# Parametric excitation of magnetoelastic waves in single crystals of $CoCO_3$ and $FeBO_3$

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Parametric excitation of quasiphonons has been observed in the easy-plane antiferromagnets $CoCO_3$ and $FeBO_3$ at all orientations of the static and microwave magnetic fields lying in the basal plane of the crystal. It has been established that the type of excited phonons depends on the angle between the static and microwave magnetic fields. This does not agree with the theory that models the system as an elastic-isotropic medium. The antiferromagnetic resonance spectrum in $CoCO_3$ has been measured in detail in weak fields. Formulas have been obtained for the parametric resonance thresholds of the phonons with account taken of the rhombohedral symmetry of the investigated crystals, thus allowing a qualitative explanation of the experimental data. The coupled magnetoelastic-wave spectrum parameters brought about by the nonlocal nature of the indirect elastic interactions have been refined. A model of quasiphonon relaxation by quantum noise from the magnetic system is proposed.

## 1. INTRODUCTION

Studies of magnetic and magnetoelastic excitations in magnetically ordered dielectrics are interesting both from the viewpoint of learning more about the properties of specific materials and their applications and from the viewpoint of understanding the most general questions of the physics of nonlinear wave processes.

One of the main methods of investigating spin and magnetoelastic waves in antiferromagnetics is the parametric-resonance method, in which the external magnetic microwave field $h\cos\omega_p t$ excites a pair of waves with antiparallel wave vectors ($\mathbf{k}$ and $-\mathbf{k}$) and with the sum of frequencies equal to $\omega_p$ (see, e.g., the monograph by L'vov[1]). Instead of the general case of "Raman" pumping one most frequently encounters the case of "degenerate" pumping, in which waves of one branch of the spectrum are generated at the half-frequency ($\omega_p/2$). The main advantages of the parametric resonance method are that it excites a narrow wave packet ($\Delta k \ll k$) and makes it possible to determine the relaxation rates ($\gamma_{1\mathbf{k}}$ and $\gamma_{2\mathbf{k}}$) of the parametric waves from the magnitude of the threshold field $h_c$ at which the instability develops. For Raman pumping

$$h_c=\min[(\gamma_{1\mathbf{k}}\gamma_{2\mathbf{k}})^{1/2}/\mu_{eff}], \tag{1a}$$

and for degenerate pumping

$$h_c=\min(\gamma_{\mathbf{k}}/\mu_{eff}). \tag{1b}$$

Here $\mu_{\text{eff}}$ is the effective coupling coefficient of the waves with the microwave field.

The present article is dedicated mainly to an investigation of the parametric resonance of magnetoelastic waves in antiferromagnets with magnetic anisotropy of the "easy plane" type, specifically the "low-temperature" antiferromagnet $CoCO_3$ (Néel temperature $T_N \approx 18$ K) and the "high-temperature" antiferromagnet $FeBO_3$ ($T_N \approx 348$ K). These crystalline materials have identical rhombohedral symmetry ($D_{3d}$), but different ratios between the velocities of the sound ($v_s$) and of the spin waves ($v_m$) (in the linear portion of the spectrum): $v_s \gg v_m$ for the first and $v_s < v_m$ for the second compound.

The distinguishing feature of easy-plane antiferromagnets is the presence of a low-activation–quasiferromagnetic ($f$) branch of the spin wave spectrum and the so-called exchange amplification effect of the magnetoelastic interactions (we refer the reader to the reviews by Ozhogin and Preobrazhenskiĭ,[2,3] where the necessary references to the original works can be found). The magnetoelastic interaction leads to strong mixing of the initially "pure" quasiferromagnetic and elastic modes, as a result of which new normal quasimagnetic and quasielastic modes arise. Thus, under the action of a variable magnetic field, which is bringing about intense oscillations of the magnetization, elastic oscillations are also excited. In what follows we will refer for simplicity to the quasiparticles corresponding to the normal quasielastic modes simply as phonons.

Depending on the mutual orientation of the static ($\mathbf{H}$) and microwave ($\mathbf{h}(t)$) magnetic fields in the basal plane of the crystal, one can distinguish two methods of parametrically exciting the waves: perpendicular pumping ($\mathbf{H}\perp\mathbf{h}$) and parallel pumping ($\mathbf{H}\parallel\mathbf{h}$). These two methods differ fundamentally by the mechanism of coupling of the microwave field to the excited phonons. In the case of perpendicular pumping the alternative field linearly excites on the line wing, homogeneous oscillations of the quasiferromagnetic branch of the spectrum. Thanks to the nonlinear magnetoelastic interaction these oscillations generate phonons above some threshold. In the case of parallel pumping, on the other hand, the energy of the microwave field is pumped into the magnetic (and from there into the elastic) subsystem both through linear excitation of the homogeneous oscillations on the line wing of the quasi-antiferromagnetic branch and through modulation of the quasiferromagnon spectrum. In other words, the coupling coefficients $\mu^{\parallel}_{\text{eff}}$ and $\mu^{\perp}_{\text{eff}}$ are formed by different interactions of the pump field with the magnetic and magnetoelastic subsystem of the crystal. For

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this reason it is interesting to examine the parametric excitation of the phonons both for perpendicular and for parallel mutual orientations of the fields.

Under perpendicular pumping conditions parametric excitation of phonons in $CoCO_3$ (at the frequency $\omega_p \approx 2\pi \cdot 50$ GHz) was observed by Borovik-Romanov and coworkers,[4-6] and in $FeBO_3$ (at the frequency $\omega_p \approx 2\pi \cdot 10$ GHz) by Wettling and coworkers.[7-9] It was found that in both antiferromagnetics, at $\mathbf{h} \perp \mathbf{H}$, transverse phonons with frequency $\omega_{\text{ph}} = \omega_p/2$ are excited above some threshold way (i.e., the case of degenerate pumping is realized); however, the dependence of the threshold field on the experimental parameters was not examined in detail in the above-mentioned studies.[1)]

Subsequently, Kotyuzhanskiĭ and Prozorova[11,12] measured the temperature and field dependences of $h_c^{\perp}$ in $FeBO_3$ at the frequency $\omega_p \approx 2\pi \cdot 35$ GHz and on the basis of the improved formula of Lutovinov and Savchenko[13] derived an estimate for the relaxation rate of phonons parametrically excited by the transverse pumping.

Parallel pumping of phonons in antiferromagnets had up until now been observed only in $FeBO_3$.[14,15] It was discovered that in this case transverse phonons with frequency $\omega_{\text{ph}} = \omega_p/2$ are also excited, and detailed measurements of the threshold $h_c^{\parallel}$ were carried out over a wide range of frequencies ($\omega_p/2\pi \approx 0.5$–$1.6$ GHz), temperatures, and magnetic fields, and the connection between the amplitude of the threshold field and the phonon relaxation rate was experimentally established. In the present article we report the first observation of parallel pumping of phonons in $CoCO_3$.

