Metal–insulator transition in amorphization of Cd–Sb

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The metastable metallic high-pressure phase of the alloy $Cd_{43}Sb_{57}$ is gradually transformed at room temperature and atmospheric pressure into an amorphous dielectric state. It was possible to observe at various stages of this transformation three different power-law variations of the conductivity with temperature. Interelectron interaction predominates at $T < 40$ K. It ensures that $\sigma \propto T^{1/3}$ in the immediate vicinity of the metal–insulator transition and $\sigma \propto T^{1/2}$ away from this transition. Above 40 K the $\sigma(T)$ dependence becomes linear. It is determined by inelastic electron scattering by the structural disorder.

1. INTRODUCTION

Metal–insulator transitions have remained for many years among the most interesting problems in solid-state physics. There are still many questions for which there are no clear answers. One of them concerns the temperature dependence of the conductivity $\sigma(T)$ near the transitions. These questions are the subject of the present paper.

Two of the causes of a metal–insulator transition the most important: the increase of the disorder in the static potential, possibly, at a constant density of the electronic states on the Fermi level (the Anderson transition), and the decrease of the electron density of states possibly, even in the complete absence of disorder (Mott transition). Although real transitions are usually caused by simultaneous action of various factors, in the analysis of experimental data it is convenient to start with a definite model. Since transition evolves in experiment against the background of amorphization of an alloy, it is natural to start out with the terminology of the Anderson model.

The Anderson transition is defined only at $T = 0$ K. It amounts to vanishing of the conductivity σ. Application of a scaling description to this transition leads in the three-dimensional case to a conductivity $\sigma(0) = (e^2/\hbar)(1/\xi)$, which contains beside the natural conductance unit $e^2/\hbar = (4.1\ \text{k}\Omega)^{-1}$ only the correlation length ξ which becomes infinite at the transition point. The value of ξ depends on some structure parameter (e.g., the degree of disorder). Such transitions are therefore investigated experimentally by comparing the characteristics of a number of samples prepared with gradual variation of one of the parameters (chemical composition, film-sputtering conditions, etc.) It is also possible to induce the transition in the sample by varying a magnetic field or subjecting the sample to an elastic stress.

Inelastic interactions come into play at $T \neq 0$ K. They are characterized by a parameter $L(T)$ with dimension of length, which can be regarded as the length at which these interactions destroy the phase coherence of the wave function.[3] If the temperature is high enough so that $L \ll \xi$, it is precisely the inelastic interaction, and not the geometric conductivity of the equipotential surface of the static potential, which determines the value of the conductivity. This means that at $L \ll \xi$ the expression for σ contains L in place of ξ. A simple interpolation equation

$$\sigma(T) = (e^2/\hbar)(c_1/\xi + c_2/L), \tag{1}$$

has been proposed[3] for the region $L \approx \xi$, with the first term indicative of the proximity to the transition and the second describing the temperature dependence. It appears that the constants c_1 and c_2 depend little on the actual material.

We must emphasize the fundamental difference between equations such as (1), in which the length L is in the denominator, from the equation for the Boltzmann conductivity of an electron gas of density n with a Fermi momentum k_F:

$$\sigma = ne^2 l/\hbar k_F, \tag{2}$$

in which the path length l is the numerator. In particular, the changes of the conductivity with temperature are opposite in these equations. It is known that a decrease of σ with decrease of T is possible in metals on account of quantum corrections to the conductivity: weak localization[4] and interelectron interaction.[5] In these equations L has the meaning of diffusion length. The quantum corrections, however, should be small compared with the total conductivity. The term $(e^2/\hbar)L$ in (1) is the principal one. It is therefore natural to call a conductivity of type (1) quantum or scaling conductivity, to distinguish it from classical or Boltzmann conductivity.

The quantum conductivity (1) is small in absolute value. It can be arbitrarily assumed that the quantum and Boltzmann conductivities are separated by the Mott value of the minimum metallic conductivity:[6]

$$\sigma_M [\Omega^{-1}\cdot\text{cm}^{-1}] \approx 0{,}03 (e^2/\hbar)(1/a) \approx 6\cdot 10^{-6}/a \tag{3}$$

(a is the parameter with dimension of length and is connected with the density n of the electrons that can participate in the conductivity: $a = n^{-1/3}$). The limiting value of (3) is obtained, apart from a coefficient, from (2) at the minimum possible value $l \approx k_F^{-1} \approx n^{-1/3}$. The latter is obtained as the highest possible value of the second term in (1) if it is assumed that $L = l$. That l is the smallest possible value of L is natural if it is assumed that L is the diffusion length and l is the pace of the diffusion.

