Effects of magnetostatic scattering fields on the dynamics and interaction of Bloch points

Yu. A. Kufaev and E. B. Sonin

A. F. Ioffe Physicotechnical Institute, Russian Academy of Sciences
(Submitted 27 May 1991)
Zh. Eksp. Toer. Fiz. **101**, 1550–1560 (May 1992)

Effects of long-range scattering fields on the magnetic structure around a Bloch point and its dynamic parameters are studied for films with a large quality factor. The interaction of two Bloch points is considered. This interaction is the sum of static, resulting in repulsion of Bloch points, and dynamic interaction, arising for moving Bloch points and described by interaction mass. An expression is derived for the frequency of free oscillations of two Bloch points as a function of external constant magnetic field which can be used alter the distance between the Bloch points.

1. INTRODUCTION

One of the topologically stable structural elements of a ferromagnet is a Bloch point (BP).[1] At present, dynamic properties of a BP, i.e., its mass and mobility, are being studied both theoretically[2] and experimentally.[3] The BP mobility μ for a large quality factor $Q \gg 1$ is given in Ref. 1, while its mass m has been found in Ref. 2.

In Ref. 3 the experimental value of the BP mobility for $Q \ll 1$ is smaller than the theoretical value given in Ref. 1. The discrepancy may be accounted for by the strong influence of magnetostatic scattering fields on the BP structure for $Q \ll 1$.

In the present study the effect of magnetostatic scattering fields on the BP magnetic structure and dynamics has been studied for $Q \gg 1$. The interaction of two BP has also been considered. The interaction of two BP can be presented as a sum of two parts: static and dynamic. Static interaction of two BP results in their repulsion. Their dynamic interaction arises when they begin to move due to the bending of the domain wall (DW) in the region around the BP. Dynamic interaction of two BP results in an interaction mass, since the magnitude of the DW bending is proportional to the velocity of the BP. The dynamic interaction can be of two types: (i) direct bending interaction and (ii) interaction of charges produced on the DW surface by its bending.

The interaction mass of two BP could be observed in an experiment in which the oscillations of two BP induced by an alternating external magnetic field are observed. This experiment is discussed at the end of the paper.

2. EFFECT OF LONG-RANGE SCATTERING FIELDS ON THE BP STRUCTURE

The BP structure is shown in Fig. 1. Here the DW is in the xz plane, the Bloch line (BL) is along the z axis, and the BP is at the origin. The magnetic scattering energy has the form

$$W_m = -\frac{1}{2}\iint \frac{\nabla \mathbf{M}(\mathbf{r})\, \nabla \mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}\, d^3\mathbf{r}'. \qquad (1)$$

The expression (1) can be reduced to a more convenient one, since the magnetization $\mathbf{M}$ changes most rapidly along the y axis, which is perpendicular to the DW:

$$W_m = 2\pi \int M_y^2\, d^3r - \frac{1}{2}\iint \frac{(\partial M_y(\mathbf{r})/\partial x)\,(\partial M_y(\mathbf{r}')/\partial x)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}\, d^3\mathbf{r}'$$
$$-\frac{1}{2}\iint \frac{(\partial M_y(\mathbf{r})/\partial z)\,(\partial M_y(\mathbf{r}')/\partial z)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}\, d^3\mathbf{r}'$$
$$+\iint \frac{(\partial M_y(\mathbf{r})/\partial y)\,(\partial M_x(\mathbf{r}')/\partial x)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}\, d^3\mathbf{r}'$$
$$+\frac{1}{2}\iint \frac{(\partial M_x(\mathbf{r})/\partial x)\,(\partial M_x(\mathbf{r}')/\partial x)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}\, d^3\mathbf{r}', \qquad (2)$$

where the first term gives the Winter approximation,[1] and the second and the third are, as shown below, the main corrections due to the long-range scattering fields. In (2) we have omitted the terms in $\partial M_z/\partial z$, since they contain $\partial\Delta/\partial z \sim 1/Q$, where Δ is the DW thickness.

We look for M_y in the form

$$M_y = M \sin\theta_0(y) \sin\varphi(x, z), \qquad (3)$$

where $\cos\theta = M_z/M$ and $\tan\varphi = M_y/M_x$. When the DW is crossed, the change in $\theta(y)$ is given by the Landau–Lifshitz solution:

$$\theta_0(y) = 2\,\mathrm{arctg}\,\exp(y/\Delta_0), \qquad (4)$$

where Δ_0 is the DW thickness.

