Surface guided electromagnetic modes in films with periodically modulated characteristics

V. A. Belyakov, V. P. Orlov, and G. I. Shilina

All-Union Scientific Research Center for the Investigation of Surfaces and Vacuum
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Using the formalism of the dynamic theory of diffraction, we investigate surface guided electromagnetic modes (SGEM) in finite-thickness layers having dielectric permittivities that are periodically modulated in the transverse direction. We analyze the conditions for the existence of TE- and TM-type SGEM and their dispersion relations as a function of the phase of the dielectric permittivity modulation for semi-infinite structures, and find how the dispersion laws of the SGEM and the conditions for their existence change in layers of finite thickness. We discuss the optimal conditions for observing SGEM as a function of the phase of the dielectric permittivity modulation; in particular, we identify a strong dependence of the SGEM attenuation on this phase, associated with finiteness of the layer thickness.

INTRODUCTION

Surface guided electromagnetic modes (SGEM) at the boundary of a semi-infinite periodic medium have been investigated by a number of authors[1–6] and were observed experimentally in Ref. 8. The physical phenomena that lead to the existence of SGEM are total internal reflection from within the periodic medium toward the boundary and diffractive reflection of the waves in the bulk of the periodic medium. Theoretical investigation of SGEM has been limited to semi-infinite media for the special case of periodic characteristics, i.e., for layered media, and several questions remain which to date have received little attention: specific features of TE and TM SGEM, their differences and general properties, and how the SGEM characteristics depend on the phase with which the periodic modulation of the properties of the medium terminates at the boundary.

As for SGEM in structures of finite thickness, only two papers[4,5] have been published on this topic. In these papers the practical importance of taking into account the finite thickness of the periodic structure was demonstrated, in particular its role in producing SGEM attenuation in these layers even when the medium is nonabsorbing. These papers show the importance of detailed theoretical investigations of SGEM in such structures in connection with their potential use as waveguides that can be controlled by small external perturbations (the latter primarily involving chiral liquid crystal films).

In this paper, we use the formalism of the dynamic theory of diffraction to carry out a detailed theoretical analysis of TE and TM SGEM, which reveals how their characteristics depend on the modulation phase of the dielectric permittivity of the periodic medium, both in semi-infinite structures and in structures of finite thickness. By identifying the dependence of the SGEM parameters on the phase of the modulation and thickness of the periodic structure, we are able to optimize the corresponding experiments.

1. SGEM IN A SEMI-INFINITE MEDIUM

1.1. Fundamental equations

We first discuss SGEM at the boundary between two media, one uniform and one periodically nonuniform. Let the upper half-space ($z > 0$) be filled with a uniform isotropic medium with dielectric permittivity ε_1, while the periodic medium fills the half-space $z < 0$ and is described by a scalar dielectric permittivity modulated along the z axis:

$$\varepsilon=\bar{\varepsilon}[1+\delta\cos(\tau z-\varphi)], \tag{1}$$

where $\bar{\varepsilon}$ is the average value of the dielectric permittivity, δ is the modulation parameter, which will be considered small in what follows, τ is a vector of the reciprocal lattice of the periodic medium ($\tau = 2\pi/d$, where d is the period), and φ is the phase of the modulation at the boundary of the periodic medium.

We will assume that total internal reflection is possible at the boundary of the periodic medium, i.e., $\varepsilon_1 < \bar{\varepsilon}$, and assume for simplicity that the magnetic susceptibility equals unity.

Expression (1) for the dielectric permittivity allows us to isolate the polarizations and solve the problem separately for TE and TM waves.

Let us seek a solution to the Maxwell equations in the form of a monochromatic wave of frequency ω propagating along the surface parallel to the x axis with wave vector **q**, using the two-wave approximation of the dynamic theory of diffraction.[7] For a TE wave, we will write the field in the uniform medium in the form

$$E_y=C_1\exp(iqx-\gamma_1 z),\qquad \gamma_1=\left(q^2-\varepsilon_1\frac{\omega^2}{c^2}\right)^{1/2},\qquad z>0 \tag{2}$$

and write the field in the periodic medium in the form

$$E_y=B\left\{\exp\left(i\frac{\varphi+\beta}{2}\right)\exp\left[-\left(\gamma+i\frac{\tau}{2}\right)z\right]+\exp\left(-i\frac{\varphi+\beta}{2}\right)\times\exp\left[-\left(\gamma-i\frac{\tau}{2}\right)z\right]\right\}\exp(iqx),\qquad z<0, \tag{3}$$

where

$$q=q_B+\Delta q,\qquad q_B^2=\varkappa_0^2-\frac{\tau^2}{4},\qquad \varkappa_0^2=\bar{\varepsilon}\frac{\omega^2}{c^2}. \tag{4}$$

