Relativistic-ponderomotive self-channeling of intense ultrashort laser pulses in a medium

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(Submitted 28 June 1991)
Zh. Eksp. Teor. Fiz. **101**, 1132–1153 (April 1992)

The self-channeling of an intense ultrashort laser pulse in a plasma as the result of a change in the refractive index due to the relativistic increase in the mass of the electrons and also due to the expulsion of electrons from the strong-field region by the ponderomotive force is analyzed. The process is described by the nonlinear Schrödinger equation. A two-dimensional, axisymmetric, and otherwise arbitrary solution of this nonlinear Schrödinger equation tends asymptotically toward the lowest spatially localized mode. The critical power for self-channeling is the same as the minimum power in the corresponding mode, $P_{cr} = 2P_{cr,c}$. In other words, it is twice the critical power for a medium with a quadratic nonlinearity. The practical realization of self-channeling of a laser pulse would make it possible to produce long, narrow regions with multiply charged ions and an ultrastrong electromagnetic field at $I \approx 10^{19}$–10^{20} W/cm^2, in the absence of free electrons. This capability would open the door to research on matter in ultrastrong electromagnetic fields. It would also be pertinent to the effort to develop an x-ray laser.

INTRODUCTION

The interaction of ultrashort (subpicosecond) laser pulses of high intensity ($10^{18} < I < 10^{21}$ W/cm^2) with gaseous media has been a field of active research over the past few years. At intensities $I \geqslant I^* = 10^{16}$ W/cm^2, the atoms at the front of such a pulse undergo a rapid nonlinear photoionization. The external light strips 10–15 electrons off each atom.[1,2] This process leads to the production of a plasma, along which the main part of the pulse propagates. A study of the interaction of the light with the matter in this case may be pertinent to two problems: (1) the physics of elementary processes in an ultraintense optical field and (2) the nonlinear refraction of a pulse in a material. The second of these questions is the topic of the present study.

The possibility of a self-focusing of an intense optical-range electromagnetic beam in a material medium, resulting in waveguide propagation of the beam, was first pointed out back in 1962, in Ref. 3. Thermal and ponderomotive mechanisms for the self-focusing were proposed in the same paper. The problem of a self-consistent transverse distribution of the field and the plasma in a light beam was solved in 1964, in Ref. 4. It was shown that the field could undergo self-localization, creating for itself a channel which would oppose a transverse divergence of the beam. The concept of a critical power for self-focusing was introduced in Ref. 5. Self-focusing was observed experimentally in Refs. 6 and 7. The basic equation for describing a steady-state self-focusing of a light beam—the nonlinear Schrödinger equation—was derived in Ref. 8. This equation was subsequently generalized to the time-varying case in dispersive media.[9] An approximate expression for the self-focusing length of a collimated beam (i.e., the distance to the first focus) was derived in Refs. 10 and 11 as a function of the beam aperture and power. The filamentation of a beam with a power well above the critical value was explained in Ref. 12.

Several ideas have grown up about possible regimes of the self-focusing of a steady-state electromagnetic beam at power levels above the critical value, $P_0 > P_{cr}$, on the basis of analysis of two-dimensional steady-state solutions of the nonlinear Schrödinger equation with nonlinearities of various types. The possible formation of a waveguide from a focus was suggested in Ref. 10. The nonlinear Schrödinger equation with a quadratic nonlinearity (a Kerr nonlinearity) was analyzed in Ref. 13 for a beam with an initially Gaussian transverse intensity profile. A solution corresponding to a multiple-focus structure was found. The number of foci was finite, roughly equal to P_0/P_{cr}. The thin filaments were interpreted as wakes of moving foci. A nonlinear Schrödinger equation with a nondissipative, saturable nonlinearity of a simple algebraic type was examined in Ref. 14. At low intensities, that nonlinearity converted into a quadratic (Kerr) nonlinearity. Numerical calculations yielded waveguide solutions which converged asymptotically over large propagation distances to normal modes of the nonlinear Schrödinger equation. The possible existence of pulsating waveguide solutions has also been conjectured. Single-focus propagation regimes were found in Ref. 15 for a dissipative Kerr medium, for beams with a hyper-Gaussian transverse intensity profile under the condition $P_0 \gg P_{cr}$.

Efforts to analyze the nonlinear Schrödinger equation run into the difficulty that for a nonlinear equation there are no general theorems which would make it possible to narrow the range of possible solutions of the equation on the basis of the type of nonlinearity. For this reason, analysis of a nonlinear Schrödinger equation becomes a complex independent problem for each type of nonlinearity.

Time-dependent self-focusing (for a long laser pulse with a time-varying intensity) can be interpreted as the picture found for a steady-state beam which is moving through space in accordance with a change in the initial intensity. Experiments carried out to observe a spatially moving multi-focus structure in a nonlinear medium were described in Ref. 16. Studies of the self-focusing of light beams and pulses in various nonlinear media are reviewed in Refs. 17 and 18.

In this paper we examine a particular propagation re-

 0038-5646/92/040604-12$05.00

gime for an intense, ultrashort laser pulse. We assume that the inequality $L_d > L_{\text{pulse}}$ holds, where $L_{\text{pulse}} = ct_{\text{pulse}}$ is the spatial length of the laser pulse, t_{pulse} is the duration of the pulse, and $L_d = r_0^2/\lambda$ is the diffraction length corresponding to the pulse aperture r_0. The particular nature of the propagation of the pulse is determined by the nonlinear change in the dielectric properties of the medium over the length of the pulse. Nevertheless, the pulse propagates in a nonlinear fashion at distances $L_{\text{prop}} \gg L_d > L_{\text{pulse}}$. We will call this propagation regime of an ultrashort pulse "self-channeling."

Below we discuss a possible picture for the self-channeling of an intense ultrashort laser pulse. As it propagates, a pulse with a temporal length of a few hundred femtoseconds which extends spatially over distances on the order of a few tens of microns because of the nonlinear variation of the refractive index, causes a change in its own structure, losing some of its energy as a result of refraction. The pulse contracts toward the axis, with the result that its intensity increases manyfold. The self-consistent system which arises—the system consisting of the electromagnetic field plus a medium with altered dielectric properties—propagates without a refraction loss over substantial distances, many times as large as the spatial length of the pulse itself. (The propagation regime of an intense ultrashort pulse could apparently be linked with a three-dimensional soliton.)

One could imagine that an ultrashort laser pulse stretching out in the longitudinal direction is cut up into a set of layers oriented perpendicular to the light propagation direction. We assume that the medium responds instantaneously and the complex amplitude of the electromagnetic field varies slowly over distances on the order of the wavelength λ along the propagation direction and over times on the order of the period ω^{-1} of the rf oscillations. The problem of the nonlinear dynamics of the light concentrated in each of these layers can then be reduced to that of solving the nonlinear Schrödinger equation for a complex field amplitude. The effect of the medium on the light propagation is reduced along this approach to the specification of some type of nonlinearity in the equation. For an analysis of ultrashort laser pulses, the two-dimensional beam solutions of the nonlinear Schrödinger equation should be interpreted as trajectories described by thin transverse sections of the pulse.

For intense short pulses with a temporal length $\tau < 1$, the inertial mechanisms for the formation of a nonlinearity of the medium are inconsequential. Falling in the category of these inconsequential mechanisms are the thermal self-focusing and the mechanisms involving the onset of turbulence in the plasma. On the other hand, noninertial mechanisms may operate. One example is the relativistic nonlinearity which stems from the change in the rest mass of the electrons which are gyrating in the intense field at velocities approaching the velocity of light.[19] With certain reservations (spelled out in the text below), we could also include here the ponderomotive nonlinearity for electrons associated with their expulsion from the strong-field region by the force associated with the rf pressure.[20] Another noninertial mechanism is the Kerr nonlinearity which stems from the nonlinearity polarization of the electron shells of ions in an intense field.[21] For light substances (H, He, N, O, etc.), atoms can be stripped to a state of complete ionization at the pulse front. In this situation, the Kerr nonlinearity should be inconsequential. The dynamics of the pulse may also be affected by the quasi-static magnetic fields (with a corresponding scale time ω^{-1}) which arise as the plasma electrons move.

