Particle acceleration and oscillator instability in dissipative media

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A mechanism is discussed for the instability produced in an oscillator moving in a conductive medium by the reaction of the oscillator's proper (nonwave) field, and manifesting itself at essentially nonrelativistic velocities. The possibility of collective instability in the particle flow, due to this mechanism, is considered. Estimates show that such an instability could exist in media with semiconductor conductivities.

Particle uniform-motion instability connected with wave emission by the particles has been actively discussed in the literature. As is known, for the instability to arise it is necessary that the "anomalous" part of emission localized in the Cherenkov cone predominate (in the sense of action on the particle) over the "normal" part propagating outside this cone. Thus, self-oscillations can be produced spontaneously in a magnetoactive plasma by to electromagnetic-wave emission,[1] and in an isotropic plasma by emission of longitudinal waves.[2] Similar effects are observed in hydrodynamics,[3,4] where the instability of motion can be associated with surface and internal wave emission. Such processes are of interest both in their own right and as "elementary acts" for stream-type instabilities, which, of course, arise under different conditions (since they are determined by the phasing of particle radiation), but, in principle, have the same "Cherenkov" character.

In the present paper we discuss a somewhat different type of "field" particle-motion instability caused by losses.[1)] This instability is due to the near field generated in a medium by induced charges. The conditions under which it arises do not depend (for a given conductivity) on the parameter U/C, where U is the particle velocity and C is the wave phase velocity, and often turn out to be much less stringent than for a transparent medium. For example, such instability is possible in an isotropic nonideal dielectric for nonrelativistic charge motion. It leads either to aperiodic particle acceleration or (in the case of average motion stabilization) to the buildup of oscillators. We discuss first, a simple model of individual instability related to the motion of a charged particle near an interface. In principle, we should solve the self-consistent problem of the particle motion in its proper field (cf. Ref. 6). However, we will not go beyond the given motion approximation, finding the force of near-field reaction on the particle. Its phase with respect to the oscillation velocity determines the possibility of buildup or suppression of oscillations. In the second part of the paper we consider the instability of flow of oscillators in a macroscopic (mean) field, caused by the "individual" buildup of oscillators in the near field.

1. CHARGE MOTION NEAR A CONDUCTIVE MEDIUM

Let a point charge be moving in vacuum parallel to a plane interface of conductive half-space at a distance d from the latter, with velocity $U + U_\sim \cos\Omega t$, so that the oscillation amplitude is $a = U_\sim/\Omega$. Considering the motion nonrelativistic, we will start from the Poisson equation for the electric-field potential

$$\varepsilon\Delta\varphi=-4\pi\rho(x,\ y,\ z,\ t), \tag{1}$$

where $\rho = e\delta(z - Ut - a\sin\Omega t)\delta(x - d)\delta(y)$ and ε is the complex dielectric constant of the medium (different for $x > 0$ and $x < 0$). Using, as usual, the Fourier-representation of the source and satisfying the boundary conditions, we can find the charge field and, as a result (similar to Ref. 7), the drag force due to the Joule losses in the conductive medium:

$$F=-\frac{e^2 i}{\pi}\sum_{l,l_1=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1-\varepsilon(kU+l\Omega)}{1+\varepsilon(kU+l\Omega)} \times kK_0(2d|k|)J_l(ka)J_{l_1}(ka)\,dk\exp[i\Omega(l-l_1)t]. \tag{2}$$