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The parameters Δq and γ are defined by

$$\Delta q=\frac{\delta}{4}\frac{\varkappa_0^2}{q_B}\cos\beta,$$
$$\gamma=\frac{\delta}{2}\frac{\varkappa_0^2}{\tau}\sin\beta, \tag{5}$$
$$-\pi<\beta<0.$$

where the relation $\cos\beta=-\alpha$ connects β with the following parameter, which is commonly used in the dynamic theory of diffraction:[7]

$$\alpha=\frac{\bar{\tau}(\bar{\tau}+2\bar{\varkappa}_0)}{\delta}. \tag{6}$$

This parameter determines the departure from the Wolf–Bragg condition in optical problems.

The condition $-\pi<\beta<0$ in (5) ensures that the field of the SGEM decays into the bulk of the periodic medium.

We obtain the dispersion equation for SGEM from the condition that the tangential components of the fields **E** and **H** be continuous at the boundary $z=0$. For the TE SGEM the dispersion relation has the form

$$(\gamma_1-\gamma)\cos\frac{\varphi+\beta}{2}+\frac{\tau}{2}\sin\frac{\varphi+\beta}{2}=0, \tag{7}$$

$$\gamma_1=\left(\eta_1\varkappa_0^2-\frac{\tau^2}{4}+\frac{\delta}{2}\varkappa_0^2\cos\beta\right)^{1/2},\qquad \eta_1=1-\frac{\varepsilon_1}{\bar{\varepsilon}}. \tag{8}$$

From Eqs. (2)–(8) it follows that the condition for the existence of SGEM is that the value of the SGEM frequencies exceeds a certain threshold value:

$$\omega_{min}=\frac{\tau c}{2(\bar{\varepsilon}-\varepsilon_1)^{1/2}}. \tag{9}$$

The parameter β, which is found from (7) for $\omega>\omega_{\min}$ by using Eqs. (2)–(5), completely specifies the TE-wave solution we are looking for. In explicit form the dependence of the parameters Δq and γ on the phase of the dielectric permittivity modulation at the boundary and frequency has the form

$$\Delta q=\frac{\delta}{4}\frac{\varkappa_0^2}{q_B}\,\frac{(\tau^2/2-\eta_1\varkappa_0^2)\cos\varphi-\tau\sin\varphi(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1\varkappa_0^2},$$

$$\gamma=-\frac{\delta}{2}\frac{\varkappa_0^2}{\tau}\,\frac{(\tau^2/2-\eta_1\varkappa_0^2)\sin\varphi+\tau\cos\varphi(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1\varkappa_0^2}. \tag{10}$$

Similarly, we can find the dispersion equation for surface waves with TM polarization. Let us write the field **H** in the form (3). The parameters Δq and γ for this polarization are defined by Eq. (5) if we replace δ in them by

$$\delta\left[1-\frac{\tau^2}{2\varkappa_0^2}\right].$$

The dispersion equation for TM SGEM has the form:

$$[(1-\eta_1)\gamma_1-\gamma]\cos\frac{\varphi+\beta}{2}+\frac{\tau}{2}\sin\frac{\varphi+\beta}{2}+\frac{\delta}{2}\frac{\tau}{2}\sin\frac{\varphi-\beta}{2}=0, \tag{11}$$

$$\gamma_1=\left[\eta_1\varkappa_0^2-\frac{\tau^2}{4}+\frac{\delta}{2}\varkappa_0\left(1-\frac{\tau^2}{2\varkappa_0^2}\right)\cos\beta\right]^{1/2}. \tag{12}$$

The dependence of the parameters Δq and γ on the phase of the dielectric permittivity modulation at the boundary and on frequency has the form

$$\Delta q=\frac{\delta}{4}\frac{\varkappa_0^2-\tau^2/2}{q_B}\times\frac{[(2-2\eta_1+\eta_1^2)\tau^2/4-\eta_1\varkappa_0^2]\cos\varphi-\tau(1-\eta_1)\sin\varphi(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1[(\eta_1-2)\tau^2/4+\varkappa_0^2]},$$

$$\gamma=-\frac{\delta}{2}\frac{\varkappa_0^2-\tau^2/2}{\tau}\times\frac{[(2-2\eta_1+\eta_1^2)\tau^2/4-\eta_1\varkappa_0^2]\sin\varphi+\tau(1-\eta_1)\cos\varphi(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1[(\eta_1-2)\tau^2/4+\varkappa_0^2]}. \tag{13}$$

The condition that Eqs. (7) or (11) have solutions for real values of Δq determines the region of existence of the SGEM.