In this paper we present a theory for the self-channeling of intense, ultrashort, circularly polarized laser pulses in an initially homogeneous, cold, subcritical plasma and also in plasma formations ("plasmoids") of finite dimensions. The diffraction of the light and its refraction by transverse variations in the refractive-index profile of the medium are taken into account. These variations arise because of relativistic and ponderomotive nonlinearities. The ponderomotive effect involving electrons is assumed to be instantaneous, while the motion of the ions is ignored. The pulse is assumed to be stretched out along its propagation direction. The theory includes a simpler derivation of the nonlinear Schrödinger equation with a relativistic-ponderomotive nonlinearity, which was first established in Ref. 22. This nonlinearity falls in the category of dissipationless nonlinearities which are saturable. The theory includes the case of an initially inhomogeneous plasma, which was not discussed in Ref. 22.

We report the results of a detailed numerical simulation of the two-dimensional problem in an (r,z) geometry for the cases of a single relativistic nonlinearity and of a combined relativistic-ponderomotive nonlinearity. In each of these two cases, we analyze the normal modes of the nonlinear Schrödinger equation in the basic regimes of two-dimensional propagation. A new result found here is that two-dimensional solutions of the nonlinear Schrödinger equation for this nonlinearity asymptotically approach the lowest-lying normal mode of the nonlinear Schrödinger equation. The squared absolute value of this mode depends only on a single transverse coordinate and is localized in the axial region of the space. The relativistic nonlinearity dominates the initial concentration of the light, while the ponderomotive effect, which begins to operate at the first focus, predominantly forms a cavitation channel of the electron density, in which light concentrates and stabilizes the solution.

1. GENERAL QUESTIONS

We begin with a brief review of research on the electrodynamics of light in matter with relativistic effects in the motion of electrons. The many studies in which the motion of the electrons has been assumed to be nonrelativistic will of course remain outside our discussion.

The propagation of plane electromagnetic waves of relativistic intensity in a plasma was apparently first studied in Ref. 19. Equations derived there describe the propagation of the waves as a function of the single argument $\omega t - kz$. The problem was reduced to Lagrange form with two integrals of motion. Certain exact and approximate solutions were found.

Later, several studies[23-26] were carried out on the acceleration of charged particles by beats of two optical waves with relativistic intensities or by the plasma wake field which arises behind a single intense laser pulse. All these studies were carried out in plane geometry. We believe that a number of extremely important problems were left unresolved.

The motion of electrons in the electromagnetic field of a given pulse has also been studied. A solution for the case of motion in the field of a monochromatic plane wave was given in Ref. 27, for example. A further analysis of these solutions was offered in Refs. 28 and 29. An attempt was undertaken in Ref. 30 to study the motion of electrons in a field of a pulse with a given shape (the intensity was a bell-shaped function of the time), but inexact equations of motion were used for the analysis there.

The threshold for the filamentation of a monochromatic plane wave of relativistic intensity was derived in Ref. 31 from the well-known linear theory for the stability for the nonlinear Schrödinger equation. It was also derived in Ref. 32 on the basis of analytic calculations.

A nonlinear Schrödinger equation describing the self-focusing of an axisymmetric laser pulse in an initially homogeneous plasma due to two effects—the relativistic nonlinearity and the transverse ponderomotive effect—was derived in Ref. 22. The lowest normal mode of the nonlinear Schrödinger equation was found in the same study; the minimum power corresponding to this mode was calculated. Two-dimensional axisymmetric solutions of the nonlinear Schrödinger equation over short propagation distances were reported. These solutions were found for the case of small deviations of the initial conditions from the normal mode which was found and in the absence of a cavitation in the profile of the electron component. Analyzing the lowest normal mode of the nonlinear Schrödinger equation, Sun *et al.*[22] showed that the rf-pressure force can expel all the electrons from a certain spatial region (we recall that this effect is known in the literature as "electron cavitation"). On the other hand, those investigators were unable to describe electron cavitation in the derivation of two-dimensional solutions of the nonlinear Schrödinger equation. The problem was apparently a matter of mathematical difficulties and the complexity of choosing a stable difference scheme to describe the solutions of the nonlinear Schrödinger equation with a nonlinearity which has a discontinuity in its first derivative. (These difficulties have been overcome in the present study, and we will discuss arbitrary two-dimensional solutions of the nonlinear Schrödinger equation, taking electron cavitation into account.)

The normal mode of the nonlinear Schrödinger equation with a relativistic ponderomotive nonlinearity was found in Ref. 33 for the planar problem. In some previous studies,[34-36] we took up the case of two-dimensional axisymmetric self-focusing when the relativistic mechanism dominates (without the ponderomotive effect). We reported results on the stabilization of self-focusing in plasmoids. Analytic estimates on the propagation of picosecond pulses in cavitation channels of the electron-density profile with walls with a supercritical electron density were carried out in Ref. 37. In Refs. 38 and 39 we reported preliminary calculations on relativistic-ponderomotive self-focusing, and we reported stabilization of the solution and the formation of a cavitation channel. The threshold conditions for the onset of the relativistic-ponderomotive self-channeling of an ultrashort laser pulse were analyzed in Ref. 40.

1.1. Physical model

According to the present understanding, the following factors influence the nonlinear dynamics of an intense ultrashort laser pulse in the medium.

1) First, there is the shaping of the leading edge of the plasmoid pulse.

2) Second, there is the nonlinear variation of the dielectric constant of the medium. At least three mechanisms which lead to nonlinear changes of this sort are associated with the plasma electrons. These are the relativistic increase in the masses of electrons oscillating at velocities comparable to the velocity of light in the intense optical field[19] and the expulsion of free electrons by the ponderomotive force from the volume occupied by the intense field.[20] Since the laser pulse is short, the heavy ions are unable to undergo any substantial change in position during the pulse. The expelled electrons are partially confined by electrostatic forces resulting from the charge separation in the plasma. In the case of pulses which are stretched out, this expulsion of electrons occurs primarily in the direction transverse to the pulse propagation direction. Let us estimate the time scales of this process. The time over which an electron moves off a distance r_0 is $t \approx r_0/c$. Taking $r_0 = 3\cdot 10^{-4}$ cm, for example, we find $t = 10^{-14}$ s. The theory derived in this paper is thus valid under the condition $t < t_{\text{pulse}}$.

Yet another mechanism for nonlinear variation in the refractive index of the medium involves the ion dipole moments which are induced by the rf field.

It is possible that the decrease in the carrier frequency of the pulse due to the loss of energy by photons in the course of nonlinear interactions with plasma waves is important.[25]

3) Defocusing mechanisms have important effects on the pulse propagation dynamics. One defocusing mechanism is the diffraction of the light by the transverse aperture of the pulse, and another is the refraction by irregularities in the profile of the electron density.

4) Dissipation of pulse energy can also play an important role. The loss due to ionization of the gas, the conversion of electromagnetic-field energy into kinetic energy of the potential motion of electrons, the generation of harmonics, nonlinear scattering, and multiphoton inverse bremsstrahlung are examples of such dissipation mechanisms.

These considerations give an idea of just how complex this problem is. The analysis below is based on the following physical phenomena: (1) the nonlinear variation in the refractive index of the medium as a result of the relativistic increase in the mass of the electrons; (2) the nonlinear variation of the refractive index due to the transverse relativistic ponderomotive effect; (3) the initial bell-shaped profile of the electron density, which serves as a model for the nonlinear ionization of atoms at the pulse front; (4) the diffraction of the pulse by the transverse aperture; and (5) the refraction of the light by transverse irregularities in the refractive index of the medium which are caused by the first three of these factors. The interaction of the laser light with the plasma is examined in this paper over distances much shorter than the characteristic dissipation lengths for the pulse energy. The length scales for ionization dissipation are tens of centimeters. We can write the length scale for the ionizational loss at $L_{\text{ion}} \approx E_L/\pi r^2 N \varepsilon_{\text{ion}}$. If the energy of the laser pulse is $E_L \approx 0.1$ J (Refs. 1 and 2), if the channel radius is $r \approx 3\cdot 10^{-4}$ cm, if the density of gas particles is $N \approx 10^{20}$ cm^{-3}, and if the ionization energy is $\varepsilon_{\text{ion}} = 1.486\cdot 10^3$ eV (nitrogen), then this length is $L \approx 14.5$ cm.

The efficiency of collisional dissipation falls off with increasing field intensity and is of minor importance.[32] The efficiency with which pulse energy is converted into plasma waves is low if the pulse length does not coincide with the period of the plasma wave.[25] We will assume that this resonance does not occur.

