Pecularities of luminescence spectrum in quantum wells in a magnetic field for integer filling factors

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In the recent experiments by B. B. Goldberg *et al.* [Phys. Rev. Lett. **65**, 641 (1990); The Application of High Magnetic Fields in Semiconductor Physics (Abstracts of the Int. Conf., Wursburg, 1990), p. 49] peculiarities of luminescent lines in quantum wells have been observed in the vicinity of integer and fractional filling factors $\nu = 1, \frac{1}{3}$, and $\frac{2}{3}$. In the present study the onset of peculiarities for $\nu = 1$ is accounted for by a change in the ground state of a two-dimensional electron gas in the vicinity of a valence hole.

In the experiment described in Ref. 1 a small number of holes in the valence (v) band have been created by illumination, and their recombination with electrons in the upper and lower spin sublevels in the conduction (c) band has been studied. Qualitatively the peculiarities in the luminescence line can be accounted for by a change in the ground state of the system near $\nu = 1$. For $\nu = 1 - \varepsilon$. $0 \leqslant \varepsilon \ll 1$, there is a small number of Fermi holes in the lower spin sublevel, which are repelled from the valence hole and affect the luminescence weakly. For $\nu = 1 + \varepsilon$ the situation changes drastically, and a small number of electrons in the upper spin level of the c-band form bound states (complexes) with valence holes. In a symmetrical quantum well, when the Coulomb interaction between electrons in the c-band is the same as between carriers in the c- and v-bands, a change in the ground state does not give rise to peculiarities in the luminescence line (this follows from the Kohn theorem,[3] which, in a symmetrical case, is also valid for an interband exciton[4]). In experiment, a tilt of the quantum-well bottom always leads to spatial separation of the carriers belonging to different bands in the direction perpendicular to heterojunction planes (the z-axis in Fig. 1). The symmetry between the c–c- and c–v-interactions therefore becomes broken, and a blue shift arises jumpwise for $\nu = 1$. The bound-state structure has been found to depend on the distance between the carriers. In narrow wells a valence hole binds two c-electrons, while in wide ones only one. Numerical solution of the Schrödinger equation shows that the energies of the two bound-state types become equal for the well width $b \sim 1.5 l_H$ [$l_H = (c\hbar/eH)^{1/2}$ is the magnetic length and H is the magnetic field].

GROUND STATES IN THE VICINITY OF $\nu = 1$

Consider the simplest model of a real quantum well, in which the asymmetry is set by different wave functions of size quantization for carriers in the c- and v-bands. In the calculations we use simple Gaussian functions

$$\xi_c = \exp\left(-\frac{z^2}{2a^2}\right)$$

for c-band carriers and

$$\xi_v = \exp\left(-\frac{(z-b)^2}{2a^2}\right)$$

for the v-band (see Fig. 1).

Let the spectra of two-dimensional carriers in the well be parabolic, the temperature zero, and the Landau level mixing negligible. Three levels are important for us. Let a_1^+ and a_2^+ be the electron creation operators in the upper and lower spin sublevels respectively, and a_3^+ the electron creation operator in the zeroth Landau level of the c-band. Furthermore, let $|0\rangle$ be the ground state of the system for $\nu = 1$:

$$a_1|0\rangle = a_2^+|0\rangle = a_3^+|0\rangle = 0.$$

In the radial gauge we choose the wave functions of two-dimensional electrons in the zeroth Landau level in the form

$$\Psi_{i,n}(r, \varphi, z) = \frac{1}{(2^{n+1}\pi n!)^{1/2}} r^n \exp(in\varphi) \times \exp(-r^2/4)\xi_i(z), \quad n \geqslant 0. \tag{1}$$

Here ζ_1, $\zeta_2 = \zeta_c$, and $\zeta_3 = \zeta_v$. The projection of the Coulomb-interaction Hamiltonian on this level is

$$H_{int} = \frac{1}{2} \sum_{i,j=1}^{3} \sum_{n_1, n_2, n_1', n_2' \geqslant 0} V_{(i,j)n_1,n_2}^{n_1',n_2'} a_{i,n_1'}^{+} a_{j,n_2'}^{+} a_{j,n_2} a_{i,n_1}. \tag{2}$$

Here

$$V_{(i,j)n_1,n_2}^{n_1',n_2'} = \langle \Psi_{i,n_1'}, \Psi_{j,n_2'} | V | \Psi_{j,n_2}, \Psi_{i,n_1} \rangle \tag{3}$$

is the matrix element of the Coulomb interaction.