An attempt to describe the experimental values of $\sigma(T)$ by starting from Eq. (1) requires identification of the physical nature of the processes that control L. It appears that in the critical region, i.e., close to the transition and at not too high temperatures), the principal role in the formation of L is played by interelectron interaction,[7,8] for which the characteristic time is[5]

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$$\tau \equiv \tau_{int} = \hbar / T,$$

and the characteristic length is

$$L \equiv L_{int} = (D\tau_{int})^{1/2} = (D\hbar/T)^{1/2}, \tag{4}$$

where D is the diffusion coefficient. Assume that with respect to the degree of disorder we are so close to the transition that the first term of (1) can be neglected. We can then calculate D self-consistently in terms of the Einstein relation. From the set of equations

$$\sigma(T) = C_2(e^2/\hbar)(T/\hbar D)^{1/2}, \quad D = D_{int}(T) = \sigma(T)/e^2 g_F$$

(g_F is the density of states on the Fermi level) we obtain with the aid of (1) and (4)

$$\sigma(T) = c_2^{2/3}(e^2/\hbar)(T g_F)^{1/3}. \tag{5}$$

According to Ref. 9, $c_2 = (2/3)\pi^3$, so that the coefficient $c_2^{2/3}$ in (5) is equal to 0.077. This coefficient is important for an estimate of the density of states near the transition.

The $\sigma \propto T^{1/3}$ dependence was observed in experiment a number of times,[10-16] but in most cases only at $T < 1$ K and using a magnetic field to approach the transition. This dependence remained in force up to 10 K in $Ge_{1-x}Au_x$ (Ref. 10) and In_2O_{3-x} (Ref. 11) up to higher temperatures.

Moving away from the transition by varying the degree of disorder, the first term of (1) becomes larger than the second. The diffusion coefficient is then controlled by this term, i.e., D in Eq. (4) ceases to depend on T. The second term of (1) becomes then proportional to $T^{1/2}$. This crossover was observed earlier only in a magnetic field.[13]

It is also possible to move away from the transition by increasing T. This decreases L_{int} according to (4), but only if it remains of the order of the elastic mean free path l_0 controlled by the static disorder. Different variants of the behavior of $\sigma(T)$ are probably possible[17] with further increase of T. One of the variant is a transition from $T^{1/3}$ to a linear dependence on T. Such an evolution of the function $\sigma(T)$ was observed experimentally only in $Ge_{1-x}Au_x$ (Ref. 10) In_2O_{3-x} (Ref. 18).

We present here the results of an investigation of the resistivity of the alloy $Cd_{43}Sb_{57}$ which decays from a metastable metallic high-pressure phase into an amorphous insulating state.[19,21] Since the decay is slow close to room temperature, it can be interrupted in intermediate stages by rapidly lowering the temperature ("quenching"). This makes it possible to plot a set of resistivity curves for various intermediate states of the sample in the temperature range 5–300 K, and with them a set of curves of superconducting transition in these states at $T = 5$–4 K. The resistivity changed by almost six orders of magnitude when the temperature was lowered.

The plan of the paper is the following.

In the next section we describe the experimental procedure. The fact that all the experiments are performed on one sample at a fixed location of the contacts improves substantially the possibility of comparing the results for different states. By stepwise heating it is possible to vary a state very insignificantly. It is therefore possible to come quite close to the transition. The possibilities here are comparable with those afforded by using a magnetic field to approach the transition.[12-14,16]

In the third section we describe the experimental results. We were able to distinguish the intermediate state between the metallic and insulating regions, and to observe a dependence of the type

$$\sigma = \sigma(0) + \alpha T^{1/3} \tag{6}$$

in the interval $T = 4$–40 K for a set of states close to the limit, and a crossover from (6) to the relation

$$\sigma = \sigma(0) + \alpha T^{1/2} \tag{7}$$

on penetrating deeper into the metallic as well as the insulating region [$\sigma(0) > 0$ in the metallic region and $\sigma(0) < 0$ in the insulating one[13]]. A transition from relations (6) and (7) to a linear dependence

$$\sigma = \sigma_0 + \beta T \tag{8}$$

was observed above 40 K.

In the fourth and final section we analyze the character of the conductivity and the nature of the elastic processes that lead to relations (6)–(8): the electron–electron interaction at $T \leqslant 40$ K and the inelastic scattering by structural disorder at higher temperatures.