Light waves of relativistic intensities can presently be generated experimentally only through sharp focusing of the beam from an excimer laser.[1,2] The typical parameter values in this case might be as follows: a peak intensity $I \approx 10^{18}$–10^{19} W/cm^2, a pulse length $\tau < 1$ ps, a pulse aperture (i.e., the transverse dimension of the pulse in the focal region) of 1–3 μm, a wavelength $\lambda = 0.248\ \mu$m (an excimer laser), and an initial gas pressure $\mathscr{P}_0 \approx 0.01$–$10$ atm in the chamber in which the interaction with the light occurs. The density of free electrons in such a system might be $N_e \approx 10^{18}$–10^{21} cm^{-3}. Under these conditions we could use the estimate $(\omega_p/\omega)^2 \ll 1$, where $\omega = 2\pi c/\lambda$ is the frequency of the laser light, and $\omega_p = (4\pi e^2 N_e/m_e)^{1/2}$ is the electron plasma frequency. For the parameter values listed above, for lengths $L \approx (100$–$1000)\lambda$ and for pulse apertures $r_0 \approx (4$–$12)\lambda$, the dominant role will be played by effects associated with the transverse nonuniformity of the pulse.

1.2. Basic equations

Let us consider the propagation of an intense, ultrashort, laser pulse in a plasma with an initial electron density distribution which is spatially nonuniform: $N_e = N_{e,0} f(\mathbf{r})$, $\max f(\mathbf{r}) = 1$. As usual, we denote the vector and scalar potentials of the electromagnetic field by $\mathbf{A}$ and φ, respectively. We denote by $\mathbf{p}_e$ the electron momentum. We assume that the ions are immobile. We can then write

$$\Box \mathbf{A} = \frac{1}{c} \nabla \frac{\partial \varphi}{\partial t} - \frac{4\pi}{c} \mathbf{j}, \tag{1}$$

$$\Delta \varphi = -4\pi \rho, \tag{2}$$

$$(\nabla, \mathbf{A}) = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + (\mathbf{v}_e, \nabla)\right) \mathbf{p}_e = -e\left(-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi + \frac{1}{c}[\mathbf{v}_e \times [\nabla \times \mathbf{A}]]\right). \tag{4}$$

$$\mathbf{j} = -eN_e \mathbf{v}_e, \quad \rho = e(N_{e,0} - N_e), \tag{5}$$

$$\mathbf{v}_e = \mathbf{p}_e/m_e, \quad m_e = m_{e,0}\gamma, \quad \gamma = (1 + |\mathbf{p}_e|^2/m_{e,0}c^2)^{1/2}. \tag{6}$$

Equations (1) and (2) are Maxwell's equations; Eq. (3) is the Coulomb gauge of the vector potential; Eq. (4) is the equation of motion of the electrons; Eq. (5) determines the current density and the charge density; Eq. (6) is the relativistic relationship between the momentum and velocity of an electron; $m_{e,0}$ is the rest mass of the electron; and $\Box = \Delta - c^{-2}\partial^2/\partial t^2$ is the d'Alembertian.

It is convenient to normalize the variables in these equations as follows:

$$\tilde{\mathbf{A}} = (e/m_{e,0}c^2)\mathbf{A}, \quad \tilde{\varphi} = (e/m_{e,0}c^2)\varphi, \quad \tilde{\mathbf{p}}_e = \mathbf{p}_e/m_{e,0}c, \quad \tilde{\mathbf{v}}_e = \mathbf{v}_e/c,$$
$$\tilde{N}_e = N_e/N_{e,0}, \quad \tilde{t} = ct. \tag{7}$$

[We immediately drop the tilde (˜).] Equation (4) can be rewritten in the equivalent form

$$\frac{\partial}{\partial t}(\mathbf{p}_e - \mathbf{A}) - [\mathbf{v}_e \times [\nabla \times (\mathbf{p}_e - \mathbf{A})]] = \nabla(\varphi - \gamma). \tag{8}$$

Following Ref. 22, we ignore the potential motion of the electrons; as a result we have $\mathbf{p}_e = \mathbf{A}$. We assume that the vector potential is circularly polarized:

$$\mathbf{A}(\mathbf{r}, t) = {}^1/_2[(\mathbf{e}_x + i\mathbf{e}_y)a(r, z, t)\exp[i(\omega t - kz)] + \text{c.c.}]. \tag{9}$$

The path traced out by an electron in the field of a linearly polarized plane wave, along the x axis, for example, is a figure-eight in the xz plane.[27] In this case of course, we need to take the electron momentum component p_z into account. In a circularly polarized plane wave, an electron traces out a circular orbit in the xy plane. This plane is perpendicular to the wave propagation direction.[27] In this case we do not need to consider the component p_z. In the field of a circularly polarized wave which is nonuniform in the transverse direction, the vector potential nevertheless acquires a z component. From the condition $\nabla \cdot \mathbf{A} = 0$ we find

$$|a_z| \approx \frac{1}{k}\frac{\partial |a|}{\partial r}.$$

This component is small ($|a_z| \ll |a|$) if the field in the transverse direction varies slowly over distances on the order of the wavelength. The A_z component leads to the appearance of a component $p_z \ll p_\perp$. In this case the theory can be derived without consideration of the small components A_z and p_z.

We assume that the length of the pulse is much greater than the lengths of the electromagnetic and plasma waves. In this case the following inequalities hold:

$$\frac{\partial}{\partial t}a, \quad \frac{\partial}{\partial z}a \ll ka, \quad k_p a,$$

where $k_p = \omega_{p,0}/c$, $k^2 = k_0^2 - k_p^2$, and $k_0 = \omega/c$. We use the notation $\omega_{p,0}^2 = 4\pi e^2 N_{e,0}/m_{e,0}$ for the unperturbed plasma frequency. On the other hand, we assume that the pulse is ultrashort in the sense that the ions are assumed immobile. Under the assumptions which we have made, Eqs. (1)–(8) become

$$\Box \mathbf{A} = k_p^2 N_e \gamma^{-1} \mathbf{A}, \tag{10}$$

$$\Delta \varphi = k_p^2 [N_e - f(\mathbf{r})], \tag{11}$$

$$\nabla(\varphi - \gamma) = 0, \tag{12}$$

$$\gamma = (1 + |\mathbf{p}_e|^2)^{1/2}, \quad \mathbf{p}_e = \mathbf{A}. \tag{13}$$

We have omitted a term $\partial\varphi/\partial t$ from (10) since γ and (therefore) φ do not contain an rf dependence for a circularly polarized wave. From (11) and (12) we find an expression for the electron density:

$$N_e = \max\{0, \ f(\mathbf{r}) + k_p^{-2}\Delta\gamma\}. \tag{14}$$

The logical expression max{0,...} imposes the condition $N_e \geqslant 0$. Equation (12) is the condition under which the ponderomotive and electrostatic forces are balanced. Combining (9), (10), and (14), we find an equation for the slowly varying complex amplitude of the vector potential:

$$\left[\frac{1}{v_g}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right]a + \frac{i}{2k}[\Delta_\perp a + k_p^2(1 - \gamma^{-1}\max\{0, f(\mathbf{r}) + k_p^{-2}\Delta_\perp\gamma\})a] = 0. \tag{15}$$

In the spirit of the assumptions made in the derivation of (15), we have ignored the second derivatives with respect to z and t. Here $v_g = c\varepsilon_0^{1/2}$ is the group velocity of the light in the unperturbed plasma, and $\varepsilon_0 = [1 - (\omega_{p,0}/\omega)^2]^{1/2}$ is the dielectric constant of the plasma. In the case at hand, of fairly long pulses, the relationship between the vector potential and the electric vector takes a particularly simple form:

$$\mathbf{E}\approx-{}^1/_2 ik_0(\mathbf{e}_x+i\mathbf{e}_y)a\exp[i(\omega t-kz)]+\text{c.c.}$$

We consider the dynamics of a thin cross section of the pulse, examining solutions of Eq. (15) along the characteristics. For this purpose we switch from the variable t to $q=t-z/v_g$. Equation (15) becomes

$$\frac{\partial}{\partial z}a+\frac{i}{2k}[\Delta_\perp a+k_p^2(1-\gamma^{-1}\max\{0,f(r)+k_p^{-2}\Delta_\perp\gamma\})a]=0. \qquad (16)$$

We examine the solutions of this equation for the case $q=$ const below (q is not included in the list of arguments of the function a). Equation (16) describes the spatially two-dimensional (r,z) dynamics of an ultrashort, circularly polarized, relativistic laser pulse in a cold, subcritical plasma. The following factors are taken into account: the relativistic nonlinearity (the factor γ), the relativistic transverse ponderomotive force (the factor $\Delta_\perp\gamma$), the inhomogeneous initial profile of the plasma electron density [the function $f(\mathbf{r})$], diffraction (the factor $\Delta_\perp a$), and refraction by irregularities in the refractive index of the medium (the term added to $\Delta_\perp a$ inside the square brackets). The possibility of complete electron cavitation is also taken into account (the logical expression).

