Quantum calculation of direct radiative and direct conversion charge exchange of mesic hydrogen with helium mesons

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The radiative or conversion mechanisms of direct muon transfer from mesic hydrogen to helium are considered. The energy distributions of the photons and electrons accompanying the charge exchange are calculated. The total charge exchange rates averaged over the Maxwellian distribution are found to be $\sim 10^6$ and $\sim 10^5\ s^{-1}$ for the radiative and Auger charge exchanges, respectively.

1. DIRECT CHARGE EXCHANGE PHOTON AND ELECTRON SPECTRA

Charge exchange of mesic hydrogen on helium is of definite interest for muon catalysis, since the hydrogen target contains helium as a product of the fusion reaction and of the decay of tritium in the *d–t* mixture. Interception of muons from the ground state of mesic hydrogen by helium nuclei was first considered by Matveenko and Ponomarev:[1]

$$(p\mu)_{1s}+\mathrm{He}^{++}\to(\mathrm{He}^{++}\mu)_{1s}+p \qquad (1)$$

(p denotes the nucleus of some hydrogen isotope).

The rate of this reaction is low ($\sim 10^6\ s^{-1}$) in view of the absence of crossing of the molecular terms $2p\sigma$ and $1s\sigma$ corresponding to the initial and final states of the system. As a result, during the time of a fast passage of the light particle (muon), the heavy particles (nuclei) should become accelerated to appreciable energies (~ 6 keV), which lowers the probability of the reaction substantially.

Another, molecular, charge-exchange mechanism was proposed by Aristov *et al.*,[2] namely, conversion on the electron of the He atom first produces a mesomolecular ion $(\mathrm{He}^{++}p\mu)$, which breaks up next into $(\mathrm{He}^{++}\mu)_{1s}$ and p, emitting a photon or an electron. The rate of this charge exchange is determined by the first stage of the process and is of the order of $10^8\ s^{-1}$.

We have considered in the present study two additional mechanisms of muon transfer to helium: directly radiative,

$$(p\mu)_{1s}+\mathrm{He}^{++}\to(\mathrm{He}^{++}\mu)_{1s}+p+\gamma, \qquad (2)$$

and a direct conversion (or Auger) mechanism

$$(p\mu)_{1s}+\mathrm{He}^{+}\to(\mathrm{He}^{++}\mu)_{1s}+p+e. \qquad (3)$$

Similar processes take place on atomic targets. Emission of a γ quantum (or an electron) introduces another small factor in the charge exchange, in view of the weakness of the electromagnetic interaction. Nonetheless, reactions (2) and (3) turn out to be competitive with reaction (1), since the greater part of the energy released at the instant of the muon hop-over is now carried away by a γ quantum (or an electron). This leads to adiabatic spreading of the nuclei, just as in the case of term crossing.

The most important in reactions (2) and (3) is the dipole transition between the terms $2p\sigma$ and $1s\sigma$ from an S-wave into a P-wave state of relative motion of the nuclei (only slow collisions are considered). The dipole moment of the $(p\mu\mathrm{He}^{++})$ system takes the form[2,3]

$$\mathbf{d}=a\mathbf{r}+b\mathbf{R}, \quad a=-\left(1+\frac{2m_\mu}{M_{tot}}\right), \quad b=\frac{3M_p-M_{\mathrm{He}}+m_\mu}{2M_{tot}}, \qquad (4)$$

where $M_{\mathrm{tot}}=M_p+M_{\mathrm{He}}+m_\mu$, $\mathbf{R}$ is the internuclear-distance vector, $\mathbf{r}$ is the muon radius vector drawn from the center of the segment R. Using the equations[4,5] for the dipole radiation, we obtain after simple calculations the following expression for the photon spectrum $\hbar=e=m=1$, $m^{-1}=m_\mu^{-1}+M_p^{-1}$:

$$\frac{d\lambda_\gamma}{d\omega}=\frac{16}{3}N(\alpha\omega)^3\frac{J_{k'k}^2}{\varepsilon k'}. \qquad (5)$$

Here $d\lambda_\gamma$ is the differential charge-exchange rate, ω is the photon energy, $\alpha=1/137$, $\varepsilon(\varepsilon')$ and $k(k')$ are the energy and momentum of the relative motion of the nuclei in the initial (final) state

$$k=(2M\varepsilon)^{1/2}, \quad k'=(2M\varepsilon')^{1/2}, \qquad (6)$$

$$M^{-1}=(m_\mu+M_p)^{-1}+M_{\mathrm{He}}^{-1}. \qquad (7)$$

N is the density of liquid hydrogen (in mesic-atom units we have $N=N_0a_m^3$, $N_0=4.25\cdot 10^{22}\ \mathrm{cm}^{-3}$), and a_m is the Bohr radius of the mesic atom). The integral J is a matrix element of the dipole transition between the initial and final states of the $(p\mu\ \mathrm{He}^{++})$ system:

$$J_{k'k}=\int_0^\infty \chi_{k'1}(R)D(R)\chi_{k0}(R)\,dR, \qquad (8)$$

$$D(R)=a\int \Phi_{1s\sigma}(R,\mathbf{r})\frac{\mathbf{Rr}}{R}\Phi_{2p\sigma}(R,\mathbf{r})\,d\mathbf{r}, \qquad (9)$$

$\Phi_{1s\sigma}$ and $\Phi_{2p\sigma}$ are the muon wave functions for the molecular terms $1s\sigma$ and $2p\sigma$. The term $b\,\mathbf{R}$ of the dipole moment (4) makes no contribution to $D(R)$ (Ref. 9).