Integrating (2) over y and allowing for (3), (4) and the energy of inhomogeneous exchange, we find the total DW energy (see Ref. 1):

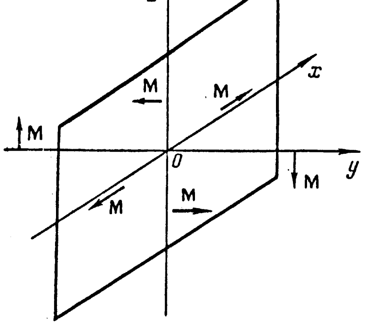

FIG. 1. Domain wall in the xz plane. The Bloch line runs along the z axis; the Bloch point is the origin of coordinates.

 0038-5646/92/050827-06$05.00

$$W=W_0+2A\Delta_0\iint\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]dx\,dz$$

$$+4\pi M^2\Delta_0\int\sin^2\varphi\,dx\,dz$$

$$-\frac{\pi^2\Delta_0^2M^2}{2}$$

$$\times\iint\frac{(\partial\sin\varphi/\partial x)(\partial\sin\varphi/\partial x')+(\partial\sin\varphi/\partial z)(\partial\sin\varphi/\partial z')}{|\mathbf{r}-\mathbf{r}'|}$$

$$\times dx\,dx'\,dz\,dz', \tag{5}$$

where A is the constant of inhomogeneous exchange.

Deriving (5), we have neglected the last two terms in (2). The first one [the fourth in (2)] is small, since the integral over y contains the functions $\partial M_y/\partial y$ and $\partial M_x/\partial x$ of opposite parity with respect to y. The last term in (2) is related to the so-called σ-charges of vertical BL,[1] but it gives corrections independent of z, the value of which does not change, when BP arise on the BL. Therefore it is of no interest to us.

The $\varphi(x,z)$ distribution is found from the equation

$$\frac{\delta W}{\delta\varphi}=0,$$

where W is given by (5). Then the equation for $\varphi(x,z)$ has the form

$$\Delta\varphi-\frac{\sin 2\varphi}{2\Lambda_0^2}-\frac{\pi^2\cos\varphi(x)}{8Q^{1/2}\Lambda_0}\left\{\frac{\partial}{\partial x}\int\frac{\partial\sin\varphi(\mathbf{r}')/\partial x'}{|\mathbf{r}-\mathbf{r}'|}d^2\mathbf{r}'+\frac{\partial}{\partial z}\int\frac{\partial\sin\varphi(\mathbf{r}')/\partial z'}{|\mathbf{r}-\mathbf{r}'|}d^2\mathbf{r}'\right\}=0. \tag{6}$$

Neglecting the integral terms, which give the effects of long-range scattering fields, we find the Winter approximation for this equation:

$$\Delta\varphi_0-\frac{\sin 2\varphi_0}{2\Lambda_0^2}=0,$$

where Λ_0 is the BL thickness.

The solution of this equation (see Ref. 1) for a BP at the origin and on a BL coinciding with the z axis, has for $|z|\gg\Lambda_0$ the form

$$\varphi(x)=\pm2\,\mathrm{arctg}\exp(x/\Lambda_0) \tag{7}$$

corresponding to uniform BL with opposite signs of the topological charge $+$ for $z<0$ and $-$ for $z>0$). Near the BP center $(x^2+z^2\ll\Lambda_0^2)$ the solution can be written as

$$\varphi_0(x,z)=\mathrm{arctg}(z/x). \tag{8}$$

Allowing for long-range scattering fields [integral terms in (6)], we can look for the solution in the form

$$\varphi=\varphi_0(x,z)+\varphi'(x,z),$$

where $\varphi'(x,z)$ is a small correction for the scattering fields for large quality factors. Its equation has the form

$$\Delta\varphi'-\frac{\sin 2\varphi_0}{\Lambda_0^2}\varphi'-\frac{\pi^2\cos\varphi_0(x)}{8Q^{1/2}\Lambda_0}\left\{\frac{\partial}{\partial x}\int\frac{\partial\sin\varphi_0(\mathbf{r}')/\partial x}{|\mathbf{r}-\mathbf{r}'|}d^2\mathbf{r}'+\frac{\partial}{\partial z}\int\frac{\partial\sin\varphi_0(\mathbf{r}')/\partial z}{|\mathbf{r}-\mathbf{r}'|}d^2\mathbf{r}'\right\}=0. \tag{9}$$

