Dislocation melting of a 2D Wigner crystal in random fields

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The effect of frozen-in random fields of various types on the translational and orientational order in a two-dimensional ($2D$) classical Wigner crystal and on the dislocation melting of the crystal is analyzed. Random fields corresponding to isotropic impurities interact with only the hard longitudinal plasma mode of the $2D$ Wigner crystal. In the harmonic approximation they do not disrupt the quasi-long-range $2D$ translation order. When dislocations are taken into account, randomly distributed Coulomb impurities lead to a reentrant low-temperature melting in addition to the shift in the temperature of the ordinary Kosterlitz–Thouless dislocation melting, provided that the concentration of these Coulomb impurities does not exceed a certain critical level. In the case of an anisotropic random field corresponding to irregularities of the substrate, the system has only a short-range crystalline order, and a true critical behavior is not possible. If the disorder in the system is only slight, however, one can observe smeared dislocation transitions, both the ordinary transition and a reentrant low-temperature transition.

1. INTRODUCTION

The Wigner crystallization of charged particles in two-dimensional ($2D$) systems is presently attracting a great deal of research interest (see, for example some conference proceedings[1,2]). In the case of electrons or helium ions at the surface of liquid helium, where the typical surface concentrations are $n < 10^9$ cm^{-2} a $2D$ Wigner crystal of electrons[3] or helium ions[4] is realized under classical conditions. In the case of $2D$ semiconductor systems, in which the typical electron concentrations are $n \sim 10^{11}$ cm^{-2}, the possibility of using a high magnetic field to induce crystallization under ultraquantum conditions has probably been realized for high-mobility electrons in GaAs/GaAlAs heterostructures. This possibility had been discussed theoretically many years ago.[5] Evidence that this possibility has been realized has come from experiments by rf spectroscopy, a magnetooptic method, and a magnetotransport method[6] (see also Refs. 1 and 2). In these systems, under quantum conditions, with a partial filling of the lower Landau level ($\nu = nhc/eV < 1$) by electrons, a competition occurs between the state with a crystalline order and the state of an incompressible quantum fluid of the Laughlin type[7] near the fractional values $\nu = 1/5$, $1/7$, and $1/9$. As the magnetic field B is raised further, quantum fluctuations are suppressed, and (at fixed values of the electron concentration n and the temperature T) a Wigner crystal should be realized under classical conditions. At sufficiently small values of ν, a classical regime prevails near the melting line. This regime may have been realized in the experiments of Ref. 8 (with $\nu < 0.1$).

The actual experiments of Ref. 9 and the numerical simulation of Ref. 10 indicate that the melting of a classical $2D$ Wigner crystal occurs by a Kosterlitz–Thouless dislocation mechanism[11–14] and that it is a continuous phase transition, not accompanied by abrupt changes in thermodynamic properties. In the present paper we examine the effect of a frozen-in disorder on the translation order, on the orientational order, and on the dislocation melting of a $2D$ Wigner crystal. Some of the results below were reported in Ref. 15.

The dislocation-melting model has been used previously to study certain problems concerning $2D$ crystals with a short-range interaction between particles. Specifically, these problems were the effect of frozen short-range impurities introduced in the crystal,[16] the effect of a random substrate relief,[17] and the effect of a random pinning force stemming from irregularities of the substrate.[18–20] The case of a $2D$ Wigner crystal requires consideration of two physically related circumstances. First, the long-range Coulomb force makes a $2D$ Wigner crystal incompressible in the long-wave limit. Standard elastic theory, ordinarily employed in studying problems of this type, is thus not directly applicable in this case. Second, for a $2D$ Wigner crystal it is physically justified and indeed important to examine random fields with long-range correlations, e.g., a field of ionized donors.

Here is an outline of the paper. The model is introduced in Sec. 2. In Sec. 3 we determine whether the power-law ordering typical $2D$ systems is preserved in the presence of random fields in the harmonic approximation (in which dislocations are ignored). In other words, we determine whether the crystal converts into a glass. The answer to this question depends strongly on the nature of the interaction of the random field with the lattice. In a $2D$ Wigner crystal there is no interaction of the random field of isotropic impurities with a Goldstone mode, i.e., with transverse phonons. In contrast, the interaction with the longitudinal plasma mode, which is harder (and which has a dispersion $\omega_q \propto q^{1/2}$ in the absence of a magnetic field), does not disrupt the existing power-law (quasi-long-range) order. The implication is that a topological phase transition—dislocation melting—may occur even in the presence of impurities. The analysis in Secs. 4 and 5 shows that randomly distributed Coulomb impurities play the role of short-range impurities for an ordinary crystal.[16] In particular, they change the low-temperature behavior substantially, leading to a reentrant melting. Impurities with a shorter-range potential do not alter the

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critical state of a $2D$ Wigner crystal.

A random field corresponding to substrate irregularities is examined in Subsec. 3.5. This field interacts with a transverse phonon mode. As a result, the $2D$ Wigner crystal, like an ordinary $2D$ crystal,[18-20] is in a disordered state. Strictly speaking, a topological phase transition is impossible. However, if the order is disrupted by the random field over a distance of macroscopic size, then the behavior of the system may exhibit some rounded features associated with a dissociation of dislocation pairs.[20]

2. MODEL

In the continuum approximation the energy of a static, inhomogeneous deformation of a $2D$ Wigner crystal with a triangular lattice is described by the Hamiltonian[21]

$$H_0=\int d^2r\left\{\frac{1}{2}\lambda u_{kk}^2+\mu u_{ik}^2+\frac{1}{2}n^2e^2\int d^2r'\,\frac{u_{kk}(\mathbf{r})\,u_{ll}(\mathbf{r}')}{\varkappa|\mathbf{r}-\mathbf{r}'|}\right\}, \tag{1}$$

where $u_{ik}=\frac{1}{2}(\partial u_i/\partial r_k+\partial u_k/\partial r_i)$ is the strain tensor; λ and μ are elastic constants (Lamé coefficients), which we treat as phenomenological constants and which may, in particular, embody a dependence on the magnetic field B $\lambda=\lambda(B,T)$, $\mu=\mu(B,T)$; n is the surface concentration of electrons; and $\varkappa$ is the dielectric constant. The Hamiltonian H_0 in (1) differs from the standard Hamiltonian of elastic theory in the presence of the last term, which corresponds to an energy component due to long-range Coulomb interactions of the charge density, $\delta\rho(\mathbf{r})=neu_{kk}$. These interactions arise because of the nonuniform strain. It is thus possible to deal correctly with the incompressibility of the $2D$ Wigner crystal in the long-wave limit [see, for example expression (15) below for the Fourier transform of the dynamic matrix $D_{ik}(\mathbf{q})$].

