Quasi-one-dimensional electron channel in a magnetic field

V. B. Shikin

Institute of Solid State Physics, Russian Academy of Sciences
(Submitted 18 October 1991)
Zh. Eksp. Teor. Fiz. **101**, 1599–1611 (May 1992)

The advantages and shortcomings of the parabolic approximation for the description of the properties of quasi-one-domain electron channels that conduct in the ballistic regime are discussed. It is noted that this approximation agrees qualitatively with the known experiments both in the absence and in the presence of a magnetic field perpendicular to the plane of the channel.

The existing theory of quasi-one-dimensional electron channels in semiconductors is being developed at present mainly for two limiting cases, each with its own advantages and shortcomings. The point is that the retaining potential that governs the parameters of the channel and whose form is as a rule unknown is approximated by two very simple asymptotes, rectangular or parabolic.

The rectangular approximation is used systematically in connection with experimental studies (see Ref. 1 and the citations in Ref. 2). It is also very suitable for a description of the behavior of the electron as a whole when it passes ballistically through a constriction of the potential relief, since it makes it possible to determine (Levinson,[3] Glazman *et al.*[4]) the degree if "smearing" of the e^2/h steps produced in the ballistic conductivity in the one-dimensional variant of the theory at $T=0$ (Landauer[5]). The rectangular approximation raises difficulties in view of the uncertainty of the connection between the channel characteristics—its width w in the saddle-point region of the retaining potential, and the number l_* of the filled subbands, as well as on the saddle itself with the governing potential V_g and the magnetic field H normal to the plane of the channel. The suggestions made by Van Wees *et al.*[6] jointly with the results of Kosevich and Lifshitz[7] yield empirical functions $w(V_g)$ and $l_*(V_g)$ which are difficult to interpret (details follow). Nor is it clear how to introduce in this approximation the Coulomb effects, which play a rather significant role.

A simple adiabatic approximation, first formulated for the channel problem by Berggren *et al.*,[8] fits much better in a self-consistent scheme that takes Coulomb effects into account. Assuming that the potential $V_e(x)$, which quantizes the single-electron motion (the x direction is perpendicular to the channel axis y) is parabolic

$$V_e(x)=V_e^0+\frac{1}{2}k_e x^2+\ldots. \tag{1}$$

it is easy to find the connection between l_* and ω_e or ω_c, where $\omega_e^2=k_e/m_*$ is the transverse frequency indicative of the degree of influence of the quantizing potential $V_e(1)$ on the electron spectrum, ω_c is the cyclotron frequency,

$$l_*^{1/2}=\frac{3\pi}{4}\left(\frac{\hbar}{2m_*}\right)^{1/2}\frac{\omega_e}{\tilde{\omega}_c^{1/2}}N_e,\quad \tilde{\omega}_c^2=\omega_e^2+\omega_c^2,\quad \omega_c=\frac{eH}{m_*c}. \tag{2}$$

Here N_e is the total number of electrons per unit length of the channel and m_* is the electron effective mass.

Knowing $l_*(H)$ from experiment (the Shubnikov–de Haas effect data) we can, fitting this dependence to Eq. (2), determine the parameters $\omega_e(V_g)$ and $N_e(V_g)$ under the condition that these parameters are independent of H. Such a program was implemented in Ref. 8 as well as in Refs. 9 and 10.

Relation (2) is useful also for the interpretation of the $l_*(V_g)$ dependence at different values of H. Experiments[6,11] with these data were reduced in the rectangular approximations. In the parabolic limit it is natural to use Eq. (2) rewritten in the form

$$l_*(H)=l_*^0\Big/\left(1+\frac{\omega_c^2}{\omega_e^2}\right),$$
$$l_*^0=\left[\frac{3\pi}{4}\left(\frac{\hbar}{2m_*}\right)^{1/2}\frac{N_e}{\omega_e^{1/2}}\right]^{2/3}. \tag{3}$$

where l_*^0 is the $l_*(V_g)$ dependence with $H=0$. This possibility will be discussed in greater detail below. We note here only that even Eq. (3) accounts qualitatively for the behavior of $l_*(H)/l_*^0$ observed in Ref. 6. In fact, the frequency ω_e decreases with increase of $V_g-V_g^*$. At a finite value of ω_c the ratio $l_*(H)/l_*^0$ should therefore decrease with increase of $(V_g-V_g^*)$. The effect should become stronger with increase of ω_c, as is indeed observed in experiment.[6] The potential V_g^* is the critical value of V_g at which the channel becomes empty.

Just as in the rectangular approximation, the parabolic approximation fails to answer some rather important questions. First, the Coulomb-interaction energy $V_c\approx e^2N_e/\varkappa$, where $\varkappa$ is the dielectric constant and N_e is taken from experimental data,[6,8–10] turns out to be substantially higher (by 3–5 times) than the Fermi energy $\varepsilon_F\approx l_*\hbar\omega_e$ determined in the same experiments. In other words, the potential $V_e(x)$ [Eq. (1)], which quantizes the electron motion along x, is incapable of retaining the electrons within the width w of the electron channel, since the Coulomb interaction should cause this channel to disintegrate. Nor is this approximation sufficient to explain the steps in the plot of the channel conductivity σ against the control voltage V_g. The effect of the magnetic field on the channel width and on N_e is likewise unclear. If the theory contains only the frequency and the magnetic field can take on values with $\omega_c\gg\omega_e$, such an influence may turn out to be quite noticeable. In this case the reduction of the experimental data of Refs. 8–10 with the aid of Eqs. (2) and (3) becomes meaningless.