2. EXPERIMENT

$Cd_{43}Sb_{57}$ is a binary alloy at which amorphization is produced in the solid state in the course of decay of a high-pressure metastable phase.[20,21] At 55 kbar pressure and high temperature there is produced in this alloy the so-called γ phase which has a simple hexagonal structure. After quenching under pressure to liquid-nitrogen temperature and removing the pressure the alloy stays in the γ phase. This is a metallic phase that becomes superconducting at $T_c \approx 4.9$ K. Heating to room temperature transforms it into the amorphous state.[19] The amorphization is accompanied by release of heat and by a 12.6% increase of the specific volume.

Further heating or prolonged (on the order of a month) storage at room temperature crystallizes the alloy. Only one stoichiometric compound exists in the crystalline state, CdSb, and is a superconductor with an approximate gap 0.5 eV. A spectrum gap exists also in the amorphous state. The excess of antimony can play then the role of an impurity in the semiconductor.

By successively stopping the amorphization by lowering the temperature at various stages and observing thereby the intermediate states of the sample, we measured the temperature dependences of the resistance $R(T)$ of the sample in these states. Each cycle consisted of the following. The sample was first cooled to helium temperature, after which $R(T)$ was measured as the temperature was raised. Measurement of $R(T)$ made it possible to determine the instant at which the amorphization resumed: the resistance began to depend on time—"float." The sample was kept at this temperature until the resistance increased by 30–40%, and then cooled rapidly to helium temperature. The number of such cycles was 23.

The sample was approximately rectangular measuring $1.2 \times 1.5 \times 6$ mm^3. The resistance was measured by the standard four-point method using pressure contacts of sharpened gold wires 0.5 mm in diameter. The measurement current ranged, depending on the resistance, from fractions of a

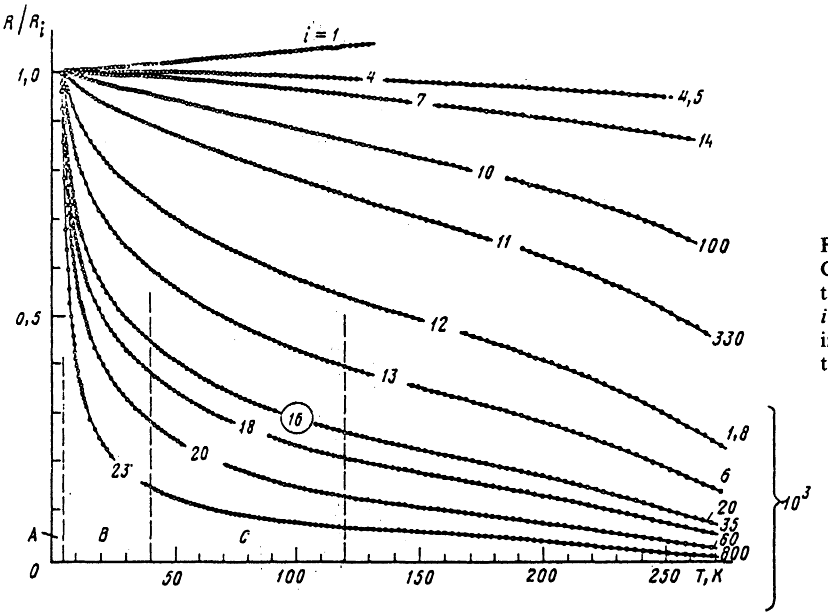

FIG. 1. Temperature dependences of the resistance of a $Cd_{43}Sb_{57}$ sample in various states i normalized to the resistance at $T \approx 5$ K. The curves are marked by the state numbers i (see the text), and the coefficients on the right are normalized relative to the first curve. A, B, and C—three temperature ranges discussed in the article.

μA to several mA; the independence of the resistance of the measurement current was checked each time. The instrumental error of $R(T)$ did not exceed 0.1%. A carbon thermometer calibrated accurate to not more than 0.01 K was used at helium temperatures. Higher temperatures were measured with a thermocouple accurate to about 0.1 K.

3. RESULTS

A general survey of the available experimental material is shown in Fig. 1, where the resistance plots of individual measurement cycles are normalized to the value of the resistance at $T \approx 5$ K, i.e., in units of $R_i(T)/R_i$ (5 K). The cycle number i is marked on each curve. The measurements of the sample in the initial state are labeled $i = 1$. To prevent cluttering, only 11 of the available 23 curves were plotted. The number $i = 16$ is special because, as we shall see below, this curve pertains to the boundary state between the metallic and insulating regions. For an arbitrary quantitative description of the curves, the ratios $R_i(5\text{ K})/R_1(5\text{ K})$ are marked on their right. They show factor by which the resistance at $T \approx 5$ K was increased by the transition from the initial to the given state. The resistance increased in the course of amorphization by almost size orders.