Consider for definiteness the region of positive z. Then Eq. (9) can be written as

$$\Delta\varphi'-\frac{\cos 2\varphi_0}{\Lambda_0^2}\varphi$$

$$=\frac{\pi^2\cos\varphi_0(x)}{8Q^{1/2}\Lambda_0}\left\{\frac{\partial}{\partial x}\int_{-\infty}^{+\infty}dz'\int_{-\infty}^{+\infty}dx'\frac{\partial\sin\varphi_0^+(\mathbf{r}')/\partial x}{|\mathbf{r}-\mathbf{r}'|}\right.$$

$$+2\frac{\partial}{\partial x}\int_{-\infty}^{0}dz'\int_{-\infty}^{+\infty}dx'\frac{\partial\sin\varphi_0^-(\mathbf{r}')/\partial x}{|\mathbf{r}-\mathbf{r}'|}$$

$$\left.+\frac{\partial}{\partial z}\int\frac{\partial\sin\varphi_0(\mathbf{r}')/\partial z}{|\mathbf{r}-\mathbf{r}'|}d^2\mathbf{r}'\right\}, \tag{10}$$

where $\varphi_0^\pm$ is the asymptotic value of $\varphi_0(x)$ as $z\to+\infty$ and $z\to-\infty$.

The first integral term in the right-hand side corresponds to scattering fields from a uniform BL, and the second and the third to the scattering fields from the BP which results in a change in the BL topological charge for negative z. The correction φ' to the Winter approximation can correspondingly be presented as the sum of two parts

$$\varphi'=\varphi'_{\mathrm{BL}}(x)+\varphi'_{\mathrm{BP}}(x,z).$$

The integrals in the right-hand side can be calculated far from the regions where magnetic charges are accumulated. When we evaluate the correction the φ'_{BL} independent of z, this approximation is valid outside the BL region, i.e., for $|x|\gg\Lambda_0$, where, with sufficient accuracy, $\cos 2\varphi_0=1$. Then we find

$$\varphi'_{\mathrm{BL}}=\pm\frac{\pi^2\Lambda_0^2}{4Q^{1/2}x^2}$$

($+$ for $x<0$ and $-$ for $x>0$).

When we find the correction due to the BP scattering fields, when the second and the third terms remain in the right hand side of Eq. (10), the integral is evaluated for any positive z, if $x^2+z^2\gg\Lambda_0^2$, which leads to an equation, which can be written in the form valid both for positive and negative z for $x^2+z^2\gg\Lambda_0^2$:

$$\Delta\varphi'_{\mathrm{BP}}-\frac{\cos 2\varphi_0}{\Lambda_0^2}\varphi'_{\mathrm{BP}}$$

$$-\,\mathrm{sign}(z)\frac{\pi^2\cos\varphi_0(x)}{4Q^{1/2}x^2}\left[1-\frac{|z|}{(x^2+z^2)^{1/2}}\right]=0. \tag{11}$$

It can be verified that the solution of this equation outside the region of size Λ_0 around the BL center is

$$\varphi'_{\mathrm{BP}}(x,z)=\mathrm{sign}(z)\frac{\pi^2\Lambda_0^2\,\mathrm{th}(x/\Lambda_0)}{4Q^{1/2}x^2}\left[1-\frac{|z|}{(x^2+z^2)^{1/2}}\right]. \tag{12}$$

Equation (12) defines the corrections for the BP scattering fields to the magnetic structure around the BP. It is assumed that the cut at which φ_0 changes abruptly from $+\pi$ to $-\pi$, is on the semiaxis $x>0$.

Knowing the correction for the BP scattering fields, it is possible to find corrections for the BP mobility μ and mass m for $Q\gg1$. The BP mobility along the z-axis is defined as[1]

$$\mu=\frac{\pi^2\gamma\Lambda_0}{\alpha\int(\partial\varphi/\partial z)^2\,dx\,dz}, \tag{13}$$

where γ is the gyromagnetic ratio, and α is the decay parameter. Then the expression for the BP mobility takes the form[1]

$$\mu=\frac{2\pi\gamma\Lambda_0}{\alpha(\ln Q+1{,}93)}. \tag{14}$$

Allowance for the scattering fields (12) gives rise to a correction of order $1/Q^{1/2}$, which will be added to $\ln Q$ in (14).