To answer the above questions we must introduce into the theory Coulomb effects. An implementation of such a program in the absence of a magnetic field was described by the author in Refs. 12 and 13. It is shown here that under the

 0038-5646/92/050852-07$05.00

conditions $a_B/w \ll 1$ (a_B is the Bohr radius) the channel parameters, namely its width w and the number N_e of the ions, are determined mainly by the retaining potential $V(x)$ which is of electrostatic origin. As for the quantizing potential $V_e(x)$ (1), it is much smaller than $V(x)$ and exists only because the proper field of the electrons in the channel are not screened completely by the quantum effects of the external potential $V(x)$. We assume in the present paper a generalization of the results of Refs. 12 and 13 to $H \neq 0$. In the first two parts of the paper we describe qualitatively the behavior of the main characteristics N_e, l_*, and ω_e of the channel as functions of V_g and **H**. The third section verifies a number of equations used in the first two sections.

1. BEHAVIOR OF $l_*(H)$ IN THE PARABOLIC APPROXIMATION

It is natural to begin the discussion of the capabilities of the parabolic approximation with an analysis of rather general consequences that should follow for the behavior of a "parabolic" channel. One of them is the statement that, given the potential V_g, the value of ω_e remains constant in a wide range of H. The corresponding data for $\omega_e(H)$ are shown in Fig. 1. Obviously, the requirement $\omega_e = \text{const}$ at a fixed value of V_g is not very exactly satisfied, especially at low values of V_g. This, incidentally, was to be expected, since the parabolic approximation, as will be shown below, is a rather arbitrary concept. It should be taken as the simplest approximation that permits a reasonable reconciliation of as many experimental facts as possible for electron channels of width w noticeably smaller than the dimension $2d$ of the "window" in the controlling electrode.

Speaking of data on the order of the results of Eqs. (2) and (3), the parabolic approximation is not much better than the rectangular one of Refs. 1 and 2, which also leads to a definite connection between w and V_g. The difference is that the frequency ω_e in (2), as well as other characteristics of the channel, such as its width w, the number l_* of the filled subbands, and the total number N_e of electrons per unit length lead here to natural definitions in terms of V_g and the nominal width $2d$ of the cut in the control electrode.

For the model of the quasi-one-dimensional electron channel shown in Fig. 2 and investigated in detail for a zero magnetic field in Ref. 14, the necessary definitions supplementing (3) are

$$N_e = 2dn_0\left[E\left(\frac{w}{d}\right) - q^2K\left(\frac{w}{d}\right)\right], \tag{4}$$

$$\varkappa V_g = 2\pi e n_0 d\left[E(q) - \frac{w^2}{d^2}K(q)\right], \quad q^2 = \frac{d^2-w^2}{d^2}, \tag{5}$$

$$\frac{2^{1/2}}{\pi^{1/2}}\int_0^1 w n_0^{1/2}\left(\frac{1-\xi^2}{d^2/w^2-\xi^2}\right)^{1/4} d\xi = \frac{k_e w^2}{2\hbar\omega_e}, \quad \omega_e^2 = \frac{k_e}{m}. \tag{6}$$

Here $\varkappa$ is the dielectric constant of the medium in which the quasi-one-dimensional channel is immersed, n_0 is the effective two-dimensional density of the domains and neutralizes the electron charge at $V_g = 0$(it is assumed that the plane filled with donors and the electron plane are not spatially separated), $K(x)$ and $E(x)$ are the corresponding elliptic integrals, the definitions (4) and (5) express w and N_e in terms of n_0, d, and V_g in the electrostatic approximation. As for the definition (6), it relates the curvature k_e from (1)

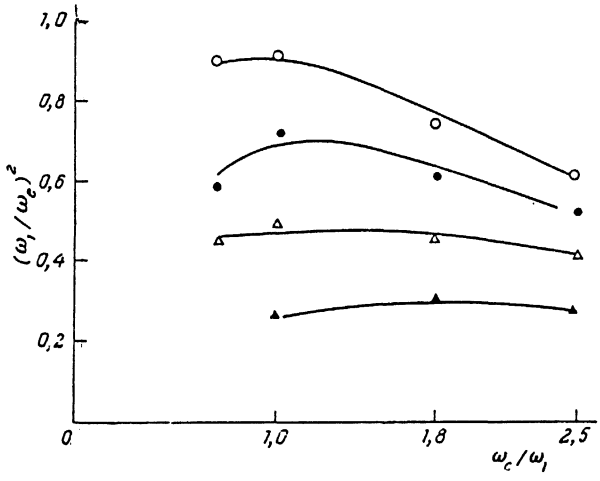

FIG. 1. Plot of $\omega_e(H)$, according to the data of Ref. 6, for different values of V_g: $V_g = 1$ V (○), 1.2 V (●), 1.4 V (△), 1.6 V (▲). The lines join points with equal values of V_g. A tendency of ω_e to be independent of H with increase of V_g can be seen. The frequencies ω_e and ω_c are referred to the cyclotron frequency ω_c at $H = 1$ T. This frequency is designated ω_1.

with the nominal parameters V_g and d of the problem in the Thomas–Fermi approximation. More detailed comments on this relation will follow Eqs. (9) and (38).