2. REGION OF EXISTENCE OF SGEM

In Fig. 2 we show the frequency ranges where SGEM exist as a function of the phase of the dielectric permittivity modulation at the boundary between the media ($z=0$) for TE and TM waves. The lower threshold (9) of existence for SGEM with respect to frequency is defined as the minimum frequency (maximum wavelength) for which the conditions of total internal reflection at the boundary and diffraction in the bulk of the periodic medium (the Wolf–Bragg condition) can still be combined. For frequencies close to $\omega_{\min}$ the width of the allowed band with respect to phase changes in a complex fashion as a function of frequency. For frequencies not close to this limiting frequency, the width of the allowed (forbidden) band practically coincides with the value π. In the zeroth approximation, the curves which bound these regions are described by the following equations: for TE waves,

$$\varphi=-2\,\mathrm{arctg}\left[\frac{2}{\tau}\left(\eta_1\varkappa_0^2-\frac{\tau^2}{4}\right)^{1/2}\right] \tag{14}$$

and for TM waves,

$$\varphi=-2\,\mathrm{arctg}\left[(1-\eta_1)\frac{2}{\tau}\left(\eta_1\varkappa_0^2-\frac{\tau^2}{4}\right)^{1/2}\right]. \tag{15}$$

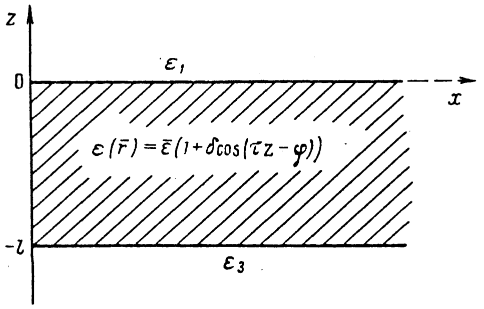

FIG. 1. Schematic illustration of the geometry of the problem.

For certain phases ($0° < \varphi < 180°$) there exists an upper frequency limit for the appearance of SGEM. For phases $-180° < \varphi < 0°$ the smallest attainable frequency for the SGEM turns out to be larger than $\omega_{\min}$ and is determined by the phase.

For every fixed frequency the regions of existence of SGEM with respect to phase overlap for TE and TM waves; however, they do not coincide. Therefore, ranges of variation of φ can be exhibited for which only one of the two modes TE or TM can be excited. Both TE and TM modes can coexist over a relatively wide range of variation of φ. The sizes of these regions are determined by relations between the parameters $\bar{\varepsilon}$, ε_1, and δ.

Figures 3 and 4 shows phase dependence of the deviation of the SGEM wave vector difference from its Bragg value (Δq) and the quantity (γ) that determines the decay of the field with the z-coordinate into the depth of the sample for various frequencies. For a given frequency there is a certain phase φ for which $\Delta q = 0$, i.e., $q = q_\beta$; in this case, the field decays most rapidly into the bulk of the medium. As the phase varies over the allowed band, the quantity γ varies from its frequency-dependent maximum value to zero. Near the edges, the field of the wave decays slowly into the depth of the sample. In the uniform medium, the decay of the SGEM field with respect to z depends only weakly on the phase of the dielectric permittivity modulation at the boundary, and is essentially determined by the frequency.

Note that the phase of the modulation determines how γ varies with increasing frequency. For certain phases (e.g., $\varphi = -60°$, $\varphi = -30°$) the rates of decay increase with increasing frequency, both in the uniform medium and in the periodic medium. However, there are some values of φ (e.g., $\varphi = 30°$, $\varphi = 60°$) for which the quantity γ first increases with increasing frequency and then begins to decrease to zero. This implies that there exists an upper frequency limit for the appearance of SGEM in this case. The field **E** penetrates into the depth of the medium, encompassing a large number of layers of the periodic structure. Whereas the electric field of a TE wave is parallel to the surface and perpendicular to the wave vector of the SGEM, the field **E** lies in the xz plane for a TM wave, although its direction in the periodic medium varies with depth. In this case, the amplitude of the field **E** decreases exponentially into the depth, while the ratio E_x/E_y varies periodically along z. In the xz plane the direction of the total field can take on all allowed values.

