The interaction between Z and $\gamma\gamma^*$ and the $Z\to\gamma\Psi$ and $Z\to\gamma\Upsilon$ decays

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The triangle diagrams $Z\to c\bar{c}\to\gamma\gamma^*$ and $Z\to b\bar{b}\to\gamma\gamma^*$ are evaluated in analytic form and utilized to study the decays $Z\to\gamma\Psi$ and $Z\to\gamma\Upsilon$ in the dispersion-relations approach. A study of the sum rules for the amplitude and its derivative lead naturally to the estimates $\mathrm{BR}(Z\to\gamma J/\Psi)\sim 10^{-5}$ and $\mathrm{BR}(Z\to\gamma\Upsilon(1S))\sim 10^{-5}$, which are larger by two orders of magnitude than those expected in the quark model. We discuss the pole in the axial anomaly. We show that the anomalous pole arises only in the limit of massless fermions and only for real photons.

1. INTRODUCTION

The purpose of this work is to study the $Z\to\gamma\Psi$ and $\gamma\Upsilon$ decays in the dispersion approach.[1] In addition I have the temerity to consider once more the question of the axial anomaly,[2,3] whose treatment seems to me to be to this day not as clear as is worthy of such a classic. For example, in a respectable textbook[4] it is asserted that the pole in the axial anomaly[5] is always present, i.e., even when the quark mass satisfies $m_q\neq 0$. This assertion is, at the very least, strange. It is hard to imagine that the invariant amplitudes which are free of kinematic singularities and are determined by the triangle diagrams, should have poles corresponding to a massless particle when $m_q\neq 0$ holds.

In Sec. 2, I evaluate in analytic form the invariant amplitudes of the triangle diagrams describing the transition axial-vector current$\to q\bar{q}\to\gamma(k_1)\gamma^*(k_2)$, $k_1^2=0$, $k_2^2\neq 0$. To my knowledge this has not been done before. I show that for $m_q\neq 0$ the invariant amplitudes have no singularities other than the dynamic cuts due to the intermediate $q\bar{q}$ states. Moreover, for $k_2^2\neq 0$, the pole for $M^2=(k_1+k_2)^2=0$ is absent also for $m_q\to 0$. Only for $k_2^2=0$ and $m_q\to 0$ does an anomalous pole arise for $M^2=0$.

Section 3 is devoted to the $Z\to\gamma\Psi$ and $Z\to\gamma\Upsilon$ decays. Here I study sum rules for the amplitude $Z\to c\bar{c}$ (or $b\bar{b})\to\gamma\gamma^*$ and its derivative. I discuss two assumptions: in Sec. 3.1 it is assumed that the resonances saturate the sum rule for the amplitude and in Sec. 3.2 it is assumed that they saturate the sum rule for the derivative of the amplitude. In the general case these are different assumptions. In the discussion under the second assumption I consider various possibilities for the contribution of the resonances to the sum rule for the amplitude, including the case where that contribution is absent. Saturation of the sum rule for the amplitude by the ground states J/Ψ and $\Upsilon(1S)$ gives

$$\mathrm{BR}(Z\to\gamma J/\Psi)=10^{-5},\ \mathrm{BR}(Z\to\gamma\Upsilon(1S))=3\cdot 10^{-5},$$

which exceeds by two orders of magnitude what is expected in quark models. Saturation of the sum rule for the amplitudes by the Ψ and Υ families gives rise to the lower bounds

$$\sum \mathrm{BR}(Z\to\gamma\Psi)\geqslant 7\cdot 10^{-6},\quad \sum \mathrm{BR}(Z\to\gamma\Upsilon)\geqslant 1{,}5\cdot 10^{-5},$$

which are reached for

$$\mathrm{BR}(Z\to\gamma J/\Psi)=5\cdot 10^{-6},\ \mathrm{BR}(Z\to\gamma\Upsilon(1S))=8\cdot 10^{-6},$$

which is also larger by nearly two orders of magnitude than quark-model predictions.