The use of Eqs. (4), (5), and (6) jointly with (3) presupposes that the parameters N_e and w are not very sensitive to the magnetic field. A proof of this statement is quite complicated, and there is still no rigorous one (see Sec. 3 below for details). Experimental considerations, however, favor this assumption. First, it follows from the data of Ref. 6 that the critical value V_g^* at which the channel becomes empty depends little on the magnetic field. Second, the different possible modifications of the quasi-one-dimensional channels[8,9] also demonstrate the independence of the electron-channel geometry on the magnetic field H. These facts are the basis for the assumption that w and N_e are insensitive to H.

In Refs. 12 and 13 we have introduced the concept of a retaining potential $V(x)$ that shapes for the most part the electron channel. This concept can also be easily used in a specific case (Fig. 2). Using the condition that an electrostatic equilibrium obtain in the electron-channel region, we have

$$V'(x) + \frac{2e^2}{\varkappa}\int_{-w}^{+w} G^1(x-s)n(s)\,ds = 0, \quad -w \leqslant x \leqslant +w. \tag{7}$$

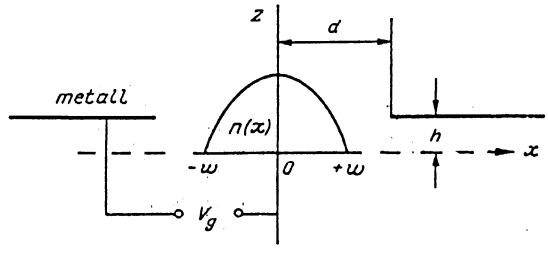

FIG. 2. Diagram of quasi-one-dimensional electron channel used in the text.

Here $n(x)$ is the electron-density distribution in the channel, corresponding to the solution (4)–(6), and $G(x)$ is the Green's function of the electrostatic problem. In the limit when $w \gg d$ the definitions of $G'(x)$ and $n(x)$ simplest to interpret are

$$G'(x) \approx x^{-1}, \quad n(x) \approx \frac{n_0}{d}(w^2-x^2)^{1/2}.$$

The potential $V(x)$ in (7) is then parabolic

$$V(x) \approx V_0 + \frac{1}{2}kx^2, \quad k = \frac{2\pi e n_0}{\varkappa d}, \quad eV_g = V_0 + \frac{e^2 N_e}{\varkappa}\ln\frac{d}{w} \tag{7a}$$

to the extent that $w \ll d$.

In terms odd $V(x)$ as given by (7) and (7a), the single-electron potential $V_e(x)$ is defined as

$$V_e(x) = V(x) + e\varphi(x), \quad -w \leqslant x \leqslant +w, \tag{8}$$

where $\varphi(x)$ is the potential of the electrons that fill the channel.

In the Thomas–Fermi approximation, when the equilibrium condition includes the energy of the zero-point oscillations

$$V(x) + e\varphi(x) + \frac{\pi\hbar^2}{m_*}n(x) = \mu, \tag{9}$$

the potential $V_e(x)$ turns out to be

$$V_e(x) = \mu - \frac{\pi\hbar^2}{m_*}n(x). \tag{9a}$$

Here μ is the position of the chemical potential.

When $n(x)$ is allowed for, this potential is not parabolic. The representation of $V_e(x)$ by expansion (1) on the interval $-w \leqslant x \leqslant +w$ can therefore not be proved in any limiting case [in contrast to the potential $V(x)$ which has a parabolic asymptote (7a) in the limit $w \ll d$]. Its use, and particularly the definition (6), must therefore be regarded as a variational principle of sorts, which permits formulation of qualitative analytic statements during the final stage of the computations.

For future convenience, we describe here the asymptotic behavior of the solution (4), (6) in the region $w \ll d$:

$$k_e = 1{,}24\frac{a_B}{w}k, \quad a_B = \frac{\varkappa\hbar^2}{m_* e^2}, \quad a_B \ll w,$$

$$l_* = 2{,}23(N_e w)^{1/2}, \quad N_e \approx \frac{\pi}{2}\frac{n_0}{d}w^2,$$

$$\frac{w^2}{d^2} \approx \frac{4\delta}{(\ln(8/\delta)+1)}, \quad \delta = 1 - \frac{V_g}{V_g^*}, \quad V_g^* = \frac{2\pi|e|n_0 d}{\varkappa}. \tag{10}$$

Here k is given by (7a).

Obviously, k_e is strongly renormalized (to the extent that $a_B/w \ll 1$) compared with the curvature k.

