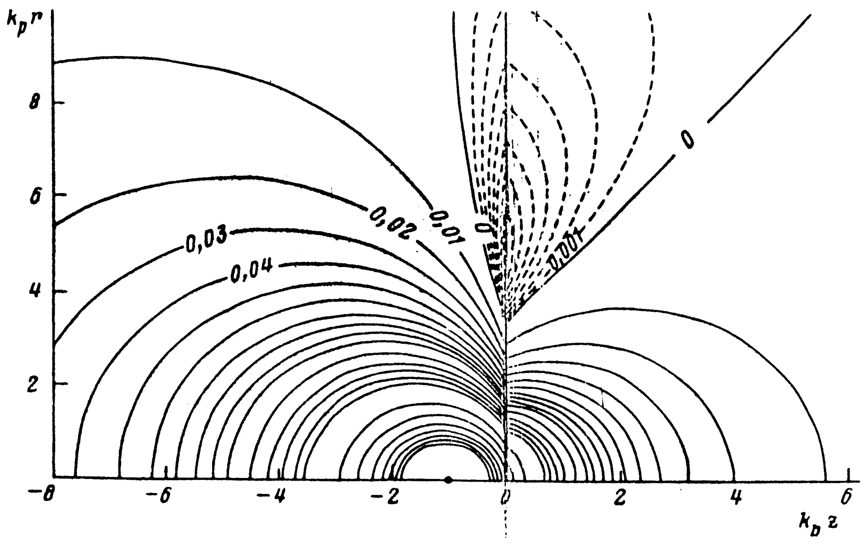

FIG. 2. Contour curves of Φ = const for a particle in vacuum at a distance $a = k_p|z| = 1$ from the interface. Solid lines—$\Phi \geqslant 0$; dashed lines—$\Phi < 0$. The interval between lines, $\Delta\Phi$, is 10^{-3} at negative values of Φ, while at $\Phi > 0$ this interval varies. It is 0.01 for $0 < \Phi \leqslant 0.1$, 0.02 for $0.1 \leqslant \Phi \leqslant 0.2$, and 0.1 for $0.2 \leqslant \Phi \leqslant 0.9$.

radius $r \lesssim k_0^{-1}$. For the same situation in the electrostatic case, there is a surface charge density only at the point $r = 0$ $[\sigma/Qk_0^2 = -2\delta(k_0^2 r^2)/\pi]$.

As the particle approaches the plasma interface, the charge density on the surface becomes redistributed. This redistribution results not from a surface current but from an inflow (or outflow) of electron fluid to the surface from the interior. It is determined by the velocity component $v_z(t;r;z = +0) = (e/mu)\varphi(t;r;z = +0)$. From (6) we find, at $t = 0$,

$$v_z(0;r;z{=}+0) = \frac{Qek_0}{mu}\left\{\frac{1}{k_0 r} - \frac{\pi}{2}[I_0(k_0 r) - L_0(k_0 r)]\right\}. \quad (8)$$

where I_0 and L_0 are, respectively, the Bessel and Struve functions of imaginary argument. The function in (8) changes sign with increasing r. At small values of r, for $Q < 0$, the velocity is positive; it goes off to infinity as $r \to 0$. With increasing r, the velocity decreases, and it goes negative at $r \gtrsim k_0^{-1}$. At large values of r, the velocity is proportional to $-(k_0 r)^{-3}$, and it approaches zero with increasing r.

As it approaches the interface, the particle is evidently attracted by the image field and accelerated. To determine the energy ΔW_1 acquired by the particle, we use expression (6) to find the electric field component $E_z = -\partial\varphi/\partial z$ at the position of the particle ($r = 0$, $z = ut$). This field determines the force which is acting on the particle; the integral of this field over time, over the interval $-\infty < t < 0$, determines the energy increment. After some straightforward calculations we find

$$\Delta W_1 = {}^1/_4\pi Q^2 k_0. \quad (9)$$

4. After the particle enters the plasma ($t > 0$) the potential is, according to (1),

$$\Phi(r;z;t) = \frac{\varphi}{Qk_p} = \begin{cases} \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - \dfrac{1}{2^{1/2}}[W_2(k_0|\xi|;k_0 r) \\ \quad + 2e^{-\nu t/2}\sin(\omega_0 t)W_1(k_0|z|;k_0 r)], \quad z<0 \\ \Phi_0 + W_2(k_p\eta;k_p r) + 2e^{-\nu t/2}\sin(\omega_p t)W_1(k_p z;k_p r) \\ \quad - \dfrac{1}{2^{1/2}}[W_2(k_0\eta;k_0 r) + 2e^{-\nu t/2}\sin(\omega_0 t)W_1(k_0 z;k_0 r)], \\ z>0, \end{cases} \quad (10)$$

