Surface electromagnetic waves at the interface of an isotropic medium and a superlattice

S. Ya. Vetrov and A. V. Shabanov

Institute of Physics of the Siberian Branch of the Russian Academy of Sciences
(Submitted November 6, 1991)
Zh. Eksp. Teor. Fiz. **101**, 1340–1346 (April 1992)

We investigate electromagnetic waves localized on the interface between an isotropic medium and a superlattice, consisting of alternating isotropic and anisotropic layers of two materials. The anisotropic layer consists of a nematic liquid crystal (NLC). The dispersion equation and an expression for the parameters controlling the degree of localization of the boundary waves are derived and investigated. Taking into account the specific nature of the NLC which distinguishes it from ordinary crystals—strong permittivity anisotropy—we show that the penetration depth of waves into the superlattice can be effectively controlled by changing the orientation of the director of the NLC (for example, by means of an external electric field). We also show that the surface electromagnetic waves corresponding to separate dispersion branches of the spectrum do not occur for arbitrary orientations of the director relative to the interface of the media.

1. Papers concerning the properties of superlattices (SLs)—layered structures consisting of alternating layers of two materials—have been widely circulated in the last few years. Superlattices are of interest in connection with both their promising practical applications and with the possibility of observing with their help different physical phenomena. Surface excitations in such systems are under active investigation; such excitations play a large role in the development of surface physics because their characteristics are sensitive to the properties of the solid surfaces. The propagation of elastic waves in semibounded layered structures is studied in Refs. 1–3. Surface spin waves and plasmons in layered systems are investigated in Refs. 4 and 5. The results of analysis of the propagation of surface electromagnetic waves (SEWs) in superlattices, consisting of alternating isotropic layers of materials, are reported in theoretical and experimental work.[6–13]

In the present paper we investigate SEWs at the interface of an isotropic medium and a superlattice, consisting of alternating isotropic and anisotropic layers of two materials. The anisotropic medium is a nematic liquid crystal (NLC), exhibiting strong permittivity anisotropy. In the next section of this paper we give a derivation of the dispersion equation for surface waves. In Sec. 3 this equation is analyzed analytically. The results of numerical analysis of the dispersion equation are described in detail in Sec. 4.

2. The layered structure which we are studying (Fig. 1) is characterized by the permittivity tensor

$$\varepsilon_{ij}=\begin{cases}\varepsilon_1\delta_{ij}, & nL<z<nL+d_1\\ \varepsilon_{ij}, & nL+d_1<z<(n+1)L\end{cases}\Bigg\}z>0,\quad \varepsilon_0\delta_{ij},\ z<0 \qquad (1)$$

where ε_0 is the permittivity of the medium occupying the half-space $z<0$; $L=d_1+d_2$ is the period of the superlattice, where d_1 and d_2 are the thicknesses of the layers of two materials with permittivities ε_1 and ε_{ij}, respectively; $n=0,1,2,\ldots$; the components of the symmetric permittivity tensor of the NLC are

$$\varepsilon_{xx}=\varepsilon_\perp\cos^2\theta+\varepsilon_\parallel\sin^2\theta,\quad \varepsilon_{yy}=\varepsilon_\perp,\quad \varepsilon_{zz}=\varepsilon_\perp\sin^2\theta+\varepsilon_\parallel\cos^2\theta,$$
$$\varepsilon_{xy}=\varepsilon_{yz}=0,\ \varepsilon_{xz}={}^1\!/_2\,\Delta\varepsilon\sin 2\theta, \qquad (2)$$

where $\varepsilon_\perp=\varepsilon''_{xx}=\varepsilon''_{yy}$, $\varepsilon_\parallel=\varepsilon''_{zz}$ are the components of the permittivity tensor along the principal axes, $\Delta\varepsilon=\varepsilon_\parallel-\varepsilon_\perp$, and θ is the angle between the director and the z axis. In what follows only the diagonal components of the tensor $\tilde{\varepsilon}$ are taken into account. This is justified for $\theta=0$ or $\pi/2$.