2. NORMAL MODES OF THE NONLINEAR SCHRÖDINGER EQUATION

For an initially homogeneous plasma ($f=1$), Eq. (16) has axisymmetric particular solutions of the type

$$a_{s,n}=U_{s,n}(\rho)\exp[i(k_p^2/2k)(s-1)z],\quad \rho=k_pr, \qquad (17)$$

where the real function $U_{s,n}$ satisfies the ordinary differential equation

$$\Delta_\perp U_{s,n}+[s+F(U_{s,n}^2)]U_{s,n}=0. \qquad (18)$$

In this section of the paper, $\Delta_\perp$ means the Laplacian in terms of the argument ρ. The nonlinear term in the last equation is

$$F(U_{s,n}^2)=-\max\{0,\gamma^{-1}(1+\Delta_\perp\gamma)\}. \qquad (19)$$

The boundary conditions on Eq. (18) are $(\partial U_{s,n}/\partial r)(0)=0$, $U_{s,n}(\infty)=0$. The first of these conditions means smoothness at the origin; the second keeps the solution spatially localized. The importance of such solutions, which are called "normal modes" of the nonlinear Schrödinger equation, (16), will be demonstrated in the following sections of this paper.

We first consider the normal modes of Eq. (16) when we retain only one nonrelativistic nonlinearity; we discard the ponderomotive term $\Delta_\perp\gamma$, and we set $f=1$. In this case the equation for the function $U_{s,n}$ becomes

$$\frac{d}{d\rho}\left\{\frac{1}{2}\left(\frac{d}{d\rho}U_{s,n}\right)^2+V(U_{s,n},s)\right\}=-\frac{1}{\rho}\left(\frac{d}{d\rho}U_{s,n}\right)^2,$$
$$V(U,s)=(s/2)U^2-(1+U^2)^{1/2}+1. \qquad (20)$$

The second boundary condition can be satisfied only if the dynamic system described by the last two expressions has three points of rest. Since one of these points is the origin, this situation is possible for values $0<s<1$. In precisely the same way as in the classical theory of a medium with a quadratic nonlinearity,[5,41] the normal modes form a countable series which is ordered in terms of the number of zeros (n) for each value of the parameter s in the specified interval. Figure 1 shows the real amplitudes of the first four modes for the case $s=0.95$.

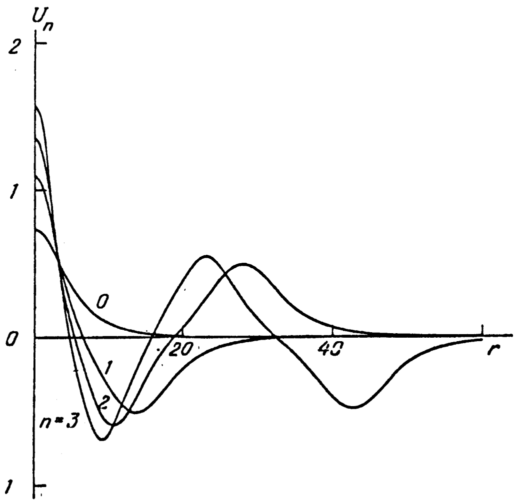

FIG. 1. Axisymmetric normal modes of the nonlinear Schrödinger equation for relativistic self-focusing with $s=0.95$.

For general problem (18), (19), the situation is similar to that for the particular case described above. The normal modes for the problem with a composite relativistic-ponderomotive nonlinearity for each $0<s<1$ also form a countable series in terms of the number of zeros. The amplitudes (and the corresponding transverse distributions of the electron density) for the zeroth, first, and second normal modes for the value $s=0.95$ are shown in Fig. 2. The higher modes, found for the first time in the present paper, have the following features: Cavitation can occur in the profile of the electron density for the higher modes even if there is no such cavitation for the zeroth mode. The higher modes may be unstable with respect to small radial perturbations of the amplitude. However, this question requires further study.

3. TWO-DIMENSIONAL MODEL

To analyze the two-dimensional problem, we renormalize the variables in Eq. (16):

$$r_1=\frac{r}{r_0},\qquad z_1=\frac{z}{2k_0r_0^2},\qquad u(r_1,z_1)=\frac{a(r,z)}{a_0},$$

where $a_0=\max|a(r,0)|$ for the given value of q. As a result, the following reduced mathematical problem arises [here $f_1(r_1)=f(r)$, and we omit the subscript 1 from r_1 and z_1]:

$$\frac{\partial u}{\partial z}+i\Delta_\perp u+iF[f_1(r),\ |u|^2]u=0,\qquad z>0, \qquad (21)$$

$$u(r,0)=u_0(r),\qquad \max_r|u_0|=1, \qquad (22)$$

$$\frac{\partial u}{\partial r}(0,z)=0,\qquad u(\infty,z)=0. \qquad (23)$$

The nonlinear term is a real-valued operator

$$F[f_1,\ \xi]=a_1[1-(1+a_2\xi)^{-1/2}\max\{0,\ f_1(r)+a_1^{-1}\Delta_\perp(1+a_2\xi)^{1/2}\}]. \qquad (24)$$

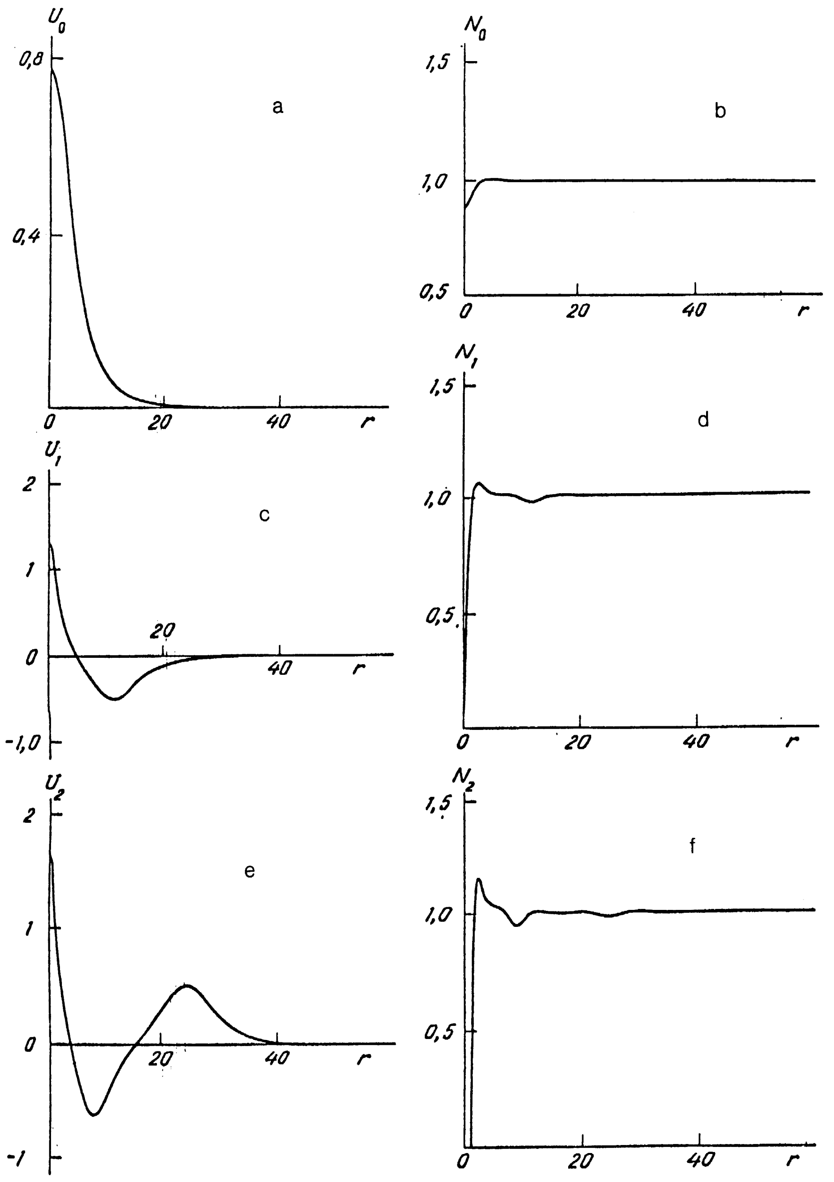

FIG. 2. Zeroth (a, b), first (c, d), and second (e, f) axisymmetric normal modes of the nonlinear Schrödinger equation for the case of relativistic-ponderomotive self-focusing, with $s = 0.95$. a, c, e—Distribution of the normalized amplitude; b, d, f—distribution of the normalized electron density.