The presence of a frozen disorder is described by the part of the Hamiltonian

$$H_1=-\int d^2r\mathbf{f}(\mathbf{r})\mathbf{u}(\mathbf{r}), \tag{2}$$

where $\mathbf{f}$ is the local density of the random force. We introduce various types of disorder in a phenomenological way, as described below (cf. Refs. 16–20).

In the first case, the force $\mathbf{f}$ is related to the potential field $\Phi(\mathbf{r})$ created by the randomly distributed impurities, with a concentration $c(\mathbf{r}')$:

$$H_1=\int d^2r\mathbf{u}(\mathbf{r})\,\nabla\Phi(\mathbf{r}),\quad \Phi(\mathbf{r})=\int d\mathbf{r}'V(\mathbf{r}-\mathbf{r}')c(\mathbf{r}').$$

This representation may be thought of switching to a continuum description from a discrete Hamiltonian of the electron-impurity interactions with an effective potential $v(\mathbf{r})$ (e.g., a potential modified by image forces):

$$\sum_{\mathbf{N}}\sum_{j}v(\mathbf{R}_{\mathbf{N}}+\mathbf{u}_{\mathbf{N}}-\mathbf{r}_j)\to\int d^2r\,n\int d\mathbf{r}'c(\mathbf{r}')[\nabla_{\mathbf{r}}v(\mathbf{r}-\mathbf{r}')]\mathbf{u}(\mathbf{r}).$$

Only those terms which are linear in the displacements $\mathbf{u}$ are retained. Here $\mathbf{R}_{\mathbf{N}}$ give the positions of the sites of the ideal $2D$ lattice, and $c(\mathbf{r}')=\Sigma_j\delta(\mathbf{r}'-\mathbf{r}_j)$ is the impurity concentration. The impurities may be in the crystal itself or in $3D$ space.

Since we are interested in the long-range asymptotic behavior, we assume, regarding the impurities, that $c(\mathbf{r})$ is a Gaussian random function. We wish to stress that by writing the force $\mathbf{f}$ as the gradient of a potential field, $\mathbf{f}=-\nabla\Phi$ (or as the divergence of the stress field σ_{ik} due to the impurities, $f_i=\partial_k\sigma_{ik}$), by integrating by parts in (2), and by ignoring the boundary contribution, we can write Hamiltonian (2) as

$$H_1=-\int d^2r\Phi(\mathbf{r})u_{kk}(\mathbf{r})\quad\left(H_1=\int d^2r\sigma_{ik}(\mathbf{r})u_{ik}(\mathbf{r})\right).$$

In other words, the Hamiltonians which we have found for the interaction with a disorder are translationally invariant: They are unchanged by a shift of the lattice as a whole, $\mathbf{u}(\mathbf{r})\to\mathbf{u}(\mathbf{r})+\mathbf{u}_0$ (see also the discussion in Refs. 16 and 19).

We will discuss two cases below.

1) The first is that of randomly distributed, frozen-in impurities which have been introduced in the $2D$ Wigner crystal. These impurities may participate in long-wave phonon displacements, but (as in Ref. 16) they cannot trade places with neighboring lattice sites over the duration of the experimental observation. For the case of a $2D$ Wigner crystal we assume that there may be a variety of electron-impurity interaction potentials $V(\mathbf{r})$. In particular, there may be long-range potentials, which would distinguish this case from that of ordinary crystals, in which the interaction forces are of short range in the continuum approximation.

We consider the influence of effective potentials from fairly wide classes: (a) integrable potentials, i.e.,

$$\mathscr{V}^0\equiv\int\limits_{r>a_0}d^2rV(\mathbf{r})<\infty$$

(a_0 is the lattice constant); and (b) long-range potentials, for which we have the Fourier transform $V(\mathbf{q})\approx\mathscr{V}/q^{2-\gamma}$ in the limit $q\to0$. This case corresponds to the behavior $V(\mathbf{r})\propto r^{-\gamma}$, with $0<\gamma<2$, as $r\to\infty$.

For the Fourier transforms of the random force $f_i(\mathbf{q})$ we have a Gaussian correlation function $\mathscr{T}_{ik}^{(1)}$:

$$[f_i(\mathbf{q})f_k(-\mathbf{q})]\equiv\mathscr{T}_{ik}^{(1)}(\mathbf{q})=n_{imp}q_iq_k|V(\mathbf{q})|^2, \tag{3}$$

where n_{imp} is the impurity concentration (in particles per square centimeter), $V(\mathbf{q})$ is the Fourier transform of the potential $V(\mathbf{r})$, and the brackets [...] mean an average over the disorder.

2) The second case is that of ionized donors which are distributed in a layer of thickness t at a minimal distance α from the plane of the $2D$ Wigner crystal. Assuming $t, a_0\ll\alpha$, we have a Gaussian correlation function similar to (3) in this case (see also Ref. 22):

$$\mathscr{T}_{ik}^{(2)}(\mathbf{q})=n_{imp}q_iq_k(2\pi ne^2/\varkappa q)^2e^{-2\alpha q}. \tag{4}$$

A disorder of another type is the random force $\mathbf{f}(\mathbf{r})$ associated with irregularities of the substrate. For this force we assume, as in Refs. 18–20, a Gaussian distribution corresponding to a white noise:

$$[f_i(\mathbf{r})f_k(\mathbf{r}')]=\sigma\delta_{ik}\delta(\mathbf{r}-\mathbf{r}'), \tag{5}$$

$$\mathscr{T}_{ik}^{(3)}(\mathbf{q})=\sigma\delta_{ik}. \tag{6}$$

We wish to stress that the correlation functions in (3), (4), and (6) have different tensor structures (in addition to

the different dependences on the wave vector q): For fields corresponding to isotropic impurities, correlation functions (3) and (4) are purely longitudinal. As a result, random fields of this type interact exclusively with the longitudinal mode of the crystal, which for a $2D$ Wigner crystal is a plasma mode. This plasma mode is harder than the transverse phonon mode and thus has no effect at all on the orientational order in the harmonic approximation.

3. LONG-RANGE TRANSLATIONAL AND ORIENTATIONAL ORDERS

3.1. Correlation functions

The presence of a long-range translational order and of an orientational order can be established by analyzing the behavior of the following correlation functions, respectively:[14]

$$C_{\mathbf{G}}(\mathbf{r})=[\langle\exp\{i\mathbf{G}(\mathbf{u}(\mathbf{r})-\mathbf{u}(0))\}\rangle], \tag{7}$$

$$C_O(\mathbf{r})=[\langle\exp\{6i(\theta(\mathbf{r})-\theta(0))\}\rangle]. \tag{8}$$

Here $\mathbf{G}$ is an arbitrary wave vector; $\theta=\frac{1}{2}\varepsilon_{ik}\partial_i u_k$; and ε_{ik} is the $2D$ antisymmetric tensor. Two averages are taken in succession in (7) and (8): one over the thermodynamic ensemble at the temperature T (we are assuming $k_B=1$) with the Hamiltonian $H=H_0+H_1$ (this average is denoted by the angle brackets) and then one over the random field (this average is denoted by the square brackets).