Here J_1 and K_0 are the Bessel function of the first kind and a modified Bessel function of the second kind respectively. For a uniformly moving charge and real ε, Eq. (2) yields a well known expression for the reactive force of radiation.[7] We, however, will assume that the medium is characterized by a constant conductivity σ,[2)] e.g. $\varepsilon(\omega) = 1 + i\sigma/\omega$. Then for $l = l_1 = 0$ Eq. (2) yields an expression for the dc component of the force

$$F_0=-\frac{e^2\sigma}{\pi}\int_{-\infty}^{\infty}\frac{k}{2kU+i\sigma}K_0(2d|k|)J_0^2(ka)\,dk, \tag{3}$$

and for $l = l_1 \pm 1$ the expression for its alternating component:[3)]

$$\tilde F=-\frac{2e^2\sigma\cos\Omega t}{\pi a}\sum_{l=-\infty}^{\infty} l\int_{-\infty}^{\infty}\frac{J_l^2(ka)K_0(2d|k|)}{2(kU+\Omega l)+i\sigma}\,dk. \tag{4}$$

We assume in what follows that $ka \ll 1$ (in the opposite case the value of F_0 is considerably affected also by the oscillating component of the motion). Then, after simple transformations, the expression for F_0 is reduced to the form

$$F_0=-\frac{e^2\sigma}{4dU}\int_0^{\infty}\frac{\tau(\exp)(-\bar\sigma\tau)}{(1+\tau^2)^{3/2}}\,d\tau = -\frac{e^2\sigma}{4dU}\left[1-\frac{\pi\bar\sigma}{2}(H_0(\bar\sigma)-N_0(\bar\sigma))\right], \tag{5}$$

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where $\bar{\sigma} = \sigma d / U$, and H_0 and N_0 are the zeroth order Struve and Neumann functions respectively.

The result of numerical calculation of Eq. (5) is shown in Fig. 1. In the limiting cases of small and large speeds Eq. (5) yields

$$F_0(\bar{\sigma}\gg1) = -\frac{e^2U}{4d^3\sigma}, \quad F_0(\bar{\sigma}\ll1) = -\frac{e^2\sigma}{4dU}. \tag{6}$$

Thus, for $\bar{\sigma}\ll1$ the drag force decreases with increase of velocity, which may give rise to various instabilities. If the charge in a conductive medium is accelerated by external field acting with a force $F_a < F_{max}$ (see Fig. 1), the drag force causes a uniform motion with velocity U_u. If, however, $F_a > F_{max}$ then, in the framework of this model, the charge is accelerated without bound.[4)]

Note that the "critical" velocity U_{cr} associated with $F = F_{max}$ is found from the condition $\sigma \approx 1$, i.e. $U_{cr} \simeq \sigma d$. This velocity is usually in the essentially nonrelativistic region (see below).

If we consider a moving oscillator, its motion on the "downward" section is accompanied, generally speaking, by buildup of oscillations. To describe this process it is necessary to find the oscillating part of the force. After simple transformations we find from (4) that

$$\tilde{F} = -\frac{4e^2\sigma\cos\Omega t}{\pi}\int_0^\infty \exp(-\sigma x)\cos\Omega x\,dx\int_0^\infty K_0(2dk)k \times J_1(2ka\sin\Omega x)\cos(2kUx)dk, \tag{7}$$

whence, if $ka\ll1$,

$$\tilde{F} = -\frac{e^2\sigma a\cos\Omega t}{8d^2U}\int_0^\infty \frac{(1-2\tau^2)\sin\bar{\Omega}\tau}{(1+\tau^2)^{5/2}}\exp(-\bar{\sigma}\tau)d\tau. \tag{8}$$

Here $\bar{\Omega} = 2d\Omega/U$. If, besides, $\bar{\Omega}\ll1$, then

$$\tilde{F} = -\frac{e^2U_\sim\bar{\sigma}\cos\Omega t}{4d^2U}\Big\{-1+\pi\bar{\sigma}[H_0(\bar{\sigma})-N_0(\bar{\sigma})] - \bar{\sigma}^2\Big[\frac{\pi}{2}(H_1(\bar{\sigma})-N_1(\bar{\sigma}))-1\Big]\Big\}. \tag{9}$$

