# Localization of edge states in a quantum "wire" in a strong magnetic field

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(Submitted 16 January 1992)
Zh. Eksp. Teor. Fiz. **101**, 2001–2009 (June 1992)

The behavior of the zero-order mode, the least localized in the longitudinal direction, is studied in conditions where the overlap of edge states is weak. It is found that the population corresponding to this mode in the $N$ th channel is much lower than that for states with numbers $n = 0,1,..., N-1$. The special role of the $N$ th channel is that only in this state is backscattering possible. Thus, by lowering the probability of backscattering the zero-order mode increases its localization scale.

## 1. INTRODUCTION

Anderson localization continues to draw great interest. In recent years this interest has been whetted by successful experiments with mesoscopic structures at low temperatures, where the quantum-mechanical nature of transport properties manifests itself. Quantization of ballistic conductivity through a point contact[1,2] has shown that the number $N$ of channels open to electrons is a measurable quantity. As noted in Ref. 3 and 4, in a disordered "wire" whose length $L$ exceeds the mean free path $l$ there remains a finite number of ballistic channels,

$$N_{eff} \sim Nl/L. \tag{1}$$

According to Landauer's formula, to each channel there corresponds a conductance $\sim e^2/h$, with the result that as $L$ grows, the conductance decreases by Ohm's law. However, at $L \sim l_c$ the number $N_{\text{eff}}$ is of the order of unity. A further increase in $L$ leads to a situation in which there are no more ballistic channels. As a result the conductivity begins to fall off exponentially,

$$\sigma \sim \exp(-L/l_c),$$

which is the manifestation of localization in a long wire.[5] Calculations[6–9] of the characteristic scale $l_c$ yield

$$l_c = \alpha N l, \tag{2}$$

where $\alpha = 1/2$, 1, and 2 for, respectively, the orthogonal, unitary, and simplex ensembles.[10,11]

A unitary ensemble corresponds to a system with a magnetic field. The results of Refs. 6–9 cannot, however, be applied directly to the case of a strong field, when very pronounced edge states, whose overlap may be assumed weak,[12,13] are formed. This paper is devoted to the calculation of $l_c$ in such a situation, and the conclusions reached qualitatively agree with the numerical calculations of Refs. 14–16.

## 2. THE QUANTUM-WIRE MODEL

We consider the simplest model of a wire in a magnetic field.[13] The appropriate two-dimensional Schrödinger equation is

$$\left\{ \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + V(y) + U(x,y) \right\} \psi = E_F \psi. \tag{3}$$

The gauge is taken in the form $\mathbf{A} = (-Hy, 0, 0)$. The potential holding the electrons within the wire is assumed to be quadratic:

$$V(y) = {}^1/_2 m\omega_0^2 y^2. \tag{4}$$

This makes it possible to write the wave functions explicitly. Suppose that a weak random potential is specified on the lattice,

$$U(x,y) = \sum_{ij} U_{ij} a^2 \delta(x-ai)\delta(y-aj) \tag{5}$$

and has the form of white noise:

$$\langle U_{ij} \rangle = 0, \quad \langle U_{ij} U_{i'j'} \rangle = U_0^2 \delta_{ii'} \delta_{jj'}. \tag{6}$$

The lattice constant $a$ is assumed small compared to the other characteristic scales in the problem.

In the absence of a random potential, the solution of Eq. (3) has the form

$$\psi = \sum_n \left\{ A_n \frac{\exp(ik_n x)}{k_n^{1/2}} \varphi_n(y-y_n) + B_n \frac{\exp(-ik_n x)}{k_n^{1/2}} \varphi_n(y+y_n) \right\}, \tag{7}$$

where we have introduced the momenta

$$k_n = \frac{\omega}{\omega_0}\{2m[E_F - \omega({}^1/_2 + n)]\}^{1/2}. \tag{8}$$

The centers of the edge states are located at the points

$$y_n = k_n l_H^2 \omega_H^2/\omega^2. \tag{9}$$

Here we have used the notation

$$\omega = (\omega_H^2 + \omega_0^2)^{1/2}, \quad \frac{1}{l_H^2} = m\omega_H = \frac{eH}{c}, \tag{10}$$

and $\varphi_n(y)$ is the $n$th wave function of a linear oscillator with a frequency $\omega$. The quasiclassical condition

$$|k_n| l \gg 1 \tag{11}$$

makes it possible to restrict oneself in sum (7) to edge states with real-valued $k_n$ and ignore other modes. Thus, the number of edge states with the same direction of velocity is equal to $N+1$, with $N$ the maximum admissible value of parameter $n$ in Eq. (8).