Simultaneous saturation by resonances of the sum rules for the amplitude and its derivative gives provocative results for the lower bounds:

$$\sum \mathrm{BR}(Z\to\gamma\Psi)\geqslant 2\cdot 10^{-4},\quad \sum \mathrm{BR}(Z\to\gamma\Upsilon)\geqslant 2{,}3\cdot 10^{-3}.$$

When these are satisfied we have

$$\mathrm{BR}(Z\to\gamma J/\Psi)=4{,}4\cdot 10^{-5},\ \mathrm{BR}(Z\to\gamma\Upsilon(1S))=1{,}4\cdot 10^{-3}.$$

Saturation of the sum rule for the derivative of the amplitude by low-lying resonances is better founded than that for the amplitude itself, because the sum rule for the derivative converges more rapidly. It is interesting that if the contribution of the resonances to the sum rule for the amplitude is altogether absent, i.e., the sum rule is saturated by the continuous spectrum, the result differs insignificantly from the case in which the two sum rules are saturated simultaneously:

$$\sum \mathrm{BR}(Z\to\gamma\Psi)\geqslant 3{,}7\cdot 10^{-5},\quad \sum \mathrm{BR}(Z\to\gamma\Upsilon)\geqslant 8{,}7\cdot 10^{-4}$$

and

$$\mathrm{BR}(Z\to\gamma J/\Psi)=1{,}3\cdot 10^{-5},\ \mathrm{BR}(Z\to\gamma\Upsilon(1S))=5{,}6\cdot 10^{-4}$$

when the equalities hold. If, while the resonances saturate the sum rule for the derivative their contribution to the sum rule for the amplitude is viewed as a free parameter, the smallest lower bounds

$$\sum \mathrm{BR}(Z\to\gamma\Psi)\geqslant 10^{-6},\quad \sum \mathrm{BR}(Z\to\gamma\Upsilon)\geqslant 4{,}4\cdot 10^{-6}.$$

can be found. These are reached for

$$\mathrm{BR}(Z\to\gamma J/\Psi)=7\cdot 10^{-7},\mathrm{BR}(Z\to\gamma\Upsilon(1S))=2{,}5\cdot 10^{-6},$$

which is larger than quark-model expectations by one order of magnitude.

2. THE AXIAL-VECTOR CURRENT $\to q\bar{q}\to$ VECTOR CURRENT (k_1), VECTOR CURRENT (k_2), $k_1^2=0, k_2^2\neq 0$

As is well known,[6] the axial-vector vertex, determined by the triangle diagrams, has the form

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$$T_{\alpha\beta\mu}=A_1k_1^{\sigma}\varepsilon_{\sigma\alpha\beta\mu}+A_2k_2^{\sigma}\varepsilon_{\sigma\alpha\beta\mu}+A_3k_{1\beta}k_1^{\sigma}k_2^{\sigma}\varepsilon_{\delta\sigma\alpha\mu}$$
$$+A_4k_{2\beta}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\alpha\mu}+A_5k_{1\alpha}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\beta\mu}+A_6k_{2\alpha}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\beta\mu}. \quad (1)$$

Gauge invariance of the amplitude

$$k_1^{\alpha}T_{\alpha\beta\mu}=k_2^{\beta}T_{\alpha\beta\mu}=0 \quad (2)$$

is ensured by the following relations:

$$A_1=k_2^2A_4+(k_1k_2)A_3,\ A_2=k_1^2A_5+(k_1k_2)A_6. \quad (3)$$

In addition

$$A_3(k_1,\ k_2)=-A_6(k_2,\ k_1),\ A_4(k_1,\ k_2)=-A_5(k_2,\ k_1), \quad (4)$$

A_3, A_4, A_5 and A_6 are invariant amplitudes free of kinematic singularities and they are well-defined. For $k_1^2=0$ (or $k_2^2=0$) they can be evaluated analytically. Let us consider the region

$$k_1^2=0,\ Q^2=-k_2^2=-E^2>0,\ 0<W^2=-M^2=-(k_1+k_2)^2,$$

which is convenient for the calculation of dispersion relations in M^2 (or E^2). The result of this calculation is

$$-A_3=A_6=\frac{1}{2\pi^2(Q^2-W^2)}\left\{\frac{Q^2}{Q^2-W^2}L_1(Q,W)+\frac{m_q^2}{Q^2-W^2}L_2(Q,W)-1\right\},$$

$$A_4=-\frac{1}{2\pi^2(Q^2-W^2)}L_1(Q,W),$$

$$A_2=\frac{1}{4\pi^2}\left\{\frac{Q^2}{Q^2-W^2}L_1(Q,W)+\frac{m_q^2}{Q^2-W^2}L_2(Q,W)-1\right\}, \quad (5)$$