There are two candidates for the ground state of the system at $\nu = 1 - \varepsilon$: it is either a hole and a fully filled lower spin sublevel $\chi_3 = a_3|0\rangle$ or a hole screened by a spin exciton

$$\chi_{1,2,3} = \sum_{n_1,n_2,n_3} A_{n_1,n_2,n_3}^{M} \delta_{n_2+n_3-n_1,M} a_{1,n_1}^{+} a_{2,n_2} a_{3,n_3} |0\rangle. \tag{4}$$

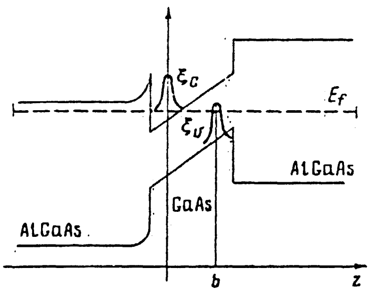

FIG. 1. Wave functions of size quantization in a quantum well for c- and v-bands.

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In Eq. (4) it is taken into account that the projection M of the angular momentum on the magnetic field is conserved. The energy $E_{1,2,3}$ of the ground state is found from the Schrödinger equation

$$\sum_{n_1,n_2,n_3}(\delta_{n_1,n_1'}V^{n_2,n_3}_{(2,3)n_2',n_3'}-\delta_{n_2,n_2'}V^{n_1',n_3}_{(1,3)n_1,n_3'}-\delta_{n_3,n_3'}V^{n_1',n_2}_{(1,2)n_1,n_2'})A_{n_1,n_2,n_3}$$
$$=(E_{1,2,3}-e_0-E_{gap})A_{n_1',n_2',n_3'}. \qquad (5)$$

Here

$$e_0=\sum_{n_1}V^{n_1,n}_{(2,2)n,n_1}$$

is the exchange energy of a hole in the fully filled Landau level and E_{gap} is the band gap width.

We diagonalize the operator in the left-hand side of Eq. (5) numerically. To find the lower level, it is sufficient to take into account a finite set of one-electron states $a^+_{i,n}|0\rangle$ with $n\leqslant N$, since the wave function of the complex decreases rapidly (for example, for $N=15$ and $N=20$ the corresponding energies differ by 4%).

The ground state is found to be the one with $M=1$ and its energy equals

$$E_{1,2,3}=-0{,}05\frac{e^2}{\varepsilon_0 l_H}+g\mu_b H+E_{gap},\quad a=b=0,$$

$$E_{1,2,3}=-0{,}016\frac{e^2}{\varepsilon_0 l_H}+g\mu_b H+E_{gap},\quad a=b=l_H.$$

Here ε_0 is the dielectric constant of GaAs, μ_b is the Bohr magneton, and g is the g-factor of GaAs. Since the energy of the unscreened valence hole is $E_3=E_{\text{gap}}$, the type of the ground state for $\nu=1-\varepsilon$ is determined by the ratio of the spin splitting and Coulomb energy. In the GaAs-based structures the spin splitting is small. For a characteristic field $H=10$ T we have

$$\frac{e^2}{\varepsilon_0 l_H}=14\text{ MeV},\quad g\mu_b H=0{,}5\text{ MeV}.$$

In a narrow well ($a=b=0$) for $H=10$ T the state $\chi_{1,2,3}$ is lower by 0.2 meV, and for $a\sim b\sim l_H$ screening is not advantageous.