The localization length $l_c$ can be calculated in two extreme cases. In the first, trivial, case, where $N = 0$,

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$$l_c = l, \tag{12}$$

and we need only find the mean free path prior to backscattering, $l$. This quantity, as follows from Eq. (38), at $N = 0$ depends on the overlap of the two remaining edge states, that is, on the magnetic field.

We are primarily interested in the second solvable case,

$$N \gg 1. \tag{13}$$

The magnetic field is assumed so strong that the overlap of the neighboring edge states is negligible:

$$y_n - y_1 \gg 1/(m\omega)^{1/2}. \tag{14}$$

Combining this with condition (13) yields

$$\omega_H \gg \omega_0 N^{1/2}. \tag{15}$$

The states with large numbers $n$ obey the quasiclassical approximation,

$$\varphi_n(y) = \left[\frac{2m\omega}{\pi p_n(y)}\right]^{1/2} \cos\left\{\int_{-\xi_n}^{y} p_n(y)\,dy - \frac{\pi}{4}\right\}, \tag{16}$$

$$p_n(y) = \{2m[\omega(n+1/2) - 1/2 m\omega^2 y^2]\}^{1/2}. \tag{17}$$

The classically accessible region can be represented as follows:

$$-\xi_n < y < \xi_n, \quad \xi_n = \left(\frac{1+2n}{m\omega}\right)^{1/2}. \tag{18}$$

Inequality (15) implies that the classically accessible regions of the edge states with $n \sim N$ do not overlap. On the contrary, they are far from each other.

### 3. THE TRANSFORM MATRIX

The random potential (5) causes an electron to hop between neighboring edge states. Hence, the amplitudes $A_n$ and $B_n$ in (7) begin to vary slowly with $x$. For a wire of length $L$ we can connect the amplitudes on the left, $A_n^L$ and $B_n^L$, with the amplitudes on the right, $A_n^R$ and $B_n^R$, by a transform matrix $t$:

$$\begin{bmatrix} A^R \\ B^R \end{bmatrix} = t \begin{bmatrix} A^L \\ B^L \end{bmatrix}. \tag{19}$$

In the case of the unitary ensemble considered here, where time-reversal symmetry is broken by the magnetic field, the following parametrization of the $t$-matrix takes place:[3,6,17]

$$t = \begin{bmatrix} u^+ & 0 \\ 0 & v \end{bmatrix} \begin{bmatrix} \operatorname{ch}\Gamma & \operatorname{sh}\Gamma \\ \operatorname{sh}\Gamma & \operatorname{ch}\Gamma \end{bmatrix} \begin{bmatrix} \bar{u} & 0 \\ 0 & \bar{v} \end{bmatrix}, \tag{20}$$

where $u$, $v$, $\bar{u}$, and $\bar{v}$ are four different unitary $(N+1)\times(N+1)$ matrices and $\Gamma$ is a diagonal real $(N+1)\times(N+1)$ matrix.

Using the explicit form (20) of the $t$-matrix, we can separate (19) into $N+1$ normal channels:

$$\begin{bmatrix} (uA^R)_n \\ (vB^R)_n \end{bmatrix} = \begin{bmatrix} \operatorname{ch}\Gamma_n & \operatorname{sh}\Gamma_n \\ \operatorname{sh}\Gamma_n & \operatorname{ch}\Gamma_n \end{bmatrix} \begin{bmatrix} (\bar{u}A^L)_n \\ (\bar{v}B^L)_n \end{bmatrix}. \tag{21}$$

The eigenvalues $\Gamma_n$ are assumed ordered in ascending values: $\Gamma_0 < \Gamma_1 < \cdots \Gamma_N$. To each of these values there corresponds transmission coefficient

$$T_n = 1/(\operatorname{ch}\Gamma_n)^2$$

for each normal channel. Knowing the rate of exponential decay of $T_n$ $(L \to \infty)$, we find the $N+1$ longitudinal scales $l_n$:

$$\frac{1}{l_n} = -\frac{d}{dx}\langle \ln T_n \rangle. \tag{22}$$

The maximum of these scales, $l_0$, is identified as the localization length $l_c$. When the wire is long enough,

$$L \gg 1/(l_n^{-1} - l_{n-1}^{-1}), \tag{23}$$

the transmission coefficients $T_n$ become exponentially distinct:

$$T_0 \ll T_1 \ll \ldots \ll T_N. \tag{24}$$

Let us now define the Markov process for the $t$-matrix. If the length $L$ of the wire is increased by adding a transverse layer of thickness $a$ on the right, the transfer matrix acquires an increment

$$\Delta t = t(L+a) - t(L) = \begin{bmatrix} -i\beta & -i\gamma \\ i\gamma^+ & i\bar{\beta} \end{bmatrix} t(L). \tag{25}$$