Formulas for the threshold amplitudes $h_c$ for parallel and perpendicular pumping of phonons were calculated in Refs. 16 and 13, respectively. The simplified model of an elastic-isotropic continuous medium was used here (in essence, the single-mode approximation), omitting the details of the anisotropy of the elastic interactions. It turned out that despite the difference in the phonon excitation mechanisms for $\mathbf{H} \| \mathbf{h}$ and $\mathbf{H} \perp \mathbf{h}$, both thresholds coincide in the limiting case when $\omega_p \ll \omega_{f0}$ and $H \ll H_D$ and can be represented in the form

$$h_c^{\parallel} = h_c^{\perp} = \frac{2}{\mu^2} \frac{\omega_{f\mathbf{k}}^4}{J_0 H_D \omega_p} \frac{\rho V_0 v_s^2}{\Theta^2} \gamma_{\mathbf{k}}. \tag{2}$$

Here and below we set $\hbar = 1$,

$$\omega_{f\mathbf{k}} = \mu [H(H+H_D) + H_\Delta^2 + H_{\Delta a}^2 + (\alpha k)^2]^{1/2} \tag{3}$$

is the frequency of the quasiferromagnetic ($f$) branch of the magnon spectrum; $H_D$ is the Dzyaloshinskiĭ field; $\mu \equiv g\mu_B$, $g$ is the Landé factor, $\mu_B$ is the Bohr magneton; $(\mu H_\Delta)^2$ is the contribution of the magnetoelastic interaction to the spin waves energy gap, and $(\mu H_{\Delta a})^2$ is the magnetic anisotropy in the basal plane of the crystal; $\alpha \equiv v_m/\mu$ is the inhomogeneous exchange constant; $J_0 = H_E/S$, $H_E$ is the homogeneous exchange field, $S$ is the spin of the electron shell; $\Theta$ is the contant of the magnetoelastic interaction; $\rho$ is the density, and $V_0$ is the volume of the basal cell of the two-sublattice antiferromagnet.

Thus, according to expression (2) the threshold field of parametric excitation of phonons should not depend on the mutual orientation of the static and microwave magnetic fields. The main task of the present work was a study of parametric resonance of phonons in cobalt carbonate and iron borate at different mutual orientations of the alternating and constant magnetic fields in the basal plane of the crystal, with the goal of comparing these thresholds, determining the phonon relaxation rates, and finding an adequate theoretical description of the experimental results.

## 2. EXPERIMENTAL TECHNIQUE

The main measurements of the threshold of phonon parametric resonance in this work were carried out on an antiferromagnetic single crystal of $CoCO_3$. The sample was cut in the shape of a disk of thickness 0.4 mm and diameter 2 mm. Some of the experiments were performed on a naturally faceted lamina of the antiferromagnet $FeBO_3$ of thickness 1.3 mm (the same sample as was investigated in Refs. 14 and 15). Both crystals had identical rhombohedral symmetry (group $D_{3d}$). After antiferromagnetic ordering the magnetic moments of the sublattices lie in the basal plane perpendicular to the threefold "difficult" axis $C_3$. The Dzyalozhinskiĭ-Moriya interaction leads to a "sloping" of the spins, as a result of which there appears a weak ferromagnetic moment lying in the basal plane. The basal plane of the sample coincided with the planes of the disk and the lamina.

Parametric excitation of the phonons was investigated with a decimeter-band spectrometer.[17] A helical cavity with loaded $Q \approx 1000$ was used as the resonant absorbing cell. The sample was fastened to a teflon holder with the help of a pocket made of cigarette paper. The cavity with the sample was placed in liquid helium (the experiment was carried out at $T = 1.5$–$4.2$ K) or in gaseous helium (at $T = 77$ K). Parametric excitation of the phonons was detected in a pulsed microwave regime by the appearance of a characteristic distortion in the shape of the pulse after passage through the cavity. The pulses had 300 $\mu$sec duration and a repetition frequency of 50 Hz. The relative accuracy of measurement of the threshold $h_c$ at a fixed pumping frequency was 5%, and the absolute accuracy of measurement was 30%.

Parametric excitation of phonons was observed at any geometry of the static and microwave magnetic fields lying in the basal planes of the crystals. The pump threshold was strongly anisotropic and depended both on the direction of the static field $\mathbf{H}$ with respect to the crystallographic twofold axes, apparently due to the presence of magnetic anisotropy in the basal plane of the $CoCO_3$ crystal,[18] and on the angle between $\mathbf{H}$ and the pump field $\mathbf{h}$.

To calculate the relaxation rate of the phonons from the threshold of paramagnetic excitation it is necessary to know the antiferromagnetic resonance (AFMR) spectrum in fields corresponding to this excitation. Detailed investigations of AFMR spectra have not yet been carried out in $CoCO_3$ in weak fields. In addition, according to the results of Refs. 19 and 20, the form of the spectrum varies strongly from sample to sample and depends on the concentration of the $Mn^{2+}$ and $Fe^{2+}$ impurities. This is why we did not use the experimental results of other authors, but rather studied AFMR only in our own samples.

The AFMR studies were carried out using a direct-amplification spectrometer operating in the frequency range 16.6–37 GHz. The sample was placed at the end face of a shorted waveguide, which made it easy to adjust the frequency of the spectrometer. The reflected signal was detected by a crystal detector and recorded on an $X$-$Y$ plotter as a function of the magnitude of the constant magnetic field.

### 3. ANTIFERROMAGNETIC RESONANCE

AFMR in $CoCO_3$ was investigated at temperatures from 1.5 to 4.2 K in magnetic fields $H$ up to 2 kOe. The resonance linewidth $\Delta H$ was $\sim 100$ Oe. A strong 60°-anisotropy in the position of the resonance line was observed upon rotation of the magnetic field in the basal plane of the crystal. Figure 1 shows the results of measurements of the angular dependence of the resonant field $H_{res}$. As was shown in Ref. 21, the magnetization easy axis (EA) (in Fig. 1 it corresponds to the angle $\varphi = 0°$) coincides with the twofold axis $C_2$. We then measured the frequency dependences $\omega_{f0}(H)$, which are shown in Fig. 2 in the coordinates $(\omega_{f0}^2, H_{res})$, at two directions of the magnetic field, one corresponding to $\varphi = 30°$ (H⊥EA) and the other to $\varphi = 0°$ (H‖EA). The AFMR spectrum in these coordinates is described by a straight line which is shifted up or down, depending on the direction of the field. The experimental results in Figs. 1 and 2 can be represented by the empirical formula

$$(\omega_{f0}/2\pi)^2\,[\mathrm{GHz}^2] = 820\,H[\mathrm{kOe}] + 50 + 60\cos 6\varphi. \tag{4}$$

Let us compare it with the theoretical formula (3) after adding to it the hexagonal anisotropy term calculated in Ref. 22. We have then for $H \ll H_D$

$$\omega_{f0}^2 = \mu^2[HH_D + H_\Delta^2 + 36H_EH_a\cos 6\varphi], \tag{5}$$

where $H_a$ is the 60°-anisotropy field in the basal plane. From relations (4) and (5) for $g = 4.0 \pm 0.1$ (Refs. 6 and 23) it follows (taking measurement error into account) that $H_D = (26.1 \pm 2.5)$ kOe, $H_\Delta^2 = (1.6 \pm 0.3)$ kOe$^2$, and $36H_EH_a = (1.9 \pm 0.2)$ kOe$^2$.

The value which we obtained for $H_D$ agrees with the static-measurement data.[24] Since the influence of the demagnetizing factor of the sample shifts the empirical AFMR spectrum quite strongly away from the real spectrum, the value of $H_\Delta^2$ obtained from experiment cannot be identified with the magnetoelastic gap $H_{\Delta ME}^2$ in the spin-wave spectrum, since the error of determining $H_{\Delta ME}^2 = (0 \pm 1)$ kOe$^2$ is large and can indicate only the interval of possible values.