The numerical parameters a_1 and a_2 are given by

$$a_1=(r_0k_p)^2,\ a_2=I_0/I_r,\ I_0=(c/4\pi)\,(k_0a_0)^2. \tag{25}$$

The "relativistic intensity" $I_r = m^2_{e,0}\omega^2_0c^3/4\pi e^2$ depends only on the frequency of the laser light.[19]

3.1. Initial conditions

In this paper we analyze the nonlinear propagation of laser light for pulses which enter the medium with an intensity which is a Gaussian or super-Gaussian function of the time and the radius:

$$I|_{z=0}=I_0(r,\ t)=I_m\exp\{-(t/\tau)^{N_1}-(r/r_0)^{N_2}\},\ N_1\geqslant 2,\ N_2\geqslant 2. \tag{26}$$

We assume that the peak intensity is $I_m \approx I_r \gg I^* = 10^{16}$ W/cm^2 (r and t are dimensional here). We write the initial transverse distribution of the pulse amplitude, at the entrance to the medium, as follows for problem (21)–(25):

$$u_0(r)=\exp(-r^{N_2}/2),\ N_2\geqslant 2. \tag{27}$$

The value of the numerical parameter a_2 in (25), which corresponds to the instantaneous intensity $I_0(t)$ of the input pulse at the beam axis is

$$a_2=\frac{I_0(t)}{I_r}=\frac{I_m}{I_r}\exp\Big[-\Big(\frac{t}{\tau}\Big)^{N_1}\Big]. \tag{28}$$

The transverse profile of the plasmoid, which models a nonlinear ionization of the gas at the pulse front, is specified to be

$$f(r)=\exp(-(r/r_*)^{N_3}),\ N_3\geqslant 2 \tag{29}$$

(r is a dimensional coordinate). The aperture of the plasmoid, r_*, is found from the relation

$$I_0(r_*,\ t_0)=I_0(t_0)\exp[-(r_*/r_0)^{N_2}]=I^*. \tag{30}$$

In the case of a Gaussian transverse intensity profile ($N_2 = 2$), for example, the aperture of the plasmoid for $I^* = 10^{16}$ W/cm^2, $I_r \approx 3\cdot 10^{20}$ W/cm^2, and $I_0(t_0) = 0.1 I_r$ is $r_* = 2.55 r_0$. It is then reasonable to adopt the homogeneous-plasma approximation, $f(r) \equiv 1$, as was shown by cal-

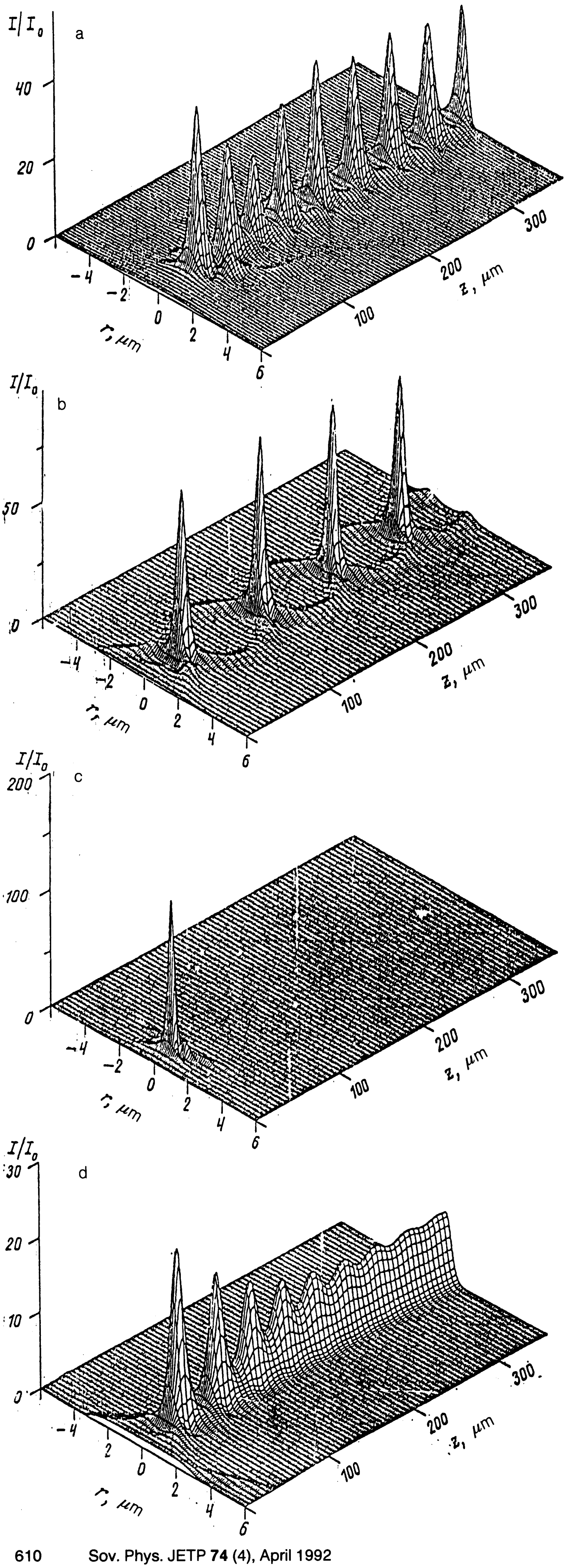

FIG. 3. Relativistic self-focusing ($I_0 = 3\cdot 10^{19}$ W/cm^2, $r_0 = 3$ μm, $\lambda = 248$ nm, $N_{e,0} = 7.5\cdot 10^{20}$ cm^{-3}). a: Intensity distribution during the propagation of a thin cross section of a pulse with a planar initial phase front [a Gaussian initial transverse distribution of the intensity, with $N_2 = 2$ in (30)], for a homogeneous plasma. b: The same, but for $N_3 = 8$. c: Regime of a single focus, for an initially focused phase front, with $R_f = R_{f,0}/2$ in (35), with $N_2 = 2$ in (30), for a homogeneous plasma. d: Formation of a quasistabilized regime along the plasmoid, with $N_3 = 8$, $r_* = r_0$, and $N_2 = 8$ in (30).

culations in Refs. 34–36. If the profile has a plateau ($N_2 = 8$), however, we find $r_* \approx 1.28 r_0$. In this case, the defocusing of the beam due to the coincidence of the apertures is important.

3.2. Relativistic self-focusing

Let us consider the solution of the problem (21)–(25) for the case in which the ponderomotive term $\Delta_\perp \gamma$ is omitted from (24):

$$F[f_1, \xi] = a_1[1 - f_1(r)(1+a_2\xi)^{-1/2}].$$

The relativistic nonlinearity outweighs the ponderomotive nonlinearity outside focal regions, i.e., under the conditions $a_1 \gg 1$, and $a_2 \approx 1$. Equations (21)–(23), with the nonlinear term written above, govern the self-focusing with a dissipationless saturation of the nonlinearity. The general features of the solution depend strongly on the values of two conserved integrals:

$$Q_1 = \int_0^\infty |u(r)|^2 r\, dr, \tag{31}$$

$$Q_2 = \int_0^\infty \left\{ \left| \frac{\partial u}{\partial r} \right|^2 - \Phi(r, |u|^2) \right\} r\, dr, \tag{32}$$

$$\Phi(r, \xi) = \int_0^\xi F(r, \eta) d\eta = a_1 \left\{ \xi - \frac{2}{a_2} f_1(r) [(1+a_2\xi)^{1/2} - 1] \right\}. \tag{33}$$

Figure 3, a–d, shows the results of two-dimensional numerical solutions of the problem of relativistic self-focusing. The parameters of the beam and the plasma in the cases shown here were as follows: $\lambda = 0.248\ \mu$m, $I_r = 1.34 \cdot 10^{20}$ W/cm^2, $I_0 = \frac{2}{9} I_r \approx 2.98 \cdot 10^{19}$ W/cm^2, $r_0 = 3\ \mu$m, and $N_{e,0} = 7.5 \cdot 10^{20}$ cm^{-3}. The corresponding values of the numerical parameters are $a_1 \approx 2.486 \cdot 10^2$ and $a_2 = 2/3$.