The matrices of Born amplitudes for forward scattering, $\beta$ and $\bar{\beta}$, and for backscattering, $\gamma$, are

$$\beta_{nn'} = \sum_j \frac{mU_{ij}a^2}{(k_n k_{n'})^{1/2}} \exp[-i(k_n - k_{n'})(L+a)]$$

$$\times \varphi_n(aj - y_n)\varphi_{n'}(aj - y_{n'}), \tag{26}$$

$$\bar{\beta}_{nn'} = \sum_j \frac{mU_{ij}a^2}{(k_n k_{n'})^{1/2}} \exp[i(k_n - k_{n'})(L+a)]$$

$$\times \varphi_n(aj + y_n)\varphi_{n'}(aj + y_{n'}), \tag{27}$$

$$\gamma_{nn'} = \sum_j \frac{mU_{ij}a^2}{(k_n k_{n'})^{1/2}} \exp[-i(k_n + k_{n'})(L+a)]$$

$$\times \varphi_n(aj - y_n)\varphi_{n'}(aj + y_{n'}). \tag{28}$$

Because the wave functions overlap only slightly, we retain only the transition-matrix elements between neighboring edge states:

$$\beta_{nn'} = \beta_{n,n-1}\delta_{n',n-1} + \beta_{n,n+1}\delta_{n',n+1}, \tag{29}$$

$$\gamma_{nn'} = \gamma_{NN}\delta_{nN}\delta_{n'N}. \tag{30}$$

The diagonal matrix elements $\beta_{nn}$ and $\bar{\beta}_{nn}$ can be ignored because they lead to a non-essential phase shift and do not send an electron from one channel to another. Averaging of the squares of the scattering amplitudes can be done via Eq. (6):

$$\langle|\beta_{n,n-1}|^2\rangle=\langle|\bar{\beta}_{n,n-1}|^2\rangle=\frac{m^2U_0^2a^3}{k_nk_{n-1}}I_{n,n-1}, \tag{31}$$

$$\langle|\gamma_{NN}|^2\rangle=\frac{m^2U_0^2a^3}{k_N^2}I_{NN}. \tag{32}$$

The integrals

$$I_{n,n-1}=\int dy\varphi_n^2(y-y_n)\varphi_{n-1}^2(y-y_{n-1}), \tag{33}$$

$$I_{N,N}=\int dy\varphi_N^2(y-y_N)\varphi_N^2(y+y_N) \tag{34}$$

have been calculated by Martin and Feng.[13] In conditions where the edge states overlap only weakly, the integrals can be evaluated by the saddle-point method:

$$I_{n,n-1}=\frac{1}{n!(n-1)!}\left(\frac{m\omega}{2\pi}\right)^{1/2}\left[(y_n-y_{n-1})^2\frac{m\omega}{8}\right]^{2n-1}\times\exp\left[-\frac{m\omega}{2}(y_n-y_{n-1})^2\right], \tag{35}$$

$$I_{NN}=\frac{1}{(N!)^2}\left(\frac{m\omega}{2\pi}\right)^{1/2}\left(y_N^2\frac{m\omega}{2}\right)^{2N}\exp(-2m\omega y_N^2). \tag{36}$$

These quantities directly determine the mean free paths:

$$\frac{1}{l_{n,n-1}}=\frac{1}{a}\langle|\beta_{n,n-1}|^2\rangle=\frac{m^2U_0^2a^2}{k_nk_{n-1}}I_{n,n-1}, \tag{37}$$

$$\frac{1}{l_{NN}}=\frac{1}{a}\langle|\gamma_{NN}|^2\rangle=\frac{m^2U_0^2a^2}{k_N^2}I_{NN}. \tag{38}$$