To facilitate the comparison with experiment, we shall use hereafter mainly not resistances but conductances $\sigma_i(T) = R_i^{-1}$. We shall analyze only the curves with $i > 11$, for only with respect to these curves do we have arguments to show that the sample can be regarded as homogeneous at the lengths that determine the conductivity in these states. We have $d\sigma/dT > 0$ all the way from critical T_c to room temperature.

In Fig. 1 are demarcated three temperature regions: a superconducting transition is observed in region A, at $T \leqslant 5$K; dependences such as (6) and (7) are observed in region B for $4 \leqslant T \leqslant 40$ K; they go over in region C at $40 \leqslant T \leqslant 120$ K into linear relations of type (8).

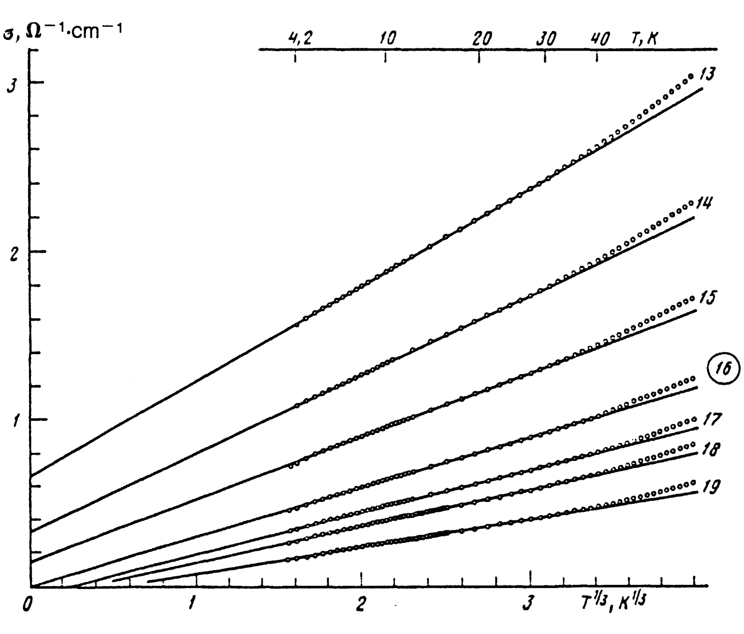

FIG. 2. Variation of the conductivity in the temperature interval B for the states $i = 13$–19. The $\sigma(T)$ dependence is described by Eq. (6).

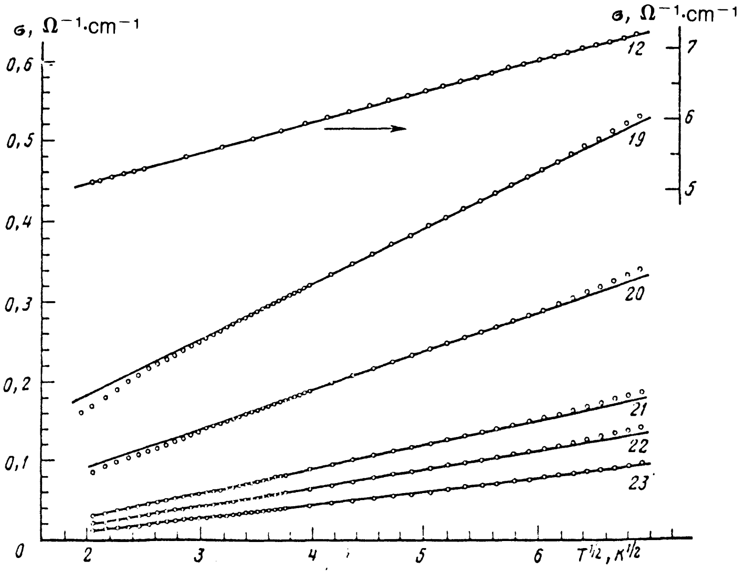

FIG. 3. The same as Fig. 2, for states $i = 12$ (right-hand scale) and $i = 19$–23 (left-hand scale). The $\sigma(T)$ dependence for these states is described by Eq. (7).

Let us examine these regions in greater detail. We begin with region B.

3.1. Intermediate temperatures $4 \leqslant T \leqslant 40$ K

The sections of the curves with i from 13 to 19 are in the interval $4 < T < 50$ K are shown in Fig. 2 with $T^{1/3}$ and σ as the coordinates. It can be seen that in this interval the function $\sigma(T)$ is well described by Eq. (6). A tendency to a systematic deviations of the instrumental points from the straight line (6) is observable only on the lower curves ($i = 18$ and 19) for the very lowest temperatures. This is the first manifestation of a transition from (6) to (7).

