Resonant scattering of neutrons by nuclei in a crystal in a laser beam

A. Ya. Dzyublik

Institute of Nuclear Research, Academy of Sciences of the Ukraine

(Submitted 13 April 1991)
Zh. Eksp. Teor. Fiz. **102**, 120–126 (July 1992)

A systematic theory is derived for the resonant scattering of neutrons by nuclei for the case in which an intense electromagnetic wave induces transitions between neighboring *s* and *p* levels of a compound nucleus. A theory for the corresponding reactions is also derived. The neutron scattering amplitudes are calculated. The cross sections for scattering and for reactions are calculated. The role of the crystalline medium is taken into account. The inelastic diffraction of neutrons in a crystal accompanied by the absorption or emission of a photon is also analyzed.

1. INTRODUCTION

The effect of laser light on the resonant scattering of neutrons by nuclei and on the corresponding reactions (radiative capture, fission, etc.) continues to hold interest. The capture of neutrons to *p* levels of a compound nucleus as a direct result of the absorption or emission of a photon of an intense electromagnetic wave was analyzed in Refs. 1–3 by a valence (one-particle) approach. Zaretskiĭ and Lomonosov[4,5] took the same approach to analyze the role played by a mixing of the *s* and *p* levels of a compound nucleus in a laser beam. The collective nature of these states was taken into account in Ref. 6. Cross sections for the inelastic scattering of neutrons by nuclei and for reactions near an *s*-wave or *p*-wave resonance, accompanied by the absorption or emission of an optical photon, were calculated in the distorted-wave approximation in Refs. 1, 2, and 4–6. In other words, the probability for a transition between states of the nucleus-plus-neutron system which are interacting with each other was calculated in first-order perturbation theory. The interaction of this system with the classical electromagnetic wave, $\hat{V}_f(t)$, was treated as the perturbation. Only that part of the wave function of the system, $\psi_a^{(+)}$, which describes the free neutron outside the nucleus, was taken into account in Refs. 4 and 5. This simplified approach proved sufficient for calculating a matrix element for $\hat{V}_f^{(t)}$ by means of the Ehrenfest theorem, with the residual interaction of nucleons being ignored. An approximate expression for the wave function of the system, $\psi_a^{(+)}$, including amplitudes for neutron capture by the nucleus, averaged over spin, was used in Ref. 6. All experimental attempts[7–9] to observe the effect have yielded negative results, because low-power lasers have been used.

In this paper we derive a systematic theory for the resonant capture of neutrons to the *s* or *p* level of a compound nucleus, followed by: a laser-induced transition of the nucleus to a neighboring excited state with the opposite parity, and the decay of that state. Our theory is derived by analogy with the theory[10] of the double γ-magnetic nuclear resonance, in which transitions between sublevels of a Mössbauer nucleus are induced by an rf field. We calculate the amplitudes for scattering, and we find the cross sections for scattering and reactions. In particular, we calculate the total cross section, which has never been calculated before. The role of the crystalline surroundings is taken into account. The method used here leads to more-accurate expressions for the cross sections for induced scattering and for reactions. We also analyze inelastic neutron diffraction in a crystal containing a resonant isotope.

2. BASIC EQUATIONS

We consider a neutron-plus-nucleus system in a crystal in a classical electromagnetic wave. The electric field of this linearly polarized wave is, in the dipole approximation, $\mathbf{E}(t) = \mathbf{E}_0 \cos \Omega t$. The quantized electromagnetic field must also be taken into account; the interaction with this field results in the emission of γ rays by the compound nucleus. We write the Hamiltonian of this system in the form

$$\mathscr{H}(q, t)=\hat{\mathscr{H}}_0(q)+\hat{V}(q, t), \tag{1}$$

where q represents all the coordinates of the system. The unperturbed Hamiltonian is

$$\hat{\mathscr{H}}_0=\hat{H}_N+\hat{H}_{ph}+\hat{H}_r-\frac{\hbar^2}{2m}\nabla_r^2, \tag{2}$$

where $\hat{H}_N$, $\hat{H}_{ph}$, and $\hat{H}_r$ are the Hamiltonians of, respectively, the nucleus, the crystal lattice, and the quantized electromagnetic field; and m is the mass of the neutron, with the radius vector $\mathbf{r}$. The perturbation operator is

$$\hat{V}(t)=\hat{V}_n+\hat{V}_r+\hat{V}_f(t), \tag{3}$$

where the operator $\hat{V}_n$ represents the interaction of the neutron with the nucleus, $\hat{V}_r$ represents the interaction of the compound nucleus with the quantized field, and $\hat{V}_f(t)$ represents the interaction of the nucleus with the classical electromagnetic wave.