The calculation in (7) and (8) is conveniently carried out in the Fourier representation, through the use of the relation

$$\langle\ldots\rangle=\langle\exp(-H_1/T)\ldots\rangle_0/\langle\exp(-H_1/T)\rangle_0 \tag{9}$$

to make the transition to a calculation of thermodynamic averages $\langle\ldots\rangle$ over the ensemble with the quadratic Hamiltonian H_0 from (1). Since (a) Hamiltonian H_1 in (2) is linear in the displacements $\mathbf{u}$ [as are the arguments of the exponential functions of correlation functions (7) and (8)] and in the random variables $\mathbf{f}$, and since (b) the terms quadratic in $\mathbf{f}$ in the numerator and denominator in (9) cancel out, all these averages reduce to Gaussian averages of the linear forms in the arguments of the exponential functions. For the correlation functions we easily find

$$C_i(\mathbf{r})=C_i^{(0)}(\mathbf{r})C_i^{(1)}(\mathbf{r})\quad(i=\mathbf{G},O),$$

where $C_i^{(0)}$ is the correlation function for the pure system, and $C_i^{(1)}$ is a multiplicative increment which stems from the disorder. For a pure $2D$ Wigner crystal we find the known asymptotic behavior[14] at $r\gg a_0$:

$$C_{\mathbf{G}}^{(0)}(\mathbf{r})\propto r^{-\eta_G^{(0)}},\quad \eta_G^{(0)}=\frac{T}{4\pi\mu}G^2, \tag{10}$$

$$C_O^{(0)}(\mathbf{r})\approx\exp\left(-\frac{9T}{4\pi\mu a_0^2}\right). \tag{11}$$

This behavior corresponds to a quasi-long-range power-law translational order and to a genuinely long-range orientational order. A distinctive feature of (10) is that the only dependence is on the absolute value of the shear modulus μ of the crystal. In a $2D$ Wigner crystal, thermal fluctuations of only the transverse phonon mode participate in the disruption of the translation order. This situation is closely related to the incompressibility as $q\to 0$ (the hardness of the longitudinal plasma mode).

In a pure $2D$ system, a change in the asymptotic behavior of the correlation functions occurs (in the absence of an anharmonicity) only if topological defects—dislocations and disclinations—are taken into account. With $T=T_m\approx\mu a_0^2/4\pi$, for example, the $2D$ Wigner crystal undergoes a topological phase transition involving a dissociation of dislocation pairs. In the process it loses its static shear hardness: It melts into a hexatic liquid-crystal phase,[14] in which a quasi-long-range orientation order prevails:

$$C_{\mathbf{G}}^{(0)}(\mathbf{r})\propto\exp\left[-\frac{r}{\xi(T)}\right],\quad C_O^{(0)}(\mathbf{r})\propto r^{-\eta_6^{(0)}}. \tag{12}$$

Here $\xi(T)$ is a finite correlation length, which diverges in a power-law fashion as $T\to T_m^+$.

If, when there is a disorder, the calculation (with phonons alone taken into account) leads to a power-law decay of $C_{\mathbf{G}}(\mathbf{r})$, then a topological phase transition is still possible, as in the pure system. The effect of a disorder on the latter transition in the case of a $2D$ Wigner crystal is analyzed below in Secs. 4 and 5.

If the disorder leads instead to an exponential decay of the correlation functions (with a length scale l, which definitely places a limit on the correlation length ξ), the meaning is that fluctuations of the random field disrupt the crystalline translational order. At low temperatures the $2D$ Wigner crystal is in a disordered "glasslike" state, and a topological phase transition cannot occur in it, strictly speaking (but see Subsec. 3.5 below and also Ref. 20).

3.2. Disorder-induced multiplicative increments

For the increments in the correlation functions we have

$$C_{\mathbf{G}}^{(1)}(\mathbf{r})=\exp\left\{-G_iG_k\int\frac{d^2q}{(2\pi)^2}D_{ij}^{-1}D_{kl}^{-1}\mathcal{T}_{jl}(\mathbf{q})\times[1-\cos(\mathbf{qr})]\right\}, \tag{13}$$

$$C_O^{(1)}(\mathbf{r})=\exp\left\{-9\int\frac{d^2q}{(2\pi)^2}q^2P_{ij}^{tr}D_{ik}^{-1}D_{jl}^{-1}\mathcal{T}_{kl}(\mathbf{q})\times[1-\cos(\mathbf{qr})]\right\}, \tag{14}$$

where the dynamic matrix of the $2D$ Wigner crystal is

$$D_{ik}(\mathbf{q})=\mu q^2P_{ik}^{tr}+(2\mu+\bar{\lambda})q^2P_{ik}^{l},\quad \bar{\lambda}=\lambda+\frac{2\pi n^2e^2}{\varkappa q}, \tag{15}$$

and the transverse and longitudinal projection operators are given by the following expressions, respectively:

$$P_{ik}^{tr}(\mathbf{q})=\delta_{ik}-\frac{q_iq_k}{q^2},\quad P_{ik}^{l}(\mathbf{q})=\frac{q_iq_k}{q^2}. \tag{16}$$

We are interested in the long-range asymptotic behavior of correlation functions (13) and (14). We can thus replace the integrands in the arguments of the exponential functions in (13) and (14) by their long-wave limits [in particular, for the Fourier transforms of the interaction potential $V(\mathbf{q})$ introduced above].

Below we report results on the long-range asymptotic behavior of correlation functions (13) and (14) found in the ordinary harmonic approximation (in which dislocations

are ignored). We repeat that the random fields corresponding to isotropic impurities do not affect the orientation order in this approximation, as follows from (14) (see also the discussion at the end of Sec. 2).

3.3. Frozen-in impurities in a 2D Wigner crystal

3.3.1. For impurities with interaction potentials which fall off more rapidly than r^{-1}, we have $C_G^{(1)}(\mathbf{r}) \to \text{const} < 1$ as $r \to \infty$. As a result, such impurities cause no qualitative change in the quasi-long-range translational order of the 2D Wigner crystal. Here we are of course tacitly assuming that the impurity concentration is low. In the opposite case, small values of the correlation functions, $C_G^{(1)} \ll 1$, would indicate a crystal–glass transition caused by short-range impurities.

3.3.2. For frozen charged Coulomb centers with a potential $V(r) = ne^2/\varkappa r$, we find a power-law decay of the correlation function:

$$C_G^{(1)}(\mathbf{r}) \propto r^{-\eta_G^{(1)}}, \quad \eta_G^{(1)} = \frac{Cn_{imp}}{4\pi n^2} G^2. \tag{17}$$

A weak angular dependence has been omitted from (17). The constant $C \leqslant 1$ incorporates correlations in the arrangement of impurities in a phenomenological way.