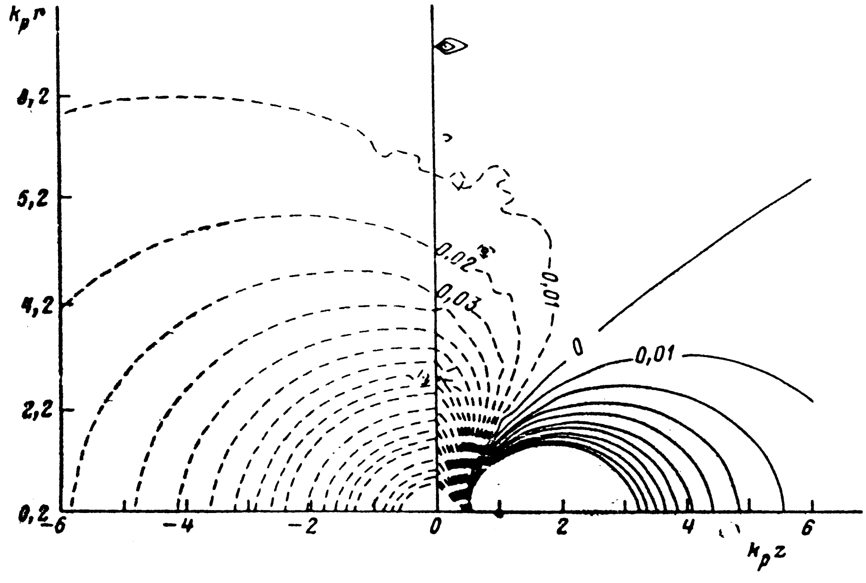

FIG. 3. Contour curves of Φ = const for a particle in the plasma at a distance $z = k_p^{-1}$ from the interface.

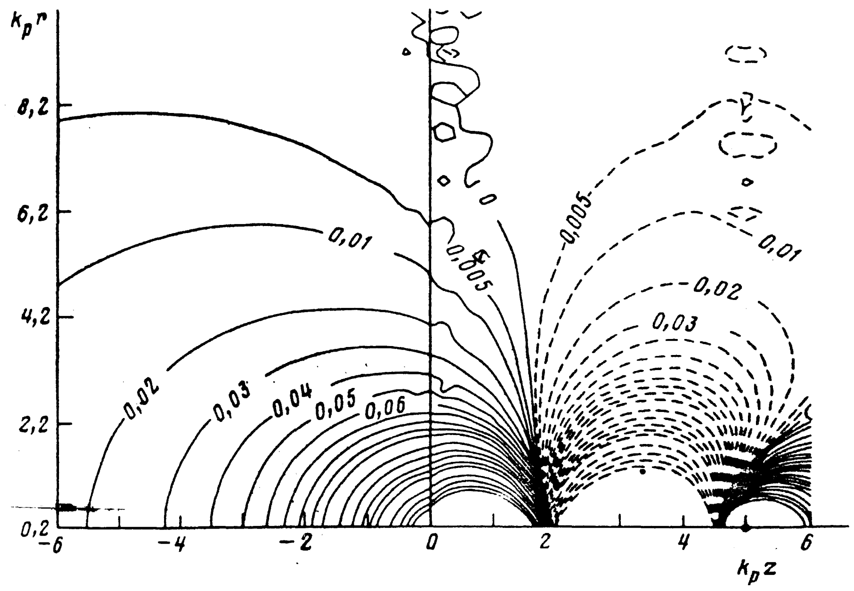

FIG. 4. Contour curves for a particle in the plasma at a distance $z = 5k_p^{-1}$ from the interface.

where Φ_0 is given by (3). Figures 3 and 4 show equipotentials for two positions of the particle with respect to the interface.

All the terms except the first in expressions (10) are associated with the presence of the interface between the plasma and the vacuum. These terms are nonzero near the interface, over a length scale $z \lesssim k_0^{-1}$. They can be divided into two groups, which differ in their behavior in time. The terms proportional to the function W_2 are zero except in the time interval $t \lesssim \omega_0^{-1}$. The other terms, which are proportional to the function W_1, describe temporal oscillations at the frequencies of surface and plasma oscillations, which are damped because of dissipation.