For arbitrary w/d, plots of k_e and w/d against $\tilde{V}_g = V_g/V_g^*$, which follow from (5) and (6), are shown in Fig. 3. It is noteworthy that $w/d \to 1$ when $\tilde{V}_g \to 0$, and not at finite values of $\tilde{V}_g$, as is actually the case, in view of the neglect of the gap h (see Fig. 2) between the metal and the

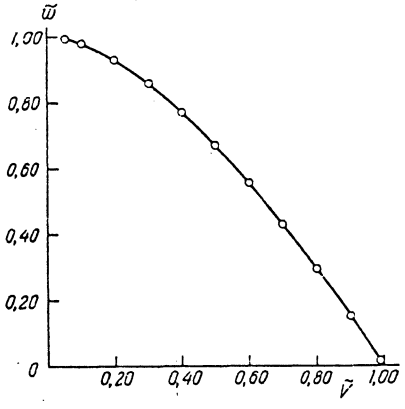

FIG. 3. Dimensionless channel width $\tilde{w} = w/d$ from (5) as a function of the dimensionless control voltage $\tilde{V} = V_g/V_g^*$ defined by Eq. (10).

$2d$-system. The divergence of $k_e(V_g)$ as $\tilde{V}_g \to 1$ has likewise no physical meaning. The theory discussed is valid in the region $a_B/w < 1$, and the divergence sets in at $a_B/w > 1$ (Fig. 4).

Using the definitions (4)–(6) and the data of Fig. 3 we can plot the function $\omega_e(V_g)$ and consequently, taking (3) into account, determine the $l_*(H)$ dependence for the conditions of Ref. 6. The corresponding values of $\omega_e(V_g)$ are shown in Fig. 5, as well as the experimental $\omega_e(V_g)$ behavior that follows from the data of Ref. 6, using Eq. (3). Obviously, the theory is far from agreement with experiment,[6] although it reflects well the tendencies. Incidentally, the question of the numbers is encountered already during the early stage of the computation. Thus, the critical value of V_g^* from (10), calculated using the experimental parameters of Ref. 6 ($d = 250$ nm, $m_* = 0.067m_e$, $\varkappa = 12$, and $n_0 = 3.56 \cdot 10^{11}$ cm^{-2}) yields $V_g^* = 0.72$ V, which 2–3 times smaller the observed[6] $V_g^* > 2$ V. It appears that the numbers are influenced by the non-one-dimensionality of the channel.[6] The same determination of V_g^* for channels that are more "one-dimensional" (Refs. 8, 9) agrees well with the observations.

A few words on the rectangular approximation for the retaining potential. As noted above, a difficulty arises in this case in the determination of the connection between w and V_g. It is caused formally by the general relation between $V(x)$ and V_g:

$$\frac{1}{\pi}\int_{-w}^{+w}\frac{\partial V}{\partial s}\frac{s\,ds}{(w^2-s^2)^{1/2}} = \frac{e^2}{\varkappa}N_e. \tag{11}$$

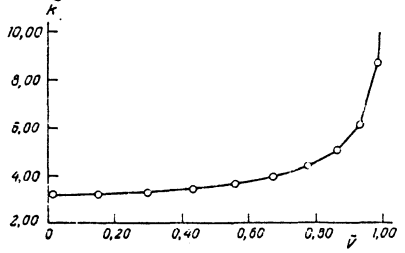

FIG. 4. Dimensionless curvature $\tilde{k} = k_e/k_0$ from Eq. (6) for a single-electron potential $V_e(x)$ as a function of $\tilde{V} = V_g/V_g^*$. The quantity k_0 is defined as $k_0 = n_0\hbar^2/m_* d^2$.

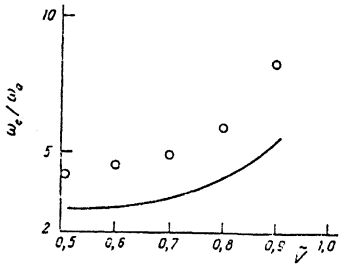

FIG. 5. Behavior of $\omega_e(V_g)$. Circles—$\omega_e(V_g)$ from Ref. 6 for a magnetic field $H = 2.5$ T. Solid line—calculated using the definition $\omega_e^2 = k_e/m_*$ and k_e from Fig. 4. The frequency ω_e refers to $\omega_0^2 = k_0/m_*$, where k_0 has the same meaning as in Fig. 4.

If the potential $V(x)$ is rectangular, the condition (11) is meaningless, since

$$\partial V/\partial x| \underset{\pm w}{\longrightarrow} \infty,$$

and the integral (11) diverges.

Even less definite is the rectangular approximation for the single-electron potential $V_e(x)$. No acceptable scheme for its construction exists as yet if local neutrality is violated, a condition obtaining in the vicinity of the electron channel.