Maxwell's equations in the anisotropic medium of the superlattice for monochromatic waves

$$\{E_{x,z};\ H_y\}=\{E_{x,z}(z),\ H_y(z)\}\exp\left[i(kx-\omega t)\right], \qquad (3)$$

propagating in the xz plane in the x direction with surface wave vector k and frequency ω, have the form

$$\left[\frac{d^2}{dz^2}-\alpha\left(k^2-\frac{\varepsilon_{zz}\omega^2}{c^2}\right)\right]E_x(z)=0,\quad \alpha=\frac{\varepsilon_{xx}}{\varepsilon_{zz}},$$
$$H_y(z)=\frac{i\omega}{c}\frac{\varepsilon_{zz}\,dE_x/dz}{k^2-\varepsilon_{zz}\omega^2/c^2},\quad E_z(z)=-\frac{kc}{\omega}H_y(z). \qquad (4)$$

Maxwell's equations in the isotropic layer of the superlattice and the medium occupying the half-space $z<0$ can be derived from Eq. (4) by making the respective substitutions $\alpha\sim 1$, $\varepsilon_{zz}\sim\varepsilon_1$ and $\alpha\sim 1$, $\varepsilon_{zz}\sim\varepsilon_0$. We construct the solutions of Eqs. (4) in the superlattice which decay exponentially away from the interface as $z\to\infty$ by means of the Floquet–Bloch theorem.

The general solution of Eq. (4) for the x component of the field in the isotropic medium of a finite superlattice has the form

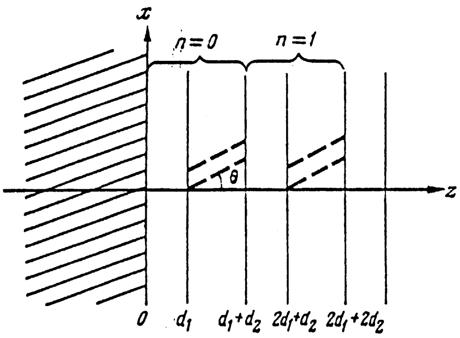

FIG. 1. Schematic representation of a layered structure bounding an isotropic medium.

 0038-5646/92/040719-04$05.00

$$E_x(z)=A_+e^{\alpha_1 z}+A_-e^{-\alpha_1 z}, \tag{5}$$

where

$$\alpha_1=(k^2-\varepsilon_1\omega^2/c^2)^{1/2}. \tag{6}$$

Using the periodicity in the z direction we represent $E_x(z)$ in the form of a Bloch wave

$$E_x(z)=e^{ik_3 z}E_x(k_3, z), \tag{7}$$

where

$$E_x(k_3, z)=E_x(k_3, z+L). \tag{8}$$

From Eqs. (5) and (7) we obtain $E_x(k_3,z)$

$$E_x(k_3, z)=e^{-ik_3 z}(A_+e^{\alpha_1 z}+A_-e^{-\alpha_1 z}). \tag{9}$$

The function $E_x(k_3,z)$ must be periodic with period L. This can be achieved by replacing z with $z-L$, when z lies in the nth layer. As a result we find

$$E_x(k_3, z)=\exp[-ik_3(z-nL)]\{A_+\exp[\alpha_1(z-nL)] +A_-\exp[-\alpha_1(z-nL)]\}, \tag{10}$$

where

$$nL<z<nL+d_1,\ n=0, \pm1, \pm2, \dots . \tag{11}$$

It is obvious that the function $E_x(k_3,z)$ does not depend on n and therefore is periodic with period L, i.e., it satisfies the periodicity condition (8). Thus using Eqs. (7) and (10) we obtain

$$E_x(z)=\exp(ik_3nL)\{A_+\exp[\alpha_1(z-nL)] +A_-\exp[-\alpha_1(z-nL)]\}, \tag{12}$$

where

$$nL<z<nL+d_1. \tag{13}$$

The expression for the field in an anisotropic medium of an infinite superlattice is obtained similarly and has the form

$$E_x(z)=\exp(ik_3nL)\{B_+\exp[\alpha_1^{1/2}\alpha_2(z-nL-d_1)] +B_-\exp[-\alpha_1^{1/2}\alpha_2(z-nL-d_1)]\}, \tag{14}$$

where

$$nL+d_1<z<(n+1)L, \tag{15}$$

$$\alpha_2=(k^2-\varepsilon_{zz}\omega^2/c^2)^{1/2}. \tag{16}$$

Expressions of the form (12) and (14) for the y component of the magnetic field can be derived on the basis of Eq. (4). The dispersion relation for bulk electromagnetic waves is determined by the boundary conditions between the different media. Real values of k and k_3 correspond to propagating waves.