Randomly distributed, frozen-in charged impurities in a 2D Wigner crystal thus play the role of short-range impurities (dilatation centers; Ref. 23, for example) which are present in a 2D crystal with a short-range interaction between particles.[16] This "freezing in" of the charged impurities, to be understood here in the sense specified above (Sec. 2), can probably be realized in a system of charged polystyrene microspheres in a 2D geometry[24] and also for electrons and helium atoms at the surface of liquid helium. In these cases the correlations in the arrangement of impurities may be extremely important and may lead to small values $C \ll 1$.

3.3.3. For long-range potentials $V(r) \propto r^{-\gamma}$ ($V(\mathbf{q}) \approx \mathscr{V}/q^{2-\gamma}$), $0 < \gamma < 1$, we find

$$C_G^{(1)}(\mathbf{r}) \propto \exp\left\{-B\left[\frac{G^2}{\varepsilon} + \left(\mathbf{G}\frac{\mathbf{r}}{r}\right)^2\right] r^{\varepsilon}\right\},$$

$$B = \frac{n_{imp}\mathscr{V}^2}{16\pi^3 n^3 e^4} \frac{\Gamma(1-\varepsilon/2)}{2^{\varepsilon}\Gamma(2+\varepsilon/2)}, \tag{18}$$

where $\varepsilon \equiv 2(1-\gamma)$, and $\Gamma(x)$ is the gamma function. The exponential decay in (18) becomes the power-law decay in (17) as $\gamma \to 1^{-0}$. This analysis, however, is purely formal: The corresponding random fields are extremely singular and apparently could not be realized physically (cf. Subsec. 4.2).

3.4. Ionized donors

For the case of ionized donors in a layer of thickness t in GaAlAs at a minimal distance α from a 2D Wigner crystal (another physical realization would consist of charged ions in an insulating layer below the surface of liquid helium; see Ref. 25 and the papers cited there), we use expression (4) for the correlation function $\mathscr{T}_{ik}(\mathbf{q})$. For $r \gg r_s, a_0, t$, where $r_s = \varkappa(\lambda + 2\mu)/2\pi n^2 e^2$ is the screening length of the 2D Wigner crystal (Subsec. 4.2), we find

$$C_G^{(1)}(\mathbf{r}) \approx \exp\left\{-\eta_G^{(1)} \ln\left[\frac{2\alpha + (4\alpha^2 + r^2)^{1/2}}{4\alpha}\right]\right\},$$

$$\eta_G^{(1)} = \frac{Cn_{imp}}{4\pi n^2} G^2. \tag{19}$$

As above, the constant $C \leqslant 1$ incorporates the correlations in the arrangement of impurities in a phenomenological way.

For the correlation function in (19) there are two regions which differ in the behavior of the correlation function. At $r \lesssim 2\alpha$ the function $C_G^{(1)}(\mathbf{r}) \approx 1$, so in this region we essentially have the translational order of the pure crystal. At $r \gg 2\alpha$, we have a power-law decay: $C_G^{(1)}(\mathbf{r}) \approx (r/4\alpha)^{-\eta_G^{(1)}}$. In the harmonic approximation, fluctuations of the random field of the charged donors thus do not alter the power-law translational order. Note, however, that at $n_{imp} \approx n$ the quantity $\eta_G^{(1)}$ in (19) is by no means small (for the principal reciprocal-lattice vector, $G_0^2 = 16\pi^2/3a_0^2$, for example, we have $\eta_G^{(1)} \approx 3.6Cn_{imp}/n$), provided that the fluctuations of the donor charge are not suppressed in some special way (so that we have $C \ll 1$), and provided that the relation $n \gg n_{imp}$ is not realized in the system, for example by photoexcitation.

3.5. Random force exerted by the substrate

Using (6), and cutting off (as in Refs. 19 and 20) the integral in the argument of the exponential function in (13), which diverges at its lower limit, at the value corresponding to the reciprocal of the size of the system, L^{-1}, we find the following expressions for the correlation functions describing the translational and orientational orders, respectively:

$$C_G^{(1)}(\mathbf{r}) \propto r^{-\eta_G^{(1)}} \exp\left[-\frac{\sigma G^2}{32\pi\mu^2}(3r_\perp^2 + r_\parallel^2)\ln\frac{L}{r}\right],$$

$$\eta_G^{(1)} = \frac{\sigma G^2}{16\pi^3 n^4 e^4}, \tag{20}$$

$$C_O^{(1)}(\mathbf{r}) \propto r^{-\eta_O^{(1)}}, \quad \eta_O^{(1)} = \frac{9\sigma}{2\pi\mu^2}, \tag{21}$$

where $r_\perp$ ($r_\parallel$) is the component of $\mathbf{r}$ which is perpendicular (parallel) to the vector $\mathbf{G}$. The fluctuations caused in the longitudinal plasma mode by the random force thus lead to a correlation-function component which falls off by a power law, while the fluctuations of the transverse (softer) phonon mode introduce an exponential decay in the correlation function for the translation order, in (20), with a length scale $l \propto \sigma^{1/2}$, and they introduce a power-law decay in the correlation function for the orientational order, in (21) (cf. Refs. 18–20). The system therefore has only a short-range crystalline order; a true critical behavior is strictly speaking not possible.

If the system has a moderate disorder, it may be thought of as a Wigner glass with a quasi-long-range orientational order. If the disorder is slight, and the relation $l \gg a_0$ holds, the system is physically a Wigner crystal with a finite correlation length. An analysis like that in Ref. 20 shows that rounded features corresponding to ordinary dislocation melting and also a low-temperature reentrant melting should be observed in the system.

4. EFFECTIVE DISLOCATION HAMILTONIAN

4.1. Dislocation part of the displacement tensor of a 2D Wigner crystal

When there are randomly distributed impurities which do not disrupt the quasi-long-range translational order of the 2D Wigner crystal in the harmonic phonon approximation, there is still the possibility of a true critical behavior: a

dislocation melting. In this case it is important to determine the effect of a disorder on this behavior, especially at low temperatures.

For this purpose we bring dislocations into the discussion, writing the displacement $\mathbf{u}$ as the sum of a dislocation part $\mathbf{u}^*$ and a phonon part ψ:

$$\mathbf{u}=\mathbf{u}^*+\psi, \quad \oint_c d\mathbf{u}^*=a_0\sum_{\alpha\in c}\mathbf{b}^\alpha, \tag{22}$$

where $\mathbf{b}^\alpha$ are the dimensionless Burgers vectors of the dislocations. As usual, we assume that the dislocations are at equilibrium:

$$\frac{\delta H}{\delta \mathbf{u}^*(\mathbf{r})}=0, \tag{23}$$

where the Hamiltonian is $H=H_0+H_1$. The component H_1, from (2), corresponds to impurities which do not disrupt the power-law translational order in the harmonic approximation. By virtue of (23) and the quadratic dependence of H on the displacements, we have $H(u^*+\psi)=H(u^*)+H(\psi)$. In other words, the phonon component and the dislocation component separate.