The point of intersection of the straight line with the ordinate axis is $\sigma(0)$. It turns out that $\sigma_{16}(0) = 0$ for the state $i = 16$. This means that this is a boundary state, in metal–insulator transition. For the curves with $i > 16$ we have $\sigma(0) = (e^2/h)(1/\xi) < 0$ (Ref. 13). The cause of the reversal of the sign of ξ at the transition is that the physical meaning of ξ is different on the two sides of the transition. This is seen most clearly with the percolation transition as the example: a medium size on one side of the transition and a medium size of the holes in an infinite cluster on the other. Were there no superconducting transition at $T < 4$ K, the curves with $i > 16$ would show dependences typical of hopping conductivity. In view of the superconducting transition, to determine $\sigma(0)$ it is necessary to extrapolate from the $T > 4$ regions, thus lowering the accuracy of $\sigma(0)$, especially for the curves with $i > 16$.

It is noteworthy that the slopes of the straight line in Fig. 2, i.e., the coefficient α of $T^{1/3}$, decreases monotonically towards the insulator state. In contrast to Refs. 11–16, a monotonic dependence of α on $\sigma(0)$ is observed.

Figure 3 shows, in coordinates $T^{1/2}$ and σ, the temperature dependences of $\sigma_i(T)$ in region B for states farther from transition region, namely $i = 12$ (right-hand scale) and $i = 19$–23 in the region of lower ones. This figure demonstrates the crossover from (6) to (7), i.e., from a temperature dependent diffusion coefficient D to an independent one. The curve $i = 19$ which deviates from the $T^{1/2}$ dependence at low temperatures is intermediate (cf. Fig. 2).

3.2. High-temperature region, $40 \leqslant T \leqslant 120$ K

We proceed now to region C, to higher temperatures. As seen from Fig. 4, the linear relation (8) is satisfied here with high accuracy. In contrast to In_2O_{3-x}, where a linear dependence of σ on T was also observed,[16] the slopes of the straight lines vary from state to state and a correlation exists between σ_0 and β. Figure 5 shows that for the states $i = 13$–19 we have with high accuracy

$$\beta \propto \sigma_0^{1/2}. \qquad (9)$$

It is curious that deviations from relation (9) set in for states 12 and 30, i.e., where a transition takes place from $T^{1/3}$ to $T^{1/2}$ (cf. Figs. 5 and 3).

3.3. Superconducting transition

Finally, let us describe briefly the results for region A, i.e., for the superconducting transition. Figure 6 is in essence

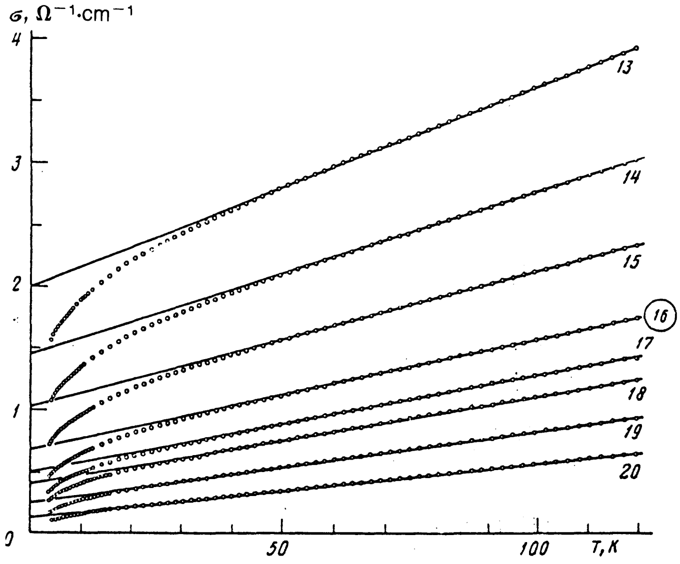

FIG. 4. Variation of conductivity in the temperature interval C for the states $i = 13$–20.