Since we are interested in the solutions of Maxwell's equations that decay as $z\to\pm\infty$, we seek them in the semibound superlattice in a form differing from Eqs. (12) and (14) only in that ik_3 is replaced with $-\beta$ ($\mathrm{Re}\,\beta>0$) (here $n=0,1,2,\dots$). In an isotropic medium with $z<0$

$$E_x(z)=A_0e^{\alpha_0 z}, \tag{17}$$

where

$$\alpha_0=(k^2-\varepsilon_0\omega^2/c^2)^{1/2}, \qquad \mathrm{Re}\,\alpha_0>0. \tag{18}$$

The expressions for different fields in different regions are matched by means of the boundary conditions.

In order to obtain the dispersion relation for SEWs it is sufficient to study the boundary conditions along three interfaces: $z=nL$, $z=nL+d_1$, and $z=0$. From the condition that the electric and magnetic fields E_x and H_y are continuous at the interfaces $z=nL$ and $z=nL+d_1$ we obtain four equations for A_+, A_-, B_+, and B_-. Eliminating B_+ and B_- from these four equations we obtain two equations for A_+ and A_-:

$$\begin{bmatrix} (1+F)(e^{\alpha_1 d_1}-e^{-\beta L}e^{-\alpha_2 d_2}) & (1-F)(e^{-\alpha_1 d_1}-e^{-\beta L}e^{-\alpha_2 d_2}) \\ (1-F)(e^{\alpha_1 d_1}-e^{-\beta L}e^{\alpha_2 d_2}) & (1+F)(e^{-\alpha_1 d_1}-e^{-\beta L}e^{\alpha_2 d_2}) \end{bmatrix}\begin{pmatrix} A_+ \\ A_- \end{pmatrix} =0, \tag{19}$$

where $F=\varepsilon_1\alpha_2/\alpha_1\varepsilon_{zz}\alpha^{1/2}$. From the condition that the electric and magnetic fields are continuous at $z=0$ it follows that

$$A_+=GA_-, \tag{20}$$

where

$$G=(1+F_0)/(F_0-1), \quad F_0=\varepsilon_1\alpha_0/\varepsilon_0\alpha_1.$$

After quite complicated algebraic transformations, we obtain from a simultaneous solution of Eqs. (19) and (20) the dispersion equation determining ω as a function of k:

$$\varepsilon_0\left(\varepsilon_1^2\frac{\varkappa_2}{\varkappa_1}-\varepsilon_{zz}^2\frac{\varkappa_1}{\varkappa_2}\right)\operatorname{th}x\operatorname{th}y+\frac{\varepsilon_{zz}}{\alpha^{1/2}}\left(\varepsilon_0^2\frac{\varkappa_1}{\varkappa_0}-\varepsilon_1^2\frac{\varkappa_0}{\varkappa_1}\right)\operatorname{th}x + \varepsilon_1\left(\frac{\varepsilon_0^2}{\alpha}\frac{\varkappa_2}{\varkappa_0}-\varepsilon_{zz}^2\frac{\varkappa_0}{\varkappa_2}\right)\operatorname{th}y=0, \tag{21}$$

where

$$\varkappa_i=\alpha_iL, \quad i=0, 1, 2; \quad x=\varkappa_1\nu_1, \quad y=\alpha^{1/2}\varkappa_2\nu_2,$$
$$\nu_1=d_1/L, \quad \nu_2=1-\nu_1$$