3. SURFACE WAVES IN A FILM

SGEM in periodic structures of finite thickness were investigated in Refs. 4 and 5. As we mentioned in the Introduction, in this case the most important peculiarity of SGEM turns out to be the appearance of attenuation even in nonabsorbing media, associated with "leaking" of the electromagnetic field through the surface of a film for which the condition of total internal reflection (TIO) is not fulfilled. In the papers we cited, the analysis of these characteristics, which was carried out for the limiting case of a thick film, was only qualitative. Furthermore, the question of how the SGEM parameters depend on the phase of the dielectric permittivity modulation at the surface of the structure was left completely untouched. However, it follows from the material presented in the previous section that the SGEM characteristics depend significantly on this phase. What is more, for certain values of the phase SGEM simply do not exist. In this section, we will analyze how the characteristics of SGEM in films depend on the value of the modulation phase at the surface, without restricting our discussion to the limiting case of a thick film.

Let us consider the SGEM of a plane parallel layer of the periodic medium with properties that coincide with those of the medium used in the previous section to discuss

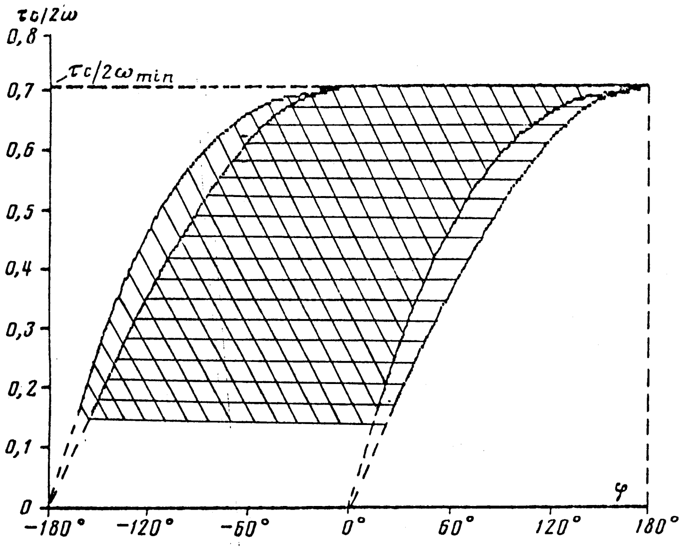

FIG. 2. Existence region of SGEM. The curves are bounded by allowed bands with respect to phase and wavelength (frequency) for the appearance of SGEM in the case of a semi-infinite medium. The horizontal crosshatching represents TE waves, the oblique crosshatching TM waves; $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$.

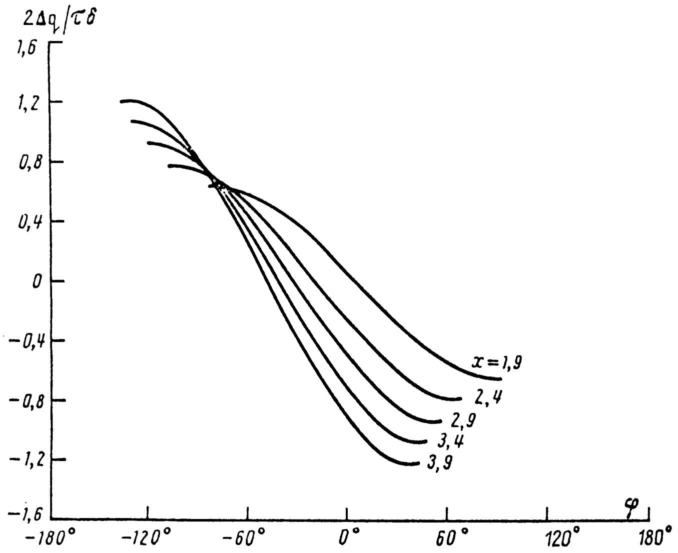

FIG. 3. Dependence of the diffraction correction Δq to the wave vector on the phase φ of the dielectric permittivity modulation at the $z = 0$ boundary for various frequencies (TE SGEM, semi-infinite medium); $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$, $x = 2\omega/\tau c$.