The BP mass along the z axis for

$$\Lambda_0\gg\beta,$$

where

$$\beta^2=\frac{\sigma_0}{2MH_z'},$$

[$\sigma_0=4(AK)^{1/2}$ is the DW energy density, and H_z' is the magnetic field gradient which creates the DW potential well ($H_z=H_z'y$)], is defined as[2]

$$m=\frac{4}{\sigma_0}\left(\frac{M}{\gamma}\right)^2\beta^2\int d^2\mathbf{r}\left(\frac{\partial\varphi}{\partial z}\right)^2. \tag{15}$$

Then the expression for the BP mass, neglecting the scattering fields, takes the form

$$m=\frac{2\pi M}{\gamma^2H_z'}\ln\frac{\Lambda_0}{r_m}, \tag{16}$$

where

$$r_m=\begin{cases}\beta, & \beta>\Delta_0,\\ \Delta_0, & \beta<\Delta_0.\end{cases}$$

Allowance for the scattering fields reduces to a correction of order $1/Q^{1/2}$, which is added to the logarithm in (16).

3. STATIC INTERACTION OF BP

The energy of static interaction of two BP is defined as the difference between scattering field energies: the energy of the BL with two BP and the uniform BL energy. For large quality factors the magnetic charge distribution can be determined in the Winter approximation. The so-called π-charges on the BL, equal to $\partial M_y/\partial y$, contribute to the energy of static interaction. They form dipoles of size Δ_0 oriented along the y-axis. The magnitude of the dipole moment **d** per unit BL length is given by the expression

$$d=M\pi^2\Lambda_0\Delta_0,$$

i.e., the BL is a dipole chain. When the BP is crossed, the dipoles reverse sign. If the BL has two BP, the dipoles situated between two BP will be oriented in the opposite direction with respect to the dipoles outside this interval (Fig. 2). This configuration of magnetic charges is a particular case of the magnetostatic structure calculated in Ref. 4. Using the results of this calculation, we find for two BP separated by a distance L that

$$W=\frac{2\pi^4M^2\Lambda_0^2\Delta_0^2}{L}, \tag{17}$$

which means that the two BP will repel each other.

If a constant external magnetic field H_y is applied along the y-axis, this field will prevent BP scattering, and the energy of static interaction will have an additional term

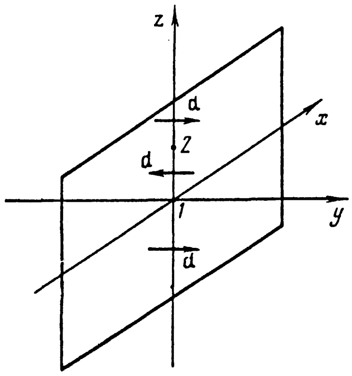

FIG. 2. Domain wall in the xz plane. The Bloch line runs along the z axis; *1* and *2* are the positions of two Bloch points; **d** is the orientation of a dipole moment on the Bloch line.

$$W=2MH_y\pi\Lambda_0\pi\Delta_0. \tag{18}$$

Minimizing the total energy of static interaction (17) and (18), we find the distance at which two BP are in the state of stable equilibrium

$$L=\pi\Lambda_0\left(\frac{M}{Q^{1/2}H_y}\right)^{1/2}. \tag{19}$$

The fact that two BP repel each other means that their annihilation is possible only if the potential barrier is overcome, for example with the help of external field H_y.

4. DYNAMIC INTERACTION OF TWO BP

When the BP move, the DW bends, resulting in dynamic interaction of the BP. Since the DW bending is determined by the BP velocity, the dynamic interaction of two BP is characterized by the interaction mass.

Two types of BP dynamic interaction are possible. The first one is connected with the overlapping of two bendings, while the second is caused by the charges produced on the DW by its bending (Fig. 3). As a result, the BP will be a quadrupole. Let us consider both types of the dynamic interaction at length.

The interaction of two BP bendings

The DW energy density σ, which is an effective Hamiltonian, depends on the DW displacement $q(x,z)^2$:

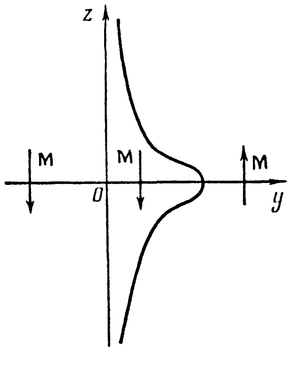

FIG. 3. Domain wall in the xz plane. The Bloch line is along the z axis: the Bloch point is the origin of coordinates; the domain wall displacement $q(x,z)$ is along the y axis [$q(-x,z)=-q(x,z)$].

$$\sigma=MH_z'q^2+{}^1/_2\sigma_0(\nabla q)^2. \qquad (20)$$

The interaction energy density $\sigma_{\rm int}$ for two BP situated on the same BL can be written in the form

$$\sigma_{\rm int}=2(MH_z'q_1q_2+{}^1/_2\sigma_0\nabla q_1\nabla q_2), \qquad (21)$$

where q_1 is the bending due to the first BP, and q_2 is the bending due to the second BP.