$\alpha=\varepsilon_{xx}/\varepsilon_{zz}$, and the equation for the exponential factor $\beta(k)$ has the form

$$e^{-\beta L}=\frac{\alpha^{1/2}}{a}\varepsilon_{zz}\frac{\operatorname{ch}x}{\operatorname{ch}y}+\frac{e^{-y}\alpha^{1/2}}{a}\Big[b\left(1-\frac{\varepsilon_1^2\varkappa_2^2}{\alpha\varepsilon_{zz}^2\varkappa_1^2}\right)\operatorname{sh}x\operatorname{th}y + b\left(1-\frac{\varepsilon_0\varkappa_2}{\alpha^{1/2}\varepsilon_{zz}\varkappa_0}\right)\operatorname{sh}x-\frac{\varepsilon_0\varkappa_2}{\alpha^{1/2}\varkappa_0}\operatorname{ch}x-\frac{\varepsilon_0^2\varkappa_2^2}{\alpha\varepsilon_{zz}\varkappa_0^2}\operatorname{ch}x\operatorname{th}y\Big], \tag{22}$$

where

$$a=\varepsilon_{zz}\alpha^{1/2}-\varepsilon_0\varkappa_2/\varkappa_0, \quad b=\varepsilon_0\varepsilon_{zz}\varkappa_1/\varepsilon_1\varkappa_0.$$

Making the substitutions $\alpha\sim1$ and $\varepsilon_{zz}\sim\varepsilon_2$ Eqs. (21) and (22) reduce, except for the notation, to the corresponding equations obtained in Ref. 6 for SEWs at the interface of the isotropic medium and the superlattice.

3. The spectrum of surface waves can be obtained analytically in the limit $x\ll1$, $y\ll1$. The dispersion relation for SEWs in this case is

$$k^2=\frac{\omega^2}{c^2}\frac{\varepsilon_0\varepsilon_1\varepsilon_{zz}[\varepsilon_0-\varepsilon_1+(\varepsilon_0-\varepsilon_{xx})d_2/d_1]}{\varepsilon_{zz}(\varepsilon_0^2-\varepsilon_1^2)+\varepsilon_1(\varepsilon_0^2-\varepsilon_{xx}\varepsilon_{zz})d_2/d_1}. \tag{23}$$

In the present approximation

$$\beta=-\frac{\varepsilon_0}{L}\left(\frac{\alpha_1^2}{\varepsilon_1\alpha_0}d_1+\frac{\alpha_2^2}{\varepsilon_{zz}\alpha_0}d_2\right). \tag{24}$$

If we take $d_1\to 0$, then from Eqs. (23), (24), and (18) there follows the well-known result of Ref. 14 for the dispersion relation and the parameters determining the damping of TM SEWs at the interface of the semi-infinite isotropic and anisotropic media:

$$k^2=\frac{\omega^2}{c^2}\frac{\varepsilon_0\varepsilon_{zz}(\varepsilon_0-\varepsilon_{xx})}{\varepsilon_0^2-\varepsilon_{xx}\varepsilon_{zz}}, \tag{25}$$

$$\beta=-\frac{\omega}{c}\frac{\varepsilon_{xx}}{\varepsilon_0}\left[\frac{\varepsilon_0^2(\varepsilon_0-\varepsilon_{zz})}{\varepsilon_{xx}\varepsilon_{zz}-\varepsilon_0^2}\right]^{1/2},\quad \alpha_0=\frac{\omega}{c}\left[\frac{\varepsilon_0^2(\varepsilon_0-\varepsilon_{zz})}{\varepsilon_{xx}\varepsilon_{zz}-\varepsilon_0^2}\right]^{1/2}. \tag{26}$$