In the c.m. frame of the compound nucleus, with z axis parallel to $\mathbf{E}_0$, we write $\hat{V}_f(t)$ as follows for the case of $E1$ transitions:

$$\hat{V}_f(t)=-d_0|\mathbf{E}_0|\cos\Omega t. \tag{4}$$

Here d_0 is the z component of the nuclear dipole moment, given by

$$d_0=e\sum_{i=1}^{z} r_i i\left(\frac{4\pi}{3}\right)^{1/2} Y_{10}(\theta_i,\varphi_i), \tag{5}$$

e is the charge of the proton; r_i,θ_i,φ_i are spherical coordinates; i specifies a proton of the nucleus in the c.m. frame; and Y_{10} is the spherical harmonic.

 0038-5646/92/070063-04$05.00

We assume that the initial state of the system is described by the wave function

$$\Psi_a(q,\ t\to-\infty)=\chi_a(q)\exp(-iE_at/\hbar),$$
$$\chi_a=|I_0M_0\rangle\exp(i\mathbf{k}_0\mathbf{r})|\mu\rangle|\{v_\gamma^0\}\rangle|0\rangle. \qquad (6)$$

Here I_0 is the nuclear spin, M_0 is the z projection of this spin, μ is the projection of the neutron spin onto the z' axis, which is parallel to $\mathbf{k}_0$, $|\{v_\gamma^0\}\rangle$ is the wave function of the crystal, with an initial number of phonons $\{v_\gamma^0\}$, the function $|0\rangle$ describes the vacuum of the quantized electromagnetic field, and E_a is the initial energy of the system, given by

$$E_a=E+\sum_\gamma\hbar\omega_\gamma\left(v_\gamma^0+\frac{1}{2}\right),\quad E=\frac{\hbar^2k_0^2}{2m}. \qquad (7)$$

Here ω_γ is the frequency of the lattice oscillator of index γ.

For the calculations it is convenient to use a composite Hilbert space $L^2_{q,t}$ of the periodic functions $\psi(q,t)=\psi(q,t+T)$, in which the scalar product of the functions $\psi(q,t)$ and $\varphi(q,t)$ is defined by[11,12]

$$\{\psi(q,t)|\varphi(q,t)\}=\frac{1}{T}\int_{-T/2}^{T/2}dt\int dq\psi^*(q,t)\varphi(q,t). \qquad (8)$$

The wave function of the system is given by[12,13]

$$\Psi_a(q,\ t)=\psi_a^{(+)}(q,\ t)\exp(-iE_at/\hbar),$$
$$\psi_a^{(+)}(q,\ t)=\chi_a+\hat{G}_0^{(+)}(E_a)\hat{T}\chi_a. \qquad (9)$$

Here $\hat{T}$ is the transition operator:

$$\hat{T}=\hat{V}'+\hat{V}'\hat{G}^{(+)}(E_a)\hat{V}. \qquad (10)$$

Here $\hat{G}_0^+$ and $\hat{G}^{(+)}$ are Green's operators with $\eta\to+0$:

$$\hat{G}_0^{(+)}(E)=\left(E+i\eta-\hat{\mathscr{H}}_0-i\hbar\frac{\partial}{\partial t}\right)^{-1},$$
$$\hat{G}^{(+)}(E)=\left(E+i\eta-\hat{\mathscr{H}}-i\hbar\frac{\partial}{\partial t}\right)^{-1}. \qquad (11)$$