To determine the meaning of the individual terms in (10), we find the surface charge density:

$$\sigma(r;t)=\frac{Qk_p^2}{2\pi}\left\{-\frac{1}{2}W_1(\omega_0 t;k_0 r)+W_1(\omega_p t;k_p r)\right.$$
$$-e^{-\nu t/2}\cos(\omega_p t)K_0(k_p r)$$
$$+e^{-\nu t/2}\sin(\omega_0 t)\left[\frac{\pi}{2}(I_0(k_0 r)-L_0(k_0 r))-\frac{1}{k_0 r}\right]$$
$$\left.-e^{-\nu t/2}\sin(\omega_p t)\left[\frac{\pi}{2}(I_0(k_p r)-L_0(k_p r))-\frac{1}{k_p r}\right]\right\}. \tag{11}$$

Expression (11) contains terms which differ in their contribution to the overall surface charge. The two last terms describe damped oscillations at respectively the surface frequency ω_0 and the plasma frequency ω_p. The total charge over the entire surface is zero in these oscillations (Appendix 2; these are "dipole" oscillations). For the oscillations at ω_0, the spatial variation of the surface charge is the same as that of the velocity component v_z of the electron fluid, (8), at the time $t = 0$.

The first term in (11) becomes Eq. (7). As the particle moves away from the interface, the corresponding charge density falls off monotonically in time at any fixed value of r. The second term in braces is zero at $t = 0$, while at $\omega_p t > 1$ it describes plasma oscillations. These two terms contribute to the total surface charge, which varies in the following way as the particle crosses the interface:

$$Q_\sigma'(t)=2\pi\int_0^\infty r\sigma(r;t)\,dr=-Q+Q\theta(t)\left[1-e^{-\nu t/2}\cos(\omega_p t)\right]. \tag{12}$$

As the particle moves into the interior of the plasma, a charge $Q_v'(t)$ is induced. Using definition (5) and expressions (10), we find

$$Q_v'(t)=2\pi\int_0^{ut}dz\int_0^\infty r\rho'(r;z;t)\,dr=Q\theta(t)\left[e^{-\nu t/2}\cos(\omega_p t)-1\right]. \tag{13}$$

It can be seen from (12) and (13) that the resultant charge induced in the plasma at each instant, $Q_v'(t) + Q_\sigma'(t)$, is equal to the charge of the particle with the opposite sign. As time elapses, this charge becomes redistributed. The charge at the surface disappears over a time $t \geqslant 2/\nu$, and the charge in the wakefield of the particle increases. No actual "overflow" of charge occurs here, of course. What actually happens is that the field of the particle, which at $t < 0$ confines the charge to the plasma surface, ceases to confine it at $t > 0$, but it induces a charge in the interior of the plasma. The magnitude of the charge induced in the interior oscillates at the plasma frequency; the surface charge oscillates at the same frequency.

Let us examine the change in the energy of a particle due to the near-surface fields (the customary polarization loss, which is determined by the potential Φ_0, is being ignored). From (10) we find the force acting on a particle and the corresponding change in the energy over a time $\infty > t > 0$:

$$\Delta W_2={}^3/_4\pi Q^2(k_p-k_0). \tag{14}$$

According to (9) and (14), the total change in the energy of the particle due to the near-surface fields $[\Delta W_1 + \Delta W_2 = \frac{1}{4}\pi Q^2 k_p (3-2^{1/2}) > 0]$ is positive; this positive sign corresponds to an acceleration of the particle. The field attracting the particle toward the plasma interface at $t<0$ is thus greater than the corresponding field at $t>0$. If we consider the effect on the particle of only those fields which have wave numbers k_0 and frequencies ω_0 which are characteristic of surface oscillations [the terms in (9) and (14) proportional to k_0], we find that the change in the energy of the particle is $\Delta W_s = -\frac{1}{2}\pi Q^2 k_0$. This result agrees with the result of Ref. 16, where the energy loss associated with the excitation of surface waves was studied.