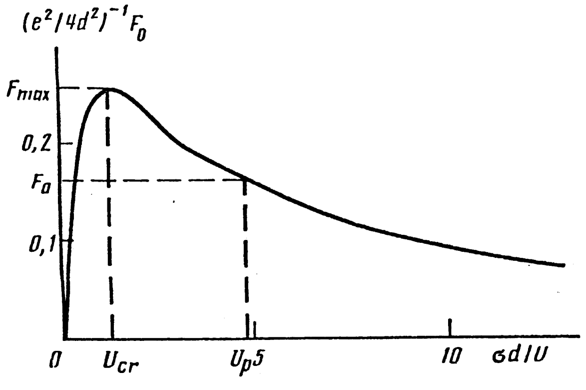

FIG. 1. Dependence of the dc component of the resistance force on the charge velocity.

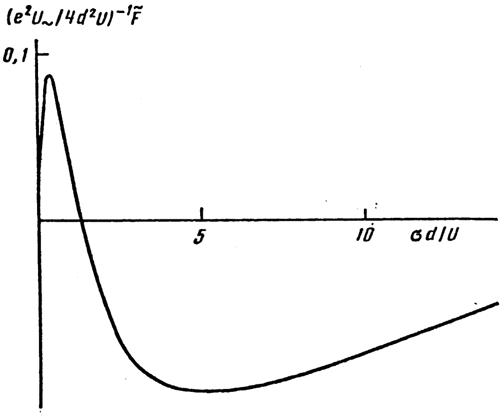

FIG. 2. Dependence of the alternating component of the resistance force on the charge velocity at $\bar{\Omega} = 0.01$.

Note that in the last case of low frequency oscillations the buildup of oscillations occurs quasistatically, namely, the oscillating part of the force is determined by the change in the force (6) acting on a uniformly moving charge: $\tilde{F}\approx -(\partial F_0/\partial U)\dot{\zeta}$, where ζ is the oscillating part of the particle displacement. Since on the decreasing section $\partial F_0/\partial U < 0$, the oscillating instability develops there. In particular, it follows from (9) that

$$\tilde{F}(\bar{\sigma}\gg1) = -\frac{e^2U_\sim}{4d^3\sigma}, \quad \tilde{F}(\bar{\sigma}\ll1) = \frac{e^2\sigma U_\sim}{4dU^2}. \tag{10}$$

The oscillating part of the drag force versus the mean velocity in the low-frequency case is shown in Fig. 2. It is seen that the oscillations become unstable for $\bar{\sigma}\leqslant1.3$.

For higher frequencies the integral in (8) has been calculated with the help of a computer. The instability-region boundary in the $\bar{\Omega}$ plane is shown in Fig. 3 (the instability region is shaded).

We will list now, without detailed discussion, the corresponding formulas for the drag force acting on a moving charge in a round cylindric channel under the condition that the channel radius R is much smaller than the characteristic field scales ($R\ll U/\sigma, U/\Omega$). In this case the component of the force F_0 differs from (5) only by a constant factor:

$$F_0 = -\frac{e^2\sigma}{RU}\Big\{1-\frac{\pi\bar{\sigma}_1}{2}[H_0(\bar{\sigma}_1)-N_0(\bar{\sigma}_1)]\Big\}. \tag{11a}$$

For the alternating component we have [cf. (8)]:

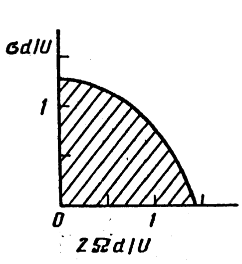

FIG. 3. Region of charge-motion instability in the plane of the parameters $\sigma d/U$ and $2\Omega d/U$ (hatched).

$$F = -\frac{e^2\sigma a \cos\Omega t}{R^2 U}\int_0^\infty \frac{1-2\tau^2}{(1+\tau^2)^{5/2}}\sin(\bar\Omega_1\tau)\exp(-\bar\sigma_1\tau)\,d\tau. \tag{11b}$$

Here $\bar\sigma_1 = \sigma R/U$, $\bar\Omega_1 = \Omega R/U$.