The wave functions χ_{kL}, which describe the relative motion of the nuclei, are solutions of the radial Schrödinger equation obtained in the framework of the effective single-channel approximation (the "simple approach").[6,8] The initial function $\chi_{k0}(R)$ is the S-wave for the $2p\sigma$ term, the final $-\chi_{k'1}(R)$ is the P wave for the $1s\sigma$ term. They satisfy the boundary conditions

$$\chi_{k0}(0)=\chi_{k'1}(0)=0,$$
$$\chi_{k0}(R)\propto\sin(kR+\delta_0), \quad R\to\infty \qquad (10)$$
$$\chi_{k'1}(R)\propto\sin\left[k'R-\frac{\pi}{2}+\frac{1}{v'}\ln(2k'R)+\delta_1\right], \quad R\to\infty,$$

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TABLE I. Rates of radiative charge exchange (10^6 s^{-1}), averaged over a Maxwellian distribution.

T, K	$p\mu+^3$He	$p\mu+^4$He	$d\mu+^3$He	$d\mu+^4$He	$t\mu+^3$He	$t\mu+^4$He
20	0,12	0,15	0,35	2,7	4,5	0,85
50	0,12	0,15	0,34	2,4	3,7	0,82
100	0,12	0,15	0,33	2,1	2,9	0,79
400	0,11	0,14	0,28	1,3	1,5	0,64
800	0,11	0,13	0,24	0,90	0,98	0,53
1000	0,10	0,13	0,23	0,80	0,84	0,50

TABLE II. Rates of Auger charge exchange (10^5 s^{-1}), averaged over a Maxwellian distribution.

T, K	$p\mu+^3$He	$p\mu+^4$He	$d\mu+^3$He	$d\mu+^4$He	$t\mu+^3$He	$t\mu+^4$He
20	0,31	0,39	0,91	6,7	12	2,1
50	0,31	0,39	0,89	6,0	9,8	2,1
100	0,30	0,38	0,86	5,2	7,8	2,0
400	0,29	0,36	0,73	3,2	4,0	1,6
800	0,28	0,34	0,64	2,3	2,6	1,4
1000	0,28	0,34	0,60	2,0	2,2	1,3

where δ_0 and δ_1 are the phase shifts, and v' is the relative velocity of the nuclei in the final state.

The connection between ω, ε, and ε' is given by the energy-conservation law

$$\varepsilon'+\omega=\varepsilon+\Delta, \qquad \Delta=U_{2p\sigma}(\infty)-U_{1s\sigma}(\infty), \tag{11}$$

where $U(\infty)$ is the energy of the term (with allowance for the adiabatic corrections) as $R\to\infty$.

For the direct Auger charge exchange (3) we obtain, proceeding as in Ref. 9, the distribution of the conversion electrons in energy:

$$\frac{d\lambda_e}{dE}=\beta\frac{64\pi}{3}Nm_e^4Z^4Q(\nu)\frac{J^2_{k'k}}{\varepsilon k'E}. \tag{12}$$

Here $d\lambda_e$ is the differential rate of Auger charge exchange, E the energy of the emitted electron, m_e the electron mass, Z the effective charge in the field of which the conversion electron moves, β the number of electrons in the target, and

$$Q(\nu)=[(1+\nu^2)(1-e^{-2\pi\nu})\exp(4\nu\,\mathrm{arcctg}\,\nu)]^{-1}, \qquad \nu=Z(m_e/2E)^{1/2}. \tag{13}$$

In the derivation of (12) and (13) we used for the electrons hydrogenlike wave functions. The energy conservation law is of the form

$$\varepsilon'+E=\varepsilon+\Delta-I, \tag{14}$$

and I is the binding energy of the electron that effects the Auger transition.

The parameters I, Z, and β for helium targets have the following values:

ion He^+: $I=54.42$ eV, $Z=2$, $\beta=1$;
atom He: $I=24.58$ eV, $Z=1.69$, $\beta=2$. (15)

2. RESULTS AND DISCUSSION

Tables I and II list the rates of the radiative charge exchange (2) and Auger charge exchange (3), averaged over a Maxwellian distribution of the mesic-atom velocities. It follows from Table I and Refs. 1 and 2 that the rate of radiative charge exchange is lower by two orders of magnitude than the rate of molecular charge exchange and is comparable with the rate of the direct charge exchange (1). Just as in molecular charge exchange, a strong energy dependence is observed in $(d\mu,t\mu)$ + He systems and is due to the presence