The magnitude of the bending q caused by one BP is given by the equation[2]

$$\frac{2M}{\gamma\sigma_0}v\frac{\partial\varphi}{\partial z}=\frac{q}{\beta^2}-\Delta q, \qquad (22)$$

where v is the BP velocity along the BL (along the z-axis), $\varphi=\varphi_0+\varphi'$, φ_0 is the solution (8) for the BP in the region $\Delta_0\ll r\ll\Lambda_0$, and φ' is a correction of order $1/Q^{1/2}$ given by (12).

As a result, Eqs. (21) and (22) yield the energy of BP bending interaction:

$$W_{\rm int}=\frac{2}{\pi\sigma_0}\left(\frac{M}{\gamma}\right)^2 v_1v_2\iint d^2r_1\,d^2r_2K_0\left(\frac{|r_1-r_2|}{\beta}\right)\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}, \qquad (23)$$

where K_0 is a modified Bessel function of the second kind, r_1 and r_2 are two-dimensional vectors in the xz-plane, $\varphi_1(r_1)$ and $\varphi_2(r_2)$ are distributions of φ connected with the first and second BP respectively, and v_1 and v_2 are the velocities of the first and second BP along the BL respectively.

Presenting the interaction energy in the form

$$W_{\rm int}=\frac{m_{\rm int}v_1v_2}{2},$$

we find from (23) the following expression for the interaction mass

$$m_{\rm int}=\frac{4}{\pi\sigma_0}\left(\frac{M}{\gamma}\right)^2\iint d^2r\,d^2r'\,K_0\left(\frac{|r-r'|}{\beta}\right)\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}. \qquad (24)$$

Consider the case $L\gg\beta$, where L is the distance between two BP. If the long-range scattering fields are neglected, φ_1 and φ_2 have the form (8) for $x^2+z^2\ll\Lambda_0^2$.

For $\beta\ll L$ in the region, determining the value of the integral (24), the inequality

$$\frac{|r-r'|}{\beta}\gg 1.$$

holds. Then the function K_0 in (24) can be replaced by the expression $(\pi/2x)^{1/2}e^{-x}$, and the expression for $m_{\rm int}$ takes on the form

$$m_{\rm int}=4\left(\frac{1}{2\pi}\right)^{1/2}\frac{1}{\sigma_0}\left(\frac{M}{\gamma}\right)^2\iint d^2\mathbf{r}\,d^2\mathbf{r}'$$
$$\times\frac{\exp(-|\mathbf{r}-\mathbf{r}'|/\beta)}{|\mathbf{r}-\mathbf{r}'|/\beta}\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}. \qquad (25)$$

Since $\partial\varphi_1/\partial z$ and $\partial\varphi_2/\partial z$ vary on the scale Λ_0, and the distance L between the BP is much larger than Λ_0 we can, evaluating the integral (25), use the dipole approximation:

$$m_{\rm int}=-4\left(\frac{1}{2\pi}\right)^{1/2}\frac{1}{\sigma_0}\left(\frac{M}{\gamma}\right)^2\frac{\exp(-L/\beta)}{(L/\beta)^{1/2}}\frac{1}{L\beta}\left(\int\frac{\partial\varphi_0}{\partial z}x\,d^2\mathbf{r}\right)^2.$$

The expression for the integral

$$\int\frac{\partial\varphi_0}{\partial z}x\,d^2\mathbf{r}$$

with (7) allowed for, has the following form

$$\int\frac{\partial\varphi_0}{\partial z}x\,d^2\mathbf{r}=\frac{\pi^3\Lambda_0^2}{4}. \qquad (26)$$

As a result, the expression for $m_{\rm int}$ for $\beta\ll L$ and without the long-range scattering fields can be written as

$$m_{\rm int}=-\frac{\pi^5}{4}\left(\frac{\pi}{2}\right)^{1/2}\frac{1}{\sigma_0}\left(\frac{M}{\gamma}\right)^2\frac{\exp(-L/\beta)}{(L/\beta)^{1/2}}\frac{\Lambda_0^4}{L\beta}, \qquad (27)$$

i.e., $m_{\rm int}$ decreases exponentially with the distance between the BP.