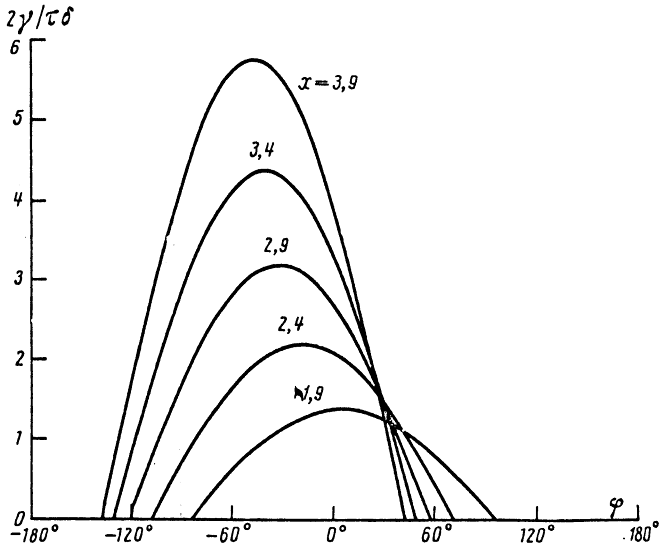

FIG. 4. Dependence of the attenuation decay rate (γ) of the SGEM field into the bulk of the periodic medium on the phase of the dielectric permittivity modulation at the $z = 0$ boundary for various frequencies (TE SGEM, semi-infinite medium); $\bar{\varepsilon} = 1.5$, $\varepsilon_1 = 1.0$, $\delta = 0.05$, $x = 2\omega/\tau c$.

the problem of SGEM at the boundary of a half-space; we will assume that the properties of the $z > 0$ medium correspond to this case as well. We also assume that for $z < -l$ (where l is the layer thickness) the homogeneous medium that surrounds the layer is characterized by a dielectric permittivity ε_3 (see Fig. 1). Let us seek the SGEM fields for TE modes in the following form: for $z > 0$ we use Eq. (2), with the same parameter values as in the previous section, while for $-l < z < 0$ we write:

$$\begin{aligned} E_y = \Big\{ & B_0 \Big[\exp\Big(i\frac{\varphi+\beta}{2}\Big) \exp\Big[-\Big(\gamma + i\frac{\tau}{2}\Big)z\Big] \\ & + \exp\Big(-i\frac{\varphi+\beta}{2}\Big) \exp\Big[-\Big(\gamma - i\frac{\tau}{2}\Big)z\Big]\Big] \\ & + B_1 \Big[\exp\Big(i\frac{\varphi-\beta}{2}\Big) \exp\Big[\Big(\gamma - i\frac{\tau}{2}\Big)z\Big] \\ & + \exp\Big(-i\frac{\varphi-\beta}{2}\Big) \exp\Big[\Big(\gamma + i\frac{\tau}{2}\Big)z\Big]\Big]\Big\} \exp[i(q_B+\Delta q)x], \end{aligned} \tag{16}$$

where B_0 and B_1 are coefficients which remain to be determined; the remaining parameters are defined by the same relations as in Eq. (3). For $z < -l$ we use Eq. (2), replacing γ_1 by $-\gamma_3$, where

$$\gamma_3 = [q^2 - (\eta_3+1)\varkappa_0^2]^{1/2}, \qquad \eta_3 = \frac{\varepsilon_3}{\bar{\varepsilon}} - 1. \tag{17}$$

Note that in contrast to the semi-infinite case, where we were able to write the SGEM field within the periodic medium using a single eigenwave solution to the corresponding optical problem, in the film we require a combination of two eigenwave solutions to describe the field, one of which decays away from the first boundary, the other away from the second.

From the condition that the fields match at the boundaries $z = 0$ and $z = -l$ we obtain the dispersion relation for SGEM in the film:

$$\begin{aligned} & e^{\gamma l}\Big[(\gamma_1+\gamma)\cos\frac{\varphi-\beta}{2} + \frac{\tau}{2}\sin\frac{\varphi-\beta}{2}\Big] \\ & \times\Big[(\gamma_3+\gamma)\cos\frac{\varphi+\beta+\tau l}{2} - \frac{\tau}{2}\sin\frac{\varphi+\beta+\tau l}{2}\Big] \\ & - e^{-\gamma l}\Big[(\gamma_1-\gamma)\cos\frac{\varphi+\beta}{2} + \frac{\tau}{2}\sin\frac{\varphi+\beta}{2}\Big] \\ & \times\Big[(\gamma_3-\gamma)\cos\frac{\varphi-\beta+\tau l}{2} - \frac{\tau}{2}\sin\frac{\varphi-\beta+\tau l}{2}\Big] = 0. \end{aligned} \tag{18}$$