Figure 3a illustrates the propagation of the cross section of the pulse corresponding to that value of t for which the relation $I_0(t) = \frac{2}{9} I_r$ holds, along the z axis in a homogeneous plasma, for a Gaussian initial transverse intensity profile ($N_2 = 2$) and for a plane phase front.

Figure 3b is the corresponding diagram for the case in which the initial transverse intensity distribution has a plateau ($N_2 = 8$).

For the values of the parameters a_1 and a_2 under consideration here, and for the initial distributions which we are considering, the relation $Q_2 < 0$ holds. In this case the following estimate is valid:

$$\max_r |u(r,z)|^2 > (4/a_1 a_2) |Q_2| / Q_1. \tag{34}$$

In other words, the solution cannot approach zero asymptotically. This fact can be proved by analogy with Ref. 12. We see that the relativistic self-focusing of intense light in a dissipationless medium under the condition $Q_2 < 0$ leads to a sequence of foci and intensity rings, which trade places with each other. Figure 3, a and b, illustrates the formation of this regime. In the particular cases shown in Fig. 3, a and b, respectively 50% and 90% of the initial power is captured.

Estimates like that in (34) cannot be found in the case $Q_2 \geqslant 0$. Let us model the case $Q_2 \geqslant 0$, considering beams which are initially focused by a lens as they enter the medium. The focal length of the lens is then $R = k_0 r_0^2 R_f$. The corresponding initial condition for the entrance amplitude of the problem then takes the form

$$u_0(r) = \exp(-r^{N_2}/2 + ir^2/2R_f), \quad N_2 \geqslant 2, \quad R_f > 0. \tag{35}$$

We assume $Q_2 = 0$ for $R_f = R_{f,0}$. The case $Q_2 > 0$ corresponds to a high degree of initial focusing of the beam: $R_{f,0} > R_f > 0$. If $R_{f,0} < R_f \leqslant \infty$, then $Q_2 < 0$. Below, the modeling of focused beams is carried out only for a Gaussian transverse distribution of the initial intensity.

Figure 3, a and b, shows two-dimensional intensity distributions for the relativistic self-focusing of beams in a homogeneous plasma with $R_f = +\infty$ and $R_f = R_{f,0}/2$, respectively. The dynamics of the transition to the single-focus regime is studied in more detail in Ref. 36. Calculations show that the value $Q_2 = 0$ is not a threshold that strictly separates these two regimes of relativistic self-focusing. The transition from a sequence of foci and intensity rings to a single focus with decreasing focal length of the lens occurs gradually, through an increase in the intensity of the first focus, a shift of this first focus toward the entrance to the medium, shifts of the other foci in the opposite direction, and a diffusive spreading of these other foci.

Figure 3d shows the distribution of the wavefront with $N_2 = 8$ along a plasmoid with $N_3 = 8$ and $r_* = r_0$. Comparison with Fig. 3, b and d, leads to the conclusion that defocusing has a substantial effect on the dynamics of the wavefront along a plasmoid whose aperture is the same as that of the pulse. Part of the beam is torn off and scattered at the periphery. For the rest of the beam, a balance is struck between self-focusing and defocusing. The power captured in the quasistabilized propagation regime is about 25% of the initial beam power.

3.3. Relativistic-ponderomotive self-focusing

Two-dimensional numerical calculations on the overall problem, (21)–(25), show that when a beam with an initially Gaussian transverse intensity distribution and a plane phase front propagates into the interior of a homogeneous plasma with $\lambda = 0.248\ \mu$m, $I_0 = \frac{2}{9} I_r = 2.98 \cdot 10^{19}$ W/cm^2, $r_0 = 3\ \mu$m, and $N_0 = 7.5 \cdot 10^{20}$ cm^{-3} (Fig. 4), cavitation arises as soon as the first focus appears at the axis. In other words, the plasma electrons are completely expelled from the region occupied by the strong field (Fig. 4b). The cavitation channel then undergoes gradual stabilization along the propagation axis. About 45% of the total power of the propagating beam is in the first focus. Part of the beam power is dissipated at the periphery, while the rest is captured in a pulsating annular structure (Fig. 4a). The power of the paraxial and annular structures ranges up to 67%. Beyond the first focus, we observe an exchange of energy between the paraxial and annular structures, until the annular structure is dissipated at the periphery in the course of the pulsations. A significant fraction of the power of the annular structure is also ultimately captured in the paraxial region. Thereafter, in the course of the stabilization of the solution, some of the power gradually drains to the periphery, and the value of the captured power approaches an asymptotic value of 46%.

In the transitional stage we see some interesting features in the profile of the electron density. The appearance of

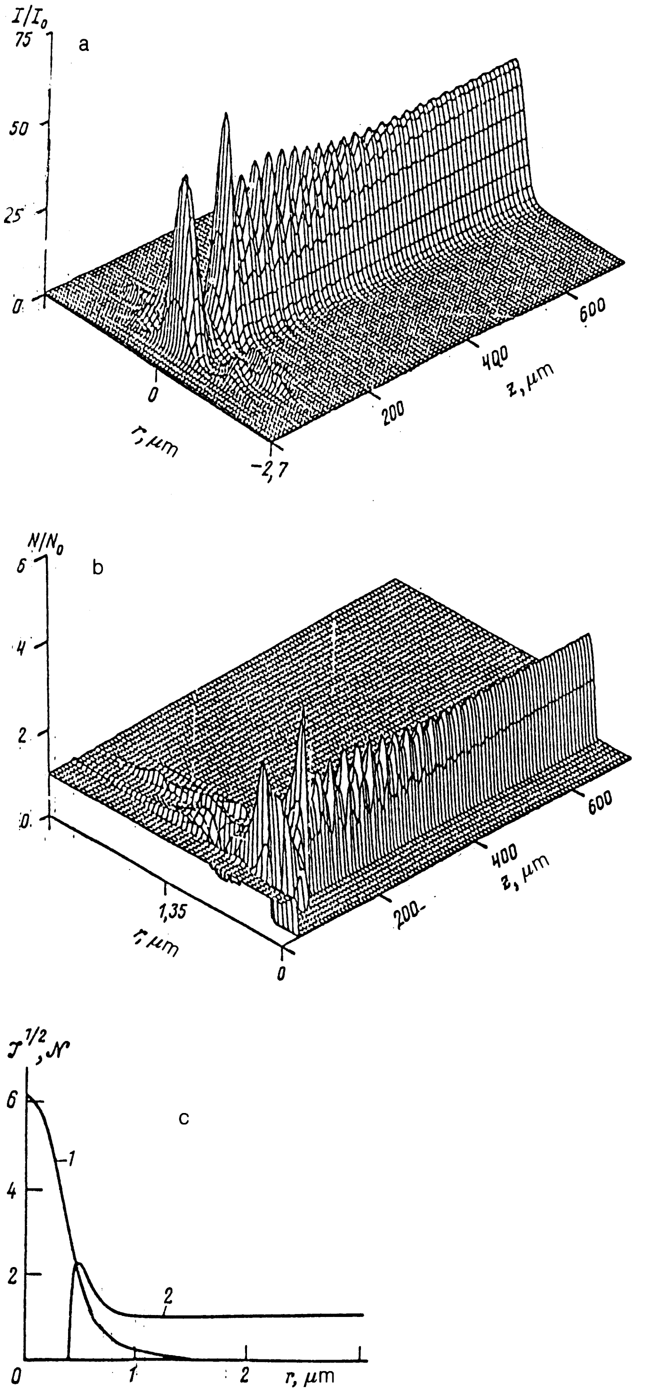

FIG. 4. Relativistic-ponderomotive self-focusing. These results correspond to the nonlinear propagation of a thin cross section of a pulse with an initially Gaussian transverse intensity profile and a plane wavefront in an initially homogeneous plasma ($I_0 = 3\cdot 10^{19}$ W/cm^2, $r_0 = 3$ μm, $\lambda = 248$ nm, $N_0 \equiv N_{e,0} = 7.5\cdot 10^{20}$ cm^{-3}). a—Distribution of the normalized intensity; b—distribution of the normalized electron density; c—radial profile of the asymptotic solution for the normalized amplitude, $\mathcal{I}^{1/2} = (I_s(r)/I_0)^{1/2}$ (1), and of the normalized electron density, $\mathcal{N} = N_s(r)/N_0$ (2), for $s = 0.554$.

an intensity ring beyond the first focus leads to a second, annular, dip on the profile of the electron density (Fig. 4b). The refraction of the light at the outer wall of this dip sends the intensity ring back into the paraxial region. The ponderomotive force thus acts as an additional mechanism to cause self-focusing of the pulse. It leads to the formation of a paraxial focal structure which is stabilized along the propagation axis.