In the Fourier representation, the dislocation part of the displacement tensor u_{ij}^* is written[21]

$$u_{ij}^*(\mathbf{q})=\left[P_{ij}^{tr}(\mathbf{q})-\frac{\bar\lambda}{2\mu+\bar\lambda}P_{ij}^{l}(\mathbf{q})\right]\eta(\mathbf{q}) -\frac{V(\mathbf{q})c(-\mathbf{q})}{2\mu+\bar\lambda}P_{ij}^{l}(\mathbf{q}), \tag{24}$$

$$\eta(\mathbf{q})=i\varepsilon_{kl}\frac{a_0 q_k\rho_l(\mathbf{q})}{q^2}, \quad \rho(\mathbf{q})=\sum_\alpha \mathbf{b}^\alpha\exp(-i\mathbf{q}\mathbf{r}^\alpha), \tag{25}$$

where $\rho(\mathbf{q})$, $V(\mathbf{q})$, and $c(\mathbf{q})$ are the Fourier transforms of respectively the dislocation density, the electron-impurity potential, and the impurity concentration.

The last term in (24), $\delta u_{ik}^* \propto V(\mathbf{q})c(-\mathbf{q})$, describes the distortion of the electron lattice by the impurities which are introduced; the screening of the impurity fields is related to this distortion. Let us discuss this effect.

4.2. Screening of charged impurities in a 2*D* Wigner crystal

We first note that the random field corresponding to impurities with fairly long-range interactions are singular. For δ-correlated Coulomb impurities, for example (this is the case which we will be discussing below), the correlation function of the random field,

$$[\varphi(\mathbf{r})\varphi(\mathbf{r}')]=n_{imp}\int\frac{d^2q}{(2\pi)^2}\exp\{i\mathbf{q}(\mathbf{r}-\mathbf{r}')\}\left(\frac{2\pi e}{\varkappa q}\right)^2 \tag{26}$$

diverges logarithmically at large distances in the 2*D* system. The mean square fluctuations of the potential at a point are proportional to the logarithm of the area of the system. There is accordingly the question of whether fields of this type can actually be achieved in real systems.[26]

We note in this connection that the total field (the field of the impurities plus the field of the deformed lattice),

$$\varphi_t(\mathbf{r})=\int d^2r'\,\frac{ec(\mathbf{r}')-ne\delta u_{kk}^*(\mathbf{r}')}{\varkappa|\mathbf{r}-\mathbf{r}'|} =\int\frac{d^2q}{(2\pi)^2}\frac{2\pi ec(\mathbf{q})}{q+r_s^{-1}}, \quad r_s=\frac{\varkappa(\lambda+2\mu)}{2\pi n^2e^2}, \tag{27}$$

which is the observable field, turns out to be regular, as can be seen from (27).

From (27) we also find an expression for the screened potential of an individual impurity of charge e at point $\mathbf{R}$:

$$\varphi_t(\mathbf{r})\approx\frac{e}{\varkappa|\mathbf{r}-\mathbf{R}|^3 r_s^{-2}}, \quad |\mathbf{r}-\mathbf{R}|\gg r_s.$$

The screening of a Coulomb center by the electrons of a 2*D* lattice thus occurs in qualitatively the same way (the potential varies $\propto r^{-3}$ at $r\gg r_s$) as the screening by free 2*D* electrons (Ref. 27, for example).

4.3. Effective replica interaction of dislocations

Using (24) and (25), we find the following result for the dimensionless dislocation Hamiltonian $H_D\equiv H(u^*)/T$ in the Fourier representation:

$$H_D=\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}\left\{\varepsilon_{ik}\varepsilon_{jl}\frac{4\mu a_0^2}{T}\frac{\mu+\bar\lambda}{2\mu+\bar\lambda}\frac{q_k q_l}{q^4}\rho_i(\mathbf{q})\rho_j \times(-\mathbf{q})+V_i^D(-\mathbf{q})\rho_i(\mathbf{q})\right\}, \tag{28}$$

$$V_i^D(-\mathbf{q})=\frac{2\mu a_0}{T}\frac{V(\mathbf{q})c(-\mathbf{q})}{2\mu+\bar\lambda}\frac{i\varepsilon_{ik}q_k}{q^2}. \tag{29}$$

The part of Hamiltonian H_D in (28) which is quadratic in ρ_i, i.e., H_D^0, can be put in the form of the potential energy of binary interactions and the sum of their "intrinsic" energies (we are thinking of configurations with $\Sigma_\alpha \mathbf{b}^\alpha=0$, which have finite energies in the thermodynamic limit):

$$H_D^0=\frac{K}{8\pi}\sum_{\alpha\neq\beta}b_i^\alpha b_j^\beta\{U_{ij}(\mathbf{r}^\alpha-\mathbf{r}^\beta)+\delta U_{ij}(\mathbf{r}^\alpha-\mathbf{r}^\beta)\}+\frac{E_c}{T}\sum_\alpha|\mathbf{b}^\alpha|^2, \tag{30}$$

$$K=\frac{4\mu a_0^2}{T}, \quad E_c\approx\frac{\mu a_0^2}{2\pi}\left(1+\ln\frac{a}{a_0}\right), \tag{31}$$

where K is a coupling constant, E_c is the energy of the dislocation core, and $a\sim a_0$ is the size of this core.

The potential in (30) is broken up into the sum of (a) the usual binary-interaction potential for 2*D* dislocations,

$$U_{ij}(\mathbf{r})=\delta_{ij}\ln\left(\frac{r}{a}\right)-\frac{r_i r_j}{r^2}, \tag{32}$$

which corresponds to a 2*D* vector Coulomb gas,[12-14] and (b) an increment[1)] $\delta U_{ij}(\mathbf{r})$, which falls off by a power law (as $r^{-3/2}$). Physically, it is obvious that the increment δU_{ij} can be ignored in the phase-transition problem, in which the remote asymptotic behavior is important (indeed, this increment always is ignored in this case).

The role played by various impurities can be analyzed qualitatively by a similar approach. For this purpose we switch to an effective Hamiltonian in which an additional interaction arises after an average is taken over the random field between dislocations. The nature of this interaction is important to the phase-transition problem.