Consider now the case $L\gg\beta$, taking into account the long-range scattering fields. In the first approximation in $1/Q^{1/2}$ the distribution of φ_1 has the form (8) for $x^2+z^2\ll\Lambda_0^2$, and φ_2 is determined by the magnetostatic "tail," i.e., by the expression (12).

For $\Lambda_0\gg\beta$

$$K_0\left(\frac{|\mathbf{r}-\mathbf{r}'|}{\beta}\right)=2\pi\beta^2\delta(\mathbf{r}-\mathbf{r}'),$$

and we find for the interaction mass (24)

$$m_{\rm int}=\frac{8}{\sigma_0}\left(\frac{M}{\gamma}\right)^2\beta^2\int\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}d^2\mathbf{r}. \qquad (28)$$

The integral in (28), after integrating over z, has the form

$$\int\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}d^2\mathbf{r}=-2\frac{\pi^2}{Q^{1/2}}\left(\frac{\Lambda_0}{L}\right)^3\int_0^\infty \mathrm{arctg}(e^{-x})\,\mathrm{th}(x)\,dx.$$

Integrating over x, we find

$$\int\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}d^2\mathbf{r}=-\frac{\pi^3\ln 2}{2Q^{1/2}}\left(\frac{\Lambda_0}{L}\right)^3.$$

For $L\gg\Lambda_0\gg\beta$, thus, $m_{\rm int}$ has the form

$$m_{\rm int}=-4\frac{\pi^3\ln 2}{Q^{1/2}\sigma_0}\left(\frac{M}{\gamma}\right)^2\beta^2\left(\frac{\Lambda_0}{L}\right)^3. \qquad (29)$$

For $L\gg\beta\gg\Lambda_0$ the expression (24) for the interaction mass can be conveniently rewritten in the following way

$$m_{\rm int}=\frac{4}{\pi\sigma_0}\left(\frac{M}{\gamma}\right)^2\int d^2\mathbf{r}\frac{\partial\varphi_2}{dz}\int d^2\mathbf{r}'K_0\left(\frac{|\mathbf{r}-\mathbf{r}'|}{\beta}\right)\frac{\partial\varphi_1}{\partial z}. \qquad (30)$$

where φ_1 has the form (8) and φ_2 the magnetostatic "tail" form (12). For $\Lambda_0\ll r\ll\beta$ the modified Bessel function of the second kind can be replaced by a logarithmic function and the dipole approximation can be used:

$$\int d^2\mathbf{r}'K_0\left(\frac{|\mathbf{r}-\mathbf{r}'|}{\beta}\right)\frac{\partial\varphi_1}{\partial z}=-\frac{\pi^3}{4}\frac{\Lambda_0^2x}{r^2}. \qquad (31)$$

For $r\gg\beta$ the integral (31) decreases exponentially and does not contribute to $m_{\rm int}$. Substituting (31) into (30), we find an estimate for the interaction mass for $L\gg\beta\gg\Lambda_0$:

$$m_{\rm int}\sim-\frac{1}{Q^{1/2}\sigma_0}\left(\frac{M}{\gamma}\right)^2\beta\Lambda_0\left(\frac{\Lambda_0}{L}\right)^3. \qquad (32)$$

If we compare the expressions for one BP mass [Eq. (21) from Ref. 2] and the interaction mass of two BP for $L\gg\beta\gg\Lambda_0$, we find

$$\frac{m_{\rm int}}{m_{\rm BP}}\sim-\frac{\Lambda_0^3\beta}{Q^{1/2}L^3\Lambda_0}.$$

Thus, for $L \gg \beta$ and the long-range scattering fields neglected, the interaction mass of two BP decreases exponentially with the distance between the BP [see Eq. (27)], whereas the long-range scattering fields taken into account result in a power law for the interaction mass decrease.

Consider now the case $\beta \gg L$. For

$$\frac{|\mathbf{r}-\mathbf{r}'|}{\beta} \ll 1$$

the modified Bessel function of the second kind in (24) can be replaced by a logarithmic one. Then the interaction mass is

$$m_{\rm int} = \frac{4}{\pi\sigma_0}\left(\frac{M}{\gamma}\right)^2 \int\int d^2\mathbf{r}\, d^2\mathbf{r}' \ln\frac{|\mathbf{r}-\mathbf{r}'|}{\beta}\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial z}. \quad (33)$$