As $d_2\to 0$ we obtain SEWs at the interface of isotropic media. It is known that the orientation of the director or, in other words, the orientation of the optic axis of the NLC can be effectively controlled, for example, by an external electric field.[15] In order to obtain an idea of how sensitive the characteristics of SEWs are to a change in the orientation of the director, we make some numerical estimates. If we have $d_2\gg d_1$, then in order to compare the characteristics of SEWs in two orientations of the director $\theta=0$ (1) and $\theta=\pi/2$ (2) we find for $\varepsilon_0<0$, $\varepsilon_\parallel\varepsilon_\perp<\varepsilon_0^2$ $(\varepsilon_{\perp,\parallel}>0)$ the ratio of the corresponding refractive indices

$$\frac{n_1}{n_2}=\left[\frac{\varepsilon_\parallel(\varepsilon_\perp-\varepsilon_0)}{\varepsilon_\perp(\varepsilon_\parallel-\varepsilon_0)}\right]^{1/2} \tag{27}$$

and

$$\frac{\beta_1}{\beta_2}=\frac{\varepsilon_\perp}{\varepsilon_\parallel}\left(\frac{\varepsilon_0-\varepsilon_\parallel}{\varepsilon_0-\varepsilon_\perp}\right)^{1/2},\quad \frac{\alpha_{01}}{\alpha_{02}}=\left(\frac{\varepsilon_0-\varepsilon_\parallel}{\varepsilon_0-\varepsilon_\perp}\right)^{1/2}. \tag{28}$$

Setting $\varepsilon_0=-3.4$ and taking BMAOB at 20 °C as the liquid crystal, when in the region of optical frequencies we have $\varepsilon=4$ and $\varepsilon=2.4$ (Ref. 15), we obtain

$$n_1/n_2=1{,}1,\quad \beta_1/\beta_2=0{,}5,\quad \alpha_{01}/\alpha_{02}=1{,}2.$$

4. Numerical analysis of the dispersion equation shows that the spectrum of SEWs is more complicated than the spectrum which can be found analytically. As an illustration we present here the computational results for the parameters given above, and $\nu_1=0.1$ and $\varepsilon_1=2.2$, which make it possible to trace the effect of the orientation of the director of the NLC on the SEW spectrum. Figure 2 shows the dispersion curves for SEWs, and the corresponding parameters which control the degree of localization of the waves at the interface of the superlattice and the isotropic medium, are presented in Figs. 3 and 4. For $kL<1$ the dispersion curve described by the relation (23) is identical to the curve in Fig. 2.

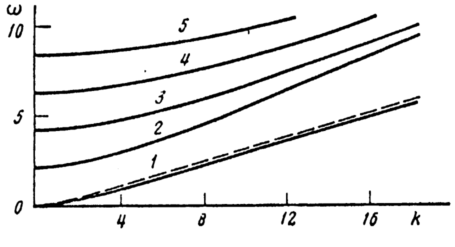

FIG. 2. Dispersion curves of surface waves; ω is given in units of c/L and k in units of $1/L$. The solid lines represent calculations with $\theta=0$, the dashed lines represent calculations with $\theta=\pi/2$, and the numbers 1–5 label the dispersion branches.

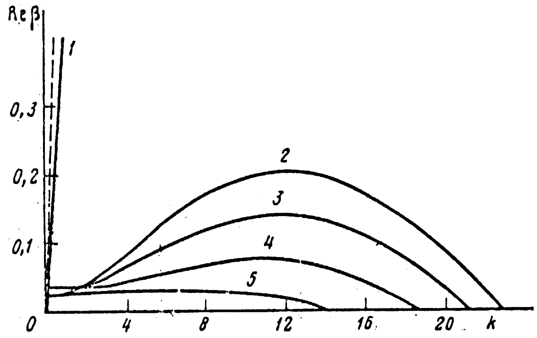

FIG. 3. The function $\mathrm{Re}\,\beta(k)$; $\mathrm{Re}\,\beta$ and k are given in units of $1/L$.

A characteristic feature of fundamental surface waves, which correspond to the dispersion curve 1 (with the exception of waves for which kL is to zero) is that they are most strongly localized in a layered medium. Thus, for example, for $L\approx 10^4$ Å for $kL\approx 10$ (when the frequencies $\omega\approx 10^{15}$ Hz) the penetration depth of fundamental excitations on the first dispersion branch into the layered medium is equal to the period of the superlattice, while the localization length of excitations from other branches is an order of magnitude larger, i.e., the energy of the principal modes is concentrated in the first few periods of an infinite periodic medium.