To average the logarithm of the partition function in the expression for the free energy over the frozen disorder, we use a replica method:[28]

$$F=-T[\ln Z]=-T\lim_{M\to 0}\frac{[Z^M]-1}{M}.$$

The quantity $[Z^M] \equiv Z_{\text{eff}}$ has the form of the partition function of an M-component system of interacting replicas:

$$Z_{eff} = \operatorname{Tr}_{\{b\}}^{(1)} \dots \operatorname{Tr}_{\{b\}}^{(M)} \exp(-H_M), \tag{33}$$

$$H_M = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} \sum_{n,l}^{M} \tilde{K}_{ln} \varepsilon_{ip} \varepsilon_{jk} \frac{q_k q_p}{q^4} \rho_i^n(\mathbf{q}) \rho_j^l(-\mathbf{q}), \tag{34}$$

$$\tilde{K}_{ln}(q) = \frac{\mu+\bar{\lambda}}{2\mu+\bar{\lambda}} K\delta_{ln} - \frac{K^2}{16a_0^2} \frac{n_{imp}|V(\mathbf{q})|^2}{(2\mu+\bar{\lambda})^2}. \tag{35}$$

The trace over the dislocation variables is to be understood as the continuum expression (Ref. 13, for example)

$$\operatorname{Tr}_{\{b\}} \dots = \sum_{\{n_p\}} \left(\prod_{p=1}^{6} \frac{1}{n_p!} \right) \left(\frac{y}{a^2} \right)^N \int d^2r_1 \dots \int d^2r_N \dots . \tag{36}$$

The parameter $y = \exp(-E_c/T)$ gives the probability for finding dislocations in the system, $N = \Sigma_{p=1}^{6} n_p$ is the total number of dislocations, and n_p is the number of dislocations of unit length in configuration $\{n_p\}$ with vector $\mathbf{b}$ directed along one of the six possible directions (for a triangular lattice with $|\mathbf{b}| = 1$). In the pure system, dislocations with $|\mathbf{b}| > 1$ are unimportant from the renormalization-group standpoint.[12,29] The impurity case is discussed in Subsec. 5.2.

For all impurities which conserve the quasi-long-range translational order in the harmonic approximation, the coupling constant $\tilde{K}_{ln}(q)$ remains finite as $q \to 0$. In the case $\tilde{K}_{ln}(q) \propto q^{\beta}, \beta > 0$ $(l \neq n)$, the additional interaction falls off by a power law at large distances and can be ignored, as in a pure system (there is no reason to expect the appearance of singularities as $M \to 0$). The potential of the interaction of such impurities with the lattice falls off more rapidly than r^{-1} as $r \to \infty$ (Subsec. 3.3.1).

For impurities with a Coulomb potential the coupling constant is

$$\tilde{K}_{ln}(0) \equiv K_{ln} = K\delta_{ln} - \bar{\sigma}K^2, \quad \bar{\sigma} = \frac{Cn_{imp}}{16a_0^2 n^2} \frac{3^{1/2} Cn_{imp}}{32n}, \tag{37}$$

and the additional interaction between dislocations (with a distinct temperature dependence $\propto T^{-2}$ and the opposite sign!) has the same functional form [see (32)] as that for the pure system.

The corresponding critical behavior (as $M \to 0$) was studied in Ref. 16. Again in the phase-transition problem, Coulomb impurities in a $2D$ Wigner crystal thus play the role of short-range crystals for $2D$ crystals with a short-range impurities (see also Subsec. 3.3.2), and the results of Ref. 16 can be transferred to our case. The most important of those results will be summarized below in Sec. 5, where we also analyze the approximate Wilson renormalization-group equations corresponding to (37).

5. SCALE DIMENSIONS OF THE IMPURITY COUPLING CONSTANTS

5.1. To determine the role played by various impurities from the renormalization-group standpoint, we make use of the well-known duality between the grand canonical sum of a $2D$ Coulomb gas (with a logarithmic interaction) and the sine-Gordon field model (Refs. 29–32, for example). To go over to the field model we write $\exp(-H_M)$ as the ratio of two functional integrals in terms of auxiliary two-component fields Ψ^l, $l = 1,\dots, M$, where l is the replica index. After the trace is taken over the dislocation variables (Ref. 30, for example), Z_{eff} becomes

$$Z_{eff} = \int \left(\prod_{l=1}^{M} D\Psi^l \right) \exp\{-H(\Psi^l)\}, \tag{38}$$

$$H = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} \sum_{n,l}^{M} q^2 \tilde{K}_{ln}^{-1} \Psi^l(\mathbf{q}) \Psi^n(-\mathbf{q}) - \omega \times \sum_{i=1}^{3} \sum_{l=1}^{M} \int d^2R \cos(\mathbf{e}_i \Psi^l(\mathbf{R})), \tag{39}$$

$$\omega = 2y/a^2, \quad \mathbf{e}_1 = (1, 0), \quad \mathbf{e}_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right), \quad \mathbf{e}_3 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2} \right). \tag{40}$$

There is an upper cutoff momentum $\Lambda \approx a^{-1}$ in (38) and (39); we have omitted the purely Gaussian normalization denominator. We have ignored the angular dependence of the interaction potential in (32); formally, we did this by making the replacement $\varepsilon_{ik}\varepsilon_{jl} \to \frac{1}{2}\delta_{ij}\delta_{kl}$ in H_M in (34). One can verify that, again in the case with impurities (which conserve the quasi-long-range order), the angular terms are intermediate in the renormalization-group sense, as in the pure system.[12,29]

For impurities with a Coulomb potential $\propto r^{-1}$ we have

$$\tilde{K}_{ln}^{-1}(0) \equiv K_{ln}^{-1} = K^{-1}[\delta_{ln} + \bar{\sigma}K/(1 - M\bar{\sigma}K)]$$

in the limit $q \to 0$. The increment $\delta K_{ln}^{-1} = \tilde{K}_{ln}^{-1}(q) - \tilde{K}_{ln}^{-1}(0)$ can be ignored (more on this below). The approximate Wilson renormalization-group equations take the following form as $M \to 0$ (see the Appendix):

$$\frac{dK^{-1}}{dl} = \frac{3}{4} c_1 y^2 \hat{K}, \quad \frac{dy}{dl} = \left[2 - \frac{\hat{K}}{8\pi} \right] y + \frac{1}{2} c_2 y^2 \hat{K}, \quad \frac{d\bar{\sigma}}{dl} = 0, \tag{41}$$

where the coupling is $\hat{K} = K - \bar{\sigma}K^2$.