3. TRANSITION MATRIX

The resonant scattering and reactions are determined by a transition matrix on a quasienergy surface:

$$\{b;n|\hat{T}|a,0\}=\sum_{c',c}V'_{bc'}\{c';n|\hat{G}^{(+)}|c;0\}V_{ca}, \qquad (12)$$

where the functions

$$|c;\ n\}=\chi_ce^{in\Omega t},$$
$$\chi_c\equiv|s(p)\rangle=|I_{s(p)}M_{s(p)}\rangle|\{v_\gamma\}\rangle|0\rangle, \qquad (13)$$

describe intermediate states, and $I_{s(p)}$ and $M_{s(p)}$ are the spin of the compound nucleus and its z projection. The Green's matrix in (12) is determined by the system of algebraic equations

$$\sum_{\varkappa''}[(E_a-\mathscr{E}_\varkappa+i\Gamma_\varkappa/2)\delta_{\varkappa\varkappa''}-\{\varkappa|\hat{V}_f(t)|\varkappa''\}]G_{\varkappa''\varkappa'}=\delta_{\varkappa\varkappa''}, \qquad (14)$$

where $\varkappa=c,n$; $\Gamma_\varkappa$ represents the widths of the s and p levels; and $\mathscr{E}_\varkappa=E_c+n\hbar\Omega$.

Working from the Wigner–Eckart theorem, and using (5), we find

$$\{s;\mp1|\hat{V}_f(t)|p;\ 0\}=-{}^1/{}_2C^{I_p1I_s}_{M_p0M_s}d^{(0)}_{sp}|\mathbf{E}_0|, \qquad (15)$$

where $C^{I_p1I_s}_{M_p0M_s}$ are the Clebsch–Gordan coefficients, and $d^{(0)}_{sp}=\langle s\|d_0\|p\rangle$ is a reduced matrix element. It can be seen from (15) that a linearly polarized wave mixes states of the compound nucleus which have identical spin projections $M_p=M_s$. The cross sections for induced scattering and reactions were calculated in Refs. 4–6 in the resonant approximation $\hbar\Omega\approx|E_s-E_p|$. However, estimates were later made for ^{139}La, for which we have $|E_s-E_p|=38.2$ eV. In other words, this energy is much larger than the photon energy of a neodymium laser, $\hbar\Omega=1.17$ eV. We accordingly use the one-photon approximation $d^{(0)}_{sp}|\mathbf{E}_0|\ll\Gamma_{s(p)}$. Over its lifetime, the compound nucleus then has time to absorb or emit only a single photon. Equations (14) are solved by an iterative procedure. In particular, we find

$$\{s;\mp1|\hat{G}^{(+)}(E_a)|p;0\}\approx\frac{\{s;\mp1|\hat{V}_f(t)|p;0\}}{(\varepsilon-E_s\pm\hbar\Omega+i\Gamma_s/2)(\varepsilon-E_p+i\Gamma_p/2)}, \qquad (16)$$

where

$$\varepsilon=E+\sum_\gamma\hbar\omega_\gamma(v_\gamma^0-v_\gamma). \qquad (17)$$

The amplitude for the sum and difference scattering of neutrons from the state $\mathbf{k}_0,\mu$ to the state $\mathbf{k}',\mu'$ by a nucleus in a crystal in which n photons are absorbed ($n<0$) or emitted ($n>0$) is related to the T matrix by[12]

$$f^{(n)}_{\alpha\beta}(\mathbf{k}_0,\mu;\mathbf{k}',\mu')=-\frac{m}{2\pi\hbar^2}\{b;n|\hat{T}|a;0\}, \qquad (18)$$

where $|\alpha\rangle$ and $|\beta\rangle$ are the initial and final states of the scatterer. The amplitude for coherent scattering of neutrons by nucleus j in the unit cell, $f^{(\mp1)}_{\rm coh}$, is found by averaging $f^{(\mp1)}_{\alpha\alpha}$ over the phonons, the spins, and the isotopes. For unpolarized targets and for unpolarized incident neutrons, with $E\approx E_p$, we find the following result in the approximation of fast collisions ($\hbar\bar{\omega}\ll\Gamma$, where $\bar{\omega}$ is a characteristic phonon frequency[14,15]):

$$f^{(\mp1)}_{coh}(\mathbf{k}_0,\ \mu;\mathbf{k}',\ \mu')_j=p_j\exp[-W_j(\mathbf{Q})](2I_0+1)^{-1}$$
$$\times\sum_{M_0}\sum_{M_p}\frac{{}^1/{}_2\mathscr{M}^*_{M_sM_0}(\mathbf{k}',\ \mu')C^{I_p1I_s}_{M_p0M_s}d^{(0)}_{sp}|E_0|\mathscr{M}_{M_pM_0}(\mathbf{k}_0,\ \mu)}{(E-E_s\pm\hbar\Omega+i\Gamma_s/2)(E-E_p+i\Gamma_p/2)}. \qquad (19)$$