In this expression the corrections to φ_1 and φ_2 due to the long-range scattering fields are not important, so we can use the expression (8). Since $\partial\varphi_1/\partial z$ and $\partial\varphi_2/\partial z$ vary over the length Λ_0, and the distance between two BP L is much larger than Λ_0, we can use the dipole approximation:

$$m_{\rm int} = -\frac{4}{\pi\sigma_0}\left(\frac{M}{\gamma}\right)^2 \frac{\left[\int (\partial\varphi_0/\partial z) x\, d^2r\right]^2}{L^2}. \quad (34)$$

From (26) we have

$$\int \frac{\partial\varphi_0}{\partial z} x\, d^2\mathbf{r}.$$

As a result, we find

$$m_{\rm int} = -\frac{\pi^5}{4\sigma_0}\left(\frac{M}{\gamma}\right)^2 \Lambda_0^2\frac{\Lambda_0^2}{L^2}. \quad (35)$$

Taking into account Eq. (21) from Ref. 2, we get

$$\frac{m_{\rm int}}{m_{\rm BP}} \sim -\frac{\Lambda_0^2}{L^2},$$

i.e., for $L \ll \beta$ the interaction mass decreases according to a power law even without the long-range scattering fields in contrast to the case $L \gg \beta$. Long-range interaction in the case considered is conditioned by slowly decreasing DW bending deformations.

Interaction of BP dynamic charges

Consider the interaction of BP dynamic charges using a vertical BL as an example. The DW deviation from the equilibrium position is given by the solution of Eq. (22):

$$q(\mathbf{r}) = \frac{M}{\pi\gamma\sigma_0} v \int d^2\mathbf{r}' K_0\left(\frac{|\mathbf{r}-\mathbf{r}'|}{\beta}\right)\frac{\partial\varphi_0}{\partial z}(\mathbf{r}'). \quad (36)$$

Consider the case $\beta \gg \Lambda_0$. This case is most typical of a real situation. Then for $\Lambda_0 \ll r \ll \beta$ the expression (36) for $q(r)$ takes on the form

$$q(\mathbf{r}) = -\frac{M}{\pi\gamma\sigma_0} v \int d^2\mathbf{r}' \ln\frac{|\mathbf{r}-\mathbf{r}'|}{\beta}\frac{\partial\varphi_0}{\partial z}(\mathbf{r}'),$$

where φ_0 is given by (8). Using the dipole approximation for $\partial\varphi_0/\partial z$ and performing the integration, we find

$$q(\mathbf{r}) = \frac{M}{\pi\gamma\sigma_0}\frac{x}{r^2} v \int \frac{\partial\varphi_0}{\partial z} x\, d^2\mathbf{r}.$$

Estimating the integral (26), we get

$$q(\mathbf{r}) = \frac{\pi^2 M}{4\gamma\sigma_0}\frac{x\Lambda_0^2}{r^2} v. \quad (37)$$

For $r \gg \beta$ the modified Bessel function can be replaced by $(\pi/2x)^{1/2}e^{-x}$. As a result, we have

$$q(\mathbf{r}) = \left(\frac{1}{2\pi}\right)^{1/2} \frac{M}{\gamma\sigma_0}\frac{e^{-r/\beta}}{(r/\beta)^{1/2}}\frac{x}{\beta r} v \int \frac{\partial\varphi_0}{\partial z} x\, d^2\mathbf{r}.$$

Using (26) for the integral

$$\int \frac{\partial\varphi_0}{\partial z} x\, d^2\mathbf{r},$$

we find that

$$q(\mathbf{r}) = \frac{1}{2}\left(\frac{1}{2\pi}\right)^{1/2} \frac{\pi^3}{2}\frac{M}{\gamma\sigma_0}\frac{e^{-r/\beta}}{(r/\beta)^{1/2}}\frac{x\Lambda_0^2}{\beta r} v,$$

i.e., the bending region $q(\mathbf{r})$ is important for $r \ll \beta$ and is given by (37), while the region $r \gg \beta$ is of no interest.