It should be noted that the spectrum $\omega_{f0}$ has the form

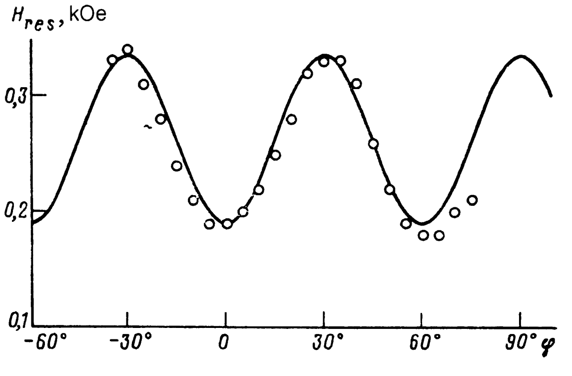


FIG. 1. Dependence of the resonant AFMR magnetic field on direction in the basal plane of the $CoCO_3$ crystal. The angles $\varphi = 0°$ and $\varphi = 60°$ correspond to H‖EA; $T = 4.2$ K, $\omega_{f0}/2\pi = 16.63$ GHz. The solid curve was calculated according to the formula $H$ [Oe] = 262–73 cos 6$\varphi$.

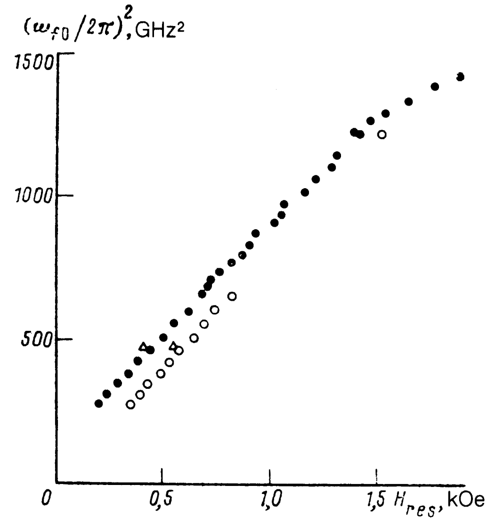


FIG. 2. Square of the AFMR frequency in $CoCO_3$ as a function of the static magnetic field $H$ for two directions of the field in the basal plane: ●—H‖EA, ○—H⊥EA, Δ—data of Ref. 19; $T = 4.2$ K.

(5) with an angular dependence in the form cos 6$\varphi$ only in fields much stronger than the spin-flop field, which is defined by the condition $H_{sf} = 36H_EH_a/H_D$ and is equal to $H_{sf} = (73 \pm 14)$ Oe. The magnetic domains previously observed in Ref. 21 can exist in weaker fields $H \lessdot H_{sf}$. In addition, in comparatively weak fields, even for $H > H_{sf}$, the magnetization **m** is not necessarily parallel to the static field **H**. However, when the external field is directed along the easy magnetization axis of one of the domains, the condition **m**‖**H** is always fulfilled for this domain and the AFMR spectrum can be described by the empirical formula (4) with cos 6$\varphi$ = 1, thus:

$$(\omega_{f0}/2\pi)^2\,[\mathrm{GHz}^2] = 820H[\mathrm{kOe}] + 110. \tag{6}$$

Since our main experimental results on parametric excitation of phonons were obtained in quite weak fields $H \approx 50$–300 Oe, a correct calculation of the connection of the phonons with the microwave pump field can be carried out only for **H**‖EA ($\varphi = 0°$) using empirical formula (6).

Our studies of the antiferromagnetic resonance of the $FeBO_3$ crystal showed that the AFMR spectrum agrees well with the results of Ref. 25. Therefore we do not cite these results in the present work.

### 4. PARAMAGNETIC EXCITATION OF PHONONS

We measured the threshold of parametric excitation of phonons as a function of the magnitude and direction of the static magnetic field, as well as of temperature, for various pumping geometries. It should be mentioned at once that pumping in $CoCO_3$ and $FeBO_3$ was observed for any orientation of the magnetic fields in the basal plane of the crystal. We also verified the effect of departure of **H** from the basis plane on the parallel pumping threshold. It turned out that the threshold $h_c^{\parallel}$ depends only on the projection of **H** on the basal plane of the crystal, to within experimental error.

Let us first consider the experimental results obtained with $CoCO_3$. Figure 3 shows the dependence of $h_c$ on the direction of **H** in the basal plane for a fixed direction of the microwave field. In other words, the dependence of the

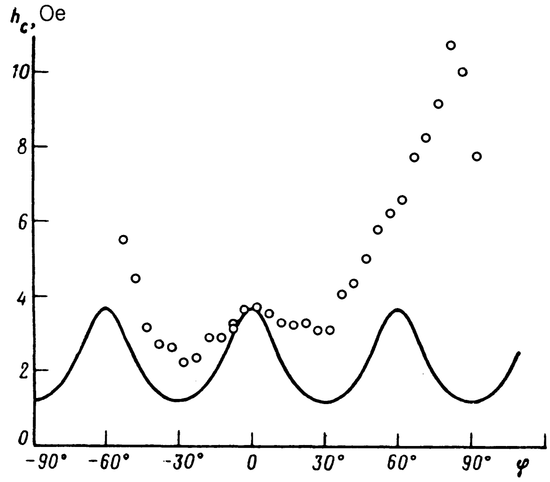


FIG. 3. Dependence of the phonon parametric excitation threshold in $CoCO_3$ on the direction of the static field **H** in the basal plane of the crystal for fixed direction of the microwave field **h**||EA. The condition **h**||**H** is fulfilled at $\varphi = 0°$. Some asymmetry of the curve indicates that there is some sloping of **h** with respect to the easy axis. $T = 1.94$ K, $H = 204$ Oe, $\omega_p/2\pi = 1430$ MHz. The solid line denotes the dependence $h_c \propto \omega_{f0}^4(\mathbf{H})$ (fitted to the experimental data at the point $\varphi = 0°$).

threshold on the angle $\varphi$ between the static and alternating magnetic fields. According to formula (2), $h_c \propto \omega_{f0}^4(\mathbf{H})$, i.e., the dependence of the threshold on $\varphi$ should have a 60° anisotropy in conformity with the AFMR spectrum (4). However, as is clear from Fig. 3, the experimental results differ qualitatively from the theoretical curve—along with the 60°-anisotropy there is observed a 180°-anisotropy of $h_c$ associated, apparently, with the mutual orientation of the microwave and static magnetic fields.

Figure 4 shows the dependences of the parallel pumping threshold on the magnitude of the static magnetic field for two directions of **H**. The fact that the threshold of phonon excitation is higher for the direction corresponding to the easy axis is in qualitative agreement with threshold formula (2) since the spin wave spectrum is higher for this direction of **H**. However, there is a quantitative discrepancy between experimental and theory for the relative increase of the

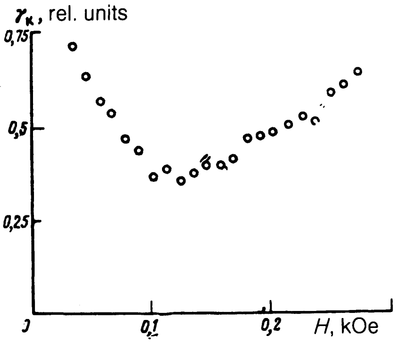


FIG. 5. Dependence of the phonon relaxation rate in $CoCO_3$ on the static magnetic field: **H**||EA, $T = 1.97$ K, $\omega_p/2\pi = 1446$ MHz.

threshold when the magnetic field is rotated from the direction perpendicular to the EA to the direction parallel to it. Thus, for $H = 1$ kOe, formula (2) gives a fourfold increase of the threshold, and for $H = 0.2$ kOe, a 1.9 times greater increase. Such a difference can be explained in principle by assuming that the phonon relaxation rate depends on the direction of the static magnetic field. The field dependence of the phonon relaxation rate calculated according to formula (2) with formula (6) taken into account, is shown in Fig. 5 for the case of **H** parallel to the $C_2$ axis. The observed growth of phonon relaxation in fields $H < 100$ Oe is evidently associated with the existence of magnetic domains for $H \leqslant H_{sf}$. To estimate the absolute value of the phonon relaxation in $CoCO_3$ it is not enough to know just the magnetoelastic coupling constant.