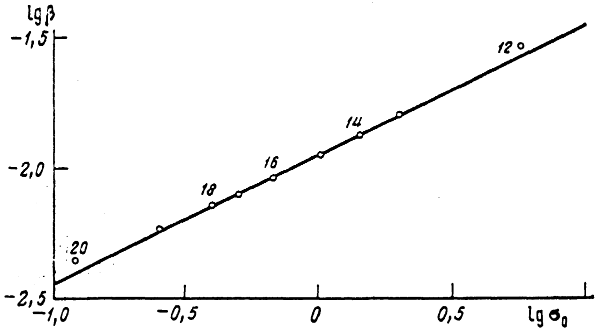

FIG. 5. Correlation of the coefficients β and σ_0 of the linear function $\sigma(T)$. The numbers i of the state are marked near the points. The number on the axes correspond to the values of σ_0 in units $(\Omega\cdot\text{cm})^{-1}$ and in units of $(\Omega\cdot\text{cm}\cdot\text{K})^{-1}$.

an extension of Fig. 1 along the T axis in the interval $1.3 < T < 5$ K. In view of the enlarged scale, it is necessary to define more precisely the meaning of the normalization factor for each curve. In contrast to Fig. 1, the resistance $R_i^{\max}$ is a maximum ahead of the superconducting transition, but the numerical differences between R_i (5 K) and $R_i^{\max}$ are very small.

Let us formulate the observations that follow from the analysis of the superconducting-transition curves.

a) Notwithstanding the difference, by almost five times, of the orders of magnitude of the sample resistance $R_i^{\max}$ in the states 1 and 19, the shift of the superconducting temperature T_c is only 25% [by T_c we mean here the start of a manifest decrease of the resistance: $R_i(T_c) \approx R_i^{\max}$].

b) The curves $i = 1$–11 have a distinct structure attesting to the presence of local inhomogeneities. Starting with curve $i = 12$, however, the structure vanishes and the curves become smooth. This means that if any inhomogeneities are preserved at all, their sizes become smaller than the characteristic scale that determines the behavior of the sample.

c) The curve $i = 16$, which marks a boundary in Fig. 2, plays the role of a boundary here, too, at $T < T_c$ on it at $dR/dT > 0$. A section on which $dR/dT < 0$ appears already on the next curve $i = 17$.

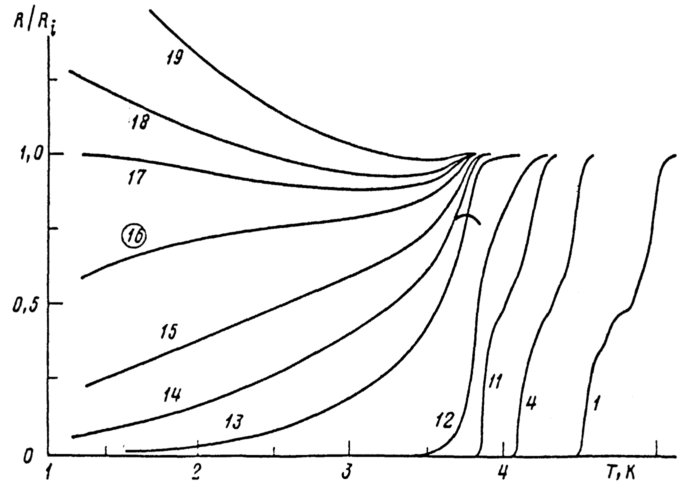

FIG. 6. Superconducting transitions in various states i of the sample, normalized to the resistance $R_i^{\max}$ (see the text).

4. DISCUSSION

Metal–insulator transitions are known to occur under a great variety of conditions. We must therefore start with an attempt to determine these conditions from the experimental results.

4.1. Homogeneity

Amorphization of an alloy is accompanied by an increase of its specific volume. During the first stage of the amorphization this should lead to formation of a homogeneous structure—a mixture of two phases. Such a structure, of the fractal type, was observed on the metallic side of the transition in amorphization of a Zn–Sb alloy.[22] In addition, the amorphization stems from density fluctuations of the individual components of the alloy. Therefore, generally speaking, it might seem that we are dealing all the time with a macroscopically inhomogeneous sample.

The shapes of the superconducting-transition curves in states $i \leqslant 10$ do indeed point clearly to the presence of an inhomogeneity. Starting with states $i = 12$, however, all the experimental data attest to a homogeneous structure: relations (6) or (7), the theoretical description of which[5] does not contain the inhomogeneity scale d at all, are observed in region B, and the superconducting-transition curves become smooth. This means that both scales, ξ and L, which according to (1) determine the conductivity, are larger than the inhomogeneity scale d. We recall in this connection that in Zn–Sb the scale d decreases as the transition is approached.[22]

4.2. Character of conductivity

It should be noted that the conductivity $\sigma_{16}(T)$ in the boundary state is very small in absolute value (see Figs. 2 and 4). This smallness, and also the fact that the derivative $d\sigma/dT$ is positive in the entire interval up to room temperature, shows that we are dealing with quantum (scaling) conductivity. Let us denote the conductivity on the boundary of the regions B and C for the state $i = 16$ by σ_{BC} and let us regard it as the characteristic value of the conductivity. Substituting $\sigma_{BC} \approx 1$ $(\Omega\cdot\text{cm}^{-1})$ in Eq. (3) for the Mott conductivity, we obtain for the average distance $a \approx n^{-1/3}$ between the carriers the estimate $a \approx 600$ Å, i.e., about 200 interatomic states. This value should be compared with the density of state discussed in the preceding subsection (see also Sec. 4.5).