From Eqs. (41) we draw the following conclusions, which agree with Ref. 16: 1) At $\bar{\sigma} > \bar{\sigma}_c = (64\pi)^{-1}$ a crystalline state is impossible. There are no fixed points if $y = 0$. The meaning is an instability with respect to the creation of dislocations. 2) At $\bar{\sigma} < \bar{\sigma}_c$ [Fig. 1 shows a schematic phase portrait of Eqs. (41); cf. Ref. 16], the impurities lead to not only changes in the critical exponents and the point of the ordinary dislocation melting but also a low-temperature reentrant dislocation melting. As a result, a crystal exists only in a bounded temperature interval. The position of the line of fixed points $y = 0$, $K_{-}^{-1} < K^{-1} < K_{+}^{-1}$, does not depend on the constants c_1 and c_2, which are not calculated in

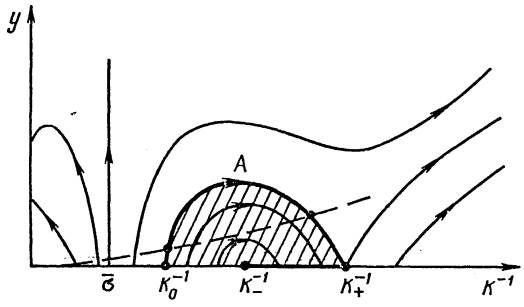

FIG. 1. Schematic phase portrait of renormalization-group equations (41) for $\bar{\sigma} < \bar{\sigma}_c$. The hatched region, with $y(l) \to 0$ as $l \to \infty$, corresponds to a crystalline phase. The phase-transition points are the points at which the initial-data line (the dashed line) $y = \exp(-E_c/T)$, where $E_c = E_c(K)$ [see (31)], intersects the separatrix $K_0^{-1} A K_+^{-1}$.

our approach (see the Appendix). This position is the same as in Ref. 16.

For all other impurities which conserve the quasi-long-range translational order (for which the potential fall off more rapidly than r^{-1}), the coupling constant at small values of q [as at $\delta K_{ln}^{-1}(q)$] is of the form $\tilde{K}_{ln}^{-1}(q) \approx W_{ln} q^{\beta}$, where $\beta > 0$ ($l \neq n$). It can be shown easily in the usual way that the scale dimensions corresponding to W_{ln} are negative in this case and equal to $-\beta$. The impurity coupling constants are inconsequential in the renormalization-group sense.[33,34]

5.2. When the system undergoes low-temperature reentrant melting, the effective replica interaction of a dislocation pair with opposite Burgers vectors becomes repulsive at low temperatures: $\hat{K} < 0$ with $K^{-1} < \bar{\sigma}$. It may thus turn out that for pairs of coupled dislocations with $|\mathbf{b}| > 1$ (which are unimportant in the absence of a disorder[12,29]) the "instability" occurs earlier than for a pair of dislocations of unit length. Let us examine this possibility. Dislocations with all possible Burgers vectors $\mathbf{b} = n\mathbf{e}_i - m\mathbf{e}_j$ ($i \neq j, n \geqslant m \geqslant 0$) make contributions of the form

$$-\sum_{n,m} \sum_{i \neq j}^{3} \sum_{l=1}^{M} \omega_{nm} \int d^2R \cos\left((n\mathbf{e}_i - m\mathbf{e}_j)\varphi^l(\mathbf{R})\right) \qquad (42)$$

to Hamiltonian (39). The scale dimensions of the coupling constants ω_{nm} ($\omega_{10} \equiv \omega = 2y/a^2$) are equal to $2 - (n^2 + m^2 + nm)\hat{K}/8\pi$. In the stability region, with $dy/dl < 0$, they turn out to be negative, and these charges are indeed inconsequential. For parameter values $K^{-1} < \bar{\sigma}$, however, which correspond to low temperatures, all charges become important, and in principle all should be taken into consideration.

6. CONCLUSION

Taking account of the compressibility of a classical 2*D* Wigner crystal in the long-wave limit, we have analyzed the effect of a frozen-in disorder of two types on the dislocation melting of such a crystal. The first type of disorder consisted of randomly distributed isotropic impurities with a variety of impurity-lattice interaction potentials, including long-range potentials. The second type of disorder was a random force with short-range correlations, stemming from irregularities of the substrate. Since the field of isotropic impurities interacts with only the hard longitudinal plasma mode of a 2*D* Wigner crystal, impurities whose potentials fall off more rapidly than r^{-1} do not affect the dislocation melting (they are inconsequential from the standpoint of the critical behavior). Randomly distributed charged Coulomb impurities act as short-range impurities in 2*D* crystals with a short-range interaction.[16] If the concentration of these impurities does not exceed the critical value $n_{\text{imp}}^{(c)} = n/2\pi 3^{1/2} C$ (n is the surface concentration of electrons; the constant $C \leqslant 1$ incorporates correlations in the arrangement of impurities in a phenomenological way), the Coulomb impurities cause not only a temperature shift of ordinary dislocation melting but also a reentrant low-temperature melting. As a result, the 2*D* Wigner crystal exists in only a bounded temperature interval. In the case $n_{\text{imp}} > n_{\text{imp}}^{(c)}$, fluctuations of the random impurity field break up the dislocation pairs, and a crystalline state with a quasi-long-range order is impossible (the translational order falls off exponentially). A situation of this sort can occur, for example, in the case of a 2*D* Wigner crystal in a heterostructure with a modulational doping, in which there is a high electron mobility (and there are also ionized donors of surface concentration $n_{\text{imp}} \approx n$ at a minimal distance $\alpha \gg n^{-1/2}$ from the plane of the 2*D* Wigner crystal, lying in a layer of finite thickness $t \leqslant \alpha$). At low temperatures, however, the regions of a short-range crystalline order may be large enough [at least no smaller than 2α; see (19)] to be detected in magnetooptic experiments.

Even when harmonic phonons alone are taken into account, the random force associated with the substrate irregularities disrupts the translational 2*D* order of the 2*D* Wigner crystal, converting this order into a short-range exponential order. In this system, which may be thought of as a Wigner glass with a quasi-long-range orientation order, a dislocation melting is not possible as a true critical behavior. If there is a slight disorder, however, and the dimensions of the crystalline domains are large in comparison with the lattice constant, the system is physically a Wigner crystal with a finite correlation length, and it can undergo some diffuse transitions involving the dissociation of dislocation pairs—either the ordinary transition or a low-temperature reentrant transition.

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APPENDIX

Here we derive renormalization-group equations (41). We write the field Ψ^l as the sum of long-wave and short-wave parts:

$$\Psi^l(\mathbf{r}) = \varphi^l(\mathbf{r}) + \vartheta^l(\mathbf{r}),$$

$$\varphi^l(\mathbf{r}) = \int_{q < \Lambda/s} \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\mathbf{r}} \Psi^l(\mathbf{q}),$$

$$\vartheta^l(\mathbf{r}) = \int_{\Lambda/s < q < \Lambda} \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\mathbf{r}} \Psi^l(\mathbf{q}). \qquad (A1)$$