Measurements of the critical field in the perpendicular-pumping geometry are presented in Fig. 6. In weak fields ($H < 0.1$ kOe) the thresholds of parallel and perpendicular pumping almost coincide; however, the dependence of the threshold $h_c^{\perp}$ on the static field is significantly weaker than that of $h_c^{\parallel}$, which makes it possible at the same microwave power to observe pumping up to large values of $H$.

Figure 7 presents the temperature dependences of $h_c^{\parallel}$ and $h_c^{\perp}$ in $CoCO_3$ for fixed field geometry and values of $H$ chosen so that the AFMR frequencies agree (to within 10%). It can be seen that the parallel pumping threshold

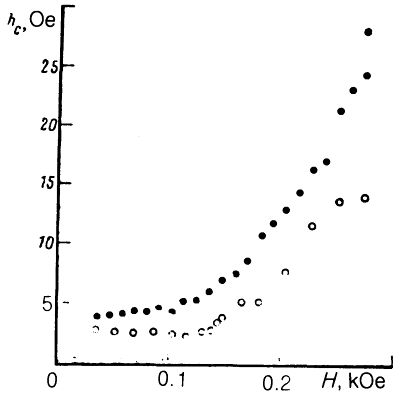


FIG. 4. Dependence of the parallel pumping threshold in $CoCO_3$ on the static magnetic field at the two directions of **H**: ●—**H**||EA, $T = 1.97$ K, $\omega_p/2\pi = 1446$ MHz; ○—**H**⊥EA, $T = 1.94$ K, $\omega_p/2\pi = 1430$ MHz.

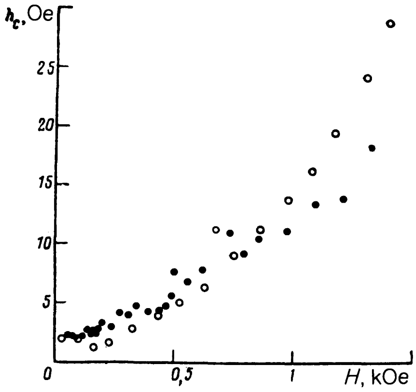


FIG. 6. Dependence of the perpendicular pumping threshold in $CoCO_3$ on the static magnetic field at the two directions of **H**: ●—**H**||EA, $T = 1.94$ K, $\omega_p/2\pi = 1430$ MHz; ○—**H**⊥EA, $T = 1.5$ K, $\omega_p/2\pi = 1570$ MHz.

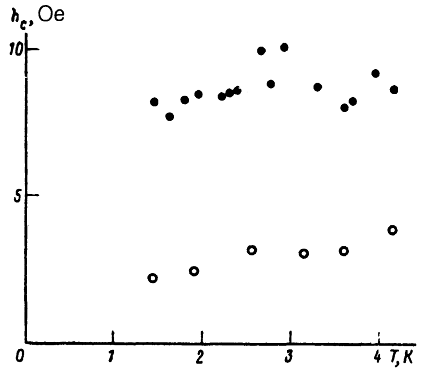


FIG. 7. Temperature dependences of the phonon parametric excitation thresholds in $CoCO_3$: ●—H‖h‖EA, $\omega_p/2\pi = 1570$ MHz, $H = 155$ Oe; ○—H⊥h‖EA, $\omega_p/2\pi = 1570$ MHz, $H = 274$ Oe.

exceeds the perpendicular pumping threshold by not less than a factor of two. In addition, a difference in the functional dependences is observed. If we assume that the magnetoelastic constants in our experiments do not depend on temperature ($T \ll T_N$), then it follows that the relaxation of the excited phonons has the same temperature dependences as in Fig. 7.

Thus, in $CoCO_3$ the thresholds $h_c^{\parallel}$ and $h_c^{\perp}$ are not only unequal but also have different functional dependences on the magnetic field and the temperature. The greatest difference in the thresholds is observed precisely in strong fields, where the inequality $\omega_p \ll \omega_{f0}$ is best fulfilled and according to formula (2) the thresholds should coincide.

We also carried out analogous measurements of $h_c^{\parallel}$ and $h_c^{\perp}$ in the antiferromagnet $FeBO_3$. It turned out here that the thresholds of parallel and perpendicular pumping also have different field dependences, but the situation observed here is the opposite of that for $CoCO_3$: the perpendicular pumping threshold has a more rapid growth in its dependence on the magnetic field, and such a behavior of the thresholds is characteristic of all directions of the magnetic field **H** in the basal plane. These dependences in the case **H**⊥EA are shown in Fig. 8. Note that in the given orientation of **H** the parallel

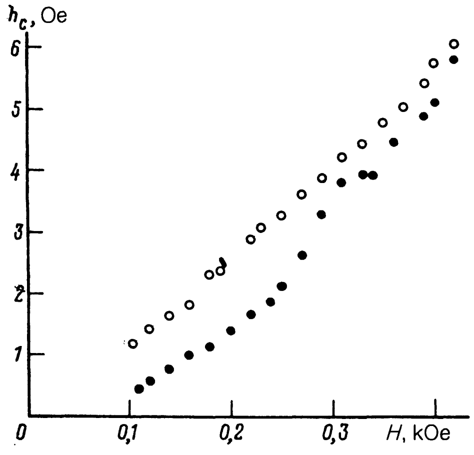


FIG. 8. Dependence of the phonon parametric excitation threshold in $FeBO_3$ on the static magnetic field for the two pumping geometries: ●—H‖EA‖h, ○—H‖EA⊥h; $T = 77$ K, $\omega_p/2\pi = 620$ MHz.

pumping threshold in weak fields is markedly lower than the perpendicular pumping threshold.

The dependence of the thresholds on the orientation of the microwave field for fixed direction of the static field is, in our opinion, of fundamental importance. As was already mentioned, according to the elastic-isotropic theory[13,16] $h_c^{\parallel} = h_c^{\perp}$ since $\mu_{\text{eff}}^{\parallel} = \mu_{\text{eff}}^{\perp}$ [see Eq. (2)], and the properties of the sample (including phonon decay) do not change if only the microwave field **h** is rotated. However, as the experiments have shown, $h_c^{\perp}$ in $FeBO_3$ is observed only in fields considerably stronger than $h_c^{\parallel}$, while in $CoCO_3$ the opposite is true: $h_c^{\parallel} > h_c^{\perp}$. Obviously then, to explain the fact of the noncoincidence of the thresholds it is not enough just to assume that phonons are excited with minimum decay, but one must also take into account the dependence of the coupling coefficient $\mu_{\text{eff}}$ on the mutual orientation of the magnetic fields. In other words, in the given pumping geometry the condition $h_c = \min(\gamma_{\mathbf{k}}/\mu_{\text{eff}})$ is satisfied, i.e., for a valid calculation of the magnitude of the threshold it is necessary to take into account not only the anisotropy of the phonon relaxation rates, but also the anisotropy of the coupling coefficient $\mu_{\text{eff}}$.