$$A_1=\frac{1}{4\pi^2}\left\{\frac{Q^2}{Q^2-W^2}L_1(Q,W)-\frac{m_q^2}{Q^2-W^2}L_2(Q,W)+1\right\},$$

$$A_5=-A_4+\frac{3}{\pi^2}Q^2\frac{d}{dQ^2}\left[\frac{1}{Q^2-W^2}L_1(Q,W)\right]+\frac{3}{2\pi^2}Q^4\left(\frac{d}{dQ^2}\right)^2\left[\frac{1}{Q^2-W^2}L_1(Q,W)\right]-\frac{3}{4\pi^2}Q^2\frac{d}{d^2Q^2}\left[\frac{1}{Q^2-W^2}L_2(Q,W)\right]+\frac{1}{2\pi^2}Q^2m_q^2\left(\frac{d}{dQ^2}\right)^2\left]\frac{1}{Q^2-W^2}L_2(Q,W)\right],$$

where

$$L_1=-\rho\ln\frac{\rho+1}{\rho-1}+\beta\ln\frac{\beta+1}{\beta-1},$$
$$L_2=-\ln^2\frac{\rho+1}{\rho-1}+\ln^2\frac{\beta+1}{\beta-1}, \quad (6)$$
$$\rho^2=1+4m_q^2/W^2,\ \beta^2=1+4m_q^2/Q^2.$$

Note that A_5 and A_4 do not contribute directly to physical quantities [not through the relation (3)], since $k_{1\alpha}$ and $k_{2\beta}$ in (1) are contracted either with the polarization vectors $(k_1^{\alpha}e_{\alpha}(k_1))=0$, $(k_{2\beta}e^{\beta}(k_2))=0$ or with the conserved currents $(k^1{}_{\alpha}j^{\alpha}(k_1))=0$, $(k_{2\beta}j^{\beta}(k_2))=0$.

In other regions $M^2=-W^2$ and $E^2=-Q^2$ the functions $L_1(Q,W)$ and $L_2(Q,W)$ are analytically continued in the following manner:[7]

$$0<-W^2=M^2<4m_q^2:$$
$$\rho\to i(-\rho^2)^{1/2},\quad \frac{1}{2}\ln\frac{\rho+1}{\rho-1}\to -i\,\mathrm{arctg}(-\rho^2)^{-1/2},$$
$$2m_q<M: \quad (7)$$
$$(-\rho^2)^{1/2}\to -i\rho,\quad \mathrm{arctg}(-\rho^2)^{-1/2}\to\frac{\pi}{2}+\frac{i}{2}\ln\frac{1+\rho}{1-\rho}.$$

$$0<-Q^2=E^2<4m_q^2:$$
$$\beta\to i(-\beta^2)^{1/2},\quad \frac{1}{2}\ln\frac{\beta+1}{\beta-1}\to -i\,\mathrm{arctg}(-\beta^2)^{-1/2},$$
$$2m_q<E: \quad (8)$$
$$(-\beta^2)^{1/2}\to -i\beta,\quad \mathrm{arctg}(-\beta^2)^{-1/2}\to\frac{\pi}{2}+\frac{i}{2}\ln\frac{1+\beta}{1-\beta}.$$

It is seen from Eqs. (5)–(8) that (both for $Q^2=-k_2^2\neq 0$ and for $k_2^2=-Q^2=0$) the amplitudes A_i contain no singularities other than the dynamic cuts for $4m_q^2\leqslant M^2<\infty$ and $4m_q^2\leqslant E^2<\infty$ due to the intermediate $q\bar{q}$ states.

For $m_q\to 0$ we obtain (omitting the physically irrelevant A_4 and A_5) expressions valid for $0<Q^2=-E^2$ and $0<W^2=-M^2$,

$$A_1=\frac{1}{4\pi^2}\left\{\frac{Q^2}{Q^2-W^2}\ln\frac{Q^2}{W^2}+1\right\},$$
$$A_2=\frac{1}{4\pi^2}\left\{\frac{Q^2}{Q^2-W^2}\ln\frac{Q^2}{W^2}-1\right\}, \quad (9)$$
$$-A_3=A_6=\frac{1}{2\pi^2(Q^2-W^2)}\left\{\frac{Q^2}{Q^2-W^2}\ln\frac{Q^2}{W^2}-1\right\}.$$