Here p_j is the probability for finding the resonant isotope at site j; $\exp[-2W(\mathbf{Q})]$ is the Debye–Waller factor; and $\mathbf{Q}=\mathbf{k}_0-\mathbf{k}'$. The amplitude for the capture of a neutron from the state $\mathbf{k},\mu$ to the p and s levels of the compound nucleus is

$$\mathcal{M}_{M_pM_0}(\mathbf{k}_0,\ \mu)=\sum_{K_p,K_0}\mathcal{D}^{I_p*}_{K_pM_p}(\alpha,\ \beta,\ 0)\mathcal{D}^{I_0}_{K_0M_0}(\alpha,\ \beta,\ 0)$$
$$\times[2C^{I_0\frac{1}{2}I_p}_{K_0\mu K_p}\mu T_p(^1/_2)+C^{I_0\frac{3}{2}I_p}_{K_0\mu K_p}T_p(^3/_2)],$$

$$\mathcal{M}_{M_sM_0}(\mathbf{k}',\ \mu')$$
$$=\sum_{K_sK_0'}\mathcal{D}^{I_s*}_{K_sM_s}(\alpha',\ \beta',\ 0)\,\mathcal{D}^{I_0}_{K_0'M_0}(\alpha',\ \beta',\ 0)\,C^{I_0\frac{1}{2}I_s}_{K_0'\mu'K_s}T_s. \tag{20}$$

Here K_0 and K_p are the projections of the spins I_0 and I_p onto $\mathbf{k}$; K'_0 and K_s are the projections of spins I_0 and I_s onto $\mathbf{k}'$; β,α, and β',α' are the spherical angles of the wave vectors $\mathbf{k}$ and $\mathbf{k}'$, respectively; $\mathcal{D}^I_{KM}(\alpha,\beta,\gamma)$ is the rotation matrix; and T_s and $T_p(j)$ are the scalar amplitudes for neutron capture from an s wave and a p wave, respectively, with a total angular momentum j (Refs. 16 and 17). These amplitudes can be expressed in terms of the partial neutron widths of the s and p levels, which depend on the energy E:

$$\Gamma^{(s)}_n(k)=2k|T_s|^2,\quad \Gamma^{(pj)}_n(k)=2k|T_p(j)|^2,$$
$$\Gamma^{(p)}_n=\Gamma^{(p\frac{1}{2})}_n+\Gamma^{(p\frac{3}{2})}_n. \tag{21}$$

4. CROSS SECTIONS

The cross section for the induced transition $a\rightarrow b$ is given by[12]

$$\sigma^{(\mp1)}_{a\rightarrow b}=\frac{2\pi}{\hbar j_0}|\{b;\mp1|\hat{T}|a;0\}|^2\delta(E_a-E_b\pm\hbar\Omega), \tag{22}$$

where j_0 is the flux density of the incident neutrons. Substituting (12) and (16) into (22), we find the cross sections for scattering and reactions induced by the electromagnetic wave. Near a p-wave resonance ($E\approx E_p$), the integral cross section for induced neutron scattering by a nucleus in a crystal is[1)]

$$\sigma^{(\mp1)}_n=\sum_{\{v_\gamma\},\{v_\gamma^0\}}g(\{v_\gamma^0\})|\langle\{v_\gamma\}|\exp(i\mathbf{k}_0\mathbf{u})|\{v_\gamma^0\}\rangle|^2\varphi^{(\mp1)}_n(\varepsilon),$$

$$\varphi^{(\mp1)}_n(\varepsilon)=\frac{\pi}{k_0^2}\frac{2I_s+1}{24(2I_0+1)}$$
$$\times\frac{\Gamma^{(s)}_n(k_{\mp1})|d^{(0)}_{sp}\mathbf{E}_0|^2\Gamma^{(p)}_n(k_0)}{[(\varepsilon-E_s\pm\hbar\Omega)^2+(\Gamma_s/2)^2][(\varepsilon-E_p)^2+(\Gamma_p/2)^2]}, \tag{23}$$

where $g(\{v^{(0)}_\gamma\})$ is the statistical distribution with respect to initial states of the crystal, $\mathbf{u}$ is the displacement of the nucleus from its equilibrium position, ε is defined in (17), and

$$k_{\mp1}=(k_0^2\pm2m\Omega/\hbar)^{\frac{1}{2}}. \tag{24}$$

The reaction cross sections $\sigma^{(\mp1)}_r$ are found from (23) by replacing $\Gamma^{(s)}_n$ by the corresponding partial widths $\Gamma^{(s)}_r$. For free nuclei we would have $\sigma^{(\mp1)}=\varphi^{(\mp1)}(E)$; $\mathbf{k}_0$ and E would be the wave vector and kinetic energy of the relative motion of the neutron and the nucleus; and m would be the reduced mass. Significantly, cross section (23) is completely independent of the angle between the vectors $\mathbf{k}_0$ and $\mathbf{E}_0$, differing in this regard from the results of Refs. 4–6.