2. WIDTHS OF TRANSITION REGIONS BETWEEN THE $\sigma(V_g)$ PLATEAUX

The widths δl_* of the transition regions between neighboring plateaux on the $\sigma(V_g)$ plot and the behavior of these regions as functions of the magnetic field is a problem in itself. Experiment[6] offers clear evidence that δl_* increases with increase of the magnetic field strength. As to the theory, we have here at present two contradictory results. Glazman and Khaetskii,[15] using the general idea[5] that δl_* is finite as $T \to 0$ because of the presence of a smooth barrier that perturbs the electron motion along the channel, attempt to explain the observed behavior of $\delta l_*(H)$. They use a rectangular quantizing potential $V_e(x,y)$ in the x direction. Buttiker, likewise in the context of a model with a saddle point in the $V_e(x,y)$ relief, cites a result contradicting the observation, wherein the width δl_* decreases with increase of the magnetic field. The single-electron potential $V_e(x,y)$ in the vicinity of the saddle was approximated by the power law

$$V_e(xy) \approx V_e^0 + {}^1/_2 k_e x^2 - {}^1/_2 q_e y^2, \qquad (12)$$

after which the problem of passage of an electron through the saddle point can be solved exactly.

It is my opinion that the conclusions of Ref. 15, in the part pertaining to the behavior of $\delta l_*(H)$, are in error, owing to the artificial definition, in Ref. 15, of the number of occupied bands in the electron channels. Consequently neither the behavior of $\delta l_*(H)$ nor the definition of l_* correlates with other studies. For example, expressions (18) and (24) in Ref. 15 for the band occupation numbers for different limiting magnetic fields are entirely different from the analogous expressions of Ref. 6 in the same model and at the same limiting field values [see Eqs. (2) of Ref. 6].

We offer our point of view, confining ourselves at the present level to a discussion of Refs. 15 and 16 and with an aim at discerning the possible causes of the observed growth of $\delta l_*(H)$ in the framework of a theory using a smooth potential $V_e(xy)$. Our main idea is that there is no need at all to introduce a saddle point to explain the experiments of Ref. 6.

The conductivity of a quasi-one-dimensional plasma in the ballistic regime at a finite temperature is defined as[3]

$$\sigma = \frac{e^2}{2\pi\hbar} \sum_l \frac{1}{1+\exp\{(\varepsilon_l-\mu)/T\}}, \qquad (13)$$

where μ is the location of the Fermi level and ε_l the discrete part of the electron spectrum. If, as customarily (see, e.g., Ref. 16), σ is plotted as a function of μ, several deductions can be made: the widths of the transition regions decrease when the temperature is lowered and the magnetic field increased. Experiments, however, deal with the behavior of $\sigma(V_g)$. To explain this effect it is therefore necessary to know the explicit dependence of the electron spectrum on V_g at fixed H. Using the results above, we can implement this program and represent the conductivity σ (13) in the form

$$\sigma = \frac{e^2}{2\pi\hbar} \sum_l \frac{1}{1+\exp(\xi_l)}, \qquad \xi_l = \frac{\hbar\tilde{\omega}_c(l-l_*)}{T}. \qquad (14)$$

Here $\tilde{\omega}_c$, k_e, and l_* are given by Eqs. (3), (6), and (3), respectively.

The derivative $\partial\sigma/\sigma V_g$ at the points $l = l_*$ is equal to

$$\frac{\partial\sigma}{\partial V_g} = \frac{e^2}{8\pi\hbar} \sum \frac{\partial\xi_l}{\partial V_g},$$

$$\frac{\partial\xi_l}{\partial V_g} = \frac{\hbar}{T}\left\{\frac{l}{2m_*(\omega_c^2+\omega_e^2)^{1/2}}\frac{\partial k_e}{\partial V_g} - \frac{\partial}{\partial V_g}(\tilde{\omega}_c l_*)\right\}. \qquad (15)$$

According to (2), the combination $\tilde{\omega}_c l_*$ in the second term of the derivative $\partial\xi_l/\partial V_g$ in (15) is independent of H:

$$l_*\tilde{\omega}_c \sim (\omega_e N_e)^{1/2}.$$

The second term makes therefore no contribution to $\delta l_*(H)$. As for the first term of this derivative, it decreases with increase of the magnetic field. On the whole, therefore, the derivative $\partial\sigma/\partial V_g$ is a decreasing function of the magnetic field at the points $l = l_*$. In other words, an increase of the magnetic field extends the lengths of the transition regions δl_* between neighboring steps, as we had to prove.

3. ORIGIN OF DOUBLE PARABOLIC APPROXIMATION

The purpose of the present section is to justify (with a definite accuracy) a number of statements used above. We have in mind the connection between the retaining $V(x)$ and quantizing $V_e(x)$ potentials and the role of the magnetic field in the determination of the parameters of the quasi-one-dimensional electron channel.