The analytic continuation to other regions of M^2 and E^2 is carried out as follows:

$$0<-Q^2=E^2:\ \ln Q^2\to -i\pi+\ln E^2,$$
$$0<-W^2=M^2:\quad \ln\frac{1}{W^2}\to i\pi+\ln\frac{1}{M^2}. \quad (10)$$

Thus, in the massless limit for $k_2^2\neq 0$ the invariant amplitudes have cuts for $0\leqslant E^2<\infty$ and $0\leqslant M^2<\infty$. The limit of the expressions (9) for $W^2\to 0$ does not exist (logarithmic singularity), and for $Q^2\to 0$ the amplitudes A_3 and A_6 acquire poles for $M^2=0$:

$$A_3=-A_6=\frac{2}{M^2}A_1=-\frac{2}{M^2}A_2=\frac{1}{2\pi^2M^2}. \quad (11)$$

For $k_2^2=k_1^2=0$ we can make use of the identities

$$k_2^{\sigma}\varepsilon_{\sigma\alpha\beta\mu}=\frac{2}{M^2}(k_{2\mu}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\beta\alpha}+k_{2\beta}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\alpha\mu}+k_{2\alpha}k_2^{\delta}k_1^{\sigma}\varepsilon_{\delta\sigma\beta\mu}),$$
$$k_1^{2}\varepsilon_{\sigma\alpha\beta\mu}=\frac{-2}{M^2}(k_{1\mu}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\beta\alpha}+k_{1\beta}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\alpha\mu}+k_{1\alpha}k_2^{\delta}k_1^{\sigma}\varepsilon_{\delta\sigma\beta\mu}),$$
$$(12)$$

which lead to the expression

$$T_{\alpha\beta\mu}=A_6(k_1+k_2)_{\mu}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\beta\alpha}+(A_4+A_6)(k_{2\beta}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\alpha\mu}-k_{1\alpha}k_1^{\delta}k_2^{\sigma}\varepsilon_{\delta\sigma\beta\mu}), \quad (13)$$

where

$$A_6 = -\frac{1}{2\pi^2 W^2}\left(1 - \frac{m_q^2}{W^2}\ln^2\frac{\rho+1}{\rho-1}\right),$$
$$A_4 = \frac{1}{2\pi^2 W^2}\left(2 - \rho\ln\frac{\rho+1}{\rho-1}\right). \quad (14)$$

Naturally, the second term in (13) is irrelevant from the physical point of view.

Consequently the anomaly in the axial-vector current is connected with the massless pole only for $m_q = 0$ and $k_1^2 = k_2^2 = 0$.

Equations (9) and (10) are a good illustration of the insufficiently rapid decrease of the invariant amplitudes of the axial-vector vertex with large momenta, which is improved by the elimination of the anomaly in renormalizable theories.

3. THE $Z\to\gamma\Psi$ and $Z\to\gamma\Upsilon$ DECAYS IN THE DISPERSION APPROACH

I now make use of the results (1)–(8) of the previous section to evaluate the amplitude

$$Z\to\gamma(k_1)\gamma^*(k_2),$$

due to the one-loop triangle diagram with intermediate heavy quarks ($Z\to c\bar{c}\to\gamma\gamma^*$ or $Z\to b\bar{b}\to\gamma\gamma^*$) for $0\leqslant k_2^2 = E^2 \leqslant 4m_q^2$ ($k_1^2 = 0$) in the rest frame of the Z boson:

$$T(Z\to\gamma\gamma^*) = M^2E\left(1 - \frac{E^2}{M^2}\right)t_q(E,M)$$
$$\times\left\{\frac{E}{M}(\mathbf{ne}(Z))(\mathbf{n}[\mathbf{e}(\gamma^*)\mathbf{e}(\gamma)]) + (\mathbf{ne}(\gamma^*))(\mathbf{n}[\mathbf{e}(\gamma)\mathbf{e}(Z)])\right\}, \quad (15)$$

where M is the mass of the Z boson,

$$M^2 = (k_1+k_2)^2,\ \mathbf{n} = \mathbf{k}_1/|\mathbf{k}_1|,$$

$\mathbf{e}(Z)$ and $\mathbf{e}(\gamma^*)$ are three-dimensional polarization vectors of the Z and γ^* in their rest frames and $\mathbf{e}(\gamma)$ is the polarization vector of the γ quantum. The amplitude $t_q(E,M)$ takes into account three identical loops corresponding to the three colors,

$$t_q(E,M) = \sigma_q\frac{e^3 3e_q^2}{4\sin 2\theta_W}(A_4 + A_6), \quad (16)$$

where $\sigma_c = 1$, $\sigma_b = -1$, $e_c = 2/3$, $e_b = -1/3$.