## 5. MAGNETOELASTIC WAVES IN AN ANISOTROPIC MEDIUM

The theory of coupled magnetoelastic waves is easiest to develop for frequency regions far below the AFMR frequency (the experimental results of this paper and of Refs. 14 and 15 were obtained under precisely such conditions). In this case the elastic subsystem can be considered as a phonon gas with effective (linear and nonlinear) interactions due to the magnetic subsystem (see Refs. 2, 3, 16, and 26). In a certain sense this situation is analogous to the case of nuclear spin waves (see Refs. 16 and 27), which arise in the NMR frequency region thanks to the Suhl-Nakamura interaction between the nuclear spins. The development of this analogy in the present work has made it possible to calculate the indirect interaction energy between the elastic deformations of the crystal (see Sec. 5.3). An analysis of the phonon system is much more complicated due to the anisotropy of the elastic and magnetoelastic interactions. We shall calculate below the amplitudes of the effective phonon interactions for a crystal of rhombohedral symmetry.

### 5.1. Energy of the magnetic and elastic subsystems

We assume that the $z$ axis points in the direction of the main—"difficult"—crystal axis $C_3$, which is characterized by the anistropy field $H_A$, and that an external magnetic field **H** points in the direction of one of the twofold axes. This axis is taken to be the $x$ axis. We write the Hamiltonian of the system of spin waves of this two-sublattice antiferromagnetic in the form[28,29]

$$\mathscr{H}_m = \mathscr{H}_m^{(2)} + \mathscr{H}_m^{(3)} + \mathscr{V}_m(t). \tag{7}$$

The quadratic part

$$\mathscr{H}_m^{(2)} = \sum_{\mathbf{k}} (\omega_{f\mathbf{k}} c_{\mathbf{k}}^+ c_{\mathbf{k}} + \omega_{a\mathbf{k}} d_{\mathbf{k}}^+ d_{\mathbf{k}}) \tag{8}$$

describes the gas of magnons of the quasiferromagnetic ($f$) and quasi-antiferromagnetic ($a$) branches of the spectrum. Here

$$\omega_{\alpha k}=\mu[2H_E H_A+H_D(H+H_D)+H_\Delta^2+(\alpha k)^2]^{1/2}, \qquad (9)$$

and $c_k^+$, $c_k$, $d_k^+$, and $d_k$ are the Bose creation and annihilation operators.

The term

$$\mathcal{H}_m^{(3)}=\frac{i\mu H}{2^{5/2}(\mathcal{N}S)^{1/2}}\sum_{1,2,3} C_1^- C_2^- D_3^- \Delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \qquad (10)$$

in Eq. (7) describes the three-magnon anharmonicity, in which

$$C_k^-\equiv(J/\omega_{fk})^{1/2}(c_k-c_{-k}^+),\quad D_k^-\equiv(\omega_{\alpha k}/J)^{1/2}(d_k-d_{-k}^+), \qquad (11)$$

$\mathcal{N}$ is the number of unit cells in the sample; $\Delta(x\neq 0)=0$, $\Delta(x=0)=1$. In expression (10) only those terms are displayed that will be of interest to us in what follows, since we will not consider anharmonicities of fourth and higher order.

The last term in expression (7) describes the interaction of an external microwave field having longitudinal ($\mathbf{h}^\parallel\|\hat{\mathbf{x}}$) and transverse ($\mathbf{h}^\perp\|\hat{\mathbf{y}}$) polarization with the magnetic system:

$$\mathcal{V}_m(t)=\mu h^\parallel \cos\omega_p t\left[i\left(\frac{\mathcal{N}S}{2}\right)^{1/2} D_0^- - \frac{\mu(H+H_D)}{4J}\sum_k C_k^- C_{-k}^-\right] - \mu h^\perp \cos\omega_p t\cdot i\left(\frac{\mathcal{N}S}{2}\right)^{1/2}\frac{\mu(H+H_D)}{J}C_0^-. \qquad (12)$$

We will describe the elastic and magnetoelastic subsystems of the crystal phenomenologically on the basis of the following expression for the energy:

$$\mathcal{E}_{em}=V_0\sum_r W(\mathbf{r}), \qquad (13)$$

in which $\mathbf{r}$ is the radius vector of the cell and

$$W={}^1/_2\rho(\dot{\mathbf{U}})^2+\mathcal{F}_e+\mathcal{F}_{em} \qquad (14)$$

is the sum of the kinetic, elastic, and magnetoelastic energies per unit volume. The explicit form of $\mathcal{F}_e$ and $\mathcal{F}_{em}$ are (see Ref. 26):

$$\begin{aligned}\mathcal{F}_e&={}^1/_2\hat{\mathcal{C}}^{(2)}\hat{u}\hat{u}={}^1/_2\mathcal{C}_{11}(u_{xx}^2+u_{yy}^2)+{}^1/_2\mathcal{C}_{33}u_{zz}^2+\mathcal{C}_{12}u_{xx}u_{yy}\\&+\mathcal{C}_{13}(u_{xx}+u_{yy})u_{zz}+(\mathcal{C}_{11}-\mathcal{C}_{12})u_{xy}^2+2\mathcal{C}_{44}(u_{xz}^2+u_{yz}^2)\\&+2\mathcal{C}_{14}[(u_{xx}-u_{yy})u_{yz}+2u_{xy}u_{xz}],\end{aligned} \qquad (15)$$

$$\begin{aligned}\mathcal{F}_{em}&=\mathbf{l}(\hat{\mathcal{B}}^{(2)}\hat{u})\mathbf{l}=\mathcal{B}_{11}(l_x^2u_{xx}+l_y^2u_{yy})+\mathcal{B}_{12}(l_x^2u_{yy}+l_y^2u_{xx})\\&+2(\mathcal{B}_{11}-\mathcal{B}_{12})l_xl_yu_{xy}+\mathcal{B}_{33}l_z^2u_{zz}+2\mathcal{B}_{44}(l_yl_zu_{yz}+l_xl_zu_{xz})\\&+2\mathcal{B}_{14}[(l_x^2-l_y^2)u_{yz}+2l_xl_yu_{xz}]\\&+\mathcal{B}_{41}[l_yl_z(u_{xx}-u_{yy})+2l_xl_zu_{xy}].\end{aligned} \qquad (16)$$

In Eqs. (15) and (16) we have made use of the following notation: $\mathbf{l}\equiv(\mathbf{M}_1-\mathbf{M}_2)/2M_0$; $|\mathbf{M}_1|=|\mathbf{M}_2|=M_0$ is the magnetization of the sublattice; $\mathbf{U}$ is the displacement vector along which the strain tensor $\hat{u}$,

$$u_{ij}={}^1/_2(\partial U_i/\partial x_j+\partial U_j/\partial x_i) \qquad (17)$$

is defined, $\hat{\mathcal{C}}^{(2)}$ is the elastic-modulus tensor, and $\hat{\mathcal{B}}^{(2)}$ is the magnetoelastic tensor.

The antiferromagnetic vector $\mathbf{l}(\mathbf{r})$ has in the approximation of interest to us the following components:

$$\begin{aligned}l_x&\approx -i(2\mathcal{N}S)^{-1/2}\sum_k \exp(i\mathbf{kr})C_k^-,\\ l_y&\approx 1+(4\mathcal{N}S)^{-1}\sum_{k,q}\exp[i(\mathbf{k}+\mathbf{q})\mathbf{r}]C_k^-C_q^-,\\ l_z&\approx 0.\end{aligned} \qquad (18)$$

The displacement vector is expressed in terms of the phonon creation and annihilation operators $b_{k\beta}^+$ and $b_{k\beta}$ in the following way:

$$\mathbf{U}(\mathbf{r},t)=(2\rho V_0\mathcal{N})^{-1/2}\sum_{k,\beta}\frac{\mathbf{e}(\mathbf{k},\beta)}{\Omega_{k\beta}^{1/2}}\exp(i\mathbf{kr})(b_{k\beta}+b_{-k\beta}^+), \qquad (19)$$

where $\mathbf{e}(\mathbf{k},\beta)$ is the unit polarization vector of the wave with frequency $\Omega_{k\beta}$, and the $\beta$ denotes the quasilongitudinal and the two quasitransverse modes.