Note that in the problems considered above the motion is nonrelativistic, and in the region where the buildup of oscillations occurs we have $\sigma/\omega \sim \sigma d/U \leqslant 1$.

Let us briefly discuss the physical mechanism of the instability. The charge motion near the interface generates on the latter a "tail" of induced charges of opposite sign. This "tail" of length $l_H \simeq U/\sigma$ is responsible for the drag force. If the particle velocity grows, the "tail" extends behind it, which may lead to the decrease in force and, as a result, to the instability. At distances much larger than l_H the charge field in the quasistatic case has a dipole character. In fact, Eq. (1) in a coordinate system moving with the charge velocity U, provided that the oscillation amplitude is small ($ka \ll 1$), can be written in the form

$$\hat\varepsilon\Delta\varphi = -4\pi e[\delta(\mathbf{r}) - z(t)\delta(\mathbf{r}_\perp)\delta'(z)], \tag{12}$$

where $z(t)$ is the oscillating part of the charge displacement, and $\hat\varepsilon$ is the dielectric constant operator. Using the Fourier representation of the field and assuming that $z = \tilde z\exp(-i\omega t)$ and $\hat\varepsilon = 1 + i\sigma/(\omega + \mathbf{kU})$, it is easy to show that at the distances $r \gg l_H$ the solution of Eq. (12) under the condition that

$$kU \ll \sigma \tag{13}$$

has the form

$$\varphi = -\left(\frac{Ue}{\sigma} + \frac{\omega\tilde z e}{\omega + i\sigma}\right)\frac{\partial}{\partial z}\frac{1}{r}. \tag{14}$$

Hence the dipole character of the charge field.

2. SPONTANEOUS BUILDUP OF ELECTRON OSCILLATIONS IN A CONDUCTIVE MEDIUM

It is interesting to discuss the instability considered above in the collective case, when there is a flow of free particles (electrons) moving in parallel channels in a dissipative medium. For a hydrodynamic description of such a system to be valid, it is necessary that there be many particles over a distance equal to the macroscopic field wavelength. On the other hand, as shown above, the characteristic scale of the induced charge "tail" is of order U/σ. Thus, under the condition

$$n_0(U/\sigma)^3 \ll 1, \tag{15}$$

where n_0 is the density of the moving electrons, we can assume that the distance between the mentioned dipoles is large in comparison with their size. Then each particle subject to the macroscopic field $\mathbf{E}'$ is also subject to the "individual" drag force $\mathbf{F}$ found above. In other words, the particle equation of motion has the form

$$\frac{d\mathbf{V}}{dt} = \frac{e}{m}\mathbf{E}' + \frac{\mathbf{F}}{m}. \tag{16}$$

To describe the dispersion properties of the system in the hydrodynamic approximation, we use, alongside with (16), the usual equations

$$\frac{\partial n}{\partial t} + \operatorname{div} n\mathbf{V} = 0, \tag{17}$$

$$\operatorname{div}\hat\varepsilon\mathbf{E}'' = 4\pi e n, \tag{18}$$

where n is the particle density, $\hat\varepsilon = 1 + i\sigma/\omega$ for a harmonic field, and $\mathbf{E}''$ is the mean field. In what follows we assume that $\mathbf{E}' = \mathbf{E}''$.[5)]

If a system is subject to a constant external field $\mathbf{E}_0$, in the stationary regime there is a flow of particles whose equilibrium velocity is found from the equation $e\mathbf{E} = -\mathbf{F}(U_0)$. We assume that U_0 lies on the section where $\partial F(U_0)/\partial U < 0$. Considering the small harmonic perturbations and linearizing Eqs. (16)–(18) with respect to the unperturbed values n_0, $\mathbf{U}_0$ and $\mathbf{E}_0$, we find in the one-dimensional case the following dispersion equation:

$$1 + \frac{i\sigma}{\omega + kU_0} - \frac{\omega_p^2}{\omega(\omega - i\gamma)} = 0, \tag{19}$$

where $\omega_p^2 = 4\pi n_0 e^2/m$ is the plasma frequency and γ is the parameter of individual instability.