Qualitative changes arise in the SEW spectrum as the orientation of the director of the NLC changes. When the director of the NLC is oriented parallel to the surface of the layer ($\theta=\pi/2$), in constrast to the case $\theta=0$, only SEWs corresponding to the first dispersion branch (see Fig. 2) are realized. In the high-frequency region of the spectrum, however, not presented in Fig. 2, other dispersion curves also exist in the case $\theta=\pi/2$. Of all the surface wave characteristics, the penetration depth of waves into a layered medium is most sensitive to the reorientation of the director.

It should be noted that the greater the permittivity anisotropy of the NLC, the more pronounced are the changes occurring in the SEW spectrum as the orientation of the director changes. If MBBA, whose permittivity anisotropy ($\Delta\varepsilon=\varepsilon_\parallel-\varepsilon_\perp=0.2$) is several times smaller than for the liquid crystal BMAOB, is studied as the NLC, then in the frequency region shown in Fig. 2 the dispersion dependences are modified only insignificantly as the angle θ changes.

In the present paper we demonstrated the existence of a number of anomalies in the properties of TM SEWs at the

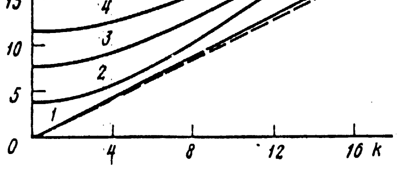

FIG. 4. The function $\alpha_0(k)$; α_0 and k are given in units of $1/L$.

boundary of an isotropic medium and a superlattice for a comparatively simple situation, when the optical axis of the anisotropic layer was oriented perpendicular or parallel to the interface. The surface solutions are much more difficult to find in the case of arbitrary orientation of the optical axis, and this problem will be studied in future investigations.

We thank E. M. Aver'yanov, R. S. Gekht, and A. F. Sadreev for a helpful discussion of this work.

[1]R. E. Camley, B. Djafari-Rouhani, L. Dobrzynski, and A. A. Maradudin, Phys. Rev. B **27**, 7318 (1983).
[2]J. Mendialdua, T. Szwacka, A. Rodriguer, and L. Dobrzynski, Phys. Rev. B **39**, 10674 (1989).
[3]S. Ya. Vetrov and V. F. Shabanov, Pis'ma Zh. Tekh. Fiz., **17**, No. 3, 34 (1991) [Sov. Tech. Phys. Lett. **17**, 169 (1991)].
[4]R. E. Camley, T. S. Rahman, and D. L. Mills, Phys. Rev. B **27**, 261 (1983).
[5]R. E. Camley and D. L. Mills, Phys. Rev. B **29**, 1695 (1984).
[6]H. Shi and C. Tsai, Solid State Commun. **52**, 953 (1984).
[7]A. V. Vinogradov and I. V. Kozhevnikov, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 405 (1984) [JETP Lett. **40**, 1221 (1984)].
[8]I. V. Kozhevnikov, Poverkhnost', No. 9, 26 (1986).
[9]L. G. Bol'shinskiĭ and A. I. Lomtev, Zh. Tekh. Fiz. **59**, 176 (1989).
[10]A. Yariv and P. Yeh, *Optical Waves in Crystals: Propagation and Control of Laser Radiation*, Wiley, New York (1984).
[11]D. Kossel, J. Opt. Soc. Am. **56**, 1434 (1966).
[12]P. Yeh, A. Yariv, and A. Y. Cho, Appl. Phys. Lett. **32**, 104 (1978).
[13]A. A. Bulgakov and V. R. Kovtun, Opt. Spektrosk. **56**, 769 (1984) [Opt. Spectrosc. (USSR) **56**, 471 (1984)].
[14]V. M. Agranovich and V. L. Ginzburg, *Spatial Dispersion in Crystal Optics and the Theory of Excitons*, Wiley, New York (1967).
[15]L. M. Blinov, *Electro-Optical and Magneto-Optical Properties of Liquid Crystals*, Wiley, New York (1983).

Translated by M. E. Alferieff