### 5.2. Parametric resonance

In the region of acoustic frequencies $\Omega_{sk}\ll\omega_{f0}\ll\omega_{\alpha 0}$ the quite complicated magnetoelastic interactions can be simplified by introducing effective phonon-phonon interactions. A convenient method of obtaining these effective interactions, which we shall use here, was proposed in Ref. 30. We assume conditions of quasi-equilibrium of the form

$$\partial(\mathcal{H}_m^{(2)}+\mathcal{H}_m^{(3)}+\mathcal{E}_{em})/\partial\psi_{jk}=0 \qquad (20)$$

in the generalized coordinates

$$\psi_{1k}\equiv C_k^-,\quad \psi_{2k}\equiv D_k^-. \qquad (21)$$

We thereby exclude the magnetic degrees of freedom and arrive at the relations

$$D_k^-\approx\frac{iH}{H_E(8\mathcal{N}S)^{1/2}}\sum_{1,2}C_1^-C_2^-\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}), \qquad (22)$$

$$\begin{aligned}C_k^-&\approx -i(2S/\mathcal{N})^{1/2}\sum_r \varkappa(\mathbf{k},\mathbf{r})[(\mathcal{B}_{11}-\mathcal{B}_{12})u_{xy}(\mathbf{r})+2\mathcal{B}_{14}u_{xz}(\mathbf{r})]\Big\{1\\&-\sum_q\sum_{r_1}\varkappa(\mathbf{q},\mathbf{r}_1-\mathbf{r})\{(\mathcal{B}_{11}-\mathcal{B}_{12})[u_{xx}(\mathbf{r}_1)-u_{yy}(\mathbf{r}_1)]\\&+4\mathcal{B}_{14}u_{yz}(\mathbf{r}_1)\}\Big\},\end{aligned} \qquad (23)$$

$$\varkappa(\mathbf{k},\mathbf{r})\equiv\exp(-i\mathbf{kr})2J_0V_0/\omega_{fk}^2.$$

It is easy to see that as a result of taking these relations into account in expression (12) terms arise which describe the interaction of the microwave field with the elastic strains of the crystal. Substituting the second-quantization displacement vector (17) into the strain tensor and writing the dynamic equations for the canonical variables $b_{k\beta}^+$ and $b_{k\beta}$, we arrive at conditions for the paramagnetic resonance threshold of the phonons. Omitting straightforward details of the computations, we obtain the effective coupling coefficients for parallel and perpendicular pumping:

$$\mu_{eff}^\parallel\approx\mu^2\frac{(2H+H_D)J_0}{\omega_{fk}^4(\Omega_{k\beta_1}\Omega_{k\beta_2})^{1/2}}\frac{\Theta^2}{\rho V_0}k^2|\mathfrak{f}_1(\mathbf{k},\beta_1)\mathfrak{f}_1(\mathbf{k},\beta_2)|, \qquad (24)$$

$$\mu_{eff}^{\perp} \approx \mu^2 \frac{(H+H_D)J_0}{(\omega_{f0}\omega_{fk})^2(\Omega_{k\beta_1}\Omega_{k\beta_2})^{1/2}} \frac{\Theta^2}{\rho V_0} k^2 |\mathfrak{f}_1(\mathbf{k},\beta_1)\mathfrak{f}_2(\mathbf{k},\beta_2)|, \tag{25}$$

where

$$\mathfrak{f}_1(\mathbf{k},\beta) \equiv \zeta(e_x n_y + e_y n_x) + e_x n_z + e_z n_x,$$
$$\mathfrak{f}_2(\mathbf{k},\beta) \equiv \zeta(e_x n_x + e_y n_y) + e_y n_z + e_z n_y$$

are functions which describe the anisotropy of the excited phonons with wave vectors **k** and $-\mathbf{k}$ ($\mathbf{k} \equiv |\mathbf{k}|\mathbf{n}$) and polarizations $\mathbf{e}(\mathbf{k},\beta_1)$ and $\mathbf{e}(\mathbf{k},\beta_2)$; $\Theta \equiv 2\mathscr{B}_{14}V_0$; $\zeta = (\mathscr{B}_{11} - \mathscr{B}_{12})/2\mathscr{B}_{14}$. Apart from the functions $\mathfrak{f}_1$ and $\mathfrak{f}_2$, expressions (24) and (25) are analogous in structure to the coupling coefficients for parallel and perpendicular pumping obtained earlier in Refs. 16 and 13, respectively. If we neglect the anisotropy of the phonon relaxation rates and drop the anistropic factors $\mathfrak{f}_1$ and $\mathfrak{f}_2$ in formulas (24) and (25), then in the interesting parameter region ($H \ll H_D, \omega_{fk} \approx \omega_{f0}$) we obtain relation (2). Taking the anistropy of degenerate pumping into account leads to formulas of the form

$$\left.\begin{matrix} h_c^{\parallel} \\ h_c^{\perp} \end{matrix}\right\} = \frac{2}{\mu^2} \frac{\omega_{fk}^4}{J_0 H_D \omega_p} \frac{\rho V_0}{\Theta^2} \min \left\{ \begin{matrix} \gamma_{k\beta} v_s^2(\mathbf{k},\beta)/\mathfrak{f}_1^2, \\ \gamma_{k\beta} v_s^2(\mathbf{k},\beta)/|\mathfrak{f}_1\mathfrak{f}_2|, \end{matrix} \right. \tag{26}$$

where $v_s(\mathbf{k},\beta)$ is the velocity of the excited phonons. Formulas (26) allow a qualitative understanding of the reason for the experimentally observed different dependences of the thresholds $h_c^{\parallel}$ and $h_c^{\perp}$ on the magnetic field **H**.

### 5.3. The phonon spectrum

Eliminating the magnetic degrees of freedom in Eqs. (7) and (14) with the help of relations (22) and (23), we arrive at the effective energy of the elastic subsystem

$$\mathscr{E}_e = V_0 \sum_{\mathbf{r}} \left[ \frac{1}{2} \rho (\dot{\mathbf{U}})^2 + \mathscr{F}_e + \Delta\mathscr{F}_e \right], \tag{27}$$

in which the term

$$\Delta\mathscr{F}_e = -\mathscr{N}^{-1} \sum_{\mathbf{k}} \sum_{\mathbf{r}_1} \varkappa(\mathbf{k}, \mathbf{r}_1 - \mathbf{r}) \mathscr{B}(\mathbf{r}) \mathscr{B}(\mathbf{r}_1), \tag{28}$$

where

$$\mathscr{B}(\mathbf{r}) \equiv (\mathscr{B}_{11} - \mathscr{B}_{12}) u_{xy}(\mathbf{r}) + 2\mathscr{B}_{14} u_{xz}(\mathbf{r})$$

describes the indirect elasticity mediated by the magnetic subsystem (we have written only the terms quadratic in $\hat{u}$). Note that the elastic potential (27), taking expression (28) into account, becomes nonlocal—the energy density at the given point depends now on the integral contribution of the elastic strains within the limits of the effective radius $\sim v_m/\omega_{f0}$ in a potential of Yukawa type.