It is seen from (5)–(8) that $t_q(E,M)$ satisfies a dispersion relation without subtractions both in M^2 and in E^2. Consequently $t_q(E,M)$ is the amplitude convenient for obtaining sum rules in the E^2 channel. Since at the present time it appears possible to test theoretically only the resonance saturation of the sum rules evaluated below, it is most convenient to derive them with the help of the following consideration. The amplitude $t_q(E,M)$ describes the full amplitude in the region $E^2\leqslant 0$ for $Z\to q\bar{q}\to\gamma\gamma^*$ accurate up to higher corrections in QCD and standard electroweak theory, i.e., accurate up to corrections of order $\alpha_S(4m_q^2)/\pi$, $\alpha_S(M^2)/\pi$ and α/π. On the other hand, the full amplitude for $Z\to q\bar{q}\to\gamma\gamma^*$ can be represented with the help of intermediate hadronic states in the E^2 channel as a sum of resonance contributions and the continuous spectrum:

$$T(Z\to\gamma\gamma^*) = M^2E\left(1 - \frac{E^2}{M^2}\right)t_H(E,M)$$
$$\times\left\{\frac{E}{M}(\mathbf{ne}(Z))(\mathbf{n}[\mathbf{e}(\gamma^*)\mathbf{e}(\gamma)]) + (\mathbf{ne}(\gamma^*))(\mathbf{n}[\mathbf{e}(\gamma)\mathbf{e}(Z)])\right\}, \quad (17)$$

where

$$t_H(E,M) = \sum_V\frac{m_V^2 e}{(m_V^2 - E^2)f_V}T_V(m_V,M) + eT_{\rm cont}(E,M). \quad (18)$$

Here V is the $(q\bar{q})$ vector quarkonium and $T_{\rm cont}(E,M)$ is the contribution of the continuous spectrum ($D\bar{D}$, $\bar{D}D^*$, $\bar{D}^*D$, $\bar{D}^*D^*$,... or $\bar{B}B$, $B\bar{B}^*$, $\bar{B}B$, $B^*\bar{B}^*$,...). There is every reason to suppose that in the region $E^2\approx 0$ we have

$$t_H(E,M)\approx t_q(E,M)$$
$$\approx\sigma_q\frac{3e\alpha e_q^2}{2\sin 2\theta_W}\frac{1}{M^2}\left(i - \frac{2}{\pi}\ln\frac{M}{m_q} + \frac{1}{\pi} + \frac{\beta}{\pi}\ln\frac{\beta+1}{\beta-1}\right). \quad (19)$$

Equation (19) incorporates the fact that $2m_q/M\ll 1$.

At the point $E^2 = 0$ I consider the sum rule for the amplitude

$$t_H(0,M) = t_q(0,M) \quad (20)$$

and its first derivative

$$\frac{d}{dE^2}t_H(E,M)|_{E^2=0} = \frac{d}{dE^2}t_q(E,M)|_{E^2=0}. \quad (21)$$

It follows from (18), (19), and (20) that

$$\sum_V\frac{1}{f_V}T_V(m_V,M) + T_{\rm cont}(0,M) \equiv T_q(Res) + T_{\rm cont}(0,M)$$
$$= T_q \equiv \sigma_q\frac{3\alpha e_q^2}{2\sin 2\theta_W}\frac{1}{M^2}\left(i - \frac{2}{\pi}\ln\frac{M}{m_q} + \frac{3}{\pi}\right). \quad (22)$$

An unusual feature of this sum rule is the presence on the right-hand side of (22) of the imaginary quantity that is the jump in the amplitude due to the intermediate $q\bar{q}$ states in the M^2 channel and which accurately approximates [up to corrections of order $\alpha_S(M^2)/\pi$] the jump due to the intermediate hadronic states.