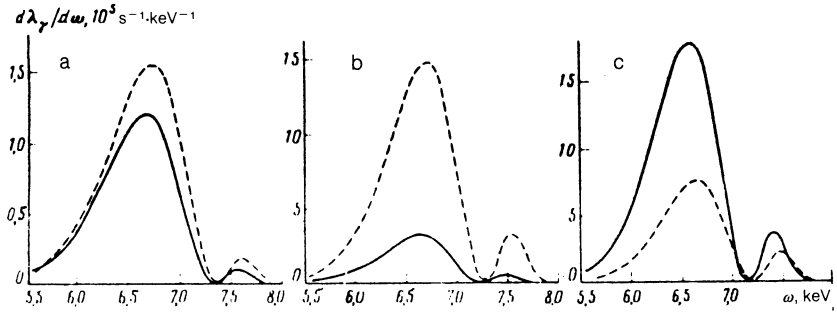

FIG. 1. Photon spectra for the systems $p\mu$ + He (a), $d\mu$ + He (b) and $t\mu$ + He (c) at $T=400$ K; solid lines—^{3}He, dashed—^{4}He.

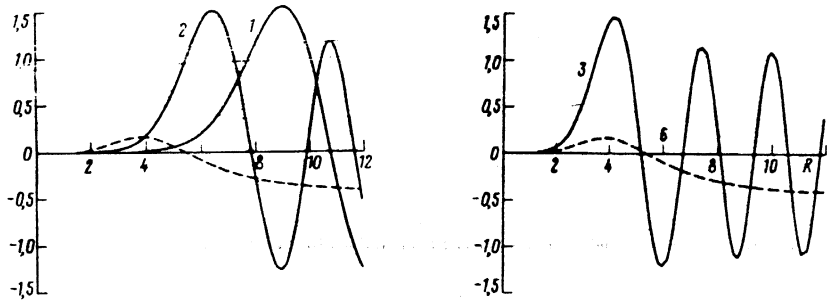

FIG. 2. Wave functions of the initial state for $\varepsilon = 0.04$ eV (dashed) and the final state (solid) for energies corresponding to the high-energy peak o the photons (curve *1*, $k_f = 0.84$), to the dip between peaks (curve *2*, $k_f = 1.01$) and to the main peak (curve *3*, $k_f = 1.28$); $k_f \equiv k'$—final momentum of the relative motion of the nuclei.

of a virtual level with low energy and with $L = 0$ (L is the total angular momentum of the system). To predict reliably the energy dependences of the reaction rates in such a situation it would be desirable to carry out a multichannel computation. The isotopic dependence is strongly pronounced, as before.

It should be noted that, notwithstanding the low radiative charge-exchange rate, this process can turn out to be substantial (and in some cases even decisive) for charge exchange on helium nuclei (He^{++}), when the molecular mechanism cannot be implemented.[1)] This statement is valid in most cases also with respect to charge exchange by the ion He^+, for only in the cases of $d\mu + {}^4He^+$ and $t\mu + {}^4He^+$ does the binding energy of the mesic molecule exceed the binding energy of the electron.[9] No mesic molecule is formed in the remaining cases.

Figure 1 shows the photon spectra averaged over a Maxwellian distribution for $T = 400$ K. Just as in the case of molecular charge exchange, each spectrum contains a broad line with $\omega \approx 6.7$ keV. In addition, they contain a relatively weak line with $\omega \approx 7.4$–7.6 keV. The origin of the second peaks in the spectra can be understood by investigating the behavior of the wave function of the final state for different values of the final momentum k' (Fig. 2). The dipole matrix element $D(R)$ is very small at $R > 12$, so that of importance to us is the behavior of the nuclear wave functions $\chi(R)$ in the region $R < 12$. The wave function χ_f of the final state for $k' = 0.84$ (which corresponds to the second high-energy peak in the spectrum of the photons) overlaps well the initial wave function χ_{in} in the region $R > 6$. This overlap is annihilated by the oscillations of χ_f when the energy is increased ($k' = 1.01$ corresponds to the dip between the peaks in the photon spectrum). Finally, with further increase of energy ($k' = 1.28$ corresponds to the principal peak of the spectrum) χ_f overlaps χ_{in} in the region $R < 6$, where $D(R)$ reaches a maximum. The locations of the peaks do not depend on the collision energy in the interval 0.004–20 eV.

Conversion-electron spectra are very similar to photon spectra and are likewise independent of the collision energy in the low temperature region. Figure 3 shows the electron spectrum in the system $d\mu + {}^3He$ for an initial energy $\varepsilon = 0.04$ eV.

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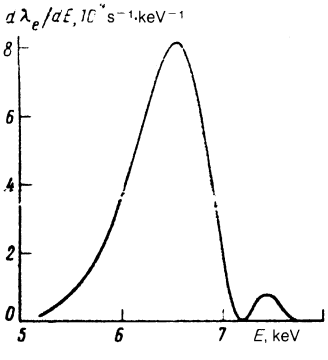

FIG. 3. Auger charge-exchange electron spectrum for the system $d\mu + {}^3He$ at $\varepsilon = 0.04$ eV.

[1)] A mesic molecule can be produced not only by conversion on an atomic electron, but also radiatively. The probability of the latter, however, is very insignificant in view of the low energy release (~ 40 eV) (Ref. 10).

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