The spectrum $\Omega_{k\beta}$ and polarization $\mathbf{e}(\mathbf{k},\beta)$ of the normal elastic modes of the system are determined by the Green-Christoffel equation

$$[\rho\Omega_{k\beta}\hat{I} - \hat{\Gamma}(\mathbf{k})]\mathbf{e}(\mathbf{k},\beta) = 0, \tag{29}$$

which is obtained after going over to the plane wave representation in the dynamic equation for **U**. In Eq. (29) $\hat{I}$ is the unit matrix, and $\hat{\Gamma}(\mathbf{k}) = [\hat{\mathscr{C}}^{(2)} + \Delta\hat{\mathscr{C}}^{(2)}(\mathbf{k})]\mathbf{kk}$ is the symmetric tensor for rhombohedral structure, first calculated in Ref. 26. The explicit form of the components of $\hat{\Gamma}(\mathbf{k})$ is given in the Appendix. It should be noted that the corrections $\Delta\hat{\mathscr{C}}^{(2)}(\mathbf{k})$ to the elastic moduli $\hat{\mathscr{C}}^{(2)}$ which we obtained is half that of Ref. 26. The reason is that there, and also in all other investigations of the effective elastic interaction in magnets (see the review in Ref. 3) the effective energy $\Delta\mathscr{F}_e$ was calculated incorrectly: no account was taken of the nonlocality of the indirect elastic interaction.

In the simplest case, in which $\mathbf{k} = (0,0,k)$, we obtain from (27) the spectra of the longitudinal and two transverse elastic waves:

$$\Omega_{k0} = (\mathscr{C}_{33}/\rho)^{1/2} k, \tag{30a}$$

$$\Omega_{k1} = [(\mathscr{C}_{44} - \mathscr{B}_k)/\rho]^{1/2} k, \quad \mathbf{e}(\mathbf{k},1) = (1,0,0), \tag{30b}$$

$$\Omega_{k2} = (\mathscr{C}_{44}/\rho)^{1/2} k, \quad \mathbf{e}(\mathbf{k},2) = (0,1,0). \tag{30c}$$

For $\mathbf{k} = (0,k,0)$ the spectrum of the field-dependent transverse wave is

$$\Omega_{k1} = [(\mathscr{C}_{66} - \mathscr{B}_k \zeta^2)/\rho]^{1/2} k, \quad \mathbf{e}(\mathbf{k},1) = (1,0,0). \tag{31}$$

Here $\mathscr{B}_k \equiv \mathscr{B}_{14}\Theta J_0/\omega_{fk}^2$ and $\mathscr{C}_{66} \equiv (\mathscr{C}_{11} - \mathscr{C}_{12})/2$. Taking the above-mentioned factor of 1/2 into account, expressions (30b) and (31) differ from the analogous expressions in Refs. 3 and 9 (there we have $2\mathscr{B}_k$ instead of $\mathscr{B}_k$). Thus the magnetoelastic constants calculated from experiment are $2^{1/2}$ times greater than those obtained from the formulas of the cited papers.

### 6. DISCUSSION OF RESULTS

Based on the observation of the dimensional effect in $FeBO_3$ (a regular decrease of the threshold field upon fullfilment of the condition $n\lambda/2 = L$, where $n$ is an integer, $\lambda$ is the wavelength of the phonon, and $L$ is the thickness of the sample) it was found in Ref. 14 that parallel pumping excites transverse phonons which propagate perpendicular to the basal plane. According to formula (24), it is precisely these phonons that have the maximum value of $\mu_{eff}$ under the given experimental conditions. The question arises what type of quasiparticles are excited when $\mathbf{H} \perp \mathbf{h}$. It should be noted that in the case of perpendicular pumping in $FeBO_3$ we also observed a lowering of the threshold at some values of $H$. But in this case the distances $\Delta H$ between the threshold minima were significantly greater (4–6 times so), the minima were markedly less pronounced, and the dependence of their positions on $H$ was random.

The increase in the magnitude of $\Delta H$ attendant to the appearance of the size effect, according to the formula of Ref. 14, may be associated with a weakening of the dependence of the speed of sound on $H$, an increase of the velocity of the excited phonons, or a decrease of the sample dimension $L$ in the formula for the size effect. It is possible to attribute the irregular nature of the minima of the threshold $h_c^{\perp}$ to the fact that the wave vector **k** of the excited phonons is not perpendicular to the plane of the (laminar) sample. In this case, because of the irregular shape of the sample in the basal plane, the condition of the size effect will be satisfied now by one, now by another dimension of the sample, which is manifested in the experiment as a chaotic appearance of the minima of the pumping threshold. The qualitative difference of the size effects for the two pumping geometries agrees with Eq. (26) above, which indicates that for fixed experimental parameters, different phonons will be excited in the different pumping geometries by virtue of the anisotropy of $\mu_{\text{eff}}$. We stress that this result cannot be explained in the simpler elastic-isotropic continuous medium model.[13,16]

No size effect was observed in our experiments for parametric phonon pumping in $CoCO_3$. Most likely this has to do with the fact that the mean free path of the quasiphonon is significantly less than the thickness of the sample. Therefore we are not able to assert that different phonons are excited also in $CoCO_3$ for $\mathbf{h}\|\mathbf{H}$ and $\mathbf{h}\perp\mathbf{H}$. However, the striking difference between the thresholds $h_c^{\|}$ and $h_c^{\perp}$ for $H \geqslant 0.2$ kOe gives a reason to suppose that the situation here is analogous to that for $FeBO_3$.

Let us turn now to a discussion of the mechanisms of phonon decay. Analysis shows that the most important intrinsic processes, calculated by diagram technique methods in Refs. 31 and 32, give substantially lower estimates of the phonon relaxation rates (with smallness parameter $\varepsilon = \Theta/\rho V_0 v_s^2$) than is required to explain the experimental results of Refs. 14 and 15 and of the present paper. The probability of extrinsic processes of phonon decay (impurities, dislocations) also contains a small parameter $\varepsilon$ (along with a small relative-defect-density parameter). Therefore in what follows we limit ourselves to one model of the appearance of decay of the quasielastic waves. The main idea of this model is that in the presence of coupled oscillations the transformation of the spectra of the initial "pure" modes is also accompanied by a "renormalization" of their relaxation parameters. This idea was successfully developed in Ref. 33 with nuclear spin waves as the example. The calculation, carried out in Ref. 33 by the method of moments, yields when applied to coupled magnetoelastic waves the following expression for the contribution of quasiferromagnon decay $\gamma_{f\mathbf{k}}^{(m)}$ to the phonon relaxation rate $\gamma_{\mathbf{k}\beta}$:

$$\gamma_{\mathbf{k}\beta} = (d\Omega_{\mathbf{k}\beta}/d\omega_{f\mathbf{k}})\gamma_{f\mathbf{k}}^{(m)}. \qquad (32)$$

Expression (32) has a simple physical meaning. Relaxation of the spin waves proceeds thanks to elementary acts of interaction of the magnons with other degrees of freedom of the crystal. This process has a random character and gives rise to quantum noise in the spin wave energy (see Ref. 34). $\gamma_{f\mathbf{k}}^{(m)}$ serves as a measure of this noise. The magnetoelastic interaction "transfers" the magnetic noise component to the elastic subsystem, which leads to phonon decay (the equivalence of low-frequency noise modulation of the oscillator spectrum to its relaxation was demonstrated in Ref. 35). Thus, formula (32) relates the characteristics $\gamma_{\mathbf{k}\beta}$ and $\gamma_{f\mathbf{k}}^{(m)}$ of noise of one and the same nature through the coupling coefficient between the subsystems. In the case under consideration for the transverse phonons with $\mathbf{k}\|C_3$ (as follows from Eqs. (26) and (29), it is precisely these phonons or phonons similar to them that are first excited) we have

$$d\Omega_{\mathbf{k}\beta}/d\omega_{f\mathbf{k}} = \frac{\Omega_{\mathbf{k}\beta}\mathscr{B}_{\mathbf{k}}}{\rho\omega_{f\mathbf{k}}v_s^2}. \qquad (33)$$

Unfortunately, at present we do not have a theory of quasiferromagnon relaxation in antiferromagnets adequate to experiment. However, we can make use of the experimental data of Ref. 12 in which the values of $\gamma_{f\mathbf{k}}^{(m)}$ in $FeBO_3$ were measured by the parametric resonance method and compare the result of renormalization of magnon decay into the phonon branch with the phonon relaxation rate.