It follows from (18), (19), and (21) that

$$\sum_V\frac{1}{f_V m_V^2}T_V(m_V,M) + \frac{d}{dE^2}T_{\rm cont}(E,M)|_{E^2=0} \equiv D_q(Res)$$
$$+ \frac{d}{dE^2}T_{\rm cont}(E,M)|_{E^2=0} = D_q \equiv -\sigma_q\frac{1}{M^2}\frac{\alpha e_q^2}{4m_q^2\sin 2\theta_W}\frac{1}{\pi}. \quad (23)$$

In the approximation (19), $2m_q/M\ll 1$ we have $\mathrm{Im}Dq = 0$. The decay width is

$$\Gamma(Z\to\gamma V)$$
$$= \frac{1}{24\pi}\left(1 - \frac{m_V^2}{M^2}\right)^3\left(1 + \frac{m_V^2}{M^2}\right)M^3 m_V^2|T_V(m_V,M)|^2$$
$$\approx\frac{1}{24\pi}M^3 m_V^2|T_V(m_V,M)|^2. \quad (24)$$

To determine $f_V^2/4\pi$, I make use of the experimental data of Ref. 8 for

$$\Gamma(V\to e^+e^-)=\frac{4\pi}{3}\frac{m_V^2}{f_V^2}\alpha^2. \quad (25)$$

As a result I obtain for the Ψ family

$$1=\Psi(3097),\ 2=\Psi(3686),\ 3=\Psi(3770),\ 4=\Psi(4040),\ 5=\Psi(4160),\ 6=\Psi(4415),$$
$$f_1:f_2:f_3:f_4:f_5:f_6=1:1,6:4,7:2,9:2,9:3.7 \quad (26)$$

and for the Υ family

$$1=\Upsilon(9460),\ 2=\Upsilon(10023),\ 3=\Upsilon(10355),\ 4=\Upsilon(10580),\ 5=\Upsilon(10860),\ 6=\Upsilon(11020),$$
$$f_1:f_2:f_3:f_4:f_5:f_6=1:1,6:1,8:2,5:2,2:3,5 \quad (27)$$

for

$$f_{J/\Psi}^2/4\pi=f_{\Psi(3097)}^2/4\pi=11,7,$$
$$f_{\Upsilon(1S)}^2/4\pi=f_{\Upsilon(9460)}^2/4\pi=125. \quad (28)$$

3.1. Sum rule for the amplitude

We consider the sum rule for the amplitude (22). Let us assume initially that the left-hand side of (22) is saturated by the ground state, i.e.,

$$T_V(m_V,\ M)=f_VT_q,$$
$$V=J/\Psi,\ \Upsilon(1S). \quad (29)$$

Using (29), (28), (24), (22), $m_c = 1.55$ GeV, $m_b = 5$ GeV, $M = 91.16$ GeV and $\Gamma_Z = 2.53$ GeV (Ref. 8), we obtain

$$\mathrm{BR}(Z\to\gamma J/\Psi)=10^{-5},\ \mathrm{BR}(Z\to\gamma\Upsilon(1S))=3\cdot10^{-5}, \quad (30)$$

which exceeds by two orders of magnitude the predictions of the quark model.[9]

How good is the assumption (29) and the result (30)? It seems to me that only experiment can answer this question. The model of dominance of the vector mesons $V=\rho$, ω, Φ in the electromagnetic current of light quarks for not too virtual γ^* quanta is a generally accepted working model. There are examples of successful application of this model also in the case of the electromagnetic current of the c quarks.[10]

At the present time it is not possible to completely clarify the role played by other intermediate states in the left-hand side of (22). However, we may attempt to estimate the situation by saturating the sum rule for the amplitude (22) by the Ψ and Υ families:

$$\sum_V\frac{1}{f_V}T_V(m_V,M)=T_q(Res)=T_q. \quad (31)$$

We remark that this takes partially into account the continuous spectrum since four members of the Ψ family and three members of the Υ family lie in the continuous spectrum of $D\bar{D}, D\bar{D}^*, D^*\bar{D}, \bar{D}^*D^*$ and $B\bar{B}, B\bar{B}^*, B^*\bar{B}, B^*\bar{B}^*$, respectively.