The amplitude of the electromagnetic wave, $\mathbf{E}_0$, can be expressed in terms of the average energy flux density of this wave:

$$\bar{S}=\frac{c}{8\pi}|\mathbf{E}_0|^2, \tag{25}$$

where c is the velocity of light. The ratio of the cross section for induced scattering of neutrons by a free nucleus, on the one hand, to the cross section for scattering in the absence of the laser light ($\sigma_{n|\mathbf{E}=0}$), on the other, can be written as follows in the case $E=E_p$:

$$R_\mp\equiv\frac{\sigma^{(\mp1)}_n}{\sigma_{n|\mathbf{E}=0}}=\frac{1}{3}\left(\frac{2I_s+1}{2I_p+1}\right)$$
$$\times\frac{\Gamma^{(s)}_n}{\Gamma^{(p)}_n}\frac{2|d^{(0)}_{sp}|^2\pi c^{-1}\bar{S}}{(E_p-E_s\pm\hbar\Omega)^2+(\Gamma_s/2)^2}. \tag{26}$$

This ratio contains the well-known[16,17] intensification factor $\Gamma^{(s)}_n/\Gamma^{(p)}_n\sim10^5$–$10^6$.

To estimate $d^{(0)}_{sp}$, we take the approach of Refs. 16 and 17. That approach has led to good agreement with neutron experiments on parity breaking. We note that the wave function $|I_{s(p)}M_{s(p)}\rangle$ is of the form of an expansion in the functions ψ_i describing various excited one-particle and collective states. The number of terms in this expansion is $\sim N=\bar{D}_0/\bar{D}$, where $\bar{D}$ is the average distance between levels of the compound nucleus, and $\bar{D}_0$ is the average distance between the one-particle resonances ($N\sim10^4$–10^6). We then have

$$d^{(0)}_{sp}\sim\overline{\langle\psi_i|d_0|\psi_j\rangle}N^{-\frac{1}{2}}. \tag{27}$$

The expectation value of the matrix element between one-particle functions is

$$\overline{\langle\psi_i|d_0|\psi_j\rangle}\sim ea, \tag{28}$$

where a is the radius of the nucleus. The component of $d^{(0)}_{sp}$ from collective components of the wave function is inconsequential, having a value $\sim1/N$.

In the optimum case of the exact resonance, with $\hbar\Omega=|E_p-E_s|$, where $a\sim10^{-12}$ cm, $N\sim10^4$, $\Gamma^{(s)}_n/\Gamma^{(p)}_n\sim10^6$, and $\Gamma_s\approx0.1$ eV, we find an estimate of the laser power from (26)–(29): $\bar{S}\sim10^{17}\cdot R_\mp$ W/cm^2. The screening of the nucleus by electrons far from electronic transitions makes the amplitude $|\mathbf{E}_0|$ near the nucleus smaller by two to four orders of magnitude than the amplitude of the incident wave.[4] Correspondingly, the laser power must be raised by four to eight orders of magnitude.

While s-wave resonances with a spin $I_0-\frac{1}{2}\leqslant I_s\leqslant I_0+\frac{1}{2}$ are excited during the capture of slow neutrons under ordinary conditions, in the case of capture to a p level, with an induced $p\rightarrow s$ transition, there may be an excitation of s levels with a spin $I_0-\frac{5}{2}\leqslant I_s\leqslant I_0+\frac{5}{2}$. There is thus the possibility in principle that s levels of a compound nucleus, which have not previously been seen, could be excited in a laser beam. The detection of γ lines from the decay of these s levels against a low background would thus make it possible to relax the requirements on the laser power.