The special features of edge states are revealed in (26)–(38), while the increments $\Delta\Gamma$, $\Delta u$, and $\Delta v$, which follow from (25), are of a very general nature:[6]

$$\Delta\Gamma_n=\tfrac{1}{2}(R_{nn}+R_{nn}^+), \tag{39}$$

$$\Delta u_{nn'}=\sum_{n_1}\{iu_{nn_1}\beta_{n_1n'}+[-\tfrac{1}{2}\delta_{nn_1}(R_{nn}-R_{nn}^+)\operatorname{cth}2\Gamma_n+(1-\delta_{nn_1})(R_{nn_1}\operatorname{sh}2\Gamma_{n_1}+R_{nn_1}\operatorname{sh}2\Gamma_n)/(\operatorname{ch}2\Gamma_n-\operatorname{ch}2\Gamma_{n_1})]u_{n_1n'}\}, \tag{40}$$

$$\Delta v_{nn'}=\sum_{n_1}\{-iv_{nn_1}\bar{\beta}_{n_1n'}+[\tfrac{1}{2}\delta_{nn'}(R_{nn}-R_{nn}^+)\operatorname{cth}2\Gamma_n+(1-\delta_{nn_1})(R_{nn_1}\operatorname{sh}2\Gamma_{n_1}+R_{nn_1}^+\operatorname{sh}2\Gamma_{n_1})/(\operatorname{ch}2\Gamma_n-\operatorname{ch}2\Gamma_{n_1})]v_{n_1n'}\}. \tag{41}$$

Here we have used the notation

$$R_{nn'}=-i(u\gamma v^+)_{nn'}. \tag{42}$$

Equations (39)–(41) are none other than the Langevin equations with random forces $\beta$, $\bar{\beta}$, and $\gamma$. Using them we can construct the Fokker–Planck equation for the distribution function $W(L;\Gamma,u,v)$. This has been done in Ref. 6. Thus, instead of Eq. (22) we can write

$$\frac{1}{l_n}=\frac{1}{a}\langle|R_{nn}|^2+\sum_{n'\neq n}(|R_{nn'}|^2+|R_{n'n}|^2)(\operatorname{ch}2\Gamma_n+1)/(\operatorname{ch}2\Gamma_n-\operatorname{ch}2\Gamma_{n'})\rangle. \tag{43}$$

In the long-wire limit [Eqs. (23) and (24)], the increments $\Delta u$ and $\Delta v$ cease to depend on $\Gamma_n$, as Eqs. (40) and (41) clearly show:

$$\Delta u_{nn'}=iu_{n,n'-1}\beta_{n'-1,n'}+iu_{n,n'+1}\beta_{n'+1,n'}+i\sum_{n_1}[\theta(n_1-n)u_{nN}v_{n_1N}^*\gamma_{NN}+\theta(n-n_1)v_{nN}u_{n_1N}^*\gamma_{NN}^*]u_{n_1n'}, \tag{44}$$

$$\Delta v_{nn'}=-iv_{n,n'-1}\bar{\beta}_{n'-1,n'}-iv_{n,n'+1}\bar{\beta}_{n'+1,n'}-i\sum_{n_1}[\theta(n_1-n)v_{nN}u_{n_1N}^*\gamma_{NN}^*+\theta(n-n_1)u_{nN}v_{n_1N}^*\gamma_{NN}]v_{n_1n'}. \tag{45}$$

In this limit Eq. (43) assumes the form

$$\frac{1}{l_n}=\frac{1}{l_{NN}}\sum_{n'}\theta(n-n')\langle|u_{nN}|^2|v_{n'N}|^2+|v_{nN}|^2|u_{n'N}|^2\rangle. \tag{46}$$

This results in the following sum rule:

$$\frac{1}{l}=\frac{1}{N+1}\sum_{n=0}^{N}\frac{1}{l_n}=\frac{1}{(N+1)l_{NN}}. \tag{47}$$

In (44)–(47) we have allowed only for the transition matrix elements between neighboring edge states [Eqs. (29) and (30)]. In the same conditions the expression for the (maximum) localization length $l_c$ becomes quite simple:

$$\frac{1}{l_c}=\frac{1}{l_{NN}}\langle|u_{0N}|^2|v_{0N}|^2\rangle. \tag{48}$$

## 4. CALCULATING THE LOCALIZATION LENGTH

Let us now calculate the average in Eq. (48). First we use (13) to simplify expressions (44) and (45) for $\Delta u_{0n}$ and $\Delta v_{0n}$:

$$\Delta u_{0n}=iu_{0,n-1}\beta_{n-1,n}+iu_{0,n+1}\beta_{n+1,n}+iu_{0N}\gamma_{NN}S_{Nn}, \tag{49}$$

$$\Delta v_{0n}=-iv_{0,n-1}\bar{\beta}_{n-1,n}+iv_{0,n+1}\bar{\beta}_{n+1,n}+iv_{0N}\gamma_{NN}^*S_{Nn}^+. \tag{50}$$