A. We consider first a situation without a magnetic field. If $V(x)$ is the potential that governs the properties of an electron channel along the y axis, the initial system of equations for the equilibrium of the channel properties of the channel are

$$\frac{\hbar^2}{2m_*}\psi_l'' + \left[\varepsilon_l - \frac{p_y^2}{2m_*} - V(x) - e\varphi(x)\right]\psi_l = 0, \qquad (16)$$

$$\varphi(x) = \frac{2e}{\varkappa}\int_{-w}^{+w} n(s)\ln\frac{d}{|x-s|}\,ds, \qquad (17)$$

$$n(x)=\frac{g}{2\pi\hbar}\int_{-\infty}^{+\infty}dp_y\sum_l\frac{|\psi_l(x)|^2}{\exp\{(\varepsilon_l+p_y^2/2m_*-\mu)/T\}+1}, \quad (18)$$

$$\int_{-\infty}^{+\infty}|\psi_l(x)|^2\,dx=1, \qquad \int_{-\infty}^{+\infty}n(x)\,dx=N_e, \qquad g=2. \quad (19)$$

Here $\psi_l(x)$ is a single-electron wave function in the l th state, $\varphi(x)$ is the electric potential due to the Coulomb interaction of the electrons, $n(x)$ is the local density of the electrons, w is the width of the electron channel, μ is the position of the Fermi level, d is the length connected with the screening of the electric fields by metallic electrodes placed along the channel, p_y is the electron momentum in the y direction, and N_e is the total number of electrons per unit channel length.

Assume that the retaining potential $V(x)$ is in the main parabolic, meaning that it is real in the region $w \ll d$:

$$V(x)\approx V_0+{}^1/_2kx^2+\ldots \quad (20)$$

In the particular case of Fig. 2 this property is explicitly manifested (see Eq. 7a) and the pertinent commentaries). In this case it is convenient to represent the electron density $n(x)$ by a series of Chebyshev polynomials $U_{2m}(x)$:

$$n(x)=\left(1-\frac{x^2}{w^2}\right)^{1/2}\left[n_0U_0\left(\frac{x}{w}\right)+n_2U_2\left(\frac{x}{w}\right)+\ldots\right]. \quad (21)$$

$$wn_{2m}=\frac{2}{\pi}\int_{-w}^{+w}n(x)\,U_{2m}\left(\frac{x}{w}\right)dx. \quad (22)$$

Using (20)–(22) and the general properties of the polynomials $U_m(x)$, in particular

$$\int_{-1}^{+1}\frac{U_{l-1}(s)\,(1-s^2)^{1/2}}{s-x}\,ds=-\pi T_m(x), \quad (23)$$

we can restate the problem (16)–(19) in the form of the nonlinear-oscillator problem

$$\frac{\hbar^2}{2m_*}\psi_l''+\left(\varepsilon_l-V_e-\frac{p_y^2}{2m_*}-\frac{1}{2}k_e^2x^2-\frac{1}{4}qx^4+\ldots\right)\psi_l=0, \quad (24)$$

$$k_e=k-2\pi\frac{e^2}{\varkappa w}n_0+\frac{6\pi e^2}{\varkappa w}n_2+\ldots, \quad (25)$$

$$q=-8\pi\frac{e^2}{\varkappa w^3}n_2+\ldots, \quad (26)$$

$$V_e=V_0+2\frac{e^2N_e}{\varkappa}\ln\frac{d}{w}. \quad (27)$$

An additional advantage of the representation (24)–(27) is the presence here of a natural limiting transition to the classical equations defining equilibrium in the electron channel. In this case [see also Eq. (7)]

$$V(x)+e\varphi(x)=\mu, \quad (28)$$

$$n(x)=\frac{2N_e}{\pi w_0}\left(1-\frac{x^2}{w_0^2}\right)^{1/2}, \qquad w_0^2=\frac{4e^2N_e}{\varkappa k}, \quad (29)$$

$$V_0+\frac{2e^2N_e}{\varkappa}\ln\frac{d}{w}=\mu. \quad (30)$$

The solution (28)–(30) can be regarded as a zeroth approximation for the system (16)–(19) or for (24)–(27) if the number N_e is large enough

$$N_ew\gg 1. \quad (31)$$

In this approximation the effective curvature k_e (25) tends to zero.

Developing further the resultant perturbation theory in the parameter $a_B/w\ll 1$ or, equivalently, $k_e/k\ll 1$, we must put, in the zeroth approximation with respect to this parameter,

$$k_e=k-\frac{2\pi e^2n_0}{\varkappa w}\neq 0, \qquad q=0, \qquad n_0=\frac{2N_e}{\pi w}. \quad (32)$$

As a result,

$$n(x)=n_0\left(1-\frac{x^2}{w^2}\right)^{1/2}, \quad (33)$$

$$\varepsilon_l=\hbar\omega_e\left(l+\frac{1}{2}\right)+\frac{p_y^2}{2m_*}+V_e, \qquad \omega_e^2=k_e/m_*. \quad (34)$$

The definition (33) of $n(x)$ follows from (21) if $n_2=0$, and the ε_l spectrum (34) follows from (24) in the limit as $q\to 0$. This is a single-particle spectrum, and the effective potential $V_e(x)$, just as $V(x)$ of (20), is parabolic with a curvature k_e (32) different from k (20). Its value is determined together with the new definition of the electron-channel width w, which differs from the classical value w_0 (29).

To determine the channel width w we must reconcile representations (18), (19) of $n(x)$ with (21), (22). It is therefore necessary to calculate the moments n_{2m} with the corresponding Chebyshev polynomials $U_{2m}(x)$ and compare these moments with one another. The number of moments is determined by the accuracy with which the $n(x)$ series (21) is set up. The condition for the solvability of the resultant homogeneous system of algebraic equations with respect to the coefficients n_{2m} is that this system include a definition of the channel width w.