4.3. Density of states

Equation (5), which describes the $\sigma \propto T^{1/3}$ dependence on the $i = 16$ curve, contains beside the constant 0.077 only one parameter, the density of states g_F, equal to

$$g_F \approx 2\cdot 10^{28}\ \text{cm}^{-3}\,\text{erg}^{-1}. \tag{10}$$

The same density of states was measured near a metal–insulator transition in n-InSb (Ref. 12), whereas the density of states obtained in n-GaAs is larger by two orders.[12,13,15]

So low a density of states means that the amorphization is accompanied by a restructuring of the electron spectrum and by formation of a gap that is smeared out by the disorder. Under these conditions the carriers are concentrated in a small region of reciprocal space. Their nature is not clear, and may be connected with the nonstoichiometry of the alloy. The smallness of k_F, which follows from the smallness of g_F, agrees with the estimate for $a \approx k_F^{-1}$.

Changes of the density of states mean that there is no genuine Anderson transition, but its combination with a Mott transition. It is also possible that near the transition the increase of the disorder is already by itself of little significance, and the main factor is the decrease of the density of states: the negligible displacements of the atoms relative to their nearest neighbors influence strongly the character of the electronic bonds.

That the electron spectrum is restructured in the vicinity of the transition is indicated also by the change of the slope θ of the straight lines $\sigma(T^{1/3})$ in Fig. 2. Since $g_F \propto (\tan)^3\theta$, it is easily seen that the transition from $i = 13$ to $i = 16$ corresponds to a decrease by almost 8 times. Using the estimate obtained above for $a \approx n^{-1/3}$, we get from the Mott criterion[6]

$$n^{-1/3} \approx 4a_n \tag{11}$$

the estimate $a_n \approx 150$ Å for the Bohr radius.

Now, having described the transition as a whole, let us analyze the observed temperature dependences in greater detail.

4.4. The $T^{1/3} - T^{1/2}$ crossover

The degree of $T^{1/3}$ in the $\sigma(T)$ dependence is the result of a self-consistent expression for the coefficient D of diffusion through the conductivity—see Eqs. (4) and (5). The diffusion is by the same electromagnetic-field fluctuations that cause the interaction. However, as soon as $\sigma(0)$ becomes large enough, it begins to control D. It follows directly from (4) that the length $L_{\rm int}$ for a diffusion coefficient that is independent of T is proportional to $T^{-1/2}$. Note that a transition to $T^{1/2}$ takes place also from the dielectric side of the transition, where formally $\sigma(0) < 0$. In this region the length ξ controls the diffusion just as effectively.

The absolute values of $\sigma(0)$ at which crossover is observed are greatly different on the two sides of the transition. The difference, however, correlates with the changes of the slope angle θ referred to above: the temperature-dependent part of the $\sigma(T)$ curve is much larger on the metallic side of the transition.

4.5. Transition to higher temperatures

Equations (6) and (7) can be used only until the quantity $L = L_{\rm int}$ in them becomes of the order of the mean free path l_0 due to scattering by the structural disorder. The $T^{1/3}$ dependence remains in force on the $i = 16$ curve up to 40 K. Here $\sigma_{16}(40\ {\rm K}) \approx 1\ (\Omega\cdot{\rm cm})^{-1}$ and $L_{\rm int} \approx l_0 \approx 500$ Å. Naturally, l_0 and k_F^{-1} are of the same order, since they are obtained in fact from the same equation.

In the equations describing the diffusion process, the diffusion length L can under no circumstances become smaller than the pace l. The second term of (1) can therefore not become larger than $(e^2/\hbar)\cdot(1/l_0)$, and cannot ensure further increase of the conductivity observed in experiment. On the other hand, there is every reason for assuming that the conductivity measured by us remains of the quantum type up to high T, i.e., it remains not explicitly dependent on the number of carriers. These statements can be joined by postulating additivity of the localization effects with the electron-interaction effect, just as in the case when these effects lead only to small corrections.[23,24] Equation (1) is then replaced by

$$\sigma(T) = \sigma(0) + \frac{e^2}{\hbar}\frac{c_2}{L_{int}} + \frac{e^2}{\hbar}\frac{c_3}{L_{in}}. \tag{12}$$

The term with $L_{\rm int}$ due to interelectron interaction ensures a temperature dependence of $\sigma(T)$ in region B. $L_{\rm int}$ reaches the limit $L_{\rm int} \approx l_0$ on the boundary of regions B and C and, neglecting the first term, we get

$$\sigma_{BC} \approx \frac{e^2}{\hbar}\frac{1}{l_0}. \tag{13}$$

This term reaches saturation in region C, $\sigma_{\rm int} = \sigma_{BC}$, and the temperature dependence of $\sigma(T)$ is governed by the inelastic-scattering diffusion length

$$L_{in} = (l_0 l_{in})^{1/2}, \tag{14}$$

where l_{in} is the distance traversed by the electron during the time between the elastic collisions.