The evolution of the unitary matrix

$$S=v^+u \tag{51}$$

is described by the closed Langevin equation

$$\Delta S_{nn'}=i\bar{\beta}_{n-1,n}S_{n-1,n'}+i\bar{\beta}_{n+1,n}S_{n+1,n'}+iS_{n,n'-1}\beta_{n'-1,n'}+iS_{n,n'+1}\beta_{n'+1,n'}+i\gamma_{NN}^{*}\delta_{Nn}\delta_{Nn'}+i\gamma_{NN}S_{nN}S_{Nn'}. \quad (52)$$

To build the Fokker–Planck equation for the distribution function $W(L;u_{0n}v_{0n})$ we must calculate the second-order increments $\Delta\Delta u_{0n}$ and $\Delta\Delta u_{0n}$. Equations (49)–(52) clearly show that

$$\Delta\Delta u_{0n}|_{n\neq N}=-\langle|\beta_{n-1,n}|^2+|\beta_{n+1,n}|^2\rangle u_{0n}, \quad (53)$$

$$\Delta\Delta u_{0N}=-\langle|\beta_{N-1,N}|^2+|\gamma_{NN}|^2\rangle u_{0N}. \quad (54)$$

Since

$$\sum_n |S_{Nn}|^2=1, \quad (55)$$

in the quadratic terms in the increments (49) and (50) we use the fact that

$$|\gamma_{NN}S_{Nn}|^2\sim\frac{1}{N}\langle|\gamma_{NN}|^2\rangle\ll\langle|\beta_{N-1,N}|^2\rangle. \quad (56)$$

For this reason the Fokker–Planck equation does not contain mixed derivatives

$$\frac{\partial^2}{\partial u_{0n}\partial v_{0n}}W(L;u_{0n},v_{0n}). \quad (57)$$

In other words, in the principal order in $1/N$ the parameters $u_{0n}$ and $v_{0n}$ are distributed independently. Hence, Eq. (48) becomes separable:

$$\frac{1}{l_c}=\frac{1}{l_{NN}}\langle|u_{0N}|^2\rangle\langle|v_{0N}|^2\rangle. \quad (58)$$

The distribution function $W(L;u_{0n})$ is described by the Fokker–Planck equation

$$a\frac{\partial W}{\partial L}=\sum_n\left\{\frac{1}{2}\left(\frac{\partial}{\partial u_{0n}}u_{0n}+\frac{\partial}{\partial u_{0n}^{*}}u_{0n}^{*}\right)\right.$$
$$\times\langle|\beta_{n,n-1}|^2+(1-\delta_{nN})|\beta_{n,n+1}|^2$$
$$+\delta_{nN}|\gamma_{NN}|^2\rangle+\frac{\partial^2}{\partial u_{0n}\partial u_{0n}^{*}}[|u_{0,n-1}|^2\langle|\beta_{n,n-1}|^2\rangle$$
$$+|u_{0,n+1}|^2\langle|\beta_{n,n+1}|^2\rangle(1-\delta_{nN})]-\left(\frac{\partial^2}{\partial u_{0n}\partial u_{0,n-1}}u_{0n}u_{0,n-1}\right.$$
$$\left.\left.+\frac{\partial^2}{\partial u_{0n}^{*}\partial u_{0,n-1}^{*}}u_{0n}^{*}u_{0,n-1}^{*}\right)\langle|\beta_{n,n-1}|^2\rangle\right\}W(L;u_{0n}). \quad (59)$$

This yields the following equation for the averages:

$$a\frac{\partial}{\partial L}\langle|u_{0n}|^2\rangle=\langle|\beta_{n,n-1}|^2\rangle(\langle|u_{0,n-1}|^2\rangle-\langle|u_{0n}|^2\rangle)$$
$$+(1-\delta_{nN})\langle|\beta_{n,n+1}|^2\rangle(\langle|u_{0,n+1}|^2\rangle$$
$$-\langle|u_{0n}|^2\rangle)-\delta_{nN}\langle|\gamma_{NN}|^2\rangle\langle|u_{0N}|^2\rangle. \quad (60)$$