In what follows we will investigate the case where SGEM are present only at the $z = 0$ boundary, i.e., $\varepsilon_1 < \bar{\varepsilon} \leqslant \varepsilon_3$. Then the dispersion equation can be written in a form having terms that coincide with the dispersion equation (7) for the semi-infinite medium, plus a correction proportional to the factor $e^{2\gamma l}$. From this it is clear that in the limit of a thick layer ($|\gamma l| \gg 1$) Eq. (18) becomes the dispersion equation for the half-space, which was investigated above. In the zeroth approximation with respect to δ, the dispersion equation for TM waves is obtained from (18) by replacing γ_1 by $(\bar{\varepsilon}/\varepsilon_1)\gamma_1$ and γ_3 by $(\bar{\varepsilon}/\varepsilon_3)\gamma_3$.

Let us investigate how the SGEM dispersion law in a layer with dielectric permittivity of the form (1) depends on the phase of the dielectric permittivity modulation at the boundary ($z = 0$) for which the TIO condition is fulfilled. For the case of layers that are not thick ($|\gamma l| \sim 1$) we were unable to derive an analytic expression for the dispersion equation; therefore the corresponding curves were found by numerical methods.

In Fig. 5 we show the results of calculating the dispersion curves for various values of the phase φ of the dielectric permittivity modulation at the layer surface ($z = 0$). From the figure it is clear that the SGEM attenuation due to the

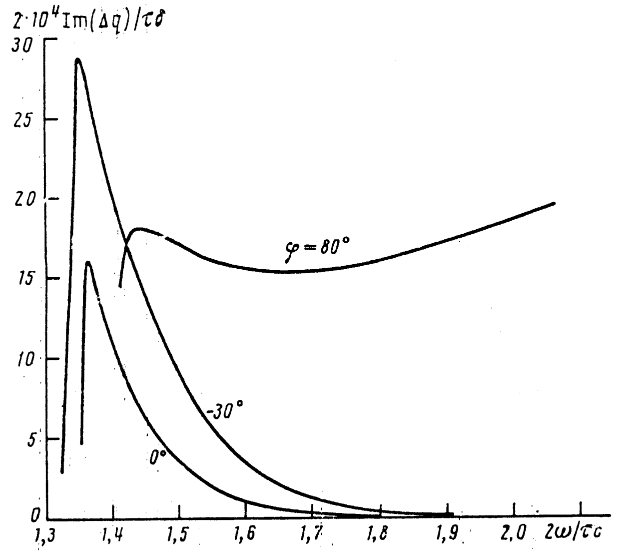

FIG. 5. Frequency dependence of the SGEM attenuation ($\mathrm{Im}(\Delta q)$) in the film for various values of the phase φ of the dielectric permittivity modulation at the $z=0$ boundary; $\bar{\varepsilon}=\varepsilon_3=1.5$; $\varepsilon_1=1.0$; $\delta=0.05$; $d=2\pi/\tau$, $l=74d/3$; $\varphi=-30°, 0°, 80°$. The values of $\mathrm{Im}(\Delta q)$ for the $\varphi=80°$ curve are scaled by a factor of 10.

finite layer thickness (i.e., the imaginary part of Δq) and its frequency behavior are strongly dependent on the phase φ. This also applies to the renormalization of the SGEM wave vector due to finiteness of the layer thickness. However, since this type of SGEM attenuation associated with the finite layer thickness is a qualitatively new effect, which is simply not present for the case of an SGEM in a half-space, it will be the focus of the analysis that follows. For thick layers ($|\gamma l|\gg 1$) an analytic expression for the SGEM attenuation and its dependence on φ can be found. In this case, the attenuation of the field is exponentially small and is determined by the following expressions: for TE SGEM,

$$\mathrm{Im}(\Delta q)=\frac{\delta}{2}\frac{\tau}{2}\nu\varkappa_0^2\sin^3\beta_0 e^{2\gamma l}\frac{1}{q_B}\left\{\frac{\tau^2}{4}+\frac{1}{2}\eta_3\varkappa_0^2 -\frac{1}{2}\frac{\eta_3}{\eta_1}\left[\cos(2\varphi+\tau l)\left(\eta_1\varkappa_0^2-\frac{\tau^2}{2}\right) +\tau\sin(2\varphi+\tau l)\left(\eta_1\varkappa_0^2-\frac{\tau^2}{4}\right)^{1/2}\right]\right\}^{-1},$$