Due to the DW bending, when the BP move on the DW, magnetic charges arise. The volume charge density ρ_v for

$$\left|\frac{\partial q}{\partial z}\right| \ll 1$$

has the form

$$\rho_V = \frac{\partial M_z}{\partial z} = \frac{2M}{\Delta_0}\frac{\partial q}{\partial z}. \quad (38)$$

Using (37), we find the surface charge density $\rho = \rho_V \Delta_0$:

$$\rho = \frac{2\pi^3 M^2 \Lambda_0^2}{\gamma\sigma_0}\frac{zx}{r^4} v. \quad (39)$$

It can be seen from (39) that the BP dynamic charge is a quadrupole of size β. The interaction energy of dynamic charges of two BP is defined as

$$W_{\rm int} \sim -\frac{M^4\Lambda_0^4}{\gamma^2\sigma_0^2}\frac{\beta^4}{L^5} v_1 v_2. \quad (40)$$

As a result, the interaction mass of two BP caused by dynamic charges is

$$m_{\rm int} \sim -\left(\frac{M}{\gamma}\right)^2 \frac{\Lambda_0^2}{\sigma_0}\frac{\Lambda_0}{L}\frac{1}{Q^{1/2}}\frac{\beta^4}{L^4}. \quad (41)$$

Using Eq. (21) from Ref. 2, we find that

$$\frac{m_{\rm int}}{m_{\rm BP}} \sim -\frac{1}{Q^{1/2}}\frac{\Lambda_0}{L}\left(\frac{\beta}{L}\right)^4, \quad (42)$$

i.e., the interaction mass caused by the charge interaction decreases by a power law to a first approximation in $1/Q^{1/2}$.

Comparing (42) with (32), we see that in the range $L \gg \beta \gg \Lambda_0$ the direct bending interaction is larger than the interaction of magnetic charges due to bending.

5. OSCILLATIONS OF TWO BP

As noted before, the static interaction of two BP results in their repulsion. Therefore, for two BP to be in the state of stable equilibrium it is necessary to apply a constant magnetic field H_y. The distance L between two BP as a function of H_y is given by Eq. (19). The Lagrangian of such a system has the form

$$L = \frac{1}{2} m_{\rm BP}\left(\frac{\partial z_1}{\partial t}\right)^2 + \frac{1}{2} m_{\rm BP}\left(\frac{\partial z_2}{\partial t}\right)^2 + \frac{1}{2} m_{\rm int}\frac{\partial z_1}{\partial t}\frac{\partial z_2}{\partial t} - 2\pi Q^{1/4} M^{1/2} H_y^{3/2}\Delta_0 (z_1 - z_2)^2,$$

where z_1 and z_2 are the deviations of the first and second BP respectively from the equilibrium position.

The equations of motion have then the form

$$m_{\rm BP}\frac{\partial^2 z_1}{\partial t^2}+\frac{m_{\rm int}}{2}\frac{\partial^2 z_2}{\partial t^2}=-4\pi Q^{1/4}M^{1/2}H_y^{3/2}\Delta_0(z_1-z_2),$$
$$m_{\rm BP}\frac{\partial^2 z_2}{\partial t^2}+\frac{m_{\rm int}}{2}\frac{\partial^2 z_1}{\partial t^2}=4\pi Q^{1/4}M^{1/2}H_y^{3/2}\Delta_0(z_1-z_2). \qquad (43)$$

Equations (43) give the free oscillation frequency of two BP

$$\omega^2=\frac{8\pi Q^{1/4}M^{1/2}H_y^{3/2}\Delta_0}{m_{\rm BP}-m_{\rm int}/2}. \qquad (44)$$

Since, according to (19), the distance L between the BP depends on H_y, allowance for the BP dynamic interaction gives rise to deviation from the dependence $\omega^2 \sim H_y^{3/2}$ following from (44) for $m_{\rm int}=0$.

6. CONCLUSION

In the present study corrections to the BP magnetic structure and dynamics along the BL due to the long-range scattering fields have been obtained. The interaction of two BP positioned in one BL has also been derived. It has been shown that BP annihilation is possible, if the potential barrier separating two BP is overcome. The interaction mass of two BP calculated above affects the free oscillation frequency of two BP allowing experimental observation of long range effects discussed in the paper. Such effects are expected to be most noticeable for $\beta \gg L$, i.e., for weak gradients H_z' of magnetic fields that secure the DW in a definite position.

[1] A. P. Malozemoff and J. C. Slonczewski, *Magnetic Domain Walls in Bubble Materials*, Academic, Orlando, 1979.

[2] Yu. A. Kufaev and E. B. Sonin, Zh. Eksp. Teor. Fiz. **95**, 1523 (1989) [JETP **68**, 879 (1989)].

[3] V. S. Gornakov, V. I. Nikitenko, and I. A. Prudnikov, Pis'ma Zh. Eksp. Teor. Fiz. **50**, 479 (1989) [JETP Lett. **50**, 513 (1989)].

[4] A. V. Nikiforov and E. B. Sonin, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 325 (1984) [JETP Lett. **40**, 1119 (1984)].

Translated by E. Khmelnitski