In the first approximation of interest to us we are dealing with one moment n_0 with the zeroth Chebyshev polynomial $U_0=1$. As a result,

$$\frac{\pi}{2}wn_0=N_e,$$

$$N_e=\frac{1}{\pi\hbar}\sum_l\int_{-\infty}^{+\infty}dp_y\exp\left(\frac{p_y^2/2m_*+l\hbar\omega_e-\delta\mu}{T}\right)+1\Bigg]^{-1}, \quad (35)$$

$$\delta\mu=\mu-V_e-\frac{1}{2}\hbar\omega_e.$$

Furthermore, recognizing that in the approximation (32) the wave functions $\psi_l(x)$ in the definition (18) of $n(x)$ are oscillatory we must use hereafter the relation between the number l of nodes of the wave function ψ_l and the width w_l of the region in which it oscillates

$$w_l^2=2\hbar l/m_*\omega_e. \quad (36)$$

In the limiting case $T\to 0$, the definition (35) of N_e reduces to

$$N_e=\frac{4(2m_*)^{1/2}}{3\pi\hbar^2\omega_e}\delta\mu^{3/2}, \qquad l_*=\frac{\delta\mu}{\hbar\omega_e}, \quad (37)$$

which coincides with N_e of (2) at $H=0$. Taking next into

account the relation (36) for $l = l_*$, so that $\delta_\mu = \frac{1}{2}k_e w^2$, we obtain from (37)

$$\frac{1}{2}kw^2 - \frac{2e^2N_e}{\varkappa}\left(1+\frac{3\pi}{4}\frac{a_B}{w}\right)=0, \qquad w=w_l|_{l=l_*}, \tag{38}$$

where a_B is the Bohr radius mentioned above.

The definition (38) of w generalizes the classical result for w to include the case $k_e \neq 0$. Obviously, Eq. (38) goes over into the definition (29) of w_0 if $k_e \to 0$.

The above results may create an impression that the series (21) for $n(x)$ converges with respect to the parameter a_B/w. One could speak then of grounds for the parabolic approximation (1) of $V_e(x)$. Actually this is not the case, and elaboration of the theory to take account of the anharmonicity shows that the coefficient n_2 is of the same order as n_0 relative to the parameter a_B/w, and one can speak only of a relative numerical smallness. Incidentally, the statement that the potential $V_e(x)$ is not parabolic has already been discussed above; see Eqs. (8) and (9) and the pertinent explanations. Comments on relation (6) are also called for. The definition of l_* used for relation (6) does not in itself require absence of a magnetic field in the parabolic approximation of the potential $V_e(x)$. In fact, the quasiclassical number of levels in the one-dimensional $V_e(x)$ well (9a) is given by

$$l_* = \frac{1}{\pi\hbar}(2m_*)^{1/2}\int_{-w}^{+w}[\mu - V_e(x)]^{1/2}\,dx$$

for an arbitrary $V_e(x)$ dependence. In this sense, the theory of a quasi-one-dimensional channel without a magnetic field can be developed without resorting to the parabolic approximation (1) of $V_e(x)$ for a one-dimensional potential. It becomes necessary, if the theory is to be analytically generalized, to include the case $H \neq 0$. To this end the correct quasiclassical number of levels in the $V_e(x)$ well (9a), with allowance for the definition

$$n(x) = n_0\left(\frac{w^2-x^2}{d^2-x^2}\right)^{1/2},$$

contained in the solution (4) and (5), becomes equal to the total number l_* of levels in the effective parabolic well, where this number is given by $l_* = \delta_\mu/\hbar\omega_e$. The end result of the above arguments is in fact Eq. (6).

B. In the absence of a magnetic field the spectrum of single-electron excitations has again the structure (34), viz.,

$$\varepsilon_l(p_y) = V_c + \hbar\tilde{\omega}_c\left(l+\frac{1}{2}\right)+\frac{p_y^2}{2M}, \tag{39}$$

$$\tilde{\omega}_c^2 = \frac{(k_c+k_e)}{m_*}, \qquad k_c = m_*\omega_c^2, \qquad \omega_c = \frac{eH}{m_*c},$$

$$k_c = \left(k - \frac{4e^2N_e}{\varkappa w^2}\right), \qquad M = m_*\frac{(k_c+k_e)}{k_c}. \tag{40}$$

Using this analogy, we have in lieu of (37)

$$N_e = \frac{4(2M)^{1/2}}{3\pi\hbar^2\tilde{\omega}_c}\delta\mu_H^{3/2} \equiv \frac{4(2m_*)^{1/2}}{2\pi\hbar^2\omega_c}\delta\mu_H^{1/2}, \tag{41}$$

$$\delta\mu_H = \mu - V_c - \frac{2e^2N_e}{\varkappa}\ln\frac{d}{w} - \frac{1}{2}\hbar\tilde{\omega}_c, \qquad l_* = \frac{\delta\mu_H}{\hbar\tilde{\omega}_c}. \tag{42}$$