Using the optical theorem,[12] we can also find the total cross section for the capture of neutrons by a free nucleus in a laser beam:

$$\sigma_t=\sigma_{t|\mathbf{E}=0}+\Delta\sigma_t, \tag{29}$$

where $\sigma_{t|\mathbf{E}=0}$ is the cross section in the absence of the laser, and

$$\Delta\sigma_t=\frac{\pi}{k_0^2}\frac{2I_s+1}{24(2I_0+1)}\Gamma_n^{(p)}|d_{sp}^{(0)}\mathbf{E}_0|^2$$
$$\times\sum_{n=\mp1}\frac{2\Gamma_p(E-E_p)(E-E_s-n\hbar\Omega)+\Gamma_s[(E-E_p)^2-(\Gamma_p/2)^2]}{[(E-E_s-n\hbar\Omega)^2+(\Gamma_s/2)^2][(E-E_p)^2+(\Gamma_p/2)^2]^2}. \tag{30}$$

5. INDUCED DIFFRACTION

An inelastic diffraction of neutrons in a crystal accompanied by the absorption or emission of a photon occurs under the Bragg conditions[13]

$$E'=E\mp\hbar\Omega,\quad \mathbf{k}'=\mathbf{k}_0-\mathbf{Q}, \tag{31}$$

where $-\mathbf{Q}=\boldsymbol{\tau}\mp\mathbf{q}\approx\boldsymbol{\tau}$, $\mathbf{q}$ is the wave vector of the electromagnetic wave in the crystal, $\boldsymbol{\tau}/2\pi$ is a reciprocal-lattice vector, and E' is the energy of the scattered neutrons. Condition (31) reduces to the equality

$$-2\mathbf{k}_0\mathbf{Q}+\mathbf{Q}^2\pm2m\Omega/\hbar=0. \tag{32}$$

If IR lasers with $\hbar\Omega\approx0.1$ eV and thermal neutrons with $E\approx0.01$ eV are used in an experiment, a diffraction of these neutrons accompanied by the absorption of a photon will occur at $\tau\sim(2m\Omega/\hbar)^{1/2}\approx7$ Å^{-1}. The Debye–Waller factor in the cross section for coherent scattering in this case is $\exp(-\tau^2\overline{u^2})\sim1$, since the resonant scattering of neutrons occurs at fairly heavy nuclei, with an rms vibration amplitude $(\overline{u^2})^{1/2}\sim0.1$ Å. Calculations carried out in the kinematic approximation show that the ratio of the flux density of inelastically diffracted neutrons, $j_d^{(\mp1)}(\boldsymbol{\tau})$, to the flux density of neutrons diffracted in the absence of a laser, $j_d(\boldsymbol{\tau})_{|\mathbf{E}=0}$, is on the order of $R_\mp$.

Using the results of Refs. 13 and 18, we now examine the dynamic scattering of neutrons by a plane-parallel plate consisting of infinite layers of unit cells of thickness d. For Laue diffraction we have

$$\frac{j_d^{(\mp1)}(\boldsymbol{\tau})}{j_0}=\frac{k'}{k_0}\frac{1}{2}\sum_{\mu,\mu'}|\mathscr{F}^{(\mp1)}(\mathbf{k}_0,\mu;\mathbf{k}'_{\mp1},\mu')|^2$$
$$\times|\Phi(\xi)\exp(i\delta'Nd)|^2, \tag{33}$$

where

$$\mathscr{F}^{(n)}(\mathbf{k},\mu;\mathbf{k}',\mu')=\frac{2\pi}{k_z(v_0/d)}\sum_j\exp(i\mathbf{Q}\boldsymbol{\rho}_j)f_{coh}^{(n)}(\mathbf{k},\mu;\mathbf{k}',\mu')_j \tag{34}$$

is the dimensionless amplitude for the scattering of neutrons by one layer of unit cells with a volume v_0, $\boldsymbol{\rho}_j$ is the radius vector of atom j in a unit cell,

$$\Phi(\xi)=(\xi d)^{-1}[1-e^{i\xi Nd}],\quad k'_{\mp1\|}=k_{0\|}+\tau_\|\pm q_\|,$$
$$\xi=k_{0\perp}+\delta-k'_{\mp1\perp}-\delta'\pm q_\perp+\tau_\perp,\quad k_{0\perp}=\mathbf{k}_0\mathbf{e}_\perp,\ldots,$$
$$\delta d=\mathscr{F}^{(0)}(\mathbf{k}_0,\mu;\mathbf{k}_0,\mu),\quad \delta'd=\mathscr{F}^{(0)}(\mathbf{k}',\mu';\mathbf{k}',\mu'), \tag{35}$$

Nd is the thickness of the plate, $\mathbf{e}_\perp$ is a unit vector perpendicular to the surface of the plate, and $k_{0\|},\ldots$ are components along the surface of the plate.