$$\sin\beta_0=-\frac{\sin\varphi[\tau^2/2-\eta_1\varkappa_0^2]+\tau\cos\varphi(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1\varkappa_0^2}. \tag{19}$$

and for TM SGEM,

$$\mathrm{Im}(\Delta q)=\frac{\delta}{2}\frac{\tau}{2}\nu\frac{1}{\eta_3+1}\left(\varkappa_0^2-\frac{\tau^2}{2}\right)\frac{1}{q_B}\sin^3\beta_0 e^{2\gamma l} \times\left\{\frac{\tau^2}{4}+\frac{\eta_3}{2(\eta_3+1)^2}\left[\varkappa_0^2-(\eta_3+2)\frac{\tau^2}{4}\right] \times\left[1-\frac{\cos(2\varphi+\tau l)[\eta_1\varkappa_0^2-(2-2\eta_1+\eta_1^2)\tau^2/4]}{\eta_1\varkappa_0^2+[\eta_1^2-2\eta_1]\frac{\tau^2}{4}} +\frac{\tau(1-\eta_1)\sin(2\varphi+\tau l)(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1\varkappa_0^2+[\eta_1^2-2\eta_1]\frac{\tau^2}{4}}\right]\right\}^{-1},$$

$$\sin\beta_0=-\frac{\sin\varphi[(2-2\eta_1+\eta_1^2)\tau^2/4-\eta_1\varkappa_0^2]+\tau(1-\eta_1)\cos\varphi(\eta_1\varkappa_0^2-\tau^2/4)^{1/2}}{\eta_1\varkappa_0^2+(\eta_1^2-2\eta_1)\tau^2/4}$$

$$\nu=-(\eta_3\varkappa_0^2+\tau^2/4)^{1/2}. \tag{20}$$

Equations (19) and (20) give the dependence of the attenuation not only on the phase of the dielectric permittivity modulation at the boundary, but on frequency as well. However, by varying the frequency at fixed phase we can convert a film that is diffractively thick ($|\gamma l|\gg 1$) to one that is diffractively thin, i.e., one for which Eqs. (19) and (20) are outside their limits of applicability. A change in the phase can result in the same kind of behavior. For example, at the phase $\varphi=-30°$ the parameter $|\gamma l|$ increases rapidly with frequency and the film becomes diffractively thick, while at the phase $\varphi=80°$ the parameter $|\gamma l|$ decreases with frequency, i.e., the film becomes diffractively thin. Therefore, before using Eqs. (19) and (20) to find the attenuation it is necessary to check that the condition $|\gamma l|\gg 1$ is fulfilled.

For these films the decay characteristics of the field into the bulk differ very little from the case of a semi-infinite medium if the film is diffractively thick ($|\gamma l|\gg 1$), but can differ strongly from the latter for diffractively thin films. Analysis of the field decay of SGEM into the bulk of the sample and the field distributions along the sample thickness show that for phases around $\varphi=-2\tan^{-1}(2\gamma_1/\tau)+\pi/2$ the decay of the field into the bulk of the film is most rapid, while diffraction of the radiation that leaks out through the second surface ($z=-l$) is minimal.

As we might expect, the SGEM attenuation depends strongly on the layer thickness. This assertion is illustrated by the plots of the attenuation versus frequency shown in Fig. 6, which we have calculated for three different values of the thickness.