Note that this equation can be derived also on the basis of the "dipole" interpretation mentioned above: if $kl_H \ll 1$ [this condition always holds if Eq. (15) is valid] Eq. (18) together with (14) is easily reduced to the form $\operatorname{div}(\hat\varepsilon\mathbf{E} + 4\pi\mathbf{P}_d) = 0$, where $P_d = en\tilde z$ is the total dipole moment associated with the particles.

For the sake of simplicity we consider only the quasistatic case, when $\gamma = (1/m)\partial F_0/\partial U = (\tilde F/m)U_\sim$ [see Eqs. (6) and (10)].

For $\gamma = 0$ (or, more accurately, for $\gamma \ll \omega_p \ll \sigma$) the dispersion equation (19) gives the beam instability in a conductive medium (see, e.g., Ref. 12). If, however, $\gamma \neq 0$, the system behavior can change substantially. Thus, for spatially homogeneous perturbations ($k = 0$), whose beam instability increment equals zero,[12] we get from (19):

$$\omega = {}^1/_2 i\{(\gamma - \sigma) \pm [(\gamma + \sigma)^2 - 4\omega_p^2]^{1/2}\}, \tag{20}$$

i.e., for $\gamma > \sigma$ aperiodic or oscillatory (depending on the value of ω_p^2) instability develops. If $\omega_p \to 0$, it has "purely individual" character.

Let now $kU \neq 0$. In this case explicit analytic expressions for the growth rate $\delta = \operatorname{Im}\omega$ can be derived only in the limiting cases. Thus, for small $\gamma(\gamma \ll \sigma)$ we find from (19) the following expression:

$$\delta = \omega_p\left\{\bar\gamma \pm \left[\frac{\{[\bar\gamma^2(1+S^2) - 1)^2 + S^2]^{1/2} - 1 + \bar\gamma^2(1+S^2)}{2(1+S^2)}\right]^{1/2}\right\}, \tag{21}$$

where $\bar\gamma = \gamma/2\omega_p$, and $S = \sigma/kU$. The dependences (21) are shown in Fig. 4 for various $\bar\gamma$. The curve for $\bar\gamma = 0$ corresponds to purely beam instability. It is seen from Fig. 4 that the individual growth rate $\gamma > 0$ leads to one more instable mode, with the range of wave numbers corresponding to the instability region of this mode becoming wider with growing γ. For $\sigma \gg \gamma \gg 2\omega_p$ Eq. (21) yields the asymptotes $\delta_1 \simeq \gamma$ and $\delta_2 \approx \omega_p^2/\gamma(1 + S^2)$. Note that $\delta_1 \gg \delta_2$, i.e., the individual instability (δ_1) predominates over the beam instability (δ_2). In the opposite limiting case, when $\gamma \gg \sigma$, Eq. (19) also gives

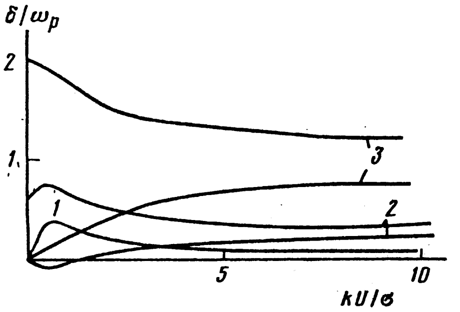

FIG. 4. Instability growth rate vs the wave number at $\sigma \gg \gamma$ for various values of the parameter $\bar{\gamma} = \gamma/2\omega_p$ curve *1*—$\bar{\gamma} = 0$; *2*—$\bar{\gamma} = 0.3$; *3*—$\bar{\gamma} = 1$.