Let us consider, to be definite, the states for which the $T^{1/3}$ law is valid. The first term of the sum (12) can be neglected for them, and the second term in region C is equal to a constant

$$\sigma_0 \approx \sigma_{BC} \propto l_0^{-1}, \tag{15}$$

while the third term is

$$\sigma(T) - \sigma_0 = (e^2/\hbar) L_{in}^{-1} \propto l_0^{-1/2}. \tag{16}$$

The experimentally observed relation

$$\beta \propto \sigma_0^{1/2} \tag{17}$$

(see Fig. 5) is a very convincing justification of such an interpretation. Moreover, it follows from Fig. 5 that relation (17) ceases to be valid as soon as we go outside the region where $\sigma(0)$ can be disregarded [curves (*12*) and (*19*)].

It follows from the foregoing that $l_{in} = \eta T^{-2}$ and that the proportionality coefficient η is independent of the state of the sample. These two statements call for a discussion.

4.6. Nature of inelastic interaction in the linear region

The T^2 law in the dependence of the collision frequency on temperature is usually associated with electron–electron collisions. However, as was established above, the interelectron interaction determines the second term in the sum (12). It would be very strange if it were also to determine the second term, and furthermore in an entirely different manner.

There is in essence also objection to such a possibility. The coefficient η of T^2 in the probability of the interelectron collisions is proportional to the density of states g_F which, as follows from Fig. 2 and Eq. (5), varies greatly from one state i of the sample to another (see Sec. 4.3). Yet the explanation, inherent in Eqs. (13)–(16), of the experimental fact (17) requires that the coefficient η be one and the same for different states of the sample.

The possibility that the T^2 law in the probability of scattering in metals may be due also to inelastic interaction with impurities and other structural inhomogeneities has already been discussed long ago. We have in mind incoherent inelastic scattering of electrons by random distributed regions of perturbation of a phonon field. This inelastic-scattering mechanism is theoretically justified in Ref. 25, and experimental evidence in its favor is contained in Refs. 26–28 (Ref. 28 has a detailed bibliography).

If such a scattering mechanism exists in the Boltzmann-conductivity region, it should be manifested also in the quantum conductivity. One of the method of distinguishing electronic and inelastic scatterings in metals was the establishment that the coefficient η is proportional to the number of impurities, i.e., to the residual resistivity σ_0^{-1}. There can be no such correlation in quantum conductivity.

The constancy of η and l_{in} shows that the randomness of the dynamic-perturbation fields near the transition point changes little. This agrees with the statement made above, that in our case the principal role is played near the transition not by an increase of the structural disorder but by an electron wave function restructuring that leads to a decrease of the density of states.

5. CONCLUSION

The results of the measurements of $\sigma(T)$ in the metastable alloy $Cd_{43}Sb_{57}$ decaying to an amorphous state can be summarized as follows.

The quantum (scaling) conductivity of material in states close to a metal–insulator transition can be described by a sum of three terms:

$$\sigma(T)=\frac{e^2}{\hbar}\left(\frac{c_1}{\xi}+\frac{c_2}{L_{int}}+\frac{c_3}{L_{in}}\right), \qquad (12')$$

the first of which describes the conductivity at absolute zero, and the two other determine its temperature dependence. The lengths ξ, L_{int}, and L_{in} in the denominators of (12′) can vary in the interval from ∞ to l_0. It was possible to observe three different power-law variations of the scaling conductivity with temperature. The relation $\sigma \propto T^{1/3}$ was observed at $T \leqslant 40$ K for the immediate vicinity of the transition and was transformed into $\sigma \propto T^{1/2}$ for transitions in either the metallic or the insulating region. Both relations are governed by interelectron interaction in the critical region. The law realized depends on the relation between the lengths ξ and L_{int}.

Above 40 K the second term of (12′) saturates, and further increase of the conductivity with temperature is due to the third term, which is controlled by the elastic scattering of the carriers by structural inhomogeneities.

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