Here s is related to an infinitesimal change of scale dl by the relation $s = 1 + dl$. We will integrate in Z_{eff} in Eq. (38) at the short-wave fields ϑ^l. Here we use a perturbation theory in the small parameter $y \ll 1$. Within terms on the order of y^2 inclusively, we find

$$\left\langle\!\left\langle \exp\left\{ \omega \sum_{i=1}^{3} \sum_{l=1}^{M} \int d^2R \cos\left(\mathbf{e}_i(\varphi^l(\mathbf{R}) + \vartheta^l(\mathbf{R}))\right)\right\}\right\rangle\!\right\rangle_0 = \exp\{\bar{H}(\varphi^l)\},$$

where $\langle\langle \ldots \rangle\rangle_0$ means an average with quadratic Hamiltonian $H_0(\vartheta^l)$, and $\bar{H}$ is given by (cf. Ref. 32)

$$\bar{H} = \omega S \int d^2R \sum_{i,l} \cos(\mathbf{e}_i \varphi^l(\mathbf{R}))$$

$$+ \frac{1}{4}\omega^2 S^2 \int d^2R \int d^2R' \sum_{i,l} \sum_{j,n} \{[\exp(-\mathbf{e}_i\mathbf{e}_j G_{ln}(\mathbf{R}-\mathbf{R}')) - 1]$$

$$\times \cos(\mathbf{e}_i\varphi^l(\mathbf{R}) + \mathbf{e}_j\varphi^n(\mathbf{R}'))$$

$$+ [\exp(\mathbf{e}_i\mathbf{e}_j G_{ln}(\mathbf{R}-\mathbf{R}')) - 1]\cos(\mathbf{e}_i\varphi^l(\mathbf{R}) - \mathbf{e}_j\varphi^n(\mathbf{R}'))\}, \quad \text{(A2)}$$

Here the propagator is

$$\delta_{ij} G_{ln}(\mathbf{R}) = \langle\!\langle \vartheta_i^l(\mathbf{R})\vartheta_j^n(0)\rangle\!\rangle_0$$

$$= K_{ln} \int_{\Lambda/s<q<\Lambda} \frac{d^2q}{(2\pi)^2} \frac{e^{i\mathbf{q}\mathbf{R}}}{q^2} \approx \frac{K_{ln}}{2\pi} J_0(\Lambda R)\, dl, \quad \text{(A3)}$$

$$S \equiv \exp\left(-\frac{G_{ll}(0)}{2\pi}\right) = \exp\left(-\frac{\bar{K}}{2\pi}\ln s\right), \quad \text{(A4)}$$

and $J_0(x)$ is a Bessel function. To obtain the renormalization of the coupling constants, we would like to use a gradient expansion to put the part $\bar{H}$ in (A2) in the form of the original Hamiltonian. However, if we used expression (A3) for the propagator G_{ln}, the integrals which arise become infinite in $\mathbf{R}$ space. When the fields are cut off smoothly [in contrast with the sharp cutoff in (A1)] we could expect[33,34] $G_{ln}(\mathbf{R})$ to fall off fairly rapidly at distances $R \sim (\Lambda/s)^{-1}$.

For a qualitative analysis of the critical behavior at low temperatures, we follow Ref. 35 (see also Ref. 36) in assuming that a smooth-cutoff procedure[2)] can be carried out. We furthermore assume that this procedure leads to the result

$$G_{ln}(\mathbf{R}) \to K_{ln} \tilde{J}_0(\Lambda R)\, dl/2\pi,$$

where the function $\tilde{J}_0(x)$ falls off rapidly at $x \geqslant 1$.

The renormalization of the coupling constants K_{ln} is given by the terms in Hamiltonian $\bar{H}$ which contain $\cos(\mathbf{e}_i\varphi^l(\mathbf{R}) - \mathbf{e}_j\varphi^n(\mathbf{R}'))$ with $i = j$, $l = n$. Using a gradient expansion to write them in the form $1 - \frac{1}{2}[(\mathbf{r}\nabla_{\mathbf{R}})\cdot(\mathbf{e}_i\varphi^l(\mathbf{R}))]^2$, and integrating over $\mathbf{r} = \mathbf{R} - \mathbf{R}'$, we find

$$\delta H_0 = \frac{3}{8} c_1 y^2 \bar{K} dl \sum_{l=1}^{M} \int d^2R (\partial_i \varphi_j^{\,l})^2. \quad \text{(A5)}$$

where $c_1 = \int dx x^3 \tilde{J}_0(x)$, and where we have used

$$\sum_{i=1}^{3} (\mathbf{e}_i)_k (\mathbf{e}_i)_l = {}^3/_2 \delta_{kl}$$

for a triangular lattice.

The renormalization of y is given in first order in y by the first term in $\bar{H}$. In second order—this is an effect which occurs for a triangular lattice[14,32] and which formally results from $\Sigma_{i=1}^{3} \mathbf{e}_i = 0$—this renormalization can again be found through a gradient expansion from terms containing $\cos(\mathbf{e}_i\varphi^l(\mathbf{R}) + \mathbf{e}_j\varphi^n(\mathbf{R}'))$ with $i \neq j$, $l = n$.

To restore the original scale we transform to the new fields $\tilde{\varphi}^l$: $\varphi^l(\mathbf{q}) = \zeta\tilde{\varphi}^l(\mathbf{q}')$, where $\mathbf{q}' = s\,\mathbf{q}$. We choose the parameter $\zeta = s^2$; this choice corresponds in the coordinate representation to $\varphi^l(\mathbf{R}) = \tilde{\varphi}^l(\mathbf{R}/s)$. It is then a straightforward matter to derive the renormalization-group equations, which are

$$\frac{dK_{ln}^{-1}}{dl} = \frac{3}{4} c_1 y^2 \bar{K} \delta_{ln}, \quad \text{(A6)}$$

$$\frac{dy}{dl} = \left(2 - \frac{\bar{K}}{8\pi}\right) y + \frac{1}{2} c_2 y^2 \bar{K}, \quad \text{(A7)}$$

where $c_2 = \int dx x \tilde{J}_0(x)$. In the limit $M \to 0$ we find (41), making use of the circumstance that (A6) is diagonal with respect to the replica indices.

[1)] It corresponds to the incorporation in (28) of a dependence of the coupling constant on the wave vector q: $\delta K(q) \equiv K(q) - K(0)$, where $K(q) = K(\bar{\lambda} + \mu)/(\bar{\lambda} + 2\mu)$, and $K \equiv K(0)$. A corresponding increment arises when the q dependence of the elastic constants $\lambda(q)$ and $\mu(q)$ is taken into account.

[2)] A procedure used in Ref. 32 to regularize the propagator G simulates the effect of a smooth cutoff. This procedure consists of replacing $1/q^2$ by $[q^2 + (\Lambda/s)^2]^{-1}$ in the propagator and switching to an integration over all momentum space. Some essential *uv* divergences arise in the process and render this model unrenormalizable at low temperatures, $K^{-1} < 1/16\pi$ (furthermore, the model itself can be defined only as a double expansion in the parameters $y \ll 1$ and $K/16\pi - 1 \ll 1$; Ref. 29). For this reason, the approximate approach of Ref. 32 works only near $K = 16\pi$.

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