An important fact, reported here for the first time, is that the amplitude distribution $u(r,z)$ tends asymptotically at large z toward the lowest normal mode $U_s(r)$ of the nonlinear Schrödinger equation. In the case at hand, we have $s \approx 0.554$. Figure 4c shows normalized asymptotic amplitudes $U_s(r)$ and a profile of the plasma electron density, $N_s(r)/N_0$. These profiles were found on the basis of two-dimensional calculations at distances up to $z \approx 900$ μm; they differ from the exact normal modes by less than 1%.

Note that for the ranges of parameter values studied here the displacement of the charges (the ponderomotive effect) has a strong influence on the nature of the nonlinear propagation of the light in the plasma beyond the first focus. The nonlinear regime consisting of a sequence of foci and intensity rings characteristic of relative self-focusing without a ponderomotive effect (Fig. 3, a and b), gives way in this case to a stabilized focal structure of the intensity. A cavitation channel forms simultaneously.

We have also carried out some two-dimensional calculations on the propagation of beams with initially plateau-shaped transverse intensity distributions and a plane or focused phase front in a homogeneous plasma and along plasmoids. In all cases, we again observed that the solutions approach an asymptotic form coresponding to the lowest normal modes of the nonlinear Schrödinger equation. The effect of the displacement of the charges (the ponderomotive effect) on the propagation of the light is so strong that self-channeling arises even in the case of extremely focused beams. Figure 5 shows the results of corresponding calculations for $R_f = R_{f,0}/2$ in (35) with a Gaussian profile [$N_2 = 2$ in (30)]. For this version, a single-focus regime arises in the case without a ponderomotive effect (Fig. 3c).

3.4. Critical power for relativistic-ponderomotive self-focusing

Our basic purpose in this subsection of the paper is to determine the conditions for the occurrence of relativistic-ponderomotive self-focusing. A study shows that these conditions include a threshold in the power and that they are different for (in particular) beams with an initially plane phase front and for initially focused (or defocused) beams.

We define the threshold for the relativistic-ponderomotive self-focusing of a beam as the threshold for the change in the large-z asymptotic behavior, from a zero profile (the subcritical case) to the profile determined by the lowest normal mode of the nonlinear Schrödinger equation.

A characteristic feature of this problem is that the power of the zeroth normal mode,

$$P_s = 2\pi \int_0^\infty U_{s,0}^2(\rho)\rho\, d\rho, \tag{36}$$

depends on the value of the parameter z; more precisely, it decreases with increasing s in the interval $0 < s < 1$. In this regard, the problem at hand differs from the classical case of a medium with a quadratic nonlinearity. Since our problem is conservative, a necessary condition for the occurrence of self-focusing is that the initial power P_0 be above the point of the lower bound (P_s) in terms of s:

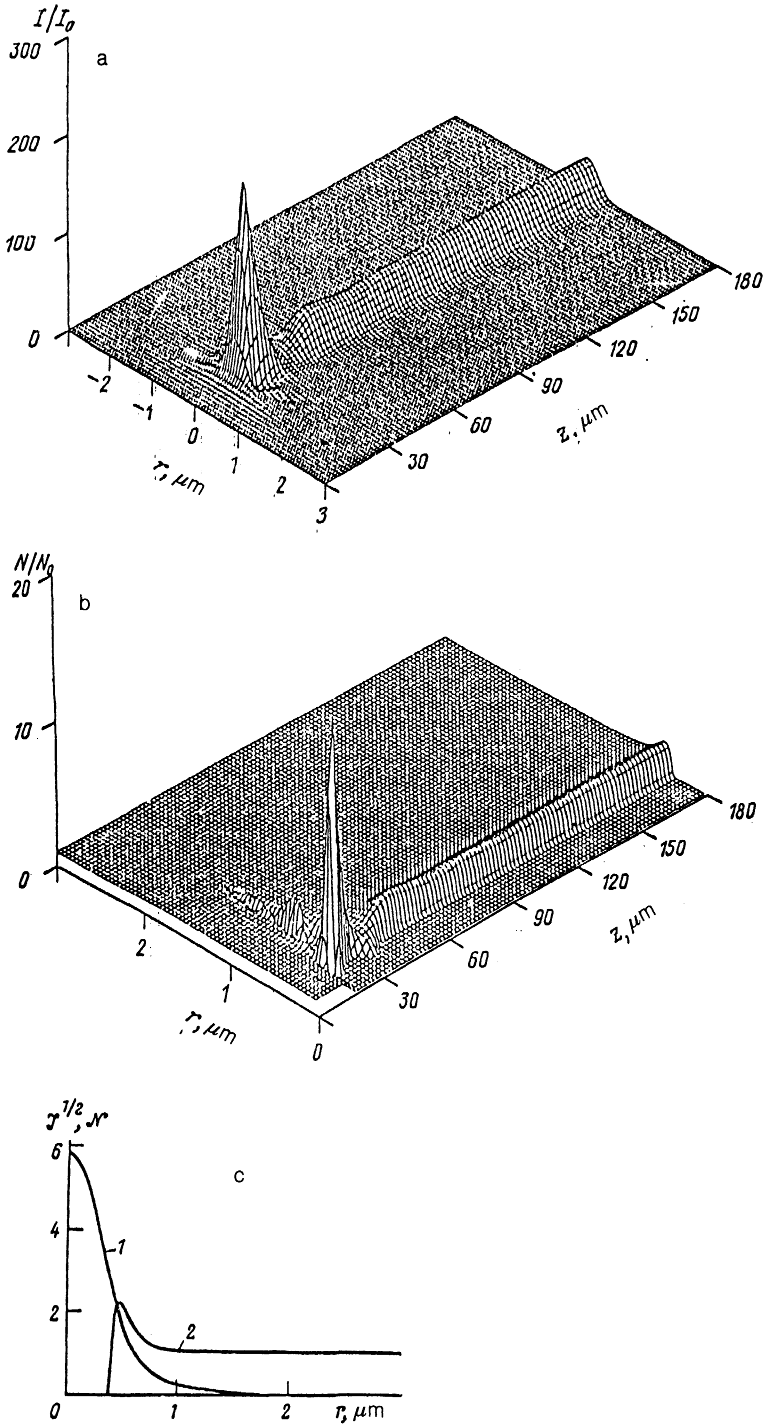

FIG. 5. Relativistic-ponderomotive self-focusing. These results correspond to the nonlinear propagation of a thin cross section of a pulse with a Gaussian ($N_2 = 2$) initial transverse intensity profile and a focused phase front [$R_f = R_{f,0}/2$ in (35)] in an initially homogeneous plasma ($I_0 = 3\cdot 10^{19}$ W/cm^2, $r_0 = 3$ μm, $\lambda = 248$ nm, $N_0 \equiv N_{e,0} = 7.5\cdot 10^{20}$ cm^{-3}). a—Distribution of the normalized intensity; b—distribution of the normalized electron density; c—radial profile of the asymptotic solution for the normalized amplitude $\mathcal{T}^{1/2} = (I_s(r)/I_0)^{1/2}$ (1) and of the normalized electron density $\mathcal{N} = N_s(r)/N_0$ (2), for $s = 0.566$.