Figure 7 shows the dependence of the SGEM attenu-

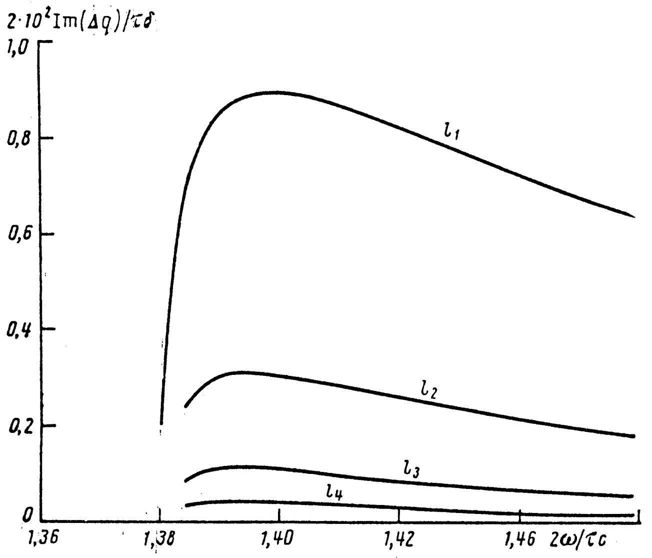

FIG. 6. Frequency dependence of the SGEM attenuation ($\mathrm{Im}(\Delta q)$) in the film for various values of the layer thickness; $\bar{\varepsilon}=\varepsilon_3=1.5$; $\varepsilon_1=1.0$; $\delta=0.05$; $\varphi=50°$, $d=2\pi/\tau$, $l_1=59d/3$; $l_2=74d/3$; $l_3=89d/3$; $l_4=104d/3$.

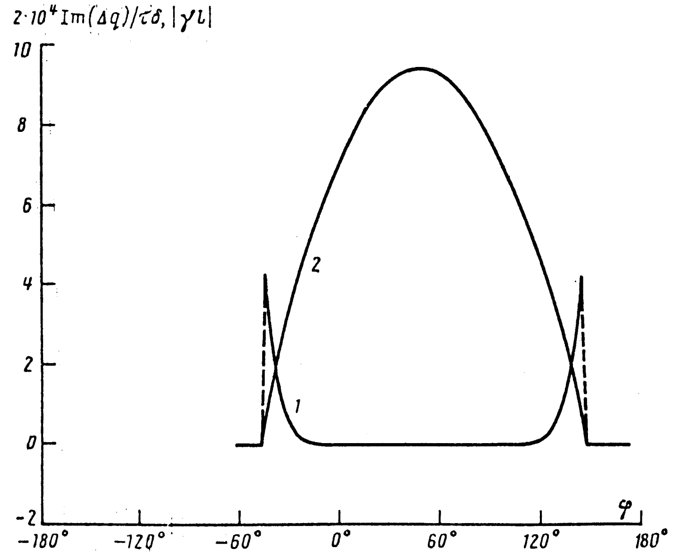

FIG. 7. Dependence of the SGEM characteristics of a film on the phase φ of the dielectric permittivity modulation at the boundary $z=0$. *1*—phase dependence of SGEM attenuation ($\mathrm{Im}(\Delta q)$), *2*—phase dependence of the parameter $|\gamma l|$, which characterizes the diffractive thickness of the layer (the thickness is measured in units of the extinction length); $\bar{\varepsilon}=\varepsilon_3=1.5$; $\varepsilon_1=1.0$, $\delta=0.05$, $l=209d/3$, $2\omega/\tau c=2.0$, $d=2\pi/\tau$.

ation (curve *1*) and the decay of the wave field into the film (curve *2*) on the phase of the dielectric permittivity modulation at the $z=0$ surface of the film.

CONCLUSION

The results we have presented here demonstrate that the SGEM characteristics depend significantly on the phase of the dielectric permittivity modulation at the surface of a periodic medium. The dependence we have identified allows us to connect the SGEM characteristics with the detailed structure of the modulation of the dielectric properties, not only for the well-investigated case of a layered medium[1-3] but also for the case of a medium whose dielectric properties are subject to modulation of more general form.

In fact, it follows from the discussion we have given here that in general the characteristics of SGEM turn out to depend significantly on the phase of all the Fourier harmonics which enter into the expansion of the dielectric permittivity, and which are responsible for diffraction scattering of the field in the periodic medium. This phase dependence, along with the effects of finiteness of the layer thickness, determines the optimum conditions for experimental observation of SGEM. In particular, it is now clear that the detailed structure of the variation in dielectric properties, specifically those properties near the boundary with the uniform medium, strongly affects the behavior of SGEM. This opens up the possibility of artificial variation of SGEM parameters by changing the profile of the dielectric permittivity at the boundary.

Thus, the results we have presented here can be used to identify the optimum SGEM parameters for a structure of given thickness and with known dielectric permittivity profile; conversely, starting with a given set of SGEM parameters, the equations can be used to identify a structure in which the SGEM will possess certain prespecified parameters.

It should also be noted that the study of SGEM in structures with controllable periodicity parameters, e.g., chiral liquid crystals[7] and media whose properties are modulated by ultrasonic waves, can be extremely useful in the experimental verification of the behavior we have discussed here, and also perhaps for future applications.

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