5. Corresponding calculations have been carried out for the case in which the particle crosses the plasma–vacuum interface in the opposite direction, i.e., from the plasma into vacuum [ε_1 is given by expression (2), and $\varepsilon_2 = 1$]. It follows from (1) that the potential of the particle while in the plasma ($t<0$) is

$$\Phi \equiv \frac{\varphi}{Qk_p} = \begin{cases} \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - W_2(k_p|\xi|;k_p r) \\ +2\theta(-\xi)e^{\gamma\xi}\sin(k_p\xi)K_0(k_p r) \\ +W_2(k_p\eta;k_p r) - \dfrac{1}{2^{1/2}}W_2(k_0\eta;k_0 r), & z<0, \\ \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - \dfrac{1}{2^{1/2}}W_2(k_0\eta;k_0 r), & z>0. \end{cases} \tag{15}$$

If the particle has left the plasma ($t>0$), we have

$$\Phi = \begin{cases} \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - \dfrac{1}{2^{1/2}}W_2(k_0|\xi|;k_0 r) + 2e^{\gamma\xi}\sin(k_p\xi)K_0(k_p r) \\ +2e^{-\nu t/2}\sin(\omega_p t)W_1(k_p|z|;k_p r) - 2^{1/2}e^{-\nu t/2}\sin(\omega_0 t) \\ \times W_1(k_0|z|;k_0 r). & z<0, \\ \dfrac{1}{k_p(r^2+\xi^2)^{1/2}} - \dfrac{1}{2^{1/2}}W_2(k_0\eta;k_0 r) - 2^{1/2}e^{-\nu t/2}\sin(\omega_0 t) \\ \times W_1(k_0 z;k_0 r), & z>0. \end{cases} \tag{16}$$

From (15) and (16) we can find the densities of the induced surface charge and space charge u, integrating them over the surface and over the volume, respectively, and we can also determine the total charge:

$$Q_\sigma' = Q\theta(t)[e^{-\nu t/2}\cos(\omega_p t) - 1],$$

$$Q_\rho' = -Q + Q\theta(t)[1 - e^{-\nu t/2}\cos(\omega_p t)]. \tag{17}$$

We see that the space charge disappears in an oscillatory manner (oscillating in time), while the surface charge arises in the same manner.

The energy loss of the particle as it crosses the interface is

$$\Delta W = \begin{cases} -{}^1/_4\pi Q^2(k_p - k_0). & -\infty<t<0 \\ -{}^3/_4\pi Q^2 k_0, & 0<t<+\infty \end{cases} . \tag{18}$$

As in the case in which the particle enters the plasma, the energy loss due to the excitation of surface waves is $\Delta W_s = -\frac{1}{2}\pi Q^2 k_0$ (Ref. 15).

From (18) we see that there are also losses determined by the boundary conditions on the wakefield ($\Delta W_v = -\frac{1}{4}\pi_1 Q^2 k_p$).

As a particle passes through a plasma slab of thickness L greater than u/ν, the effect of each of the interfaces on the field of the particle and on the energy loss can be treated independently. According to (9), (14), and (18), we can then add a term $\Delta W_f = -\pi Q^2 k_0 (1-2^{-1/2})$, associated with the boundaries, to the polarization law. This term is smaller in magnitude by a factor of $(1-2^{-1/2})^{-1}$ than that which follows from the results of Ref. 15.

6. As a fast charged particle enters a plasma from vacuum, the polarization processes thus develop in the following way. As it approaches the plasma interface, the particle induces at the surface of the plasma a charge of the opposite sign, equal in absolute value to its own charge, Q. As the particle comes nearer the interface, the region in which the surface charge is localized shrinks, and at the time the particle crosses the interface the size of this region is on the order of u/ω_0, where ω_0 is the frequency of the surface oscillations. The redistribution of charge density stems from fluxes of electrons to the plasma surface from the interior and in the opposite direction.

After the particle enters the plasma, a dynamic screening of the Coulomb field of the particle arises. A wakefield charge oscillating at the plasma frequency ω_p is induced over a distance u/ω_p behind the particle. The total magnitude of this wakefield charge depends on the distance which the particle has traversed in the plasma. If damping is ignored, this charge varies periodically from 0 to $-2Q$. As the particle moves, the electric field near the interface thus oscillates at the frequency ω_p, causing electrons to move either toward the plasma interface or away from it. One might say that the wakefield excites oscillations of the surface charge which completely screen the wakefield in vacuum, where this field is zero. As the particle moves a distance $\sim u/\nu$ into the plasma, the wakefield at the interface disappears, as do the oscillations of the surface charge at the plasma frequency. The total charge induced in the wakefield medium becomes equal to $-Q$, and it is the same as the charge induced by the particle at the plasma interface as it approached this interface. In a sense, there is an overflow of the induced surface charge into space charge.