$$P_0 > \inf_{0<s<1} P_s = \lim_{s\to 1-0} P_s.$$

We can determine the value of this exact lower bound, which we will call the "critical power":

$$P_{cr} = \inf_{0<s<1} P_s = \lim_{s\to 1-0} P_s. \qquad (37)$$

Using

$$U_{s,0}(\rho) \underset{s\to 1-0}{\approx} U_0(\rho), \qquad (38)$$

where $U_0(\rho)$ is a positive, monotonically decreasing solution (it has no zeros at finite ρ) of the boundary-value problem

$$\nabla_\perp^2 U_0 - \varepsilon U_0 + {}^1/_2 U_0^3 = 0, \quad (dU_0/d\rho)(0) = 0, \quad U_0(\infty) = 0,$$

where we have introduced $\varepsilon = 1 - s$. Making the standard replacement for the case of a cubic nonlinearity, we verify that we have

$$U_0(\rho) = (2\varepsilon)^{1/2} g_0(\varepsilon^{1/2}\rho), \qquad (39)$$

where g_0 is the well-known Townes mode, which is a positive, monotonically decreasing solution (which has no zeros at finite ρ) of the boundary-value problem

$$\nabla_\perp^2 g_0 - g_0 + g_0^3 = 0, \quad (dg_0/d\rho)(0) = 0, \quad g_0(\infty) = 0. \qquad (40)$$

Using (37)–(40), we find

$$P_{cr}=\inf_{0<s<1} P_s=\lim_{s\to 1-0} P_s=2\pi\int_0^\infty U_0^2(\rho)\rho\,d\rho$$
$$=4\pi\int_0^\infty g_0^2(\rho)\rho\,d\rho=2P_{cr,c}, \tag{41}$$

where

$$P_{cr,c}=2\pi\int_0^\infty g_0^2(\rho)\rho\,d\rho$$

is the classical critical power for cubic Kerr self-focusing.[5]

Solving problem (40) numerically, it is sufficient to determine P_{cr}: $P_{cr}\approx 2\pi\cdot 3.72451\approx 23.4018$. A value $P_{cr}/2\pi=3.72$ was found numerically in Ref. 22 as the approximate value

$$\lim_{s\to 1-0}\int_0^\infty U_{s,0}^2(\rho)\rho\,d\rho.$$

Switching to dimensional variables, we find the following expression for the power P_{cr} (in Watts):

$$P_{cr,1}=\frac{m_{e,0}^2c^5}{e^2}\left(\frac{\omega}{\omega_{p,0}}\right)^2\int_0^\infty g_0^2(\rho)\rho\,d\rho$$
$$\approx 1{,}6198\cdot 10^{10}\left(\frac{\omega}{\omega_{p,0}}\right)^2\approx 1{,}62\cdot 10^{10}\frac{N_{cr}}{N_e}, \tag{42}$$

where the numerical constant is a minor refinement of that in Ref. 22.

There is of course the question of sufficient conditions for the relativistic-ponderomotive self-focusing. The calculations of Ref. 40 show that self-focusing of beams which enter an initially homogeneous plasma with a plane phase front occurs if the power of these beams exceeds the critical power (42). A sufficient condition for the relativistic-ponderomotive self-focusing of beams with an arbitrary phase profile in an initially homogeneous plasma is that the Hamiltonian of the corresponding purely relativistic problem[40] be negative for the initial transverse distribution of the amplitude of the laser pulse.

6. CONCLUSION

We have studied the self-channeling of an intense, ultrashort laser pulse in a cold, subcritical plasma by solving a nonlinear Schrödinger equation incorporating a relativistic-ponderomotive nonlinearity. Let us summarize the results.

It has been found theoretically that a substantial fraction of the power of an arbitrary two-dimensional solution of the nonlinear Schrödinger equation with a relativistic-ponderomotive, dissipationless, saturable nonlinearity concentrates asymptotically in an axial focal region. At the same time, a cavitation channel forms on the profile of the electron density. The light concentrates in this channel. Mathematically, this effect corresponds to the asymptotic approach of an arbitrary two-dimensional solution of the nonlinear Schrödinger equation to the lowest normal mode of this equation. The square of the absolute value of this mode is spatially localized. The corresponding mathematical fact that a two-dimensional solution of the nonlinear Schrödinger equation approaches a normal mode of this equation in the case of a dissipationless, saturable instability of simple algebraic form had been established previously.[14] This study has thus revealed yet another, and more complex, example which illustrates that a two-dimensional solution of the nonlinear Schrödinger equation approaches the lowest normal mode of this equation.

We have analyzed the normal modes of nonlinear Schrödinger equations with a relativistic nonlinearity and with a relativistic-ponderomotive nonlinearity. The higher modes of this equation have been found here for the first time. Also for the first time, we have carried out extensive numerical calculations on a two-dimensional problem for various initial transverse distributions of the intensity and the phase. We have discussed cases in which light propagates in the interior of an initially homogeneous plasma and in which the light propagates along plasmoids. Calculations for a situation without a ponderomotive effect reveal a set of two-dimensional solutions corresponding to nonlinear propagation of the light. These solutions are possible for relativistic self-focusing. They consist of a sequence of foci and intensity rings, a single-focus regime (for beams which are focused to a sufficiently great extent) and a regime of quasi-stabilized propagation (along a plasmoid). In the case of the relativistic-ponderomotive nonlinearity (the complete problem), the primary regime of two-dimensional propagation is one in which the solution asymptotically approaches the lowest normal mode of the nonlinear Schrödinger equation. This tendency has been observed even for extremely focused beams.

We have few comments regarding the critical power. Sun *et al.*[22] introduced the concept of the critical power for relativistic-ponderomotive self-focusing as the lower limit on the power concentrated in the zeroth normal mode of the nonlinear Schrödinger equation. Before that definition can be accepted as correct, however, it is necessary to prove that two-dimensional solutions converge on the zeroth normal mode. Sun *et al.*[22] tacitly avoided this question, so until now their definition of the critical power could not be regarded as having a solid foundation. In the present paper we have made a special study of this question. We have demonstrated by numerical methods that fairly arbitrary two-dimensional solutions do indeed converge on the zeroth normal mode of the nonlinear Schrödinger equation with a relativistic-ponderomotive nonlinearity. Consequently, the surmise by Sun *et al.*[22] that it is legitimate to introduce a critical power in accordance with Eq. (37) has been proven correct in the present paper. Furthermore, it has been found possible to prove relation (41), a curious one, which relates the critical power for the relativistic-ponderomotive self-focusing to the critical power for the Townes mode, (40).

We would like to call attention to the following circumstance. We have discussed two-dimensional solutions of the nonlinear Schrödinger equation which can be interpreted in two ways. First, they can be thought of as solutions for steady-state beams applied to the entrance to a nonlinear medium. Second, they can be thought of as trajectories traced out by thin cross sections of a laser pulse which varies in time. The second of these interpretations of the two-dimensional solutions is correct in a study of the self-channeling of ultrashort pulses.

If we imagine that the initial pulse has been sliced up into a set of thin layers oriented perpendicular to the light propagation direction, and if we then solve the two-dimensional nonlinear Schrödinger equation for each such layer, we can reconstruct the spatially three-dimensional dynamics of the overall pulse. Results found above provide support for the hypothesis that a three-dimensional soliton can exist. In other words, they support the possibility that there can exist a self-consistent physical object consisting of "light plus a medium with altered dielectric properties" and that this object is capable of propagating substantial distances without a refractive loss. In order to pursue this hypothesis, however, it will be necessary to solve a problem which incorporates the second and cross derivatives $\partial^2/\partial t^2$, $\partial^2/\partial z^2$, $\partial^2/\partial t\partial z$ in Eq. (15) as well as physical dissipation of the pulse energy.

We note in conclusion that a practical realization of the relativistic-ponderomotive self-channeling of an intense, ultrashort laser pulse would make it possible to increase the intensity of this pulse by a large factor, to 10^{20}–10^{21} W/cm^2, and to produce matter in the form of multiply charged ions in an ultrastrong electromagnetic field. There would be essentially no electrons inside the cavitation channel. Such a situation would be of great interest from the standpoint of fundamental physics and also from the standpoint of applications, e.g., from the standpoint of the x-ray laser problem.[42]

We wish to thank G. A. Askar'yan for useful discussions.

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Translated by D. Parsons