Viewing (31) as a constraint and using (24) we obtain

$$\min\sum_V\Gamma(Z\to\gamma V)=\frac{1}{24\pi}M^3|T_q|^2a^{-1},$$
$$a=\sum_V\frac{1}{f_V^2m_V^2}. \quad (32)$$

When the lower bound (32) is reached,

$$\Gamma(Z\to\gamma V)=\frac{1}{24\pi}M^3|T_q|^2(f_Vm_Va)^{-2}. \quad (33)$$

For the Ψ family

$$\min\sum\mathrm{BR}(Z\to\gamma\Psi)=7\cdot10^{-6},\quad \mathrm{BR}(Z\to\gamma J/\Psi)=5\cdot10^{-6}. \quad (34)$$

For the Υ family

$$\min\sum\mathrm{BR}(Z\to\gamma\Upsilon)=1,5\cdot10^{-5},\quad \mathrm{BR}(Z\to\gamma\Upsilon(1S))=8\cdot10^{-6}. \quad (35)$$

By making use of the results of Sec. 2 we can verify that the dispersion integral for T_q is determined by the region $2m_q\leqslant E\lesssim M$, which is hardly a low-energy region. However this circumstance cannot be viewed as an objection to the sum rule (31) since the hadronic spectrum should not (and does not) locally repeat the quark spectrum. Nevertheless it is useful to study the sum rule (23) for the first derivative. It is well known that in the dispersion integral for the derivative of the amplitude the relative importance of the contribution of the low-lying states is significantly enhanced as compared to their contribution to the amplitude itself. We remark that 90% of the dispersion integral for D_q in (23) is determined by the region of low energies $2m_q\leqslant E\leqslant 6m_q$.

3.2. The sum rule for the first derivative of the amplitude

We consider the resonance contributions to the sum rule for the amplitude and its first derivative, $T_q(Res)$ and $D_q(Res)$, respectively, as two constraints and use (24) to obtain

$$\min\sum_V\Gamma(Z\to\gamma V)=\frac{1}{24\pi}M^3\times\frac{|T_q(Res)|^2g+|D_q(Res)|^2a-2\,\mathrm{Re}(T_q^*(Res)D_q(Res))d}{ag-d^2},$$
$$d=\sum_V\frac{1}{f_V^2m_V^4},\qquad g=\sum_V\frac{1}{f_V^2m_V^6}. \quad (36)$$

The lower limit is reached when

$$T_V=\frac{1}{f_Vm_V^2}\frac{T_q(Res)(g-d/m_V^2)-D_q(Res)(d-a/m_V^2)}{ag-d^2}. \quad (37)$$

Let us suppose that the sum rule for the derivative (23) is saturated by the resonances

$$\sum_{V}\frac{1}{f_V m_V^2}T_V(m_V, M)=D_q(Res)=D_q. \quad (38)$$

We consider various possibilities for the resonance contribution to the amplitude $T_q(Res)$.

1) Let $T_q(Res) = T_q$, i.e., both the sum rule for the derivative of the amplitude and the sum rule for the amplitude itself are saturated by the resonances. Making use of (38), (37), (36), (31), and (24) we obtain for the Ψ family

$$\min\sum \mathrm{BR}(Z\to\gamma\Psi)=2\cdot 10^{-4}, \quad \mathrm{BR}(Z\to\gamma J/\Psi)=4{,}4\cdot 10^{-5}, \quad (39)$$

and for the Υ family

$$\min\sum \mathrm{BR}(Z\to\gamma\Upsilon)=2{,}3\cdot 10^{-3}; \quad \mathrm{BR}(Z\to\gamma\Upsilon(1S))=1{,}4\cdot 10^{-3}. \quad (40)$$

2) We now suppose that the contribution of the resonances to the amplitude can be ignored, i.e., $T_q(\mathrm{Res}) = 0$. It then follows from (38), (37), (36), and (24) that for the Ψ family

$$\min\sum \mathrm{BR}(Z\to\gamma\Psi)=3{,}7\cdot 10^{-5}, \quad \mathrm{BR}(Z\to\gamma J/\Psi)=1{,}3\cdot 10^{-5}, \quad (41)$$

and for the Υ family

$$\min\sum \mathrm{BR}(Z\to\gamma\Upsilon)=8{,}7\cdot 10^{-4}, \quad \mathrm{BR}(Z\to\gamma\Upsilon(1S))=5{,}6\cdot 10^{-4}. \quad (42)$$

Unsurprisingly, the results are not substantially different in the two cases.