The three relations in (41) and (42), which contain the four variables N_e, ω_e, l_*, and w, are insufficient for the solution of this system. We need an additional connection between these quantities to close the system of the definitions. This relation is an analog of Eq. (36) and stems from the following considerations. If the magnetic field is strong enough, so that all the electrons are on the Landau ground level, then the single-electron wave functions have no zeros as in the absence of a magnetic field. Nevertheless, a connection between the single-electron spectrum and the channel width w does exist and can be written in the form

$$w \approx \frac{cp_y^{max}}{eH}, \qquad \frac{(p_y^{max})^2}{2M} = \delta\mu_H, \tag{43}$$

where p_y^{max} is the maximum momentum of the electron as it moves along the y axis, and M is given by (40). Assuming next that $k_c \gg k_e$ we obtain in this limit just as for $H = 0$

$$\delta\mu_H = {}^1/_2 k_c w^2. \tag{44}$$

As a result we get for w from (41) and (44) a definition similar to (38). In other words, in two limiting cases, $H = 0$ and a strong magnetic field, the definitions of w coincide.

CONCLUSION

In summary, a systematic description has been presented for the available experimental facts on quasi-one-dimensional electron channels in the ballistic regime, in the case $H \neq 0$, using a doubly parabolic approximation. The presence of the parabolic approximation (7a) of $V(x)$ can be confirmed here in the limiting case $w/d \ll 1$. When it comes to simplifying the form of the potential $V_e(x)$ [Eq. (1)], this procedure is not consistent, so that the results of the developed theory are in the main qualitative. Nonetheless, this is the first scheme that makes it possible to relate with one another the observed quantities with allowance for the Coulomb effects that accompany the formation of the electron channel in systems with a divided control electrode.

Among the specific results, we note primarily a justification of the assumption that the width w of the electron channel and the number N_e are independent of the magnetic-field intensity [Eqs. (41)–(44) and the pertinent comments]. Using this statement, one can resort in the analysis of the experimental data to the definition (2) of l_* and to its modification (3). Together with Eqs. (4)–(6), which relate ω_e with V_g, we have a self-consistent set of definitions sufficient to describe the available experimental data. By way of example, Fig. 5 shows a comparison of the calculated ω_e and of the same frequency extracted from experimental data.[6] The qualitative agreement between the theory and the experiment is obvious.

A special question that arises in the discussion of the data of Ref. 6 concerns the behavior of the widths δl_* of the transition regions between the plateaux on the $\sigma(V_g)$ dependence. A mechanism not connected with the presence of a saddle point in the path for the production of this quasi-one-dimensional electron channel.

The theory can develop along the following lines. In the case of really quasi-one-dimensional channels, such as in Refs. 9 and 10, it is of interest to refine the solution of the system (16)–(19) in the sense of determining the further U_{2m} coefficients of the expansion (21) of $n(x)$. For systems exhibiting a stepwise behavior of the conductivity,[1,6] account must be taken of the finite length of the electron chan-

nel in the y direction. All these complication require the use of numerical methods.

[1] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson *et al.*, Phys. Rev. Lett. **60**, 848 (1988).
[2] C. W. J. Beenakker and H. van Houten, *Quantum Transport in Semiconducting Nanostructures*, Academic, 1991.
[3] I. B. Levinson, Pis'm Zh. Eksp. Teor. Fiz. **48**, 273 (1988) [JETP Lett. **48**, 301 (1988)].
[4] L. I. Glazman, G. B. Lesovik, D. E. Khmel'nitskii, and R. I. Shekhter, *ibid.* **48**, 218 (1988) [**48**, 238 (1988)].
[5] R. Landauer, Z. Phys. B **68**, 217 (1987).
[6] B. J. van Wees, L. P. Konwenhoven, H. van Houten *et al.*, Phys. Rev. B **38**, 3625 (1988).
[7] A. M. Kosevich and I. M. Lifshitz. Zh. Eksp. Teor. Fiz. **20**, 743 (1956).
[8] K. F. Berggren, T. J. Thornton, D. N. Newson, and M. Pepper, Phys. Rev. Lett. **57**, 1769 (1986).
[9] F. Brinkop, W. Hansen, J. Kothaus, and K. Bloog, Phys. Rev. B **37**, 6547 (1988).
[10] T. Demel, D. Heitmann, P. Grambow, and K. Ploog, Appl. Phys. Lett. **53**(22), 2176 (1988).
[11] D. A. Wharam, T. J. Thornton, R. Nesbury *et al.*, J. Phys. C **21**, 1209 (1988).
[12] V. B. Shikin, Pis'ma Zh. Eksp. Teor. Fiz. **50**, 188 (1989) [JETP Lett. **50**, 212 (1989)].
[13] V. B. Shikin, T. Demel', and D. T. Khaĭtman, Zh. Eksp. Teor. Fiz. **96**, 1406 (1989) [Sov. Phys. JETP **69**, 797 (1989).
[14] I. A. Larkin and V. B. Shikin, Phys. Lett. **A151**, 335 (1990).
[15] K. U. Glazman and A. V. Khaetskii, J. Phys. Cond. Matter **1**, 5005 (1989).
[16] M. Buttiker, Phys. Rev. B **47**, 7906 (1990).

Translated by J. G. Adashko