On the right-hand side of the terms with $\beta$, which are due to forward scattering, resemble the difference diffusion operator. Hence, on distances $L\gg l_{nn-1}$ there is diffusion balancing in the population of channels with numbers $n=0,1,\ldots,N-1$:

$$\langle|u_{0n}|^2\rangle|_{n\neq N}=\frac{1}{N}. \quad (61)$$

At the same time the special feature of Eqs. (59) and (60) for the $N$th channel lead to a situation in which the channel population attains the lower time-independent value

$$\langle|u_{0N}|^2\rangle=\frac{1}{N}\langle|\beta_{N,N-1}|^2\rangle/(\langle|\beta_{N,N-1}|^2\rangle+\langle|\gamma_{NN}|^2\rangle). \quad (62)$$

Thus, the least localized zero-order mode decreases its backscattering, lowering the population of the $N$th edge state.

In this connection we call attention to the experiment described in Ref. 18, where a selective transition to equilibrium between the current-carrying edge states was observed. It was found that out of the $N$ accessible channels the current to the $N-1$ channel corresponding to the lower bands rapidly evens out while the fraction of current that goes into the $N$th channel greatly diminishes when the center of the respective Landau level approaches $E_F$. This phenomenon is better considered as the separation of the $N$th channel from the other channels[19] than as a general effect valid for all edge states. This behavior agrees with the results (61) and (62), which point to the special role of the $N$th channel, the only place backscattering can occur.

Returning to Eq. (58) and allowing for the fact that $v_{0n}$ is described by formulas similar to (59)–(62), we arrive at the following expression for the localization length:

$$l_c=N^2l_{NN}(1+l_{N,N-1}/l_{NN})^2. \quad (63)$$

Let us briefly examine this result. If the Fermi level lies somewhat below the bottom of the current $(N+1)$st band, we have $l_{NN}\gg l_{N,N-1}$ and

$$\frac{1}{l_c}=\frac{m^2U_0^2a^2}{N^2k_N^2}l_{NN}. \quad (64)$$

But if $E_F$ lies somewhat above the bottom of the specified band, two new edge states with number $N+1$ form. At this moment the mean free paths $l_{N+1,N+1}$ and $l_{N+1,N}$ vanish because of the singularity owing to $k_{N+1}\to 0$ in the denominators of (37) and (38). Here our approximation (11) is, generally speaking, invalid. In the final expression (63), however, the singularity at $k_{N+1}=0$ cancels out:

$$\frac{1}{l_c}=\frac{m^2U_0^2a^2}{N^2k_N^2}\frac{l_{N+1,N}^2}{l_{N+1,N+1}}. \quad (65)$$

Note that the coefficient in (64) coincides with that in (65). The two expressions differ only in the combination of overlap integrals (33) and (34). This makes it obvious why (64) and (65) differ. Prior to the formation of the $(N+1)$st edge state, the electron must hop over a greater distance (64) in order to backscatter. On the other hand, formula (65) corresponds to a double "jump" between the tightly bound $(N+1)$st states that have emerged. This forces $1/l_c$ to increase rapidly from the value (64) to (65). Further raising

of the Fermi level results in a smooth decrease in $1/l_c$ to a value specified by (64) but with a new number $N+1$. This sawtoothed behavior agrees qualitatively with the results of numerical calculations reported in Refs. 14–16.

## 5. CONCLUSION

We have considered a model with weak overlap of edge states. The specific form of the retaining potential (4) is not important if we can keep to electron hopping between neighboring states. The overlap integrals (33) and (34) for the chosen model, however, must be evaluated. We have also established for such a system the behavior of the zero-order mode, which is the least localized in the longitudinal direction, and found that the stronger the backscattering in the $N$th channel the lower the population in this channel. In other words, the zero-order mode avoids backscattering which is the reason for the further weakening of its localization. At the same time, forward scattering leads to an effective electron diffusion through channels with numbers $n = 0,1,...,N-1$. The discrepancy between the population of the $N$th edge state and that of the other edge states has been observed in experiments.[18] It is this phenomenon that causes the localization length (63) to be greater than the result that follows from (2) obtained in Refs. 6–9.


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Translated by Eugene Yankovsky