If $\mathrm{Im}\xi=0$, then $\Phi(\xi)=N$ at $\xi=0$, and we have $j_d\propto N^2$. With increasing $|\xi|$, the function $|\Phi(\xi)|^2$ undergoes damped oscillations. As $N\to\infty$, we have $|\Phi(\xi)|^2\to2\pi N\delta(\xi)$. The quadratic N dependence of j_d has been discussed in the kinematic approximation ($\delta=\delta'=0$) in several places (e.g., Ref. 19). The condition $\xi=0$ is the same as the Bragg condition (31) in this case. If the spread $\Delta k_0\ll(Nd)^{-1}$, then we have $j_d\propto N^2$. The relative energy spread of the incident neutrons in this case is $\Delta E/E\ll(k_0Nd)^{-1}$; i.e., the behavior $j_d\propto N^2$ prevails only if the plate is sufficiently thin.

[1)] In the case $E\approx E_s$, the indices s and p should be interchanged in (23).

[1]D. F. Zaretskiĭ and V. V. Lomonosov, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 541 (1979) [JETP Lett. **30**, 508 (1979)].
[2]D. F. Zaretskiĭ and V. V. Lomonosov, Zh. Eksp. Teor. Fiz. **81**, 429 (1981) [Sov. Phys. JETP **54**, 229 (1981)].
[3]A. Ya. Dzyublik, Zh. Eksp. Teor. Fiz. **82**, 977 (1982) [Sov. Phys. Usp. **55**, 573 (1982)].
[4]D. F. Zaretskiĭ and V. V. Lomonosov, Zh. Eksp. Teor. Fiz. **84**, 1951 (1983) [Sov. Phys. JETP **57**, 1136 (1983)].
[5]D. F. Zaretskiĭ and V. V. Lomonosov, Yad. Fiz. **41**, 655 (1985) [Sov. J. Nucl. Phys. **41**, 417 (1985)].
[6]V. A. Plyuĭko, Yad. Fiz. **44**, 362 (1986) [Sov. J. Nucl. Phys. **44**, 231 (1986)].
[7]V. P. Vertebnyĭ, V. F. Razbudeĭ, S. V. Sidorov *et al.*, Yad. Fiz. **36**, 3 (1982) [Sov. J. Nucl. Phys. **36**, 1 (1982)].
[8]F. Bechvarzh, P. Zeman, M. Kralik *et al.*, Yad. Fiz. **36**, 1364 (1982) [Sov. J. Nucl. Phys. **36**, 791 (1982)].
[9]S. M. Mironov and G. V. Muradyan, Yad. Fiz. **40**, 1393 (1984) [Sov. J. Nucl. Phys. **40**, 885 (1984)].
[10]A. Ya. Dzyublik, Phys. Status Solidi (b) **104**, 81 (1981).
[11]H. Sambe, Phys. Rev. A **7**, 2203 (1973).
[12]A. Ya. Dzyublik, *Quantum Theory of Systems in Alternating Fields* [in Russian], Nauk. dumka, Kiev, 1992.
[13]A. Ya. Dzyublik, Zh. Eksp. Teor. Fiz. **85**, 1658 (1983) [Sov. Phys. JETP **58**, 965 (1983)].
[14]G. T. Trammel, Phys. Rev. **126**, 1045 (1962).
[15]Yu. Kagan and A. M. Afanas'ev, Zh. Eksp. Teor. Fiz. **50**, 271 (1966) [Sov. Phys. JETP **23**, 178 (1966)].
[16]V. E. Bunakov and V. P. Gudkov, Z. Phys. A **303**, 285 (1981).
[17]O. P. Sushkov and V. V. Flambaum, Usp. Fiz. Nauk **136**, 3 (1982) [Sov. Phys. Usp. **25**, 1 (1982)].
[18]A. Ya. Dzyublik, Phys. Status Solidi (b) **93**, 515 (1979).
[19]V. G. Baryshevskii and A. M. Zaitseva, Phys. Status Solidi (b) **157**, 129 (1990).

Translated by D. Parsons