two modes, "individual" and beam, whose respective growth rates are

$$\delta_1 = \begin{cases} \gamma/2, & \omega_p > \gamma/2, \\ [\gamma + (\gamma^2 - 4\omega_p^2)^{1/2}]/2, & \omega_p < \gamma/2, \end{cases} \tag{22a}$$

$$\delta_2 = -\frac{\sigma k^2 U^2 (\gamma^2 + k^2 U^2 - \omega_p^2)}{(k^2 U^2 - \omega_p^2)^2 + \gamma^2 k^2 U^2}. \tag{22b}$$

It is seen that $\delta_1 \gg \delta_2$, i.e., under these conditions the individual instability also prevails. Furthermore, beam instability exists only in a limited range of wave numbers $(0 < kU < (\omega_p^2 - \gamma^2)^{1/2})$, i.e. only if $\omega_p > \gamma$. The dependences (22) are shown in Fig. 5.

For the realization of the instability described above the conductivity and the channel diameter should satisfy definite conditions. These conditions hold, in particular, for semiconductors ($\sigma \simeq 10^{12}$–10^{14} s^{-1}), where it is possible to make sufficiently narrow (with a diameter of tens of Ångströms) channels.[13] Thus, for $\sigma = 10^{13}$ s^{-1}, $R = 40$ Å, $U = 2 \cdot 10^7$ cm/s we have $\gamma = e^2\sigma/mRU^2 = 1.5 \cdot 10^{13}$ s^{-1}. Then, in the example considered, the growth rate $\delta \simeq 0.25 \cdot 10^{13}$ s^{-1}, and the characteristic spatial gain is $L \sim U/\delta \simeq 800$ Å, i.e., transit through a thin film is sufficient. For the dipole-independence condition (15) to be satisfied, the electron density n_0 should be less than 10^{17} cm^{-3} or $\omega < 2 \cdot 10^{13}$ s^{-1}.

We believe that the instabilities considered above are of general and, at the same time, practical interest, since their realization does not require relativistic motion of particles.

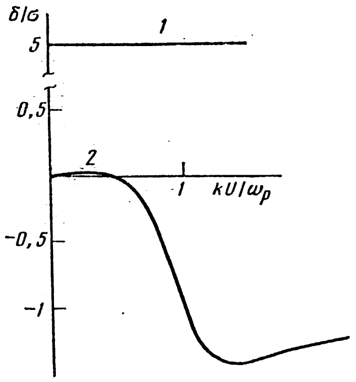

FIG. 5. Instability growth rate vs the wave number at $\sigma = 0.1\gamma$, $\gamma^2 = 0.75\omega_p^2$: *1*—growth rate of individual instability, *2*—growth rate of purely beam instability.

[1)] Oscillator motion in a medium with effective absorption associated with the scattering by inhomogeneities of the dielectric constant has been studied in Ref. 5. However, this dissipation mechanism does not lead to oscillator instability.

[2)] This approximation is valid, in particular, for plasma-like media at frequencies $\omega \ll \nu$ or $d\nu/U \gg 1$, where ν is the collision frequency.

[3)] It is clear that only these two components of the force perform work on the particle (see, e.g., Ref. 8).

[4)] Essentially, the presence of runaway electrons in plasma with the Coulomb collisions is also associated with a similar mechanism,[9] since at nonrelativistic velocities the Coulomb-scattering cross section and the drag force fall with increase of velocity. Another example is the particle deceleration due to ionization losses: as is well-known, the drag force for nonrelativistic particles also falls as the velocity increases.[10]

[5)] Without going into the details of the usually complicated and confused question of the relation between the effective, E_{eff}, and mean, $E' = E''$, fields (see, e.g., Ref. 11), note that in our case these fields are not equal: according to (16), $\mathbf{E}_{\text{eff}} = \mathbf{E}' + \mathbf{F}/e$. This difference is due here to the induced polarization of the conductive medium.

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