Surface oscillations with a frequency $\omega_0 = \omega_p/2^{1/2}$ are also excited near the interface. The total surface charge, varying at the frequency ω_0, is zero; one might say that these are oscillations of a dipole type. Such oscillations arise because the component of the electron velocity perpendicular to the interface, v_z, is nonzero at the time at which the particle enters the plasma ($t=0$). A surface charge thus arises at a later time at the interface, and the spatial variation of this charge reproduces the variation in the velocity.

As the particle leaves the plasma, the charge of the wakefield overflows into image charge. Surface oscillations are again excited.

In analyzing the electric field of the particle, we used the quasistatic approximation and ignored vortical electric fields. The condition for the validity of this approximation for the question of the polarization loss is the inequality $u \ll c$

(Ref. 10). For the question of interest here—the fields generated by the particle—the condition for the validity of the quasistatic approximation is related to the distance (R) from the observation point to the particle or to the interface. Analysis shows that the vortical fields can be ignored completely if $R \lesssim u/\omega_p$. In the interval

$$\frac{c^2}{\omega_p u} > R > \frac{u}{\omega_p}$$

the vortical fields may have the same strength as the quasistatic fields. Finally, at $R > c^2/\omega_p u$, the vortical fields are the dominant fields; in particular, they determine the transition radiation.[17]

Although our analysis is valid only for a cold, nondegenerate electron gas, there is the possibility that polarization processes of the type discussed here also occur in other media. In particular, it is possible that by examining the effect of near-surface fields on the dynamics of the particles of a dicluster passing through a thin foil one might find a more complete explanation for the existing experimental data.[20]

We wish to thank B. M. Bolotovskiĭ for useful discussions of the questions involved here and S. B. Tarakanov for carrying out numerical calculations.

APPENDIX 1

The function $W_2(a;b)$, defined by (4), has the following properties:

$$W_2(0;\, b) = {}^1\!/_2\pi[I_0(b) - L_0(b)],$$

where I_0 and L_0 are respectively the Bessel and Struve functions of imaginary argument. If $b < 1$, then

$$W_2(0;\, b) \approx \frac{\pi}{2}\left(1 - \frac{2b}{\pi} + \frac{b^2}{4} - \frac{2b^3}{9\pi} + \frac{b^4}{4^3} - \dots\right).$$

If $b > 1$, then

$$W_2(0;\, b) \approx \frac{1}{b} + \frac{1}{b^3} + \dots\ .$$

For $b = 0$ we have $W_2(a;0) = \sin a \operatorname{ci} a - \cos a \operatorname{si}(a)$, where $\operatorname{si}(a)$ and $\operatorname{ci}(a)$ are the integral sine and cosine, respectively, and

$$W_2(a;\, 0) = \begin{cases} {}^1\!/_2\pi + a\ln a + (C-1)a - {}^1\!/_4\pi a^2 + O(a^3), & a<1, \\ 1/a - 2/a^3 + O(a^{-5}), & a>1. \end{cases}$$

The function W_2 satisfies the equation $\Delta W_2 = 0$ for $a,b > 0$ and has no extrema in this region. In addition, W_2 satisfies the equation

$$\frac{\partial^2}{\partial a^2} W_2(a;\, b) + W_2(a;\, b) = \frac{1}{(a^2+b^2)^{1/2}},$$

which gives us yet another representation for W_2:

$$W_2(a;\, b) = \int_a^\infty \frac{d\xi \sin(\xi - a)}{(\xi^2+b^2)^{1/2}}.$$

APPENDIX 2

We consider the function

$$F(z) = \frac{1}{z} - \frac{\pi}{2}[I_0(z) - L_0(z)].$$

We write it in the form

$$F(z) = \frac{1}{z} - \int_0^{\pi/2} d\theta \exp(-z\cos\theta)$$

$$= \frac{e^{-z}}{z} - \int_0^1 dt\, e^{-zt}\left[\frac{1}{(1-t^2)^{1/2}} - 1\right].$$

Using this expression, we can evaluate the integral:

$$\int_0^\infty zF(z)\,dz = 1 - \int_0^{\pi/2} d\theta \frac{1-\cos\theta}{\sin^2\theta} = 1 - \int_0^{\pi/4} \frac{dx}{\cos^2 x} = 0.$$

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Translated by D. Parsons