3) It is very difficult to find any objections against the saturation by the six resonances of the rapidly convergent sum rule for the derivative (38). We therefore clarify what is the contribution of the resonances to the sum rule for the amplitude $T_q(Res)$ for which the lower bound (36) is minimal if $D_q(Res) = D_q$ [see (38)]. The minimum of the expression (36) is reached when

$$T_q(Res)=(d/g)D_q(Res). \quad (43)$$

There

$$\min\left\{\min\sum_{V}\Gamma(Z\to\gamma V)\right\}=\frac{1}{24\pi}M^3\frac{|D_q(\mathrm{Res})|^2}{g},$$

$$\Gamma(Z\to\gamma V)=\frac{1}{24\pi}M^3\frac{|D_q(Res)|^2}{f_V^2 g^2 m_V^6}. \quad (44)$$

For $D_q(Res) = D_q$ we obtain for the Ψ family

$$\min\left\{\min\sum \mathrm{BR}(Z\to\gamma\Psi)\right\}=10^{-6}, \quad \mathrm{BR}(Z\to\gamma J/\Psi)=7\cdot 10^{-7}, \quad (45)$$

and for the Υ family

$$\min\left\{\min\sum \mathrm{BR}(Z\to\gamma\Upsilon)\right\}=4.4\cdot 10^{-6}, \quad \mathrm{BR}(Z\to\gamma\Upsilon(1S))=2\cdot 10^{-6}. \quad (46)$$

In this manner, even in the worst case, the predictions of the dispersion analysis exceed the expectations from the quark model[9] by one order of magnitude.

We note that

$$T_c(Res)=(d/g)D_c=0.44\,\mathrm{Re}\,T_c, \quad T_b(Res)=(d/g)D_b=0{,}8\,\mathrm{Re}\,T_b. \quad (47)$$

It is hard to imagine that Im $Tq(Res)$ is altogether absent. But even if it has a small value the lower bounds (45) and (46) are noticeably increased. For example, it follows from (36) that Im $T_c(Res) = 0.3$ Im T_c and Im $T_b(Res) = 0.08$ Im T_b [along with (47) for the real parts] would bring us back to the lower bounds (34) and (35), respectively.

4. CONCLUSION

One might be tempted to consider the sum rule for the second derivative of the amplitude. It seems to us that this is inappropriate. The point is that radiative QCD corrections lead to the appearance of intermediate states of light quarks ($d\bar{d}$, $u\bar{u}$ and $s\bar{s}$) in the E^2 channel, which contribute to the second derivative an amount of relative order $(\alpha_s/\pi)^3(15/4)(m_{c,b}/m_{u,d,s})^4$, i.e., an amount which is not small. In the case of the sum rule for the first derivative this correction is of order $(\alpha_s/\pi)^3(3/2)(m_{c,b}/m_{u,d,s})^2$, i.e., insignificant. The masses $m_{u,d,s}$ are, of course, constituent masses.

It seems to me that this analysis shows clearly that the estimates $\mathrm{BR}(Z\to\gamma J/\Psi)\sim 10^{-5}$ and $\mathrm{BR}(Z\to\gamma\Upsilon(1S))\sim 10^{-5}$ arise quite naturally. I therefore do not share the pessimism[11] regarding the possibility of observing the decays $Z\to\gamma J/\Psi$ and $Z\to\gamma\Upsilon(1S)$ with high luminosity at LEP.

The expected angular distribution $W(\theta)$ for the reaction $e^+e^-\to Z\to\gamma V$ follows from (15) and (17):

$$W(\theta)=\frac{3}{8}\frac{1+\cos^2\theta+2m_V^2/M^2\sin^2}{1+m_V^2/M^2}\approx\frac{3}{8}(1+\cos^2\Theta), \quad (48)$$

where Θ is the angle between the momentum of the γ quantum and the axis of the beam.

Note that we have applied recently a similar dispersion approach[12] to the study of the decays of heavy Higgs bosons $H\to\gamma\Psi$, $\gamma\Upsilon$, and also of the decays Ψ, $\Upsilon\to\gamma H$ (or axion).

Lastly, one more remark. All of the formulas obtained in Sec. 2 and the expression (15) in Sec. 3 are valid for the transition of an axial-vector current into two gluons $[g(k_1)g^*(k_2)]$ and for the amplitude $Z\to q\bar{q}\to g(k_1)g^*(k_2)$, respectively. The practical consequences of this remark need to be considered at greater length.

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