For $\nu=1+\varepsilon$ the possible ground states are either an interband exciton with zero momentum

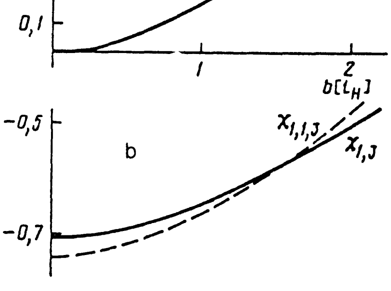

FIG. 2. a—Luminescence line shift. b—Energies $\chi_{1,1,3}$ and $\chi_{1,3}$ of complexes as functions of the well width ($a=l_H$).

$$\chi_{1,3}=\sum_n a^+_{1,n}a_{3,n}|0\rangle \qquad (6)$$

or a $\chi_{1,1,3}$ complex containing two c-electrons in the upper spin sublevel and a valence hole

$$\chi_{1,1,3}=\sum A'_{n_1,n_2,n_3}a^+_{1,n_1}a^+_{1,n_2}a_{3,n_3}|0\rangle.$$

Here and below E_{ij} denotes the energy of a complex consisting of carriers in the levels $i,j,\cdots$. The energies have the following form

$$E_{1,1,3}=-1{,}30\frac{e^2}{\varepsilon_0 l_H}+2g\mu_b H+E_{gap},$$

$$E_{1,3}=-1{,}25\frac{e^2}{\varepsilon_0 l_H}+2g\mu_b H+E_{gap},\quad a=b=0,$$

$$E_{1,1,3}=-0{,}66\frac{e^2}{\varepsilon_0 l_H}+2g\mu_b H+E_{gap},$$

$$E_{1,3}=-0{,}64\frac{e^2}{\varepsilon_0 l_H}+g\mu_b H+E_{gap},\quad a=b=l_H.$$

In Fig. 2 the calculated energies $E_{1,3}$ and $E_{1,1,3}$ are plotted as functions of the well width b. It is interesting that for $b>1.5\,l_H$ the lowest in energy is the interband exciton. It is easy to estimate the binding energy $\Delta E(L)$ of the interband exciton and an electron when the distance L between them is large, $L\gg l_H$, b:

$$\Delta E(L)\sim\gamma p^2-\frac{p}{L^2}+\frac{b}{L^3}, \qquad (7)$$

where p is the exciton momentum.[5,6] In (7) the energy is in units of $e^2/\varepsilon_0 l_H$, and the length in units of l_H. Varying over p, we find

$$\Delta E(L)\sim-\frac{1}{2\gamma L^4}+\frac{b}{L^3}.$$

It is seen that for $b\neq0$ we really have repulsion (at least for large distances).

PECULIARITIES OF THE LUMINESCENT LINE

Consider the recombination of a hole with an electron in the lower spin sublevel of the c-band. Selection rules allow transitions without a change in the angular-momentum projection on magnetic field. Thus, the final state for the transition from $\chi_{1,2,3}$ is

$$\chi_{1,2,2}=\sum A''_{n_1,n_2,n_3}a^+_{1,n_1}a_{2,n_2}a_{2,n_3}.$$

In the symmetrical case (for $b=0$). owing to the electron-hole symmetry of the Hamiltonian (2), the Schrödinger equations for initial and final states differ only by a general energy shift. The transitions are only to the ground state

$$E_{1,2,3}-E_{1,2,2}=E_3-E_2=E_{1,1,3}-E_{1,1,2}=E_{1,3}-E_{1,2}=E_{gap}-e_0 \qquad (8)$$

and no peculiarities in luminescence arise.

In the asymmetrical case ($b\neq0$) the electron-hole symmetry is broken and the wave functions of the ground states of the initial and final complexes becomes different. Numberical analysis shows that, nevertheless, for $b\leqslant 2l_H$ we have predominantly transitions into the ground state. Their respective energies are

$$E_3-E_2=E_{gap}-e_0,$$

$$E_{1,1,3}-E_{1,1,2}=E_{gap}-e_0-0{,}09\frac{e^2}{\varepsilon_0 l_H},$$

$$E_{1,3}-E_{1,2}=E_{gap}-e_0-0{,}06\frac{e^2}{\varepsilon_0 l_H}.$$

As a result, for $\nu=1$ we observe a blue shift (a change $\chi_3\to\chi_{1,1,3}$ in the ground state) of order $\sim 0.09\, e^2/\varepsilon_0\, l_H$ (1.26 meV for 10 T). In the wells with $d>1.5\, l_H$ the ground state for $\varepsilon>0$ is an interband exciton, therefore the shift is smaller (Fig. 2).

Experimental results are described in Refs. 1 and 2. The observed blue shifts for $\nu=1$ have an amplitude of order 1 meV which, if we allow for the approximate form of the size quantization functions, fully agrees with our estimates.

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