It follows from the experiments of Ref. 15 that the phonon relaxation rate in $FeBO_3$ is proportional to their frequency. In other words, the $Q$ factor of the phonons does not depend on the frequency and at $T = 77$ K has the values $Q_{\text{ph}} \equiv \omega_{\text{ph}}/2\gamma_{\mathbf{k}\beta} \approx 1.8 \cdot 10^3$ and $3.5 \cdot 10^3$ for the two samples investigated in Ref. 15. If we renormalize the spin wave relaxation rate obtained in the experiments in Ref. 12 according to formulas (32) and (33), then at $T = 77$ K we obtain for the phonons with frequency $\omega_{\text{ph}} = 2\pi \cdot 3.7$ GHz the value $\gamma = 0.73$ MHz, i.e., $Q_{\text{ph}} \approx 2.5 \cdot 10^3$, which is in good agreement with the results of Ref. 15.

We cannot estimate the absolute value of the phonon relaxation rate in $CoCO_3$ at the present time, since the magnetoelastic constants are unknown. Comparison of the functional dependences $\gamma_{\mathbf{k}}(H,T)$ obtained from measurements of the phonon pumping threshold and from the renormalization of magnon decay is also still not possible due to the lack of experimental data.

To wrap up this section, let us dwell for a moment on the results of AFMR studies in $CoCO_3$. It can be stated with certainty that our results are in good agreement with the data of Ref. 19 (points from Ref. 19 for a sample in the shape of a thin disk are shown in Fig. 2). Unfortunately, Ref. 19 does not give the frequency dependence of $\omega_{f0}^2(H)$, from whose slope it is possible to determine $H_D$ knowing the value of the spectroscopic splitting factor $g = 4.0 \pm 0.1$. According to our data $H_D = (26.1 \pm 2.5)$ kOe, which is in splendid agreement with the results of magnetostatic measurements.[18,24] It is interesting to note that in Refs. 23 and 26, in which the AFMR spectra were investigated in strong fields $H \geqslant H_D$, the experimental results are not described by the theoretical formula (5) with $H_D = H_D^{\text{stat}}$.

## CONCLUSIONS

1. Parametric excitation of phonons by an alternating magnetic field in the antiferromagnets $CoCO_3$ and $FeBO_3$ is observed for any orientation of the static and microwave magnetic fields lying in the basal plane of the crystal.

2. Different phonons are excited in different pumping geometries. The critical amplitude of the microwave field also depends on the angle between the static and alternating magnetic fields, i.e., on the pumping mechanism.

3. The set of experimental results for various phonon pumping geometries can be qualitatively explained by formula (26), which takes account of the competition between the anisotropies of phonon relaxation and excitation. The theory which considers parametric phonon resonance in the model of an elastic-isotropic medium is inapplicable.

4. It has been shown that the effective elastic energy arising from the magnetoelastic interaction is nonlocal—the energy density at a given point depends on the integral contribution of the elastic strains within the limits of the effective radius $\sim v_m/\omega_{f0}$.

5. Phonon decay in $FeBO_3$ can be explained by the hypothesis that the relaxation of the spin waves coupled with the sound influences the decay of the quasiphonon branch.

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## APPENDIX

The components of the symmetric tensor $\hat{\Gamma}(\mathbf{k})$ are equal to

$$\Gamma_{11}=\mathscr{C}_{11}k_x^2+\mathscr{C}_{66}k_y^2+\mathscr{C}_{44}k_z^2+2\mathscr{C}_{14}k_yk_z-\mathscr{B}_{\mathbf{k}}(\zeta k_y+k_z)^2,$$
$$\Gamma_{12}=(\mathscr{C}_{12}+\mathscr{C}_{66})k_xk_y+2\mathscr{C}_{14}k_xk_z-\mathscr{B}_{\mathbf{k}}\zeta k_x(\zeta k_y+k_z),$$
$$\Gamma_{13}=(\mathscr{C}_{13}+\mathscr{C}_{44})k_xk_z+2\mathscr{C}_{14}k_xk_y-\mathscr{B}_{\mathbf{k}}k_x(\zeta k_y-k_z),$$
$$\Gamma_{22}=\mathscr{C}_{66}k_x^2+\mathscr{C}_{11}k_y^2+\mathscr{C}_{44}k_z^2-2\mathscr{C}_{14}k_yk_z-\mathscr{B}_{\mathbf{k}}(\zeta k_x)^2,$$
$$\Gamma_{23}=\mathscr{C}_{14}(k_x^2-k_y^2)+(\mathscr{C}_{13}+\mathscr{C}_{44})k_yk_z-\mathscr{B}_{\mathbf{k}}\zeta k_x^2,$$
$$\Gamma_{33}=\mathscr{C}_{44}(k_x^2+k_y^2)+\mathscr{C}_{33}k_z^2-\mathscr{B}_{\mathbf{k}}k_x^2,$$

where

$$\mathscr{C}_{66}=(\mathscr{C}_{11}-\mathscr{C}_{12})/2,\ \mathscr{B}_{\mathbf{k}}=\mathscr{B}_{14}\Theta J_0/\omega_{f\mathbf{k}}^2.$$

We present the elastic constants of the elasticity moduli (in units of $10^{11}$ erg/cm$^3$) for $CoCO_3$ ($\rho=4.13$ g/cm$^3$) (Ref. 37) and $FeBO_3$ ($\rho=4.28$ g/cm$^3$) (Ref. 9):

| | $\mathscr{C}_{11}$ | $\mathscr{C}_{12}$ | $\mathscr{C}_{13}$ | $\mathscr{C}_{14}$ | $\mathscr{C}_{33}$ | $\mathscr{C}_{44}$ |
|---|---|---|---|---|---|---|
| $CoCO_3$ | 27,1 | 11,6 | 8,8 | −0,6 | 16,7 | 5,3 |
| $FeBO_3$ | 44,5 | 14,5 | 14,0 | 2,0 | 30,5 | 9,5 |

The magnetoelastic constants for $FeBO_3$ (taking into account the factor $2^{1/2}$, see Sec. 5.3) at $T=77$ K are[9] $\mathscr{B}_{14}=19.7\cdot10^{-6}$ erg/cm$^3$ and $(\mathscr{B}_{11}-\mathscr{B}_{12})/2=16.9\cdot10^{-6}$ erg/cm$^3$.

[1)] A theoretical analysis of the experimental results of Refs. 7–9 was